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Consequences of Model Choice in Predicting Horizontal Merger Effects

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Abstract

How practitioners model competition influences the predicted effects of a merger. We consider three models that are commonly used to evaluate horizontal mergers: Bertrand price setting, second score auction, and Nash bargaining. We first show how these models relate to one another, and specifically that the Bertrand and second score auction models can both be nested within a bargaining framework. Second, we show through numerical simulations how the predicted merger effects vary with model choice. Third, we show that two commonly used strategies for obtaining demand parameters can yield markedly different outcomes across the three models. Finally, we show how model and calibration strategy choices affect the magnitude of predicted harm in the 2012 Bazaarvoice/Power Reviews merger.

Keywords: bargaining models; merger simulation; horizontal markets
JEL classification: L41, L13, K21

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1 Introduction

Antitrust practitioners frequently analyze the effect of a proposed horizontal merger using merger simulation.[1] In order to simulate a merger's effects, practitioners must decide which models of competition best characterize the market in question, and then decide how to obtain demand and cost parameters required by the chosen models.

In business-to-business markets, where firms procure inputs from other firms, three models of differentiated product competition are often used by antitrust practitioners to assess competitive interactions: Bertrand price setting, second score auction, and Nash bargaining.[2] However, it is often not obvious which of the three models is best suited to a particular business-to-business context, and often an argument can be made for the applicability of each one.[3] This article demonstrates how the choice of model can play an important role in the predicted merger effects, even given the same observable inputs. The article also shows how the strategy used to obtain the model inputs can influence the predicted merger effects.

The specific context we consider is a business-to-business market with a small number of sellers and a large number of buyers, a common occurrence in many wholesale and procurement markets. Examples include: a retailer contracting for supply from a private label product manufacturer; a restaurant contracting with a food ingredient supplier; a company contracting for office supplies at corporate offices. In this context, we examine the horizontal competitive effects of mergers between two sellers of differentiated goods or services. We quantify the differences in predicted price effects resulting from a merger simulation of the

[1]Antitrust practitioners also employ indicia like the Herfindahl-Hirschman Index, Upward Pricing Pressure Index, and Compensating Marginal Cost Reduction to gauge merger effects. Because these indicia are all derived from particular models of competition, the analysis presented in this paper applies to these indicia as well.

[2]For example, a Bertrand price setting model was used by the plaintiffs in H&R Block (2011) and Aetna (2017), second score auction models were used by the plaintiffs in Sysco (2015), Bazaarvoice (2014), and Anthem (2017), while Nash bargaining models have been used by the defense in Anthem (2017).

[3]Bajari, McMillan, and Tadelis (2008) discuss why firms might choose auctions or negotiations for procurement. And see for example Sweeting et al. (2020) regarding the analysis of procurement of private label breakfast cereal by retailers: "Several reasonable models that are commonly used by antitrust practitioners may fit the procurement setting… Unfortunately… these models often generate meaningfully different results, which they did in this matter' ' (p. 772).

same market characteristics (prices, shares, number of firms) when changing only the model of competition.[4] Assuming that customer demand is given by the logit distribution allows us to propose a simple framework with parameters that can be calibrated with minimal data requirements (Werden and Froeb (2002)). The model then allows for the quantification of effects of a hypothetical merger as the change in pricing incentives when going from two independent firms to two jointly owned firms.

We show theoretically that a Nash bargaining game can nest the Bertrand model of competition, and that the models are equivalent when buyer bargaining power is zero. We also show that a second score bargaining model can nest the second score auction, and that again the models are equivalent when buyer bargaining power is zero. Previous literature has noted each of these relationships separately; we show how a general bargaining framework relates to both a Bertrand model and a second score auction, and we show exactly how the payoffs specified can yield a bargaining process that generalizes Bertrand price setting, and which alternative payoff specifications will yield a generalization of the second score auction. We then use numerical simulation to quantify the magnitude of the differences in consumer, producer, and total welfare across the models after a merger of two sellers.

We also consider how predicted merger outcomes vary under two approaches for obtaining demand parameters. The first approach, which we denote as the *fixed demand parameters* approach, assumes that the demand parameter estimates are known. The demand parameters, prices, and market shares are used as inputs, and the firm marginal costs required for an equilibrium to hold are then determined by the model of competition. For example, a practitioner might simulate the results of a merger using parameters from demand estimation with appropriate exogenous variation.[5] The second approach, which we call the

[4]The Gross Upward Pricing Pressure Index or "GUPPI' ' approximates the price effects from a horizontal merger between firms playing a Bertrand Pricing Game (Farrell and Shapiro (2010)). Taragin and Loudermilk (2019) show how the GUPPI-based on the Bertrand model can substantially over or under-predict merger effects, depending upon the true model.

[5]This approach, for instance, is the primary implementation in the *mergersim* Stata package (Björnerstedt and Verboven (2014)). Valletti and Zenger (2021) refer to this approach as "merger simulation based on demand estimation' '

calibrated demand parameters approach, uses as inputs the prices and market shares, and also assumes that at least some firm costs are also observed. Then for a particular market, and given a model of competition, this approach entails calibrating demand parameters that are consistent with those market characteristics and the model of competition.[6] We consider both approaches to treating the demand parameters because they are both commonly used, depending on the information available during a merger investigation.

We find that these two approaches can yield substantially different merger effects even assuming the same model of competition. Specifically, when fixing demand parameters, median consumer harm is 39% lower in a Bargaining model than a Bertrand model, and 44% lower in an auction relative to a Bertrand model. By contrast, when calibrating demand parameters, median consumer harm is 15% greater in a Bargaining model than a Bertrand model, and 29% lower in an auction relative to a Bertrand model. Thus it is not only model choice that matters, but also the decision about which data elements are taken as inputs.

The reason that model choice matters differently under each approach is because under the fixed demand parameters approach, the same demand parameters are held fixed across all considered models, causing calibrated pre-merger equilibrium margins and marginal costs to vary across models. By contrast, under the calibrated demand parameters approach, the demand parameters and inside good margins will vary. Thus, the consequences for model choice differ across the two approaches to the demand parameters. In Section 3, we show the theoretical relationships of model parameters across models for both the fixed demand and calibrated demand approaches. We show that with fixed demand parameters, different models lead to different predictions about upstream firm margins. With the calibrated demand approach, different models leads to different price coefficients and thus different demand elasticities.

To give a concrete illustration of the consequences of model choice, we examine the 2012

[6] This approach is the primary implementation in the *Antitrust* R package (Taragin and Sandfort (2021)).

merger between Bazaarvoice and Power Reviews. Treating the demand parameters as fixed across all models, the Bertrand model yields the greatest consumer harm; simulating this merger under other models yields consumer harm estimates that are anywhere between 26% to 68% of the magnitude of the Bertrand model. Calibrating the demand parameters, the Bargaining model yields slightly greater harm than the Bertrand model, but the second score auction and second score bargaining model still yield lower harm than the Bertrand model.

Previous literature has noted, but not systematically explored, relationships among these models. Werden and Froeb (1994) introduces the Bertrand price-setting game with Logit demand for horizontal merger analysis. Grennan (2013) uses a bargaining model and notes that the model generalizes the standard Bertrand-Nash price-setting model. Froeb and Tschantz (2002) introduces the second price auction model for merger analysis, and Miller (2014) shows how that model can be reframed as a second score auction when products are differentiated. Miller (2014) also introduces a Nash bargaining model that can generalize the second score auction, but does not specify how that bargaining model relates to an analogous bargaining model that nests the Bertrand model, as we do in this paper. Miller and Sheu (2020) describe Bertrand and second score models of competition and their use in merger review. The present article proposes a framework that can be used to compare outcomes across all three models, and specifies how the Bargaining model can be adjusted to nest the Bertrand model or the second score auction. We also quantify the magnitude of the consequence of these model choices for predicted merger effects.

2 The Bargaining Game

Denote the supplier/seller/wholesaler by w and the buyer/customer/retailer by r. Each supplier offers one or more products that are indexed by j. Profits are denoted by Π. Π_j^w denotes the profits to supplier w only due to the sales of product j. And similarly Π_j^r is the buyer's utility from being supplied by product j.

We begin with a general form of the bargaining equation:

$$\max_{p_j} \left[\Pi_j^r(p_j) - \widetilde{\Pi_j^r}\right]^{\lambda_j^r} \cdot \left[\Pi_j^w(p_j) - \widetilde{\Pi_j^w}\right]^{(1-\lambda_j^r)} \tag{1}$$

which yields a general form of first order condition:

$$(1-\lambda_j^r) \cdot \left[\Pi_j^r(p_j) - \widetilde{\Pi_j^r}\right] \cdot \frac{\partial \Pi_j^w}{\partial p_j} + \lambda_j^r \cdot \left(\frac{\partial \Pi_j^r}{\partial p_j}\right) \left[\Pi_j^w(p_j) - \widetilde{\Pi_j^w}\right] = 0$$

$\widetilde{\Pi_j^r}$ denotes the utility to the buyer in the case of a breakdown in negotiation between the buyer and the supplier. Likewise, $\widetilde{\Pi_j^w}$ is the payoff to the supplier in disagreement. λ_j^r is the bargaining power of the buyer, and in the most general formulation can vary by buyer-product combinations; for most of this paper, however, we will restrict it to be the same across both inside products and buyers ($\lambda_j^r = \lambda$, for all $j, r \neq 0$).

2.1 Nash Bertrand Bargaining Model

As we shall see, a bargaining model with $\lambda_j^r = 0$ is equivalent to the Bertrand model of competition. When $\lambda_j^r = 1$, the merger effect falls to zero.

2.1.1 Model Setup

Denote a seller n that owns a portfolio of products W_n that includes product j. c_j is the marginal cost of supplying product j. To nest the Bertrand model in this bargaining framework, define:

- $\Pi_j^w = \sum_{j \in W_n} (p_j - c_j) \cdot s_j(p)$
- $\widetilde{\Pi_j^w} = \sum_{k \in W_n/j} (p_k - c_k)\tilde{s_k}$
- $\Pi_j^r = \frac{1}{-\alpha} \log(1 + \sum_j \exp(\delta_j + \alpha p_j))$
- $\widetilde{\Pi_j^r} = \frac{1}{-\alpha} \log(1 + \sum_{k \neq j} \exp(\delta_k + \alpha p_k))$

$s_j = \frac{\exp \delta_j + \alpha p_j}{1+\sum_k \delta_k + \alpha p_k}$ denotes the Logit probability that a buyer either chooses inside product j for its supply agreement, or instead selects an outside option with p_0, δ_0 normalized to 0. In this context, s_j is interpreted as a probability of being chosen in any given negotiation, and thus the profits should be interpreted as expected profits. This exact same model can be used in other contexts where s_j should be interpreted as a share of quantity sold, and these derivations will still hold with only a change in the interpretation of s_j.

Further, let $\tilde{s_k} = \frac{s_k}{1-s_j}$ denote the disagreement choice probabilities when product j is excluded from the choice set. The buyer payoffs are multiplied by $\frac{1}{-\alpha}$ to translate buyer utility into dollar units, and $\alpha < 0$ is the marginal utility of income. Note that this implies that:

- $\Pi_j^r - \widetilde{\Pi_j^r} = \frac{\ln(1-s_j)}{\alpha}$
- $\frac{\partial \Pi_j^r}{\partial p_j} = -s_j(p)$
- $\frac{\partial \Pi_j^w}{\partial p_j} = \left(s_j + (p_j - c_j) \cdot (\alpha s_j (1 - s_j)) - \sum_{k \in W_n/j} (p_k - c_k) \cdot \alpha s_j s_k \right)$

We then have all the necessary terms to specify the FOC from the bargaining model, which recall from above is given by:

$$(1 - \lambda_j^r) \cdot \left[\Pi_j^r(p_j) - \widetilde{\Pi_j^r} \right] \cdot \frac{\partial \Pi_j^w}{\partial p_j} + \lambda_j^r \cdot \left(\frac{\partial \Pi_j^r}{\partial p_j} \right) \left[\Pi_j^w(p_j) - \widetilde{\Pi_j^w} \right] = 0$$

From this FOC, it can be seen that this bargaining model will nest the Bertrand solution. When $\lambda_j^r = 0$, and the supplier has all of the bargaining power, the FOC simplifies to:

$$\left[\Pi_j^r(p_j) - \widetilde{\Pi_j^r} \right] \cdot \frac{\partial \Pi_j^w}{\partial p_j^w} = 0$$

But in this setup, the buyer always will have some positive payoff, meaning that $\Pi_j^r - \widetilde{\Pi_j^r} > 0$ will always hold. As a consequence, this FOC is equal to zero if and only if $\frac{\partial \Pi_j^w}{\partial p_j^w} = 0$, which is exactly equal to solution from the Bertrand FOC. This must hold, even for multi-product firms, because Π_j^w is equivalent to the Bertrand objective function, and $\widetilde{\Pi_j^w}$ is not a function of p_j and thus drops out of the partial derivative.

There are a few differences between the bargaining setup here and that of other papers such as Draganska, Klapper, and Villas-Boas (2010), Gowrisankaran, Nevo, and Town (2015), Ho and Lee (2017), Miller and Sheu (2020), and Sheu and Taragin (2021). While those papers model negotiation between a set of upstream firms and a set of downstream firms, the model in this paper can be thought of as the same model but with only a single downstream monopolist. This more limited setting allows for the model to directly have a simple Bertrand price-setting game as the limiting model as the bargaining power shifts to the wholesaler.[7] Having a single retailer in the bargaining model shuts down the channel for one retailer's negotiation to affect another retailer, which is present in the other models. Secondly, rather than explicitly model the final consumer demand for the product of the downstream firm, we model the downstream firm (the retailer) with a reduced-form profit function that reflects the integration of downstream consumer demand and the retailer production technology. One advantage of employing a reduced-form approach is that it has fewer input requirements, since the downstream firm margins and final consumer demand need not be separately estimated.

2.1.2 Logit Equations for Simulation

Plugging in the values defined above into the general formulation of the bargaining problem, result in:

[7] We think this model appropriate for settings such as procurement. Although there is a monopolist retailer in the market, there will be many procurement or bargaining events. Wholesalers in this model make offers to maximize their expected profits, and once the preference shocks are realized, the retailer will choose their most preferred option, and the price will be observed. Because retailers are identical (other than the logit shock), the observed price will be the same every time a given wholesaler is chosen. Over many such events, the shares are the frequencies with which each wholesaler is chosen.

$$\max_{p_j} \left[\frac{\ln(1-s_j)}{\alpha}\cdot M\right]^{\lambda_j^r} \cdot \left[(p_j - c_j)\cdot s_j \cdot M - \sum_{k\in W_j/j}(p_k - c_k)\cdot \frac{s_k s_j}{(1-s_j)}\cdot M\right]^{(1-\lambda_j^r)} \tag{2}$$

The logit assumptions then allow for the derivation of the first order condition with respect to price that we use in the merger simulations:

$$\left[(p_j - c_j) - \sum_{k\in W_j}(p_k - c_k)\cdot s_k\right] = \left[\frac{\lambda_j^r}{(1-\lambda_j^r)}\cdot\frac{s_j}{(1-s_j)} - \ln(1-s_j)\right]^{-1}\cdot\left[\frac{\ln(1-s_j)}{\alpha}\right] \tag{3}$$

This first order condition holds for each product j, and these can be stacked and then expressed using matrix notation. Define the margin for each product as $m_j \equiv p_j - c_j$, and define the function $f(s_j)$ as the right hand side of Equation 3. Letting Ω denote a matrix of product ownership,

$$m - \Omega(m \circ s) = F(s)$$

This is the equation directly implemented for the simulations.

2.2 Second score bargaining Model

This section shows that a second score bargaining model can nest a second score auction model, and specifies exactly what modifications must be made to the classic formulation discussed above. Analogous to the relationship between the original bargaining model and the Bertrand outcome, the second score bargaining framework when $\lambda_j^r = 0$ is equivalent to the second score auction. The predicted effects of a merger reach zero when $\lambda_j^r = 1$.

2.2.1 Model Setup

Three modifications need to be made to the bargaining model specified above so that the model can nest the second score auction solution:

1. We now assume that in disagreement, the second-best option gets the contract *at cost*. As a consequence, supplier payoff in disagreement is zero, so $\widetilde{\Pi_j^w} = 0$.

2. Buyer payoffs are specified to reflect that the equilibrium price will be such that buyer utility equals the surplus that could have been created from the second-best offer.

3. Shares are now a function of costs and not prices. So before, $s_j(p)$, and now $s_j(c)$.

Shares might be a function of costs and not equilibrium prices in cases when the contract price will not directly affect downstream demand. Rather, the negotiated prices would serve only to split a fixed amount of agreement surplus, and then the choice probabilities are a function only of costs, with more efficient suppliers being more likely to win contracts.

In the second score bargaining problem, define:

- $\Pi_j^w = (p_j - c_j) \cdot s_j(c)$
- $\widetilde{\Pi_j^w} = 0$
- $\Pi_j^r = \frac{1}{-\alpha} E[\max(\delta_k + \alpha p_k)|\text{j wins}] = \frac{1}{-\alpha}\left(\delta_j + \alpha \cdot p_j + \gamma - \log(s_j)\right)$
- $\widetilde{\Pi_j^r} = \frac{1}{-\alpha} E[\max_{k \neq j}(\delta_k + \alpha c_k)|\text{j wins}]$

We set $\widetilde{\Pi_j^w} = 0$ because we assume that in disagreement, the buyer gets the second best option at cost. This means that the supplier will receive zero payoff in disagreement.

The buyer payoffs are specified so as to follow the payoffs in a second score auction.[8] In that model, the seller's dominant strategy is to bid their true costs, and the buyer chooses the seller that will create the most total surplus. In this sense, this outcome of this model is

[8]For further details of the second score auction and why the wholesaler's dominant strategy is to bid their true costs, see Miller (2014).

efficient. The total surplus is then split between buyer and seller through the price, which is set so that the buyer utility equals the surplus that could have been created by the second-best seller. Thus, if the second-best option is a very similar value to the buyer as the best option, the buyer will capture most of the surplus; if the second-best option is a distant substitute to the best option, the seller will capture most of the surplus.

In the more general second score bargaining framework we propose here, a bargaining parameter splits the seller's share of the surplus from the second score auction. When the seller has all of the bargaining power ($\lambda_j^r = 0$), the surplus split and equilibrium price exactly match the second score auction. When the buyer has all of the surplus ($\lambda_j^r = 1$), then the buyer is able to extract all available surplus from the best option, and so there are no merger effects because the same best option is still chosen and at the same price.

Because the price from an auction event is only observed for the winning seller, the retailer payoffs are conditioned on a particular seller having won the auction. Π_j^r takes the form given above as this is the expression for the buyer utility from product j, conditional on j being the best option.

The buyer disagreement payoff $\widetilde{\Pi_j^r}$ reflect the value of the second best surplus, conditional on j being the best option. However, that conditional second best surplus cannot be expressed directly in a closed-form equation. Therefore, we use an implication from the logit distribution to express the conditional second best surplus in terms of objects which do have a closed form solution. Specifically, it is a property of the logit distribution that $E[\max(V_k)|\text{j wins}] - E[\max_{k\neq j}(V_k)|\text{j wins}] = \frac{1}{s_j}(E[\max(V_k)] - E[\max_{k\neq j}(V_k)])$. Or in other words, the difference between the conditional total surplus and the conditional second best surplus is equal to the difference between the unconditional total surplus and the unconditional second best surplus, divided by the share of the option being conditioned on. This means that the conditional second best surplus can be expressed in terms of other objects for which closed form solutions are available under the Type 1 extreme value distribution.

In particular:

$$E[\max_{k\neq j}(\delta_k + \alpha c_k)|\text{j wins}] =$$
$$\frac{1}{-\alpha}\left(\left(\delta_j + \alpha \cdot c_j + \gamma + \log(\frac{1}{s_j})\right) - \left(\frac{1}{\sum_{k\in W_j} s_k} Emax - \frac{1}{\sum_{k\in W_j} s_k}\widetilde{Emax}_{k\notin W_j}\right)\right)$$

where $Emax$ denotes the familiar log-sum term from logit preferences, $Emax \equiv \log(\sum_j \exp(\delta_j + \alpha c_j))$.

Thus the equation we use for the buyer disagreement payoff $\widetilde{\Pi_j^r}$ has two pieces. The first piece is the total surplus available conditional on choice j being the best option; the second piece is the unconditional first best surplus ($Emax$) minus the unconditional second best surplus ($\widetilde{Emax}_{k\notin W_j}$). The value $Emax$ is the logsum term based on the logit assumption, and $\widetilde{Emax}_{k\notin W_j}$ is the logsum term excluding product j as well as any other products in the same portfolio as j.

Defining payoffs this way implies that

- $\Pi_j^r - \widetilde{\Pi_j^r} = \frac{1}{-\alpha}\left(\alpha(p_j^w - c_j^w) + \left(\frac{1}{\sum_{k\in W_j} s_k} Emax - \frac{1}{\sum_{k\in W_j} s_k}\widetilde{Emax}_{k\notin W_j}\right)\right)$

The remaining terms required in the FOC are then derived:

- $\frac{\partial \Pi_j^r - \widetilde{\Pi_j^r}}{\partial p_j} = -1$
- $\frac{\partial \Pi_j^w}{\partial p_j} = s_j(c)$

We then have all the necessary terms to express the general bargaining FOC, reproduced here from above:

$$(1-\lambda_j^r)\cdot\left[\Pi_j^r(p_j) - \widetilde{\Pi_j^r}\right]\cdot\frac{\partial \Pi_j^w}{\partial p_j} + \lambda_j^r \cdot \left(\frac{\partial \Pi_j^r}{\partial p_j}\right)\left[\Pi_j^w(p_j) - \widetilde{\Pi_j^w}\right] = 0$$

Now when $\lambda_j^r = 0$, the FOC simplifies to

$$\Pi_j^r - \widetilde{\Pi}_j^r = 0$$

because it is always the case that

$$\frac{\partial \Pi_j^w}{\partial p_j} > 0$$

This gives a solution exactly equal to the second score auction FOC when $\lambda_j^r = 0$.

2.2.2 Logit Equations for Simulation

Similar to before, we make the assumption of logit demand to obtain a simple form of the first-order condition that we use in the simulations.

We can use the general form FOC to derive an expression of the FOC in terms of margins. To begin, first define the constant term as:

$$EmaxDiff \equiv \frac{Emax - \widetilde{Emax}_{k \notin W_j}}{\sum_{k \in W_j} s_k}$$

Then, substituting in the objects into the FOC and some rearrangement yields the FOC in terms of margins and model parameters. This is the equation we use directly for the simulations.

$$m_j = \frac{(1 - \lambda_j^r) \cdot EmaxDiff}{-\alpha} \tag{4}$$

3 Model relationships

For a given approach to obtaining demand parameters, the choice of model will have implications for other model parameters, such as predicted margins, costs, or price sensitivities. Specifically, in the *fixed demand parameters* approach, demand parameters are assumed to be known. Those demand parameters, prices, and market shares are used as inputs, and the firm marginal costs required for an equilibrium to hold are then determined by the model of competition. In the *calibrated demand parameters* approach, at least one firm's marginal costs need to be observed. The inputs are prices, market shares, and marginal costs. Then for a particular market, and given a model of competition, demand parameters are calibrated to be consistent with the inputs and the model of competition.

In this section, we show relationships of model parameters across models for both the fixed demand parameter and calibrated demand parameter strategy. The reason that the treatment of the demand parameters matters is because how these model parameters vary across models is different under the fixed demand parameters versus the calibrated demand parameters approach. In the next section we use numerical simulations to explore how the differences in these model parameters affect merger outcomes.

First, we explore the relationship in the models' predicted margins and marginal costs when the demand parameters are fixed across models.

Proposition 3.1 (Fixed demand parameters)**.** *Assume as known and fixed the demand parameters α, δ and market prices and shares. Let $m_j^b, m_j^{nb}, m_j^{2a}, m_j^{2b}$ denote the the equilibrium pre-merger margin for a single-product firm j under the Bertrand, Nash bargaining, second score auction, and second score bargaining models with Logit demand, respectively. Let $c_j^b, c_j^{nb}, c_j^{2a}, c_j^{2b}$ denote the corresponding constant marginal cost for the firm. For any observed market share $1 > s_j > 0$, price $p_j > 0$, price coefficient $\alpha < 0$, and bargaining power parameter $\lambda \in (0,1)$,*

$$\begin{aligned} m_j^b &> m_j^{nb}, \\ m_j^b &> m_j^{2a} > m_j^{2b} \end{aligned}$$

and

$$\begin{aligned} c_j^b &< c_j^{nb}, \\ c_j^b &< c_j^{2a} < c_j^{2b} \end{aligned}$$

Furthermore, when $\frac{\lambda}{1-\lambda} < \log(1-s_j)\frac{1-s_j}{s_j} + 1$*, the full ordering across models is:*

$$m_j^b > m_j^{nb} > m_j^{2a} > m_j^{2b},$$
$$c_j^b < c_j^{nb} < c_j^{2a} < c_j^{2b}$$

Proof. See appendix. □

Note that with the exception of the Nash bargaining game's margins (m_j^{nb}) and marginal costs (c_j^{nb}), all the inequalities hold for multi-product firms as well. Also, $\frac{\lambda}{1-\lambda} < \log(1-s_j)\frac{1-s_j}{s_j} + 1$ never holds when $\lambda \geq 0.5$.

This proposition shows that given the same demand parameters, margins will be highest in a Bertrand model, followed by Nash bargaining, second score auction, and finally second score auction bargaining. As we will see in the next section, this ordering of margins by magnitude matches the average ordering of the magnitude of consumer harm in numerical simulations under the fixed demand parameter approach. This result is important to realize because even fixing the demand parameters, different models of competition can yield different predictions about firm margins; if one particular model is predicting margins that contradict available information about the industry, that might suggest that a different model of competition may be more appropriate.

The ordering of marginal costs follows from the ordering of margins because in this approach, observed prices are assumed to equal pre-merger equilibrium prices implied by each model. The marginal costs will only matter for merger analysis in this approach when marginal cost efficiencies are being evaluated.

Next, we explore the relationship between price coefficients α implied by each model when observed margins, prices, and shares are used to infer α.

Proposition 3.2 (Calibrated demand parameters)**.** *Assume as known and fixed the market prices and shares, and marginal cost of at least one firm in the market. Let* $\alpha^b, \alpha^{nb}, \alpha^{2a}, \alpha^{2b}$ *denote the price coefficient* $\alpha < 0$ *under the Bertrand, Nash bargaining, second score auction and second score bargaining models with Logit demand, respectively. Then for any observed* *market share* $1 > s_j > 0$*, price* $p_j > 0$*, margin* $m_j > 0$*, and bargaining power parameter* $\lambda \in (0, 1)$*,*

$$\alpha^b < \alpha^{nb},$$
$$\alpha^b < \alpha^{2a} < \alpha^{2b}$$

Furthermore, when λ *is large enough relative to the share to satisfy* $\frac{\lambda}{1-\lambda} > \frac{1}{s_j} + \frac{1-s_j}{s_j}\log(1-s_j)$*, then the full ordering across models is:*

$$\alpha^b < \alpha^{nb} < \alpha^{2a} < \alpha^{2b}$$

Proof. See appendix. □

When α approaches zero, demand in that model becomes more inelastic. Therefore, the ordering of the magnitude of α also illustrates an important driver of differences in merger effects across models in the calibrated demand parameters approach. In models with an outside option that makes for a decent substitute, demand elasticity is high, and harm from a merger in that model will be small. When demand is inelastic, the outside option is a poor alternative for the firms in the market, and harm from a merger has scope to be much larger. This proposition shows that even for the same inputs, the calibration of α will be in part driven by the model of competition.

4 Numerical Simulations

We use numerical simulation to explore how merger outcomes differ across the four aforementioned models: Bertrand, second score auction, Nash bargaining, and second score bargaining. In particular, we are interested in using these simulations to answer three questions. First, how do consumer, producer and total surplus vary both within and across models, holding demand parameters constant across models? Second, how does surplus vary both within and across models when the demand parameters are calibrated to be consistent with the model? Third, how do market outcomes vary as the bargaining power parameter vary?

To answer these questions, we simulate about 450,000 markets with logit demand, and then use the data from these generated markets to simulate horizontal merger effects under the four different models using two different demand parameter calibration strategies: the *fixed demand parameters* strategy, which uses the demand parameters from the Nash Bertrand model, and the *calibrated demand parameters* strategy, which calibrates demand parameters from the equilibrium margin condition of each model. For details of the numerical simulation exercise, see Appendix A.

Each market has three data elements: (i) prices and shares, (ii) costs, and (iii) demand parameters. When generating the raw data for the numerical simulations, we generate all three elements for each market using a Bertrand model, so that these elements are consistent with each other within that competitive environment. To compare merger effects across models, only two elements can be taken as inputs, while the third element needs to be recovered to be consistent with the model and the two input elements. In the fixed demand parameters approach, for each market we take the (i) prices and shares and (iii) demand parameters as inputs, and back out (ii) costs to be consistent with the given model of competition. In the calibrated demand parameters approach, we take (i) prices and shares and (ii) one firm's cost as the data inputs, and calibrate the (iii) demand parameters to be consistent with the given model of competition.[9]

Although we generated the data elements with a Bertrand model, this is not a test of model misspecification, as conditional on prices and either demand parameters or a single margin and shares, model-specific product marginal costs are just-identified and the Bertrand, second score, and Nash bargaining models are therefore observationally equivalent pre-merger. Accordingly, in this exercise, we take each model as true when conducting the merger simulations with that model. The only consequence of using Bertrand to generate the data is that both the fixed demand and calibrated demand approaches will give the exact same

[9] There is a third approach, which is to take (ii) costs and (iii) demand parameters as inputs, and recover (iii) prices and shares for each model. We do not consider that approach in this paper.

merger effects for the Bertrand model, because the data element recovered in each approach for Bertrand is equivalent in value to the input used in the other approach.

Figure 1 displays box and whisker plots summarizing the extent to which horizontal mergers affect consumer, producer and total surplus across the Nash bargaining, auction, and second score bargaining models. Outcomes are expressed relative to the Bertrand outcome, as the percent change relative to Bertrand, or $\frac{Effect_{model} - Effect_{Bertrand}}{Effect_{Bertrand}}$. The Bargaining and second score bargaining categories show effects for models with a bargaining parameter ranging from 0 to 1 in 0.1 increments.

Within each panel, red (left) plots depict outcomes using the fixed demand parameters strategy, where the same demand parameters are used across all models in a given simulation. The blue (right) plots depict outcomes using calibrated demand parameters strategy, where the demand parameters are model-specific. Whiskers depict the 5^{th} and 95^{th} percentiles of a particular outcome, boxes depict the 25^{th} and 75^{th} percentiles, and the solid horizontal line depicts the median.

The red plots in the "Consumer Harm" panel show that with fixed demand parameters, the Bertrand model typically generates more merger harm than either the Nash bargaining, second score auction, or second score bargaining models. In particular, relative to the Bertrand model, median consumer harm from horizontal mergers is 39% less under Bargaining, 44% less under second score auction, and 63% less under second score bargaining. The ordering of consumer harm across these models matches the ordering of margins predicted by each margin shown in the previous section. By contrast, the blue plots show that with model-specific calibrated parameters, the Bertrand model generates more consumer harm than the Nash bargaining model in 58% of markets, more consumer harm than the second score auction model 99% of markets, and more consumer harm than than the second score bargaining model in 26% of markets. Moreover, with model-specific calibrated demand parameters, the inter-quartile range is more than twice as large relative to models where the same set of demand parameters are used. This may be caused by, as shown in the previous section, the calibrated price coefficient varies by model, and that appears to lead to larger variations in merger effects than in the fixed demand approach.

Turning to the "Producer Benefit" panel, the red plots suggest that for the same demand parameters, there is no systematic relationship between the models. Relative to Bertrand, median producer surplus is about 15% lower under Nash bargaining, but 5% higher under second score auction and 37% lower under second score bargaining. However, it is difficult to know what to infer from the median as 26% of the Nash bargaining mergers have greater surplus than Bertrand, 46% of the second score auction mergers have less surplus than Bertrand, and 22% of second score bargaining mergers have greater surplus than Bertrand. By contrast, the blue plots indicate that with model-specific calibrated parameters, median producer surplus is 70% higher under the Nash bargaining model, 30% higher under the second score auction, and 22% higher under the second score bargaining model than the Bertrand model.

Finally, the "Total" panel reveals that under both demand parameter assumptions, Nash bargaining models typically generate less harm. For the fixed demand parameters, median total harm was 67% less under Nash bargaining than Bertrand, while for model-specific demand parameters, median harm was 44% less. The -100% difference in total surplus between the Bertrand model and both the second score auction and second score auction bargaining models is entirely due to the fact that under these two models consumer harm exactly equals producer benefit, implying no change in total surplus.

4.1 Bargaining Parameter

Having examined how these models differ, we next investigate the important role that the bargaining parameter plays in the outcomes of these simulated mergers. Figure 2 displays box and whisker plots summarizing how the value of the bargaining power affects consumer, producer and total surplus in a merger for the Nash bargaining (top panel) and second score bargaining (bottom panel) models. This figure displays the same merger simulations shown in Figure 1 for the bargaining models, but broken out by the bargaining parameter. The bargaining parameter for all inside goods is the value on the x-axis, while the bargaining power parameter for the outside good is held fixed and equal to 0.5. The left panel in Figure 2 reveal that relative to the Bertrand model, median consumer harm always decreases. In particular, note that under the Nash bargaining model with fixed demand parameters, the bargaining power parameter is a good predictor of relative consumer harm. For example, under the Nash bargaining model, median consumer harm when the bargaining power parameter is 0.5 is 46% lower than the harm under the Bertrand model. The same cannot be said either for the Nash bargaining model with model-calibrated parameters or for the second score bargaining model. For example, when the bargaining power parameters is 0.5, median consumer harm from the Nash bargaining model with model-calibrated demand parameters is 25% greater than the Bertrand model. Likewise, under the second score bargaining model, consumer harm is 72% less than the harm under the Bertrand model under the fixed demand parameters, but about 29% less than the Bertrand model with model-calibrated parameters.

The middle panel of Figure 2 reveals an analogous pattern for producer surplus. There are, however, two major differences. First, for sufficiently low bargaining weights, relative producer benefit is frequently greater under both bargaining models than under Bertrand. For example, under the same demand parameters, when $\lambda = 0.1$, 51% of the Nash bargaining markets and 56% of the second score bargaining markets yield greater surplus than Bertrand. However, increasing λ to 0.5 reverses this trend, with surplus under Bertrand greater than Bargaining in 78% of markets and greater than second score bargaining in 95% of markets. A similar pattern occurs with calibrated demand parameters, though at higher values of λ. Second, relative to consumer harm, producer benefit exhibits substantial variation even after conditioning on bargaining weights. For example, under the same demand parameters, at $\lambda = 0.1$, the producer surplus inter-quartile range is 10% for Nash bargaining markets and 53% for second score bargaining markets, compared to the 2% consumer surplus interquartile and 8% for second score bargaining. This variation in producer surplus is largely driven by the differences in the different models' implied marginal costs, which persist despite the fact that all models employ the same demand parameters.

Like consumer surplus, the total surplus under Nash bargaining increases as λ increases. However, unlike consumer surplus, the bargaining power is a poorer predictor of total harm than consumer harm. For example, under the Nash bargaining model with the same demand parameters, median consumer harm when the bargaining power parameter is 0.5 is 73% smaller under the Nash bargaining model than the Bertrand model. Moreover, as the bargaining parameter increases to 0.7, consumer harm becomes 90% smaller. Because consumer harm and producer benefit exactly offset each other under the second score bargaining model, conditioning on the bargaining parameter provides no additional information.

5 Bazaarvoice Revisited

Here, we revisit the 2012 merger between Bazaarvoice and Power Reviews, two customer rating and reviews platforms, in order to explore how predicted merger effects can vary based on model and calibration strategy in an actual merger investigation (see Miller (2014)).

Table 1 summarizes the quantity shares, prices, margins and bargaining parameters for the three platforms that we use for this exercise: Bazaarvoice, Power Reviews, and an aggregated "Self-Supply/Other" alternative. We assume that λ equals 0.5 for all firms in the bargaining models.[10]

[10]With only one observed margin, λ is not separately identified from the demand parameters.

Table 1: Bazaarvoice/Power Reviews Simulation Inputs

Firm	Share (%)	Price ($)	Margin (%)	λ
Bazaarvoice	40	154,000	34	0.5
Power Reviews	28	84,080		0.5
Self/Other	32	37,183		0.5

Source: prices, shares, and margins are from Miller (2014) backup.

Figure 3 displays the simulation results for all four models. The "Fixed Parameters" panel (left) uses the same demand parameters from all the models, specifically the parameters calibrated from assuming Bertrand competition. Here we see results that are broadly consistent with Figures 1 and 2. The Bertrand model yields about twice the consumer harm as the auction and bargaining models and more that three times the harm of the second score bargaining model. By contrast, the "Calibrated Parameters" panel (right), which uses demand parameters calibrated by each model, tells a different story. Consumer harm is now greatest under the Nash bargaining model. Moreover, consumer harm under both bargaining models is more than twice as large using model-calibrated parameters as the same parameters. Finally, because of the assumption that λ is the same for all firms, the model-calibrated parameters yield identical results for the second score auction and second score bargaining models.

6 Conclusion

We have described three models of competition commonly used in merger analysis, and clarified the relationship among the models under commonly used strategies for obtaining demand parameter estimates. The R package 'Antitrust: Tools for Antitrust Practitioners' contains code to calibrate and simulate all the models discussed in this paper using either calibration strategy. Further research remains to be done on how to better incorporate model uncertainty into the merger effects estimates, as well as which contexts best suit the different models.

Our paper generates two key sets of results of particular importance to economists using merger simulation tools for policy analysis. The first is that for purposes of evaluating horizontal merger effects, how a practitioner models the strategic interaction between firms matters. In particular, we show that merger effects not only vary substantially across Bertrand, auction, and bargaining models, but also within bargaining models as well. For the two bargaining models considered, the assumed model of competition has different implications for how a merger can affect total surplus. The second score bargaining model has the agents bargaining over a fixed pie of surplus, while the Bertrand-based Nash bargaining model has agents bargaining over a total surplus amount that will change with the resulting equilibrium negotiated prices. This is because the choice probabilities in the Bertrand-based bargaining model are functions of prices, whereas the second score bargaining model are functions of marginal costs. Thus depending on whether the surplus is fixed, or whether the demand and therefore surplus will change with prices, could be informative about which model is appropriate in a given context.

The second is that the decision of how to treat the demand parameters matters a great deal for how predicted merger effects vary across models. In the fixed demand approach, the demand parameters, prices, and market shares are used as inputs, and the firm marginal costs required for an equilibrium to hold are then determined by the model of competition. With that approach, we find that the Bertrand model predicts the greatest amount of harm from a merger, while the Nash bargaining model predicts less consumer harm. In the calibrated demand approach, the inputs are prices, market shares, and at least one firm cost, and the demand parameters required for an equilibrium to hold are then determined by the model of competition. In contrast to the fixed demand approach, the calibrated demand approach results in the Nash bargaining model predicting greater harm than the Bertrand model. Moreover, while the second score auction always predicts less harm from a merger than a Bertrand model, under the fixed demand parameter approach the second score bargaining model predicts even less harm that it does under the calibrated demand approach. Using calibrated demand parameters, the second score auction and second score bargaining models predict equivalent harm.

In sum, both the choice of model of competition and the treatment of demand parameters will affect the conclusions of a merger analysis.

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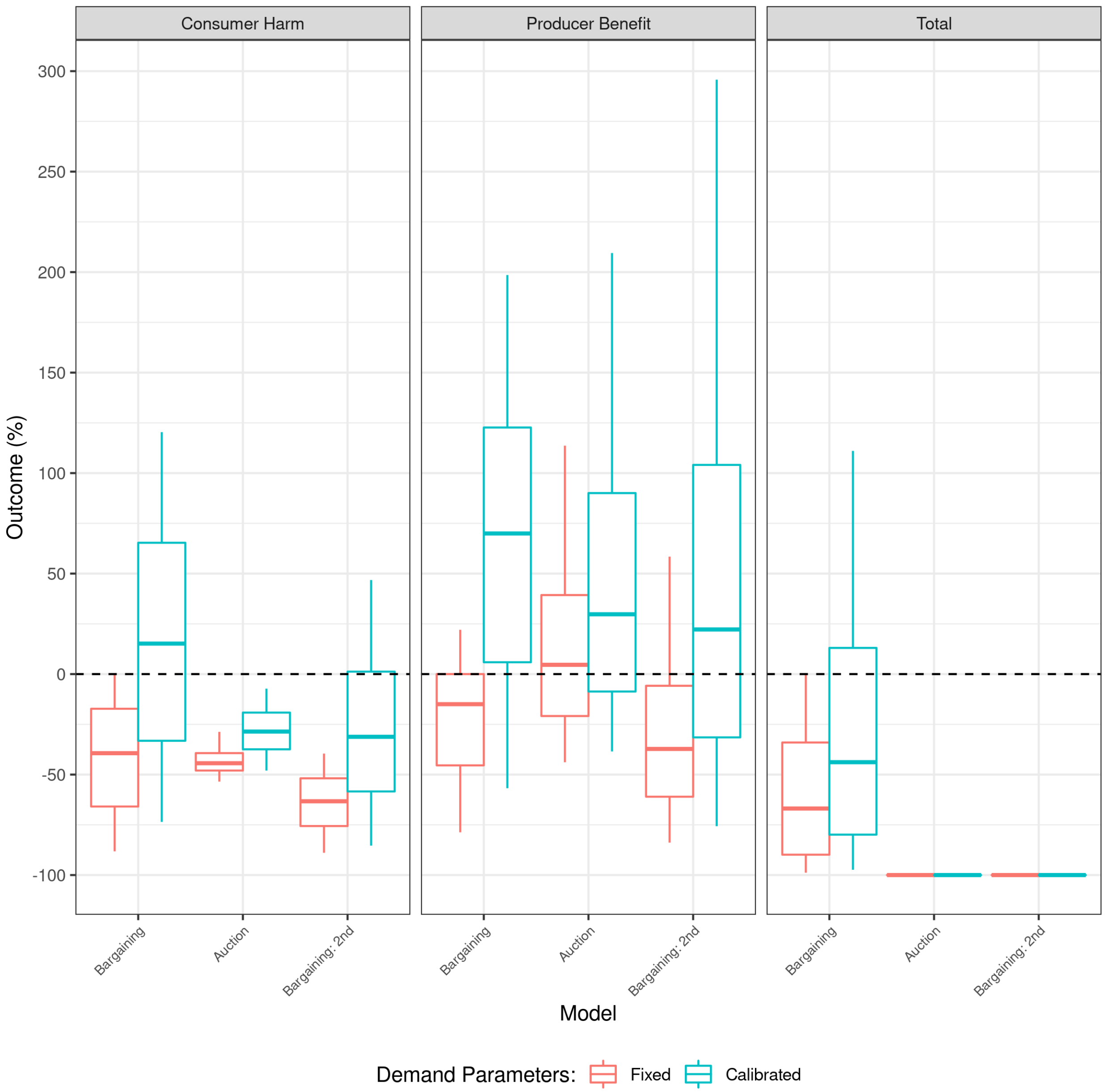

Figure 1: The figure displays box and whisker plots summarizing the extent to which horizontal mergers affect consumer, producer and total surplus across the Bargaining, Auction, and second score bargaining models. For the Bargaining and second score bargaining models, the effects shown reflect models with bargaining parameters ranging from 0 to 1 in 0.1 increments. Outcomes are expressed relative to the Nash Bertrand model outcome, as $(Effect_{model} - Effect_{Bertrand})/(Effect_{Bertrand})$. Red (left) plots depict outcomes using demand parameters calibrated from the Bertrand Model, while the blue (right) plots depict outcomes using demand parameters calibrated from the corresponding model. Whiskers depict the 5^{th} and 95^{th} percentiles of a particular outcome, boxes depict the 25^{th} and 75^{th} percentiles, and the solid horizontal line depicts the median.

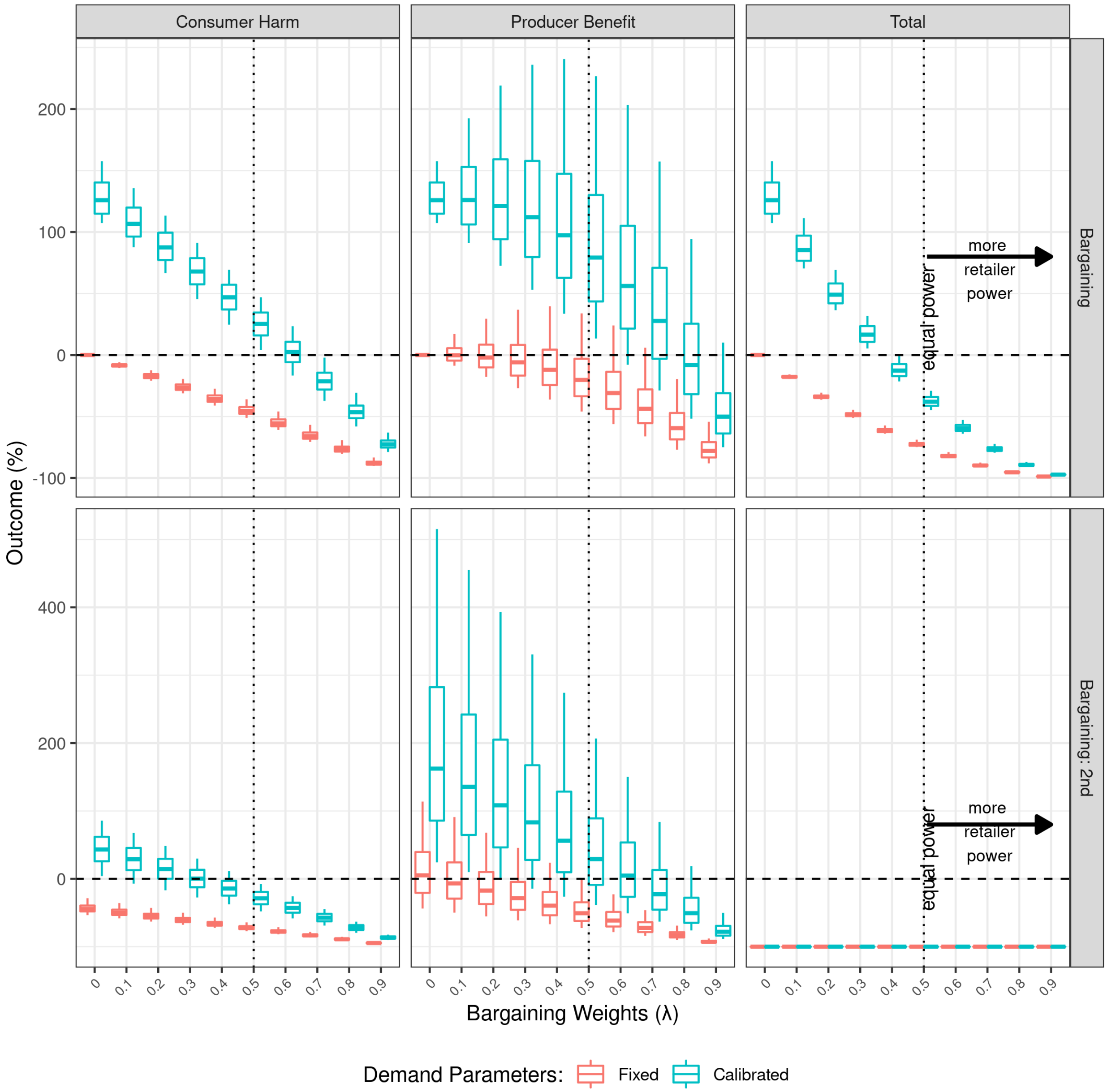

Figure 2: The figure displays box and whisker plots summarizing how the value of the bargaining power parameter affects consumer, producer and total surplus across the Bargaining, and second score bargaining models. The bargaining parameter of one firm is fixed at $\lambda_1 = 0.5$, while the bargaining parameter for the other firms is given by the x-axis. Outcomes are expressed relative to the Nash Bertrand outcome, as $(Effect_{model} - Effect_{Bertrand})/(Effect_{Bertrand})$. Red (left) plots depict outcomes using demand parameters calibrated from the Bertrand Model, while the blue (right) plots depict outcomes using demand parameters calibrated from the corresponding model.Whiskers depict the 5^{th} and 95^{th} percentiles of a particular outcome, boxes depict the 25^{th} and 75^{th} percentiles, and the solid horizontal line depicts the median.

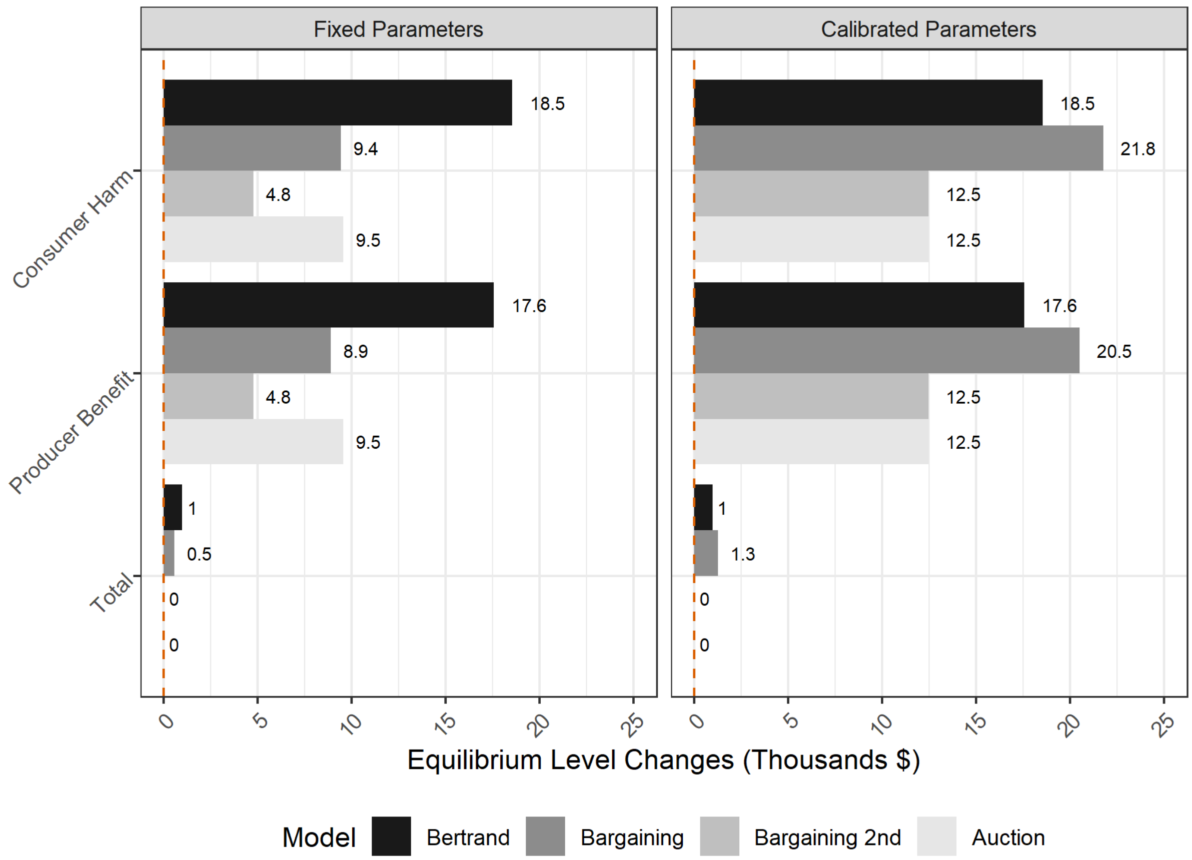

Figure 3: Simulated outcomes across models. "Fixed parameters" panel (left) depicts simulation results using fixed demand parameters across all models. "Calibrated parameters" panel (right) depicts simulation results using demand parameters calibrated from each model.

A Numerical Simulation

Here we describe our two-part strategy for numerically simulating the market data underpinning Figures 1 and 2. First, we simulate pre-merger market prices and shares from the Nash Bertrand pricing game with Logit demand. We then use the simulated pre-merger market characteristics to simulate merger effects for each model under both the fixed and calibrated demand parameter strategies.

A.1 Data Generating Process (DGP)

For each simulated market, first we draw the number of firms $N = \{3, ..., 7\}$ and separately sample $N + 1$ market shares for both the inside and outside goods. The outside share s_0 is drawn from a uniform distribution bounded between .1 and .6, while conditional inside shares are drawn from a symmetric Dirichlet distribution with concentration parameter equal to 2.5 and then multiplied by $1 - s_0$ to calculate the unconditional shares.[11]

Second, we assume that in the pre-merger state, all firms, including the outside firm, are playing a Nash Bertrand pricing game with Logit demand, and that pre-merger, the outside firm sets a price p_0 of \$100, and earns a per-unit margin m_0 drawn from a uniform distribution bounded between \$10 and \$90.[12]

Third, we simulate N equilibrium pre-merger prices. To accomplish this, we use the Bertrand Lerner condition $m_0 = (\alpha(1 - s_0))^{-1}$ to recover the price coefficient α, and then use the sampled unconditional shares, the Lerner condition, and α to calculate Bertrand margins for the N inside goods. For each inside firm, we randomly draw marginal costs from a uniform distribution bounded between 85% and 115% of the outside firm's marginal costs. We then recover pre-merger equilibrium prices by adding these marginal costs to the corresponding

[11]When the concentration parameter equal 1, the Dirichlet samples from a uniform distribution over an open standard K-1 simplex. In general, a concentration parameter above 1 favors dense symmetric distributions from the simplex.

[12]Post-merger, the outside firm is assumed to fix its price at pre-merger levels.

margin.

Finally, with the price coefficient α and prices and shares, mean values δ_j can be recovered from the Logit share equation.

A.2 Calibration Strategies

Having simulated market characteristics, we then recover missing inputs for each market using two approaches to treating demand parameters. Under the *fixed demand parameters* approach (red box and whisker plots), we treat the demand parameters from the DGP as the true underlying parameters and conduct merger simulations changing the model of competition but holding these parameters fixed. Under the *calibrated demand parameters* approach (blue box and whisker plots), we instead treat the market prices, shares, and one product margin from the DGP as observed, use those inputs to calibrate demand parameters for each model separately, and then conduct merger simulations holding market characteristics fixed but with demand parameters varying across models.

The nature of the two different treatments of the demand parameters means that each approach uses different inputs from the DGP. The fixed demand parameter approach takes as the input the prices and shares from the DGP, and the corresponding α and δ_j for that market. Model-specific marginal costs are recovered from the first-order conditions of the corresponding model, providing the final requirement for a merger simulation exercise. In the calibrated demand approach, the market prices and shares from the DGP and the drawn margin of the outside good m_0 are taken as the inputs. The model-specific α is recovered from the outside firm's equilibrium margin equation.[13] Given α, the product-level Logit mean valuations δ_j may be recovered from the Logit share equation and the observed shares and prices. Finally, model-specific inside good marginal costs are recovered from observed prices and the equilibrium margin equation, providing the required input for merger simulations

[13]For the second score auction model, this is $m_0 = \frac{\log(1-s_0)}{\alpha s_0}$. For the Bargaining models, we assume either equation 3 or 4 holds for the outside firm in the pre-merger state, with $\lambda_0 = 0.5$.

under this approach.

A.2.1 Identification when calibrating demand parameters

Suppose product-level prices, margins, and shares are observed for a single time period. Then there are $2 * |J|$ equations available for estimation ($|J|$ first order conditions and $|J|$ choice probabilities). If the bargaining weights freely varied across products, the model would be under-identified, as there would be $2 * |J| + 1$ parameters to estimate with only $2 * |J|$ equations. If addition assumptions are made to restrict λ, then the model can be identified. For example, if it is assumed that the bargaining parameter does not vary at all across the j products, and $\lambda_j^r = \lambda \, \forall j$, then the model is over-identified and can be calibrated with inputs from only one time period. In this case, there would be only $|J| + 2$ parameters (α, λ, and δ_j for $j \in J$), and these parameters can be calibrated using a minimum distance estimator that stacks the J first order conditions and J choice probabilities, subject to the constraints that $\lambda \in (0, 1), \alpha < 0$.

Calibration proceeds in a fashion similar in second score bargaining, but with the first order condition given in Equation 4. One consequence of the form of that FOC is that in the case when the bargaining power parameter is assumed to be constant across products, then the parameter values and therefore merger effects will be identical to the effects from the second score auction.

A.3 Merger Simulation

For all models we simulate a merger by randomly selecting two inside products and change the ownership matrix accordingly. We then re-solve each model for post-merger equilibrium prices.

For each N, we simulate 100,000 markets, yielding 500,000 markets. For the Nash Bargaining and second score bargaining models, we explore how changing the bargaining parameter for all N inside firms affects merger outcomes by splitting the 500,000 markets into ten groups of 50,000 and then assuming that the inside firms in each group have a bargaining power parameter value between 0 and 1 in 0.1 increments.

We exclude simulated markets where any firm has negative marginal costs, the merger is unprofitable, or where a Hypothetical Monopolist over all inside products could not impose a 5% price increase on one of the merging parties' products. We also exclude unconcentrated markets (pre-merger HHI $<$ 1,500) and markets where the merger yields small changes in concentration (HHI changes $<$ 100) as these markets are "unlikely to have adverse competitive effects" (2010 Horizontal Merger Guidelines). Doing so eliminates approximately 10% of the simulated markets, yielding about 452,000 markets for our analysis. Table 2 summarizes the HHI, change in HHI, as well as market-wide price changes, firm-specific price changes, and welfare changes (Bertrand only). Also reported is the distribution of market elasticity $\epsilon = \alpha s_0 \overline{p}$ where $\overline{p}$ is the share weighted average of the prices.

Table 2: Merger simulation summary. Effects are for the Bertrand model only. Consumer, producer, and total surplus are reported as a percentage of pre-merger revenues.

Variable	50%	5%	25%	75%	95%
HHI	2,604	1,698	2,078	3,417	4,510
HHI Change	687	151	350	1,328	2,804
Industry Price Change (%)	1.9	0.3	0.8	4.2	11.7
Merging Party Price Change (%)	4.5	0.9	2.4	8.1	16.6
Consumer Harm (%)	1.9	0.3	0.9	4.2	11.3
Producer Benefit (%)	1.0	0.1	0.4	2.2	6.2
Total (%)	0.8	0.1	0.3	1.9	5.8
ϵ	-0.9	-3.7	-1.6	-0.6	-0.3

B Proofs

B.1 Proof for Proposition 3.1 (Fixed demand parameters).

Fix α, δ, p, and s. Let $0 \leq \lambda \leq 1$.

First, we show that $c_j^b < c_j^{nb}$. From equation 3, we have that for a single-product firm the margin can be expressed as:

$$(p_j - c_j) - (p_j - c_j) \cdot s_j = (p_j - c_j)(1 - s_j) = \frac{\frac{1}{\alpha} \cdot \log(1 - s_j)}{-\log(1 - s_j) + \frac{\lambda}{1-\lambda} \frac{s_j}{1-s_j}}$$

Equivalently,

$$m_j \equiv (p_j - c_j) = \frac{\frac{1}{\alpha} \cdot \log(1 - s_j)}{-(1 - s_j)\log(1 - s_j) + \frac{\lambda}{1-\lambda} \cdot s_j} \tag{5}$$

In Bertrand, $\lambda = 0$, and so the term $\frac{\lambda}{1-\lambda} \cdot s_j$ in the denominator drops out. In a bargaining model with any $\lambda > 0$, this term in the denominator is positive. Thus, the right-hand side of Equation 5 is larger in the Bertrand model than in any Nash Bargaining model. Thus, $m_j^b > m_j^{nb}$, $\forall \lambda$. Since prices are fixed, $c_j^b < c_j^{nb}$, $\forall \lambda$.

Second, we show that $c_j^{2a} < c_j^{2b}$. From equation 4, we have that the margin can be expressed as:

$$m_j \equiv (p_j - c_j) = \frac{(1 - \lambda_j^r) \cdot EmaxDiff}{-\alpha} \tag{6}$$

In the second score auction, the margin is given by this equation with $\lambda = 0$. In a second score bargaining model with any $\lambda > 0$, the margin is given directly by this equation. Since $0 < (1 - \lambda) < 1$, and $EmaxDiff$ is the same under both models, it must be the case that $m_j^{2a} > m_j^{2b}$ for all λ. Since prices are fixed, this implies that $c_j^{2a} < c_j^{2b}$.

Finally, we show when $m^{nb} > m^{2a}$. Recall the expression for single-product firm margins in these two models:

$$m_j^{nb} = \frac{\frac{1}{\alpha} \cdot \log(1 - s_j)}{-(1 - s_j)\log(1 - s_j) + \frac{\lambda}{1-\lambda} \cdot s_j} \tag{7}$$

$$m_j^{2a} = \frac{1}{\alpha} \cdot EmaxDiff = \frac{1}{\alpha} \cdot [\frac{Emax - \widetilde{Emax}_{k \notin W_j}}{s_j}] \tag{8}$$

And note that for a single-product firm, $[Emax - \widetilde{Emax}_{k \notin W_j}] = -\log(1 - s_j) = \log(1/(1 - s_j))$. To see this, note that for single-product firms:

$$Emax = \log \left(\sum_{k=0}^{J} e^{\delta_k + \alpha p_k} \right)$$

$$\widetilde{Emax}_{k \notin W_j} = \log \left(\sum_{k \neq j} e^{\delta_k + \alpha p_k} \right)$$

$$\begin{aligned} -\left[Emax - \widetilde{Emax}_{k \notin W_j} \right] &= \log \left(\frac{\sum_{k \neq j} e^{\delta_k + \alpha p_k}}{\sum_{k=0}^{J} e^{\delta_k + \alpha p_k}} \right) \\ &= \log \left(\frac{\sum_{k=0}^{J} e^{\delta_k + \alpha p_k} - e^{\delta_j + \alpha p_j}}{\sum_{k=0}^{J} e^{\delta_k + \alpha p_k}} \right) \\ &= \log(1 - s_j) \end{aligned}$$

Therefore, $Emax - \widetilde{Emax}_{k \notin W_j} = -\log(1 - s_j) = \log(1/(1 - s_j))$.

Now to compare margins under Nash Bargaining (nb) and the second score auction (2a),

$$
\begin{aligned}
& m_j^{nb} > m_j^{2a} \\
\iff & \frac{\frac{1}{\alpha} \cdot \log(1-s_j)}{-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j} > \frac{1}{\alpha} \cdot (-1) \cdot \left[\frac{Emax - \widetilde{Emax}_{k \notin W_j}}{s_j} \right] \\
\iff & \frac{\log(1-s_j)}{-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j} < (-1) \cdot \left[\frac{Emax - \widetilde{Emax}_{k \notin W_j}}{s_j} \right] \\
\iff & \log(1-s_j) \cdot s_j < (-1) \cdot \left[Emax - \widetilde{Emax}_{k \notin W_j} \right] \cdot \left[-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j \right] \\
\iff & \log(1-s_j) \cdot s_j < (-1) \cdot \left[-\log(1-s_j) \right] \cdot \left[-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j \right] \\
\iff & s_j > -(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j \\
\iff & s_j + (1-s_j)\log(1-s_j) > \frac{\lambda}{1-\lambda} \cdot s_j \\
\iff & 1 + \frac{(1-s_j)}{s_j}\log(1-s_j) > \frac{\lambda}{1-\lambda} \\
\iff & \frac{(1-s_j)}{s_j}\log(1-s_j) > \frac{\lambda}{1-\lambda} - 1 \\
\iff & \log(1-s_j) > \frac{s_j}{(1-s_j)} \left[\frac{\lambda}{1-\lambda} - 1 \right] \\
\iff & -\log(1-s_j) < \frac{s_j}{(1-s_j)} \left[1 - \frac{\lambda}{1-\lambda} \right] \\
\iff & \frac{\lambda}{1-\lambda} < \log(1-s_j)\frac{1-s_j}{s_j} + 1
\end{aligned}
$$

B.2 Proof for Proposition 3.2 (Calibrated demand parameters).

Fix p_j, c_j, s_j. We will show under what conditions $\alpha^b < \alpha^{nb} < \alpha^{2a} < \alpha^{2b}$ will hold.

First, we show that $\alpha^b < \alpha^{nb}$ for all $\lambda \in (0,1)$. Because prices and costs are fixed across models, margins are also fixed across models. Thus we have that:

$$
\begin{aligned}
m_j^{nb} &= m_j^b \\
\implies \frac{\frac{1}{\alpha^{nb}} \cdot \log(1-s_j)}{-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j} &= \frac{\frac{1}{\alpha^b} \cdot \log(1-s_j)}{-(1-s_j)\log(1-s_j)} \\
\implies \alpha^b \cdot (-(1-s_j)\log(1-s_j)) &= \alpha^{nb} \cdot (-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} s_j)
\end{aligned}
$$

Because $\frac{\lambda}{1-\lambda} s_j > 0$ This implies that $\alpha^b < \alpha^{nb}$.

Second, we show that $\alpha^{2a} < \alpha^{2b}$ for all $\lambda \in (0, 1)$.

$$
\begin{aligned}
m_j^{2a} &= m_j^{2b} \\
\implies \frac{EmaxDiff^{2a}}{-\alpha^{2a}} &= \frac{(1-\lambda)EmaxDiff^{2b}}{-\alpha^{2b}} \\
\implies \alpha^{2b} EmaxDiff^{2a} &= \alpha^{2a}(1-\lambda) EmaxDiff^{2b} \\
\implies \alpha^{2b}[-\log(1-s_j)] &= \alpha^{2a}(1-\lambda)[-\log(1-s_j)] \\
\implies \alpha^{2b} = \alpha^{2a}(1-\lambda) \implies \alpha^{2b} &> \alpha^{2a}
\end{aligned}
$$

Finally, we show the condition under which $\alpha^{nb} < \alpha^{2a}$.

$$
\begin{aligned}
m_j^{nb} &= m_j^{2a} \\
\implies \frac{\frac{1}{\alpha^{nb}} \cdot \log(1-s_j)}{-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j} &= \frac{-\log(1-s_j)}{-\alpha^{2a}} \\
\implies \alpha^{2a} &= \alpha^{nb} \cdot \left[-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda} \cdot s_j\right]
\end{aligned}
$$

This means that

$$
\begin{aligned}
& \alpha^{nb} < \alpha^{2a} \\
\iff & \left[-(1-s_j)\log(1-s_j) + \frac{\lambda}{1-\lambda}\cdot s_j\right] > 1 \\
\iff & \frac{\lambda}{1-\lambda} > \frac{1}{s_j} + \frac{1-s_j}{s_j}\log(1-s_j)
\end{aligned}
$$