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I. Introduction

This paper presents a model in which a firm chooses between selling a monopolized product alone or in bundles to identical consumers. It formalizes bundling in a new way[2] and examines optimal bundle construction[3] as well as optimal pricing. The model is used to analyze the special case of goods demanded in fixed proportions.[4]

With goods demanded in fixed proportions, bundling is profitable if the market price of the second good exceeds the firm's

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1 Division of Industry Analysis, Bureau of Economics, Federal Trade Commission. The opinions expressed in this paper are those of the author and do not reflect those of the Commission, individual Commissioners, or other staff members. Daniel Alger, Richard Craswell, Alan Fisher, Garth Saloner, Steven Salop, Richard Schmalensee, and Carl Shapiro provided helpful comments on earlier drafts.

2 Compare Adams and Yellin (1976) and Schmalensee (1982).

3 Paroush and Peles (1981) also examine the issue of bundle construction. However, in their model, demands for the two products are independent and the purpose of bundling is to price discriminate among consumers of different types. Because all consumers are identical, price discrimination is not a motive for bundling in the model presented here.

4 The circumstances in which it is profitable to tie goods used in variable proportions are considered by Blair and Kaserman (1978), Bowman (1957), Burstein (1960), and Gelman (1983). The polar case of profitable bundles involving goods for which demands are independent is considered in Adams and Yellin (1976) and Schmalensee (1982).

marginal cost. This may occur, for instance, if the second market is monopolistically competitive.

That monopolists may have an incentive to bundle products used in fixed proportions contradicts the assertions in the legal literature on tying arrangements.[5] We show that it is only profitable to bundle products used in fixed proportions when it is also efficient. Hence, we agree with scholars who argue that such ties should be allowed.[6] Although the outcome of this model does not change the policy conclusion, understanding the monopolist's incentive to tie is nevertheless instructive. Examining the fixed proportions case shows that bundling is analogous to vertical

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5 See, for example, Sullivan (1977), pp. 446-47:

> "If . . . the item on which there is a patent or other monopoly (say, hammerheads) were used in fixed proportions with the tied item (say, hammer handles), the tie, though extending the monopoly from heads to handles, would not result in any greater monopoly profits . . . . [t]hough it displaces the prior sellers of handles, it does not increase the aggregate price for hammers or increase profits or reduce output."

Ward Bowman (1957) p. 20, agrees, asserting that a bolt monopolist has no incentive to tie the sale of nuts to bolts:

> "Every increase in the price of nuts, even if the monopolist could produce them as cheaply as competitors, would require [a] reduction in the price of bolts by a compensating amount. If the monopolist acted otherwise, he would be creating a situation which reduced his monopoly return."

6 See Posner (1976), Bork (1978), Sullivan (1977), and Bowman (1957).

integration to the extent that it enables a monopolist of one product to control another market.[7]

In this paper, we assume that a monopolist is the sole supplier of one product and that it has the capability of supplying the second product if it chooses. We derive the monopolist's profit-maximizing strategies for selling the monopolized product alone and in bundles. We prove that a monopolist will choose not to bundle the two products if the second product's unbundled price is below the monopolist's marginal cost. If the second product's unbundled price equals the monopolist's marginal cost, the monopolist is indifferent between bundled and unbundled sales. If the second product's unbundled price exceeds the monopolist's marginal cost, the monopolist earns higher profits from bundling. This increase in profits occurs even if the monopolist were previously selling the second product at the market price. Bundling results in a lower aggregate price for the two products and the output of each product is higher as well.

The conclusion summarizes these results, discusses their policy implications, and compares bundling to vertical integration.

II. The Model

If products are used in fixed proportions, the demand for each product depends on both prices. If the monopolized product

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7 Warren-Boulton (1978) showed that monopolists have an incentive to vertically integrate when products are used in fixed proportions to overcome the problem of dual-markups.

is only sold in bundles which also contain the second product, demand for additional units of the second product depends on the proportion contained in the bundles relative to the proportion in which the products are used. If there are insufficient units of the second product in the bundle, consumers purchase additional unbundled units.

We denote the two products used in fixed proportions as X and Y. For convenience, we assume that the factor of proportionality is equal to one. That is, using lowercase letters to denote quantities, X and Y are demanded in equal proportions such that $x = y$.

We assume a patent monopolist produces X at constant marginal cost c_x and has the technology to produce Y at constant marginal cost c_y.[8] We assume that there are no complementarities in production.

Let p be the market price of a unit of X, let q be the market price of a unit of Y, and let r denote the price of a bundle containing units of X and Y.

For products used in fixed proportions, the consumer derives no additional utility from purchasing additional units of X if $x > y$ nor vice versa. If the products are sold separately, consumers will purchase equal amounts of the two goods. Their demand for each product will depend on the prices of both, or

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8 Throughout this paper, we assume marginal costs are constant. This simplifies the math but does not change any of the central results.

$$x = y = f(p+q) \tag{1}$$

We assume that if bundles are offered by the monopolist, X is available only in bundles but Y continues to be sold separately.

Let e denote the proportion of Y to X in each bundle (i.e., $e = \frac{Y}{X}$). For convenience, we normalize bundles to contain one unit of X. Thus, x represents both the units of X and the number of bundles sold.

We assume that consumers are able to purchase additional units of Y in the market at price q and, if $e < 1$, they do so to keep their consumption of the two goods in equal proportions. Hence, if units of X must be purchased in bundles, we can write the demand for bundles as

$$x = x(r+\max(1-e,0)q) \tag{2}$$

In deciding how to sell its product, the monopolist has a choice of two strategies. The monopolized product can be sold alone or in bundles containing units of the second product. If it bundles, the monopolist must also choose the composition of the bundle e. The monopolist sells bundles if doing so is more profitable than single product sales.

We analyze the two strategies below and show that bundling is optimal if the second product's unbundled price exceeds the monopolist's marginal cost. We also show that given that bundling is optimal, it is most profitable to construct bundles containing the two goods in the proportions in which they are used.

Let π^u denote the monopolist's profits when selling X as a single (unbundled) product. Using the demand function given by equation (1), we can write the single-product monopolist's profit maximization problem as

$$\max_{p} \pi^u = [p - c_x]x(p+q) \tag{3}$$

Differentiating equation (3) yields the usual first-order condition

$$x(p^*+q) + [p^* - c_x]x'(p^*+q) = 0 \tag{4}$$

where p^* denotes the profit-maximizing value of p.

We write the single-product monopolist's maximized profits as

$$\pi^u(p^*,q) = [p^* - c_x]x(p^*+q) \tag{5}$$

Let π^b denote the monopolist's profits when selling bundles containing e units of product Y. Using the demand function given in equation (2), we can write the tying monopolist's profit maximization problem as

$$\max_{re} \pi^b = [r - c_x - c_y e]x(r+\max(1-e,0)q) \tag{6}$$

Differentiating equation (6) with respect to e, we have

$$\frac{\partial \pi^b}{\partial e} = \begin{cases} -[r - c_x - c_y e]qx' - c_y x(r+(1-e)q) > 0 & \text{for } 0 < e < 1 \\ 0 & e > 1 \end{cases} \tag{7}$$

at the profit-maximizing (r,e) pair.

Because the level of demand is the same for e > 1 as for e = 1 at any r but bundles with a higher production of Y cost more to produce, the monopolist must earn higher profits at e = 1 than at $e > 1$. Hence, the monopolist will always set $e \leq 1$. Substituting $1-e$ for $\max(1-e,0)$ in equation (6) and differentiating with respect to r, we have

$$\frac{\partial \pi^b}{\partial r} = x(r+(1-e)q) + [r - c_x - ec_y]x' = 0 \quad \text{for } 0 \leq e \leq 1 \qquad (8)$$

Substituting equation (8) into equation (7), we have

$$\frac{\partial \pi^b}{\partial e} = [q - c_y]x(r+(1-e)q) \qquad (9)$$

Solving equations (8) and (9) simultaneously for $0 \leq e \leq 1$ gives us the monopolist's profit-maximizing price and bundle composition choices, which we denote as r^* and e^*. The monopolist's maximized profits from bundling can be written as

$$\pi^b(r^*,e^*,q) = [r^* - c_x - c_y e^*]x(r^*+(1-e^*)q) \qquad (10)$$

Whether bundling or selling product X alone is more profitable depends on the relationship of the monopolist's marginal cost c_y to the product Y's unbundled price q. There are three cases to consider: $q < c_y$, $q = c_y$ and $q > c_y$.

Case 1: $q < c_y$

If $q < c_y$, then, by equation (9), $\frac{\partial \pi^b}{\partial e} < 0$.

Hence, the monopolist chooses to construct bundles with as low a proportion of Y as possible and sets $e = 0$. Of course, if $e = 0$, "bundles" only contain product X and are equivalent to unbundled sales.

Thus, the monopolist does not bundle the two goods if the second product's unbundled price is below the monopolist's marginal cost.

Case 2: $q = c_y$

If $q = c_y$, equation (7) holds with equality for all $0 \leq e \leq 1$. Since profits do not depend on e when product Y is sold at its marginal cost, we can set e at any convenient value. Substituting $e^* = 1$ into equation (9), we eliminate the unbundled price of the second good from the equation and we can write the maximized profits from bundling as $\pi^b(r^*) = [r^* - c_x - c_y]x(r^*)$.

Let $t^* = r^* - c_y$. Substituting for r^* in $\pi^b(r^*)$, we have $\pi^b(r^*) = [t^* - c_x]x(t^*+c_y)$. But, as equation (3) states, p^* maximizes $[p - c_x]x(p+q)$. Hence, for $q = c_y$, t^* only maximizes π^b if $t^* = p^*$. Thus, we must have $r^* = p^* + c_y$. This implies that

$$\pi^t(r^*) = \pi^u(p^*,c_y) \qquad \text{for } q = c_y. \tag{11}$$

Thus, when the monopolist's marginal cost just equals the unbundled price of the second product, the monopolist earns the same profits bundling or selling the monopolized product alone. Therefore, when the unbundled market for product Y is competitive, or when the monopolist is a marginal supplier, bundling has no advantage over single-product sales.

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9 We assume that although its costs are below the market price, the monopolist cannot capture the entire second market simply by undercutting the unbundled price. This may occur in several circumstances. For example, if the second market is monopolistically competitive, the typical firm wishes to increase its output but cannot do so because its pricecuts are matched by other firms. Brand loyalty or similar barriers may also prevent the firm from entering the market for unbundled units. (See Farrell (1982), Salop (1979) and Schmalensee (1978) for models in which entry is deterred when the market price is above the entrant's cost.)

Case 3: $q > c_y$9

By equation (9), we see that profits are strictly increasing in the range $0 < e < 1$ for $q > c_y$. Hence, the monopolist will include as high a proportion of Y as possible in the bundles and set $e = 1$. Substituting $e = 1$ into the first order conditions given by equations (8) and (9), we see that if $q > c_y$ the optimal price of r^* is independent of the unbundled price q.

From equation (4), we see that the profit-maximizing price p for single-product sales is a function of the unbundled price of the second product or, $p^* = p^*(q)$. Partially differentiating $\Pi^u(p^*,q)$, as given by equation (5), with respect to q, we have

$$\frac{\partial \Pi^u}{\partial q} = [x + (p^* - c_x)x']p^{*'} + (p^* - c_x)x' \qquad (12)$$

The expression in brackets is the first order condition for single product sales, given by equation (4), and thus is equal to 0 at $p^* = p^*(q)$. Since $[p^* - c_x]x' < 0$, we have $\frac{\partial \Pi u}{\partial q} < 0$. This implies that $\Pi^u(p+q) < \Pi^u(p+c_y)$ for $q > c_y$. Thus, by equation (11), we have $\Pi^t(r^*) > \Pi^u(p+q)$ for $q > c_y$.

Hence, if the second product's unbundled price exceeds the monopolist's marginal cost, bundling is more profitable than single-product sales.

When $q > c_y$, the monopolist's decision to bundle does more than merely transfer profits to the monopolist from other, equally efficient producers. It also results in a lower aggregate price, higher output, and higher aggregate profits. We show this as follows.

Totally differentiating equation (5) gives us the slope of $p^* = p^*(q)$, or

$$\frac{dp^*}{dq} = \frac{-[x' + (p - c_x)x'']}{[2x' + (p - c_x)x'']} \tag{13}$$

The denominator of equation (13), which equals $\frac{\partial^2 \pi^u}{\partial p^2}$, is negative by concavity of the profit function. The numerator may be positive or negative but is smaller in absolute value than the denominator. Thus, we have $-1 < \frac{dp^*}{dq} < 1$. This tells us that the aggregate price consumers pay increases as the price of the second good increases, or $\frac{d(p^*(q)+q)}{dq} > 0$. Hence, for $q > c_y$, we have $p^*(q) + q > p^*(c_y) + c_y$. We have previously shown that the bundled price equals the sum of the unbundled prices when the second good is available at marginal cost, or $r^* = p^*(c_y) + c_y$. Therefore, we have $r^* < p^*(q) + q$ for $q > c_y$.

Thus, consumers pay less under bundling. Since consumers only consider the aggregate price in deciding how much of each product to purchase, lower prices under bundling mean that more of both products is demanded, or $x(r^*) = y(r^*) > f(p^*(q)+q)$ for $q > c_y$.

Of course, at $q > c_y$, the monopolist has the incentive to undercut the market price by setting its price for unbundled units q_m below the market price ($q_m < q$) in order to supply all or part of the unbundled market. If the monopolist can capture the entire second market with a price cut, the monopolist can duplicate the bundling price by setting $p = r^* - q_m$ and thus earn $^b(r^*)$ on sales to consumers demanding the goods in fixed proportions. However, this result depends crucially on the monopolist's being the only source of the second product. With monopolistic competition or entry barriers, this may not be the case.

If consumers of the monopolized product may purchase the second product from another source, the monopolist does not take full account of the effect on the second product's profits when setting the price of the monopolized product. Under such circumstances, the monopolist has an incentive to set $p > r^* - q$ for $q > c_y$. Hence, bundling results in higher monopoly profits and a lower aggregate price whenever the monopolist cannot capture the entire second market.

For fixed proportions demand, the incentive to bundle is fully analogous to the incentive to vertically integrate.[10] Both bundling and integration are methods of control enabling the monopolist of one product to capture profits otherwise earned by sellers of the second product. Each control enables the monopolist to control the implicit prices of both products and to earn all the profits generated by sales to consumers of the monopolized product. With the controls the monopolist maximizes total profits rather than profits from only one product or stage in the vertical

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[10] Blair and Kaserman (1978) have previously shown that tying and vertical integration are analogous for the variable proportions case.

chain. When the market price of the second product is above the monopolist's cost, bundling and vertical integration both result in the consumer's paying a lower aggregate price for goods used in fixed proportions.

III. Conclusion

We have shown that ties may be profitable when products are used in fixed proportions. In particular, if the unbundled price of the second product exceeds the monopolist's marginal cost, the monopolist earns greater profits from bundling than from single-product sales.

Bundling does not merely transfer profits from independent vendors of the second product to the monopolist. The monopolist only bundles when its cost of providing the tied product is below the market price. Since bundling lowers the total costs of providing both products to consumers, the monopolist wishes to sell more units. Hence, when the monopolist bundles, it sets a lower aggregate price for the two products. This leads to a higher output of each product and higher consumer welfare. Aggregate profits (those of independent vendors plus those of monopolist) may also rise.

Bundling products used in fixed proportions may thus generate efficiency benefits. Hence, such ties by firms with market power should not prevent "per se".[11] The efficiency gain from bundling results from there being a single monopoly markup, rather than two, when products are sold together.

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11 For products used in fixed proportions, firms can currently get around the prohibition in two ways. First, many products used in fixed proportions involve a physical tie, either through assembly (e.g., hammerheads and hammershanks) or through design (e.g., nuts and bolts). Second, the courts have sometimes viewed two products used in close association as a single product. (See Harris (1980) on this second issue).

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