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Kenneth Kelly

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Kenneth Kelly
Bureau of Economics
Federal Trade Commission
Washington, D.C. 20580

Economists are in general agreement that a primary source of increased per capita income has been changes in technology. The question of whether or not society devotes sufficient resources to the production of new technology is then an important one.

An influential effort to answer this question was made by Kenneth Arrow, who concluded ". . . we expect a free enterprise economy to underinvest in invention and research (as compared with an ideal) because it is risky, because the product can be appropriated only to a limited extent, and because of increasing returns in use," (p. 619).

The claim that firms will spend less than is socially optimal for R&D because they cannot fully appropriate the benefits from it has been challenged by Guy Arvidson and Jack Hirshleifer, who show that the private value of an invention can exceed its social value, leading to an overinvestment in research. However, no attempt has been made to formally examine Arrow's claim that the riskiness of the innovation process causes firms to underinvest in R&D.[1]

Arrow begins his analysis by considering an economy in which there is a complete set of contingent commodity markets, i.e., markets for futures contracts contingent on the realization of a particular state of the world. The competitive equilibrium of such an economy can, under certain conditions, be shown to be Pareto optimal.

He then considers an economy with no provisions for transferring risk bearing. "The firm and its owners cannot relieve themselves of risk bearing in this model. Hence any unwillingness or inability to bear risks will give rise to a nonoptimal allocation of resources, in that there will be discrimination against risky enterprises as compared with the optimum," (pp. 611-12). While real economies do not have complete sets of futures markets there are alternative institutional arrangements to permit the reallocation of risk bearing. However, Arrow argues that such arrangements cannot be utilized in the case of research. He concludes that "(S)ince it is a risky process, there is bound to be some discrimination against investment in inventive and research activities," (p. 616).

This argument does not take into account the effect of research efforts by other firms. A successful invention by a competitor or by a firm outside the industry can have a drastic impact on a firm's sales and profits, even to the point of driving it out of business. Innovation can serve to protect the firm against such losses. Joseph Schumpeter stressed the use of R&D as a means of defending monopoly profits as well as a means of obtaining them as being a major incentive for innovation.[2]

When the threat of reduced profits from innovations by other firms is substantial, R&D can serve as a means of insurance against a decline in profits as well as a gamble for higher profits. In these circumstances it is not clear what effect attitudes towards risk have on research spending. While risk averse individuals gamble less than risk neutral ones they also have a higher demand for insurance. It is therefore possible that the unwillingness of firm managers to bear risks causes them to spend more rather than less for research and development.

This paper attempts to analyze the effect of risk aversion on research and development spending. In addition, the effect of other economic variables on R&D spending when firm managers are risk averse is examined.

I. The Risks Involved in Innovation

Research and development is a type of production activity. A useful approach to understanding the innovation process and the role of uncertainty in that process is to examine R&D as we would any other production activity.

The output of the R&D process is knowledge--the knowledge of how to build a new or existing product. The output is then by definition uncertain, since if it were known, there would be no need to produce it.

There can be uncertainty about the relationship between inputs and outputs, i.e., technological uncertainty. For example, there can be a number of possible approaches to solving a technical problem, only a fraction of which will ultimately turn out to be feasible. Each may have to be investigated at a positive cost to determine whether it is feasible or not. Depending on the order in which each is investigated, the cost of solving the problem may vary by a rather high order of magnitude. Both the time and the cost of producing the invention will be uncertain.

A firm can also face market uncertainty in the production of knowledge in that the value of the output could be uncertain. The qualitative nature of the output may be uncertain: the degree to which a process innovation lowers costs or the characteristics of a product innovation. Knowledge about demand for the invention may be incomplete. Its value may also depend on exogenous forces (e.g., the value of a petroleum product substitute will depend on the price of crude oil). Finally, the value of the invention may depend upon the introduction of inventions by other firms.

Each of these uncertainties has been found to be important empirically. Cost and time overruns on military research and development projects are well known. In their study of the drug industry Edwin Mansfield and his collaborators found comparable overruns for pharmaceutical R&D.

In a study of 19 industrial laboratories in four different industries Mansfield and his colleagues found that only 56 percent of the research and development projects begun were considered to be technical successes, with the industry averages ranging from 32 to 73 percent. Of those projects that were technically successful, 55 percent were commercialized and only 40 percent of these were commercially successful in the sense of earning economic profits. Combining these figures indicates that only 12 percent of the R&D projects in the study were ultimately successful.

In a study of development costs in the pharmaceutical industry Ronald Hansen found that only one in eight new chemical entities that reach the stage of human testing are eventually marketed. This figure does not include those new chemical entities that are eliminated at earlier stages (e.g., animal testing) of the innovation process.

In the following section, a model is developed in which a risk averse firm faces technological uncertainty in R&D production and market uncertainty in that the value of the invention will depend on innovation by other firms.

II. The Model

A. The Objective of the Firm

Consider a firm that is currently producing a product on which it is earning a rate of profit, exclusive of research and development spending, of Π_1. The firm also has the option of engaging in an R&D project that, if successfully completed, will raise the firm's rate of profit to Π_2.

This firm also faces the threat that someone will introduce a new product or process that will result in the firm's rate of profit falling from Π_1 to Π_0. Such an invention could come from a current competitor of the firm, from a firm not presently in the industry, or even from an independent inventor. It is assumed that the firm will terminate its R&D efforts should this rival invention be introduced before its project is completed. Scenarios under which the firm would choose to terminate the project include

(i) the cost of inventing around the other firm's patent(s) is sufficiently high as to make further R&D an unattractive investment.

(ii) the first entrant advantages of the innovator make the increase in profit from imitation so low as to make continued research unattractive.

(iii) if R&D must be financed internally, Π_0 may be too low to permit R&D spending at a rate high enough to make completing the project profitable.[3]

Should the firm complete its R&D project before the introduction of an invention by a rival, it removes the threat of a reduction in profit and hence earns Π_2 in perpetuity. This would be the case if the firm's rival also ceases its research efforts once the firm completes its project.

Let t_1 be the date at which the firm completes its research and development and introduces its invention, and let t_2 be the date at which a rival introduces its innovation. (Although the firm will cease its R&D once a rival innovation is introduced, it will still be useful to define t_1 to be the time at which the project would have been completed, had it been continued.) The firm views t_2 as an exogenous random variable. In particular, the firm views the probability that the rival innovation will occur in a given time interval of fixed length to be constant over time, i.e., t_2 is distributed exponentially

$$F_2\,(t) = 1 - \exp(-\lambda t),$$

$$f_2(t) = dF_2\,(t) = \lambda \exp(-\lambda t),\ t>0,$$

where λ is a positive constant.

Let $\Pi(t)$ be the rate of profit at time t (either Π_0, Π_1, or Π_2) and let $r(t)$ be research spending at time t. The firm (or its manager) is assumed to have a utility function, $U(t) = U(\Pi(t)-r(t))$, that is a function of the cash flow at time t. The goal of the firm is to maximize the expected value of $\int_0^\infty U(\Pi(t)-r(t)) \exp(-\rho t)\, dt$, where ρ is the discount rate ($0 < \rho < 1$).

A particular functional form for U that gives a useful parameterization for examining the effect of attitudes towards risk on R&D spending is the constant absolute risk-aversion utility function, $U(t) = -\exp(-\gamma(\Pi(t)-r(t)))$, where γ is a positive constant. The Arrow-Pratt measure of risk aversion, $-U''/U'$, will then be γ for all nonnegative values of $\Pi-r$.

B. The Technology of the Innovation Process[4]

It is assumed that the firm knows Π_0 and Π_2 with certainty. The time and cost of the research and development program to produce the invention are not, however, known with certainty.

We introduce the variable h(t), total accumulated effort at time t, as a measure of the stock of knowledge the firm has acquired about the project by time t. The level of accumulated effort necessary to complete the project is not known to the firm, but it does have beliefs about h that can be expressed probabilistically. F(h) is the firm's assessment of the probability that its project will be completed with accumulated effort h or less.

Research spending at rate r yields the firm effective effort v(r), the rate at which its knowledge about the project is increasing. Effective effort v(r), the rate at which additions are made to the stock of knowledge, is related to the stock of knowledge, h(t), by the following relationship:

$$h(t) = \int_0^t v(r(s))\, ds.$$

The function v(r), the production function for new knowledge, is assumed to have the properties that we expect production functions for commodities to possess. That is, $v(r)$ is assumed to have continuous first and second derivatives, and there exists a value of r, r^+, such that for all $r \varepsilon [0, r^+)$ $v'(r)>0$ and $v''(r)<0$, $v'(r^+) = 0$, and for all $r>r^+$ $v'(r)<0$. As the firm increases its research spending, the rate at which additions to knowledge are made increases, but at a declining rate. This rate of increase reaches a maximum at $v(r^+)$, and a rate of research spending higher than r^+ yields a negative marginal return. These assumptions imply that there will be an inverse, convex relationship between the time and the cost of carrying out an R&D project. There is a good deal of empirical support for such a relationship (see the survey by Kamien and Schwartz (1975)).[5]

It is assumed that the firm believes that h is distributed exponentially

$$F(h) = 1 - \exp(-h),$$

$$f(h) = dF(h) = \exp(-h), \quad h>0.$$

Given that t_2 is also distributed exponentially, the optimal rate of research spending by the firm will be constant over time until the project is completed.[6] Let r^* be this optimal rate of spending. Then $h(t) = v(r^*)t$, and t_1 will be distributed exponentially

$$F_1(t) = 1-\exp(-v(r^*)t),$$

$$f_1(t) = dF_1(t) = v(r^*)\exp(-v(r^*)t), \quad t > 0.$$

It is assumed that there are no externalities, informational or otherwise, between the research efforts of the firm and any of its rivals, so that the firm takes h and t_2 and hence t_1 and t_2 to be stochastically independent.

C. The Optimal Rate of Research Spending

At any time t, three mutually exclusive events are possible: the firm has successfully completed its research ($t_1<t_2$ and $t_1<t$), a rival has introduced its invention and the firm has stopped its research effort ($t_2<t_1$ and $t_2<t$), or no firm has introduced an invention and the firm under consideration is continuing its R&D ($t<t_1$ and $t<t_2$).

Since t_1 and t_2 are assumed to be independently distributed, their joint probability density function will be

$$dF_1(t_1)\ dF_2(t_2) =$$

$$dF(t_1, t_2) = v(r)\lambda\exp(-(v(r)\ t_1 + \lambda t_2)).$$

The probability that the firm is successful at time t is then

$$\int_0^t \int_{t_1}^\infty v(r)\ \lambda\exp(-(v(r)t_1 + \lambda t_2))\ dt_2\ dt_1$$

$$= \int_0^t v(r)\ \exp(-(v(r) + \lambda)t_1)\ dt_1$$

$$= \frac{v(r)}{v(r) + \lambda} \quad (1 - \exp(-(v(r) + \lambda)t)$$

The probability that the firm will be preceded by a rival will be

$$\int_0^t \int_{t_2}^\infty = v(r)\ \lambda\exp(-(v(r)\ t_1 + \lambda t_2))\ dt_1\ dt_2$$

$$= \frac{\lambda}{v(r) + \lambda} \quad (1 - \exp(-(v(r) + \lambda)t).$$

The probability that neither event occurs and the firm is continuing its R&D at time t is then $\exp(-(v(r) + \lambda)t)$.

The firm's problem is to choose the value of r that maximizes

$$\int_0^\infty - e^{-\rho t} e^{-\gamma(\Pi_1 - r)} \exp(-(v(r) + \lambda)t)\, dt$$

$$+ \int_0^\infty - e^{-\rho t} e^{-\gamma \Pi_2} \frac{v(r)}{v(r)+\lambda} (1 - \exp(-(v(r)+\lambda)t)\, dt$$

$$+ \int_0^\infty - e^{-\rho t} \frac{e^{-\gamma \Pi_0} \lambda}{v(r) + \lambda} (1 - \exp(-(v(r)+\lambda)t)\, dt$$

$$= \underset{r}{\text{Max}} \Big[\frac{- e^{-\gamma(\Pi_1 - r)}}{v(r) + \lambda + \rho} - \frac{v(r)\ e^{-\gamma \Pi_2}}{(v(r) + \lambda)\ \rho}$$

$$+ \frac{v(r)\ e^{-\gamma \Pi_2}}{(v(r)+\lambda+\rho)(v(r)+\lambda)} - \frac{\lambda e^{-\gamma \Pi_0}}{(v(r)+\lambda)\rho}$$

$$+ \frac{\lambda e^{-\gamma \Pi_0}}{(v(r)+\lambda+\rho)(v(r)+\lambda)}\Big]$$

$$= \underset{r}{\text{Max}} \Big[\frac{-e^{-\gamma(\Pi_1 - r)}}{v(r) + \lambda + \rho} - \frac{v(r)\ e^{-\gamma \Pi_2}}{(v(r)+\lambda+\rho)\rho} - \frac{\lambda e^{-\gamma \Pi_0}}{(v(r)+\lambda+\rho)\rho}\Big]$$

Taking the partial derivative of the bracketed expression with respect to r and setting the result equal to zero gives

$$0 = \frac{v'(r)\ e^{-\gamma(\Pi_1 - r)}}{(v(r) + \lambda + \rho)^2} - \frac{\gamma e^{-\gamma(\Pi_1 - r)}}{(v(r) + \lambda + \rho)}$$

$$- \frac{v'(r)(2v(r) + \lambda + \rho)\ e^{-\gamma \Pi_2}}{(v)r) + \lambda + \rho)^2 \rho}$$

$$+ \frac{v'(r)\ \lambda\ e^{-\gamma \Pi_0}}{(v(r) + \lambda + \rho)^2 \rho} \equiv R(r^*; \gamma, \lambda, \rho \ldots),$$

where $R(r^*) = 0$ is an implicit expression for r^*, the optimal level of research spending.

Solving $R(r^*) = 0$ for $v'(r)$ gives

$$v'(r^*) = \frac{\gamma\rho(v(r^*) + \lambda + \rho)e^{-\gamma(\Pi_1 - r^*)}}{\rho e^{-\gamma(\Pi_1 - r^*)} - (2v(r^*) + \lambda + \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0}} \cdot \quad (1)$$

Since $v'(r^*)$ is positive at an internal solution (i.e., $0<r^*<r^+$), and the numerator of (1) is positive, it must be the case that the denominator of (1) is positive.

III. Comparative Statics

A. The Effect of Changes in Risk Aversion

Our concern is with the effect that attitudes towards risk have on R&D spending, i.e., the sign of

$$\frac{dr^*}{d\gamma} = \frac{-\frac{\partial R}{\partial \gamma}}{\frac{\partial R}{\partial r^*}}$$

Since $\partial R/\partial r^*$ must be ≤ 0 by the second order condition for maximization the sign of $dr^*/d\gamma$ will be the same as the sign of $\partial R/\partial \gamma$. Taking the partial derivative of R with respect to γ gives

$$\begin{aligned}\frac{\partial R}{\partial \gamma} &= \frac{v'(r^*)\Pi_2(2v(r^*)+\lambda+\rho)e^{-\gamma\Pi_2}}{(v(r^*)+\lambda+\rho)^2\,\rho} \\ &+ \frac{(\gamma(\Pi_1-r^*)-1)e^{-\gamma(\Pi_1-r^*)}}{v(r^*)+\lambda+\rho} \\ &- \frac{v'(r^*)(\Pi_1-r^*)e^{-\gamma(\Pi_1-r^*)}}{(v(r^*)+\lambda+\rho)^2} \\ &- \frac{v'(r^*)\lambda\Pi_0\, e^{-\gamma\Pi_0}}{(v(r^*)+\lambda+\rho)^2\,\rho}\end{aligned}$$

Substituting for v'(r) from (1) gives

$$\frac{\partial R}{\partial \gamma} = \frac{\gamma\rho\Pi_2(v(r^*)+\lambda+\rho)(2v(r^*)+\lambda+\rho)\, e^{-\gamma\Pi_2}\, e^{-\gamma(\Pi_1-r^*)}}{(\rho e^{-\gamma(\Pi_1-r^*)}-(2v(r^*)+\lambda+\rho)e^{-\gamma\Pi_2}+\lambda e^{-\gamma\Pi_0})(v(r^*)+\lambda+\rho)^2\rho}$$

$$+ \quad \frac{(\gamma(\Pi_1-r^*) -1)\ e^{-\gamma(\Pi_1-r^*)}}{v(r^*) + \gamma + \rho}$$

$$- \quad \frac{\gamma\rho(\Pi_1-r^*)\ (v(r^*) + \lambda + \rho)\ e^{-2\gamma(\Pi_1-r^*)}}{(\rho e^{-\gamma(\Pi_1-r^*)} - (2v(r^*) +\lambda+ \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0})\ (v(r^*) +\lambda+ \rho)^2}$$

$$- \quad \frac{\gamma\rho\lambda\Pi_0\ (v(r^*) + \lambda + \rho)\ e^{-\gamma(\Pi_1-r^*)}\ e^{-\gamma\Pi_0}}{(\rho e^{-\gamma(\Pi_1-r^*)} - (2v(r^*) + \gamma + \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0})\ (v(r^*) +\lambda+\rho)^2\rho}$$

$$= \frac{N_1}{(\rho e^{-\gamma(\Pi_1-r^*)} - (2v(r^*) + \lambda + \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0})(v(r^*) + \lambda + \rho)} \quad ,(2)$$

where

$$N_1 = (2v(r^*) +\lambda+ \rho)(\gamma(\Pi_2-\Pi_1 + r^*) + 1)e^{-\gamma(\Pi_1-r^*)}\ e^{-\gamma\Pi_2}$$

$$- \rho e^{-2\gamma(\Pi_1-r^*)} + \lambda(\gamma(\Pi_1-r^*-\Pi_0)-1)\ e^{-\gamma\Pi_0}\ e^{-\gamma(\Pi_1-r^*)}.$$

Since the denominator of (2) is positive, the sign of $\partial R/\partial\gamma$ and hence $dr^*/d\gamma$ will be the same as the sign of N_1.

For any value of γ, there will be values of the other parameters for which N_1 will be positive or negative.

By choosing values for Π_1 and Π_0 such that $\Pi_1-\Pi_0 < 1/\gamma$, the term $\lambda(\gamma(\Pi_1 r^*-\Pi_0)-1)e^{-\gamma\Pi_0}\ e^{-\gamma(\Pi_1-r^*)}$ will be negative. Since $(2v(r^*) +\lambda+ \rho)(\gamma(\Pi_2-\Pi_1+r^*) + 1)e^{-\gamma(\Pi_1-r^*)}e^{-\gamma\Pi_2}$ goes to zero as Π_2 goes to infinity, by making Π_2 arbitrarily large, we can ensure that N_1, and hence $dr^*/d\gamma$ will be negative.

Similarly, by choosing values of Π_1 and Π_0 such that $(\Pi_1-\Pi_0-r^+) > (\rho/\lambda + 1)/\gamma$, N_1 and $dr^*/d\gamma$ will be positive. Therefore, an increase in risk aversion can lead to either a decrease or an increase in research spending.

The firm has two reasons for engaging in R&D--the "carrot" of a higher rate of profit should it successfully complete the project before rival precedence, and the "stick" of permanently reduced profits from rival innovation. As Π_2 and Π_0 become large relative to Π_1, the size of the carrot is made larger relative to the stick. As the carrot comes to provide the main incentive for innovation, R&D becomes more of a gamble for higher profits than a means of insuring against the loss of profits. When the gambling aspect becomes more important than the insurance aspect, an increase in risk aversion will decrese research spending.

By the same sort of reasoning, an increase in $\Pi_1-\Pi_0$ raises the penalty the firm receives should a rival invention be introduced before its R&D is completed. As the insurance incentive becomes greater than the gambling incentive, an increase in risk aversion will have the effect of increasing the optimal rate of research spending.

There are therefore circumstances in which a risk averse firm will spend more for R&D than would a risk neutral firm in those same circumstances.

B. The Effect of Changes in the Rate of Profit

It has been claimed that a high level of profits can serve to encourage innovation. As explained before, Schumpeter emphasized the need for firms to defend their current profits from the threat of rival innovation as being an important incentive for innovation. There is at least anecdotal evidence for the claim that R&D spending must be financed by internal means. A higher level of profits can then serve as a means of increasing liquidity and encouraging research spending. However, in their survey, Kamien and Schwartz (1975) conclude "[I]n sum, the empirical evidence that either liquidity or profitability are conducive to innovative effort or output appears slim," (p. 26). It is therefore of interest to see what effect changes in profit rates have on research spending in the this model, and what light the results can shed on these findings.

An increase in Π_1, with Π_0 and Π_2 fixed, will change the incentives for carrying out R&D. The reward (carrot) for innovation is decreased while the penalty attached to rival precedence (stick) is increased. It is important to distinguish between the case in which only the current profit is changed, and that in which Π_2 and Π_0 are also changed so as to keep $\Pi_2-\Pi_1$ and $\Pi_1-\Pi_0$ constant.

Taking the partial derivative of R with respect to Π_1 gives

$$\frac{\partial R}{\partial \Pi_1} = \frac{-\gamma v'(r^*)e^{-\gamma(\Pi_1-r^*)}}{(v(r^*) + \lambda + \rho)^2} + \frac{\gamma 2\ e^{-\gamma(\Pi_1-r^*)}}{(v(r^*) + \lambda + \rho)}.$$

Substituting for $v'(r^*)$ from (1) gives

$$\frac{\partial R}{\partial \Pi_1} = \frac{\gamma e^{-\gamma(\Pi_1-r^*)}}{(v(r^*) + \lambda + \rho)^2} [\gamma(v(r^*) + \lambda + \rho)$$

$$- \frac{\gamma\rho(v(r^*) + \lambda + \rho)e^{-\gamma(\Pi_1-r^*)}}{\rho e^{-\gamma(\Pi_1-r^*)} - (2v(r^*) + \lambda + \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0}}]$$

$$= \frac{\gamma^2\ e^{-\gamma(\Pi_1-r^*)}\ (\lambda e^{-\gamma\Pi_0} - (2v(r^*) + \lambda + \rho)\ e^{-\gamma\Pi_2})}{(v(r^*) + \lambda + \rho)(\rho e^{-\gamma(\Pi_1-r^*)} - (2v(r^*) + \lambda + \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0})}.$$

$\partial R/\partial \Pi_1$, and hence $dr^*/d\Pi_1$, will be negative for small values of λ and positive for large values of λ. (The denominator will be positive by equation (1).) The parameter λ is a measure of the probability of rival precedence, i.e., the threat of the stick. A low value of λ means the threat of reduced profits from a rival innovation is remote, so it is the promise of increased profits that is the important reason for investing in R&D. If this is the case, an increase in Π_1 reduces this incentive, and the optimal level of R&D spending is lower. Similarly, as λ increases so does the threat of reduced profits from a rival innovation, and the defensive incentive for R&D increases relative to the incentive of higher profits. When the stick is more important than the carrot, an increase in Π_1 raises the defensive incentive by a magnitude greater than the reduction in the profit incentive, and optimal R&D spending increases.[7]

The partial derivatives of R with respect to Π_0 and Π_2 are

$$\frac{\partial R}{\partial \Pi_0} = \frac{-\,v'(r^*)\gamma\lambda e^{-\gamma\Pi_0}}{(v(r^*)+\lambda+\rho)^2\rho} < 0,$$

$$\frac{\partial R}{\partial \Pi_2} = \frac{v'(r^*)\gamma(2v(r^*)+\lambda+\rho)e^{-\gamma\Pi_2}}{(v(r^*)+\lambda+\rho)^2\rho} > 0.$$

To consider the effect of an increase in Π_1 that leaves the size of the reward $(\Pi_2 - \Pi_1)$ and penalty $(\Pi_1 - \Pi_0)$ the same, we add $\partial R/\partial\Pi_1$, $\partial R/\partial\Pi_0$, and $\partial R/\partial\Pi_2$.

$$\frac{\partial R}{\partial \Pi_1} + \frac{\partial R}{\partial \Pi_0} + \frac{\partial R}{\partial \Pi_2} = \frac{N_2}{(v(r^*)+\lambda+\rho)^2\rho},$$

where $N_2 = \rho\gamma e^{-\gamma(\Pi_1 - r^*)}[\gamma(v(r^*)+\lambda+\rho) - v'(r^*)]$

$$-\;v'(r^*)\gamma\lambda\, e^{-\gamma\Pi_0} + v'(r^*)\gamma(2v(r^*)+\lambda+\rho)e^{-\gamma\Pi_2}.$$

Substituting for $v'(r^*)$ from (1) gives

$$\frac{\partial R}{\partial \Pi_1} + \frac{\partial R}{\partial \Pi_0} + \frac{\partial R}{\partial \Pi_2} = 0.$$

A change in the current level of profit that leaves the dollar amounts of the increase in profit from success and the decrease in profit from rival precedence the same will have no effect on the optimal level of R&D spending.

C. The Effect of Changes in Rivalry

The parameter λ in a measure of the probability of innovation by a rival in a given time period. It can be interpreted as a measure of the degree of technological rivalry the firm faces.

The effect of a change in the degree of rivalry on R&D spending has been examined in several studies, with various conclusions. Kamien and Schwartz (1976) have found that there is a level of technological rivalry that maximizes a firm's research spending. In Glenn Loury's model an increase in rivalry causes a decrease in the equilibrium level of R&D by each firm. However, Tom Lee and Louis Wilde have shown that this result is sensitive to the assumption that R&D costs are contractual and do not vary with the time required to complete of the project. When this condition is relaxed, an increase in rivalry is shown to increase the firm's optimal R&D expenditure.

The partial derivative of R with respect to λ is

$$\begin{aligned}\frac{\partial R}{\partial \lambda} &= \frac{-2v'(r^*)e^{-\gamma(\Pi_1-r^*)}}{(v(r^*)+\lambda+\rho)^3} + \frac{\gamma e^{-\gamma(\Pi_1-r^*)}}{(v(r^*)+\lambda+\rho)^2} \\ &+ \frac{v'(r^*)[-(v(r^*)+\lambda+\rho)e^{-\gamma\Pi_2} + 2(2v(r^*)+\lambda+\rho)]e^{-\gamma\Pi_2}}{(v(r^*)+\lambda+\rho)^3\rho} \\ &+ \frac{v'(r^*)(v(r^*)-\lambda+\rho)e^{-\gamma\Pi_0}}{(v(r^*)+\lambda+\rho)^3\rho} \\ &= \frac{N_3}{(v(r^*)+\lambda+\rho)^3\rho},\end{aligned}$$

$$\text{where } N_3 = -2\rho v'(r^*)e^{-\gamma(\Pi_1 - r^*)} + \rho\gamma(v(r^*) + \lambda + \rho)e^{-\gamma(\Pi_1 - r^*)}$$
$$+ \quad v'(r^*)(3v(r^*) + \lambda + \rho)e^{-\gamma\Pi_2}$$
$$+ \quad v'(r^*)(v(r^*) - \lambda + \rho)e^{-\gamma\Pi_0} .$$

Substituting for $v'(r^*)$ from (1) gives

$$\frac{\partial R}{\partial \lambda} = \frac{\gamma e^{-\gamma(\Pi_1 - r^*)}(v(r^*)\ e^{-\gamma\Pi_2} + (v(r^*) + \rho)e^{-\gamma\Pi_0} - \rho e^{-\gamma(\Pi_1 - r^*)}}{(v(r^*) + \lambda + \rho)^2\ (\rho e^{-\gamma(\Pi_1 - r^*)} - (2v(r^*) + \lambda + \rho)e^{-\gamma\Pi_2} + \lambda e^{-\gamma\Pi_0})} > 0,$$

so that an increase in rivalry leads to an increase in the optimal rate of R&D spending by a firm.

IV. Summary and Concluding Remarks

The objective of this paper was to examine the effect attitudes towards risk have on research and development spending. It was shown that an increase in risk aversion could increase as well as decrease R&D spending. Hence, Arrow's claim that risk averse firms will spend less on R&D than risk neutral ones is not always true.

As mentioned above, overinvestment in R&D can occur where some of the rewards to innovation are pecuniary. Since the condition under which this occurs are also those where there can be a positive relation between risk aversion and research spending, the results here suggest that risk aversion could serve to increase R&D spending in those instances where it is already excessive and decrease it when there is underinvestment in new technology.

A standard defense for the assumption of profit maximizing behavior in the theory of the firm is that the forces of competition will result in the survival of only those firms that behave as if their goal was profit maximization. In an industry in which technological competition is important, firms that spend relatively more than their competitors on R&D would have a survival advantage. If risk aversion causes firms to spend more than risk neutral firms do for research, evolutionary forces would favor risk averse over risk neutral (i.e., profit maximizing) firms.

This consideration in turn raises questions about the relationship between market structure and innovation. How does technological rivalry vary over an industry's life cycle? If it is high early in an industry's development, but declining as concentration increase and the industry matures we might see the survival of firms that overinvest in technology early in the product life cycle and underinvest later on as rivalry decreases.

FOOTNOTES

1 While not addressing the positive economic question of whether risk aversion causes firms to spend less than they otherwise would for research and development, Harold Demsetz has criticized Arrow's conclusion that such a result is socially undesirable as an example of the "people could be different" fallacy.

2 For an analysis of these effects for a risk neutral firm see the article by Morton Kamien and Nancy Schwartz (1978b).

3 See Kamien and Schwartz (1978a) for explanations of why R&D must be financed internally.

4 The model of the R&D process presented here was first introduced by Robert Lucas.

5 An analogy with investment in physical capital with adjustment costs may be helpful in understanding the model. Accumulated effort $h(t)$ is a measure of the capital stock (here, a type of human capital) at time t. Research spending r is the investment spending measured in dollars. Effective effort $v(r) = dh/dt$ is then investment spending after adjustment costs.

6 A formal proof of this statement is given by Kenneth Kelly. An intuitive explanation can be given here. A feature of the exponential distribution that has made its use so popular in operations research is its "memorylessness." For instance, $P(t_2 > t + \bar{t} \mid t_2 > \bar{t}) =$

$$\frac{P(t_2 > t + \bar{t})}{P(t_2 > \bar{t})} = \frac{e^{-\lambda(t + \bar{t})}}{e^{-\lambda\bar{t}}} = e^{-\lambda t} = P(t_2 > t),$$

so that the fact that a period of time of length $\bar{t}$ has passed during which no rival innovations have appeared does not affect the probability that such an innovation will appear in a time period of length t. Since h is also distributed exponentially, and none of the parameters of the system change over time, if a period of time elapses in which no firm innovates, the problem the firm faces remains the same. Hence its optimal rate of research spending should be the same at the end of the period as it was at the start. Since this will be true for a time period of any length, the optimal rate of research spending will be constant over time. The assumption that h and t_2 are exponentially

(footnote 6 continues)

(footnote continued)

distributed then reduces what would be a stochastic dynamic programming problem of finding an optimal spending path over time, contingent on the nonappearance of a rival innovation, to a straightforward calculus problem.

7 $\partial r/\partial \Pi_1$ will also be positive for very large values of Π_2. A possible economic interpretation of this is that while the main incentive for R&D is the chance of earning higher profits, the change in this incentive is comparatively small, so that the net effect of a change in both incentives is positive.

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