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PREMIUMS FOR HIGH QUALITY PRODUCTS AS RENTS TO REPUTATION

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<u>Premiums for High Quality Products as</u>

<u>Rents to Reputation</u>

by

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It has long been recognized that a firm which has a good reputation owns a valuable asset. This is often referred to as the "goodwill" value of the firm's brand name or loyal customer patronage. This paper develops a model which explores the implications of firm-specific reputations in a perfectly competitive environment.

The idea of reputation only makes sense in an imperfect information world. A firm has a good reputation if consumers believe their products to be of high quality. If product attributes were perfectly observable prior to purchase, then previous production of high quality items would not enter into consumers' evaluations of the firm's product quality. Instead, quality beliefs could be derived solely from inspection.

When product attributes are difficult to observe prior to purchase, consumers may plausibly use the quality of products produced by the firm in the <u>past</u> as an indicator of present or future quality. In such cases, the firm's decision to produce high quality items is a dynamic one: the benefits to doing so accrue in the future via the effect of building up a reputation.

Since reputation is a capital asset, it is natural that some rental income should accrue to it. These quasi-rents exactly compensate the firm for the costs incurred in building up its reputation so that there are zero profits ex-ante. In fact, were there no flow of profits to be earned from having a high reputation, it would not pay to maintain the reputation. Instead of continuing to produce high quality items, a firm with a good reputation could cut quality and take a short-run gain as a result. This "fly-by-night" strategy would always be attractive were "profits" not being earned by maintaining reputation. These "profits" are really just rents in return for building up the reputation initially. But the above discussion makes it clear that they are also necessary to prevent the firm from preferring to milk its reputation.

These ideas are utilized in the analysis below to derive an equilibrium price-quality schedule under imperfect information. Prices of high quality products must exceed their cost in order to provide the flow of quasi-rents to reputation. The welfare consequences of this price-cost gap are investigated. In particular, a welfare analysis of minimum quality standards is presented. The higher the minimum quality standard, the less a firm can earn while milking its reputation. Since the premiums for high quality items are exactly large enough to forestall this milking, they are lower, the higher is the minimum quality standard. Thus raising minimum quality standards benefits consumers who like to consume high quality items. Balanced against this is the direct effect of excluding products other consumers would like to use.

There are a number of effects which are ruled out in order to focus on reputation as a quality-assuring mechanism. For example, the assumption that costs of production are not time interdependent is a strong one. One main reason for the fact that different firms product different quality items is that some producers find it easier to produce quality products. This will typically be due to some capital good on the cost side. Examples include expensive machinery in an auto repair shop which makes it less costly to provide good service, or training on the part of a skilled worker which has the same effect (i.e., reducing the marginal cost of quality). We might expect the capital goods to signal quality in such cases. I rule them out to focus only on a capital good on the demand side, namely reputation.

Similarly, a product line of high quality may already be designed and in production. This reduces the savings from a reduction in quality (which would involve some redesign efforts). This latter could be modelled as a cost to changing quality. However, so long as the firm can exit the market without taking losses, it will have to be the rents to reputation which forestall the temptation to do so. Firm specific capital other than reputation could serve some of this function as well.

Finally, I rule out guarantees as a quality-assuring mechanism. This is not because I believe them to be unimportant, but simply imperfect. A washing machine may have a one year guarantee, but the consumer is expecting a lifetime of 10 years from it. Therefore, there is room for potential quality cutting by the seller, the guarantee not withstanding. For a variety of moral hazard and adverse selection reasons, perfect guarantees are not feasible. Anytime the sellers could make some quick profits by reducing quality the analysis below will apply.

The paper is organized as follows: first I analyze in full the case where the seller chooses a quality of product each period but does not also have sales as a control variable. After defining and computing the equilibrium here, I analyze the welfare effects of improved information and minimum quality standards. Then I present a continuous time model where firms can choose sales as well as quality levels. This permits identification of some additional effects of imperfect information. In particular, firms do not operate at efficient scale. Finally, I provide a summary and conclusion.

General Set-Up and Definition of Equilibrium

The model to follow describes the situation discussed above. It is necessarily dynamic, since reputation formation occurs over time. It is set in discrete time, where the period is the length of production. For example, if the product is constructing a building, the period of time is however long it takes to put one building up.

Each period each seller can choose what quality product to provide. In this model the number of products a given firm produces per period is held fixed (at one). For a model where quantity is also a control variable of the firm, see the final section. The price a seller can charge is determined by his reputation, R, and is denoted by p(R). This emodies the perfect competition assumption. Since in equilibrium consumers know the price-quality schedule, no firm can exert control over its price; consumers will not purchase from any firm which offers a price-reputation pair above the schedule. The cost of production depends on quality and is called c(q). I assume $c'(q) > 0$ and $c''(q) > 0$. Sellers choose quality over time to maximize the present value of their profits.

Reputation formation will initially be assumed to be of a very simple form: a seller's reputation this period is exactly his quality last period:

(1) $R_t = q_{t-1}$.

I will discuss modifications of (1) below. It simply reflects the fact that quality cannot be observed prior to purchase, and hence sellers can, at least for one period, cheat on their customers by reducing quality.

Consumers differ in both their taste for quality, Θ, and in their underlying evaluation of the good v. Consumers purchase either 0 or 1 unit of the good. If a consumer of type (Θ, v) consumes a unit of quality q he achieves net utility $\Theta q + v - p$, where p is the price paid.

Finally, there is a minimum quality q_0. This may be given several interpretations, but the simplest is that it is illegal to sell items of quality below q_0. I will discuss q_0 at length below. The distribution of demands is such that q_0 is actually produced (we will see below that this will be the case for the optimal minimum quality standard).

Entry is permitted, but new firms must prove themselves in order to build up a reputation. Initially, they must sell their product at price $p(q_0)$. This assumption is necessary for any equilibrium to exist. In fact if new firms could sell for any price higher than the cost of producing minimum quality items, then entrants could make positive profits by producing items of quality q_0 and selling them for one period and then exiting.

Equilibrium

Equilibrium is a price-quality (or, equivalently, a price-reputation) schedule p(q) such that

A. Each consumer, knowing $p(q)$, chooses his most preferred product on the schedule to consume (if he uses the product at al

B. Markets clear at every quality level (this determines the number of active firms in equilibrium).

C. A firm with reputation R finds it optimal to produce quality $q=R$ rather than to deviate (that is, consumers' expectations regarding quality are fulfilled).

D. No new entry is attractive.

Heuristic Derivation of Equilibrium Price-Quality Schedule

In this section I will derive the equilibrium price-quality schedule from elementary arguments. Just as the price of a good in perfect competition (in the long run, with perfectly elastic input supplies) is determined by the supply side alone - namely minimum average cost - so is the price which prevails at each quality level determined solely on the supply side in this model.

There are two conditions which are used to derive the schedule- conditions C. and D. from the definition of equilibrium above. First consider the condition that a firm with reputation q does not wish to milk its reputation. One way to milk reputation is to cut quality to the minimum, take short-run gains, and exit the market. This would yield profits of $p(q) - c(q_0)$. The alternative strategy of maintaining quality forever yields present discounted profits of $\frac{1+r}{r}(p(q) - c(q))$. In order that milking not be attractive we must have

$$\frac{1+r}{r}[p(q) - c(q)] \geq p(q) - c(q_0) \quad \text{i.e.}$$

(2) $p(q) \geq c(q) + r(c(q) - c(q_0))$.

As expected, the firm must be able to earn profits by maintaining quality in order not to wish to run down its reputation.

Turning to the free-entry condition, equilibrium requires that entry not be attractive. The profits to an entrant who produces quality q forever are

$$p(q_0) - c(q) + \frac{1}{r}[p(q) - c(q)].$$

Condition D then becomes

$$p(q_0) - c(q) + \frac{1}{r}[p(q) - c(q)] \leq 0 \quad \text{or}$$

(3) $p(q) \leq c(q) + r[c(q) - p(q_0)].$

Finally, it must be the case that

(4) $p(q_0) = c(q_0).$

Basically, there is no informational problem for products of quality q_0. If $p(q_0) < c(q_0)$ no firm would supply quality q_0. If $p(q_0) > c(q_0)$ any entrant could profitably undercut sellers of quality q_0 by simply offering a product of quality q_0 at a price between $p(q_0)$ and $c(q_0)$. Since consumers of quality q_0 know they will not face lower quality than q_0, they will be happy to buy from the entrant at the lower price.

Substituting $p(q_0) = c(q_0)$ into (3), we see that (3) is the reverse inequality of (2). Therefore, these two conditions together fully determine p(q), which is given by

(5) $p(q) = c(q) + r(c(q) - c(q_0)).$

Figure 1 shows the equilibrium p(q) schedule and its relationship to the perfect information schedule c(q).

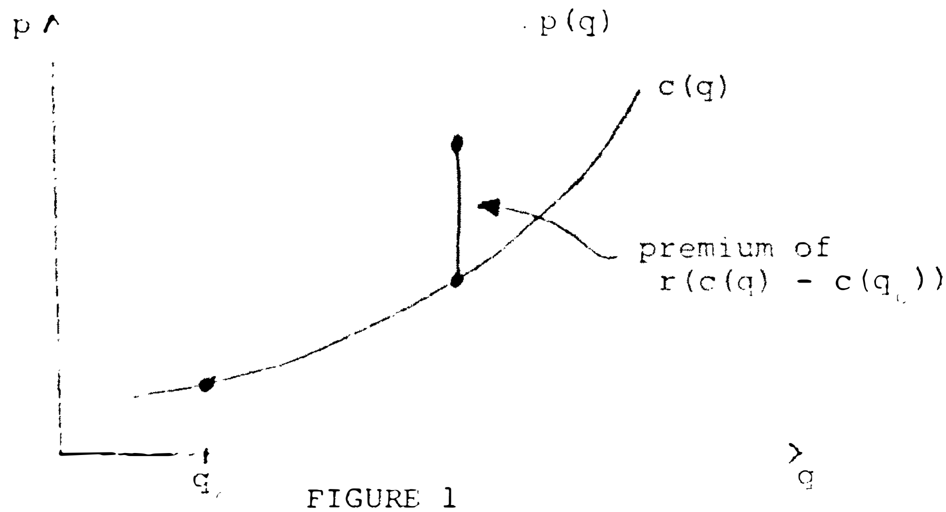

FIGURE 1

Equilibrium p(q) Schedule

Formal Derivation of p(q)

Consider a firm with initial reputation R_0. The firm can choose quality in each period to maximize present discounted profits:

$$\max_{q_0, q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(R_t) - c(q_t)]$$

$$\text{such that } R_t = q_{t-1}$$

$$R_0 \text{ given}$$

Here $\rho = \frac{1}{1+r}$ is the discount factor. This problem can be re-written as

$$\max_{q_0, q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(q_{t-1}) - c(q_t)]$$

$$\text{with } q_{-1} \text{ given.}$$

Differentiating with respect to q_t gives

$$\rho^{t+1} p'(q_t) - \rho^t c'(q_t) = 0 \quad \text{or}$$

$$\rho\, p'(q_t) = c'(q_t) \quad \text{i.e.}$$

(6) $p'(q_t) = (1+r)c'(q_t)$.

For an arbitrary p(q) schedule, so long as the second-order conditions held everywhere, there would be a unique solution to (6) and all firms would choose to produce the quality which yielded the solution. Such a price schedule could not be an equilibrium because it would violate market clearing at other quality levels.

If (6) is used to <u>define</u> $p(q)$, however, then all quality levels will by definition satisfy the steady-state condition. Therefore for any initial reputation, a firm would find it optimal to maintain its reputation.[1] This is exactly the condition needed for an equilibrium in which a variety of products is sold.

The differential equation for $p(q)$ given by (6), along with the boundary condition (4) admits only (5) as its unique solution.

Since it is optimal for a firm with reputation R_0 to maintain quality, it is easy to figure out the asset value of reputation. This is just the present value of profits accruing to having the reputation, when the firm follows its optimal regime from that point on. We can compute this value as

$$V(R_0) = \frac{1+r}{r}\,[p(R_0) - c(R_0)]$$

$$= \frac{1+r}{r}\,[r(c(R_0) - c(q_0))]$$

$$V(R_0) = (1+r)\,(c(R_0) - c(q_0)).$$

Of course this is increasing in R_0. Also, $V(q_0) = 0$ as is necessitated by free entry with initial reputation q_0. Note that it is also increasing in r; we will see below that this implies that improved information decreases the asset value of reputation. Finally it is decreasing in q_0 so an increase in the minimum quality standard would cause a capital loss for firms with good reputations.

I should emphasize that these profits are only ex-post profits. The asset value of reputation R_0 exactly equals the cost of building up that reputation. Ex ante there are zero profits.

Information Provision and the Interpretation of r

At this point I would like to indicate how the equilibrium $p(q)$ schedule depends on information flows in the market. This will be important for studying the welfare consequences of improved information.

Information in this model is embodied in the reputation adjustment equation. As alternatives to (1), consider

(7) $R_t = q_{t-n}$

(8) $R_t = \gamma R_{t-1} + (1-\gamma)\ q_{t-1}.$

To see how these specifications alter the equilibrium schedule, I simply restate the optimal control problem discussed above with these alternative equations of motion of the state variable, R.

Looking first at (7) we get

$$\max_{q_0, q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(R_t) - c(q_t)]$$

$$R_t = q_{t-n}$$

$$q_{-1}, \ldots q_{-n} \text{ given}$$

re-writing we have

$$\max_{q_0 q_1, \ldots} \sum_{t=0}^{\infty} \rho^t [p(q_{t-n}) - c(q_t)\ ]$$

$$q_{-1}, \ldots q_{-n} \text{ given}$$

Differentiate with respect to q_t to get

$$\rho^{t+n} p'(q_t) - \rho^t c'(q_t) = 0 \quad \text{or}$$

(9) $p'(q_t) = (1+r)^n c'(q_t).$

This replaces (6) when $R_t = q_{t-n}$.

For r small we get the approximation

(9)' $p'(q_t) \cong (1+rn)\ c'(q_t)$

Therefore increasing r in equation (5) can be thought of as increasing the length of time before quality is observed.

While (7) reflects a lag in observing quality, (8) captures two possible effects in reputation formation: The first is that consumers do not completely alter their judgment of the firm on the basis of one period's quality. Rather they may slowly adjust reputation towards observed quality. The second concerns the probability of observing true quality. Some product attributes are difficult to detect even after purchase - e.g., safety features. If γ is the probability that the true quality is not observed (in which case reputation is unaltered) then (8) will hold. The earlier case, (1), corresponds to $\gamma=0$ so that there is rapid or certain reputation adjustment.

To derive the steady-state necessary condition when $0<\gamma<1$, it is enough to compute the change in profits from a one-shot blip in quality at time 0 followed by a return to producing q every period. The effect of such a deviation is:

$$V = \sum_{t=0}^{\infty} \rho^t [p(R_t) - c(q_t)]$$

$$\left.\frac{dV}{dq_0}\right|_{\substack{R_t=q \\ q_t=q}} = -c'(q) + \rho p'(q)\frac{dR_1}{dq_0} + \rho^2 p'(q)\frac{dR_2}{dq_0} + \cdots$$

$$= -c'(q) + p'(q)\sum_{t=1}^{\infty} \rho^t \frac{dR_t}{dq_0}\ .$$

Now $R_t = \gamma R_{t-1} + (1-\gamma)q_{t-1}$

So $\dfrac{dR_1}{dq_0} = 1-\gamma$

$$\frac{dR_2}{dq_0} = \frac{dR_2}{dR_1}\frac{dR_1}{dq_0} = \gamma(1-\gamma)$$

And $\frac{dR_t}{dq_0} = \gamma^{t-1}(1-\gamma)$.

Substituting into $\frac{dV}{dq_0}$ we have

$$\frac{dV}{dq_0} = -c'(q) + p'(q)\sum_{t=1}^{\infty}\rho^t(1-\gamma)\gamma^{t-1}$$

$$= -c'(q) + (1-\gamma)\rho p'(q)\sum_{t=0}^{\infty}\rho^t\gamma^t$$

$$= -c'(q) + \frac{(1-\gamma)\rho p'(q)}{1-\rho\gamma}$$

If q is to be a steady-state quality level this expression must be zero. So

$$p'(q) = \frac{1-\rho\gamma}{\rho(1-\gamma)}c'(q)$$

$$= [1 + \frac{1-\rho}{\rho(1-\gamma)}]c'(q)$$

(10) $$p'(q) = (1 + \frac{r}{1-\gamma})c'(q).$$

When $\gamma=0$ this reduces to our original expression. Slow reputation adjustment (forgiving consumers) or difficult to detect attributes raise γ and, in the analysis below, can be treated by raising r.

Finally, perhaps the most important interpretation of r is as frequency of purchase. Taking as given the market discount rate per unit time, i, if the period is of length T, then $\rho=e^{-iT}$ or

$r = e^{iT} - 1$. Large T, namely infrequent production periods, is another interpretation of a large r. As one would expect, informational problems are more severe the larger is r.

Summarizing, large values of r can be interpreted as

(a) Infrequent production (i.e., lengthy production process)

(b) Long lags in detection of quality

(c) Slow updating of reputations or

(d) Difficult to detect quality attributes.

In the welfare analysis of r below, r can be thought of as a policy variable since information provision activities can influence r through several of the above channels.

Description of Equilirbium p(q) Schedule

From Figure 1 and equation (5) it is easy to see some of the qualitative characteristics of the equilibrium price - quality schedule. First notice that the premium paid for high quality products, $r(c(q) - c(q_0))$ is larger the higher the quality involved. So the imperfect operation of reputation as a quality - conveying mechanism is more severe for higher quality items.

Notice also

Theorem 1. As $r \to 0$, the equilibrium price-quality schedule approaches the perfect information schedule.

This reflects the fact that as $r \to 0$ the flow of profits necessary to forestall cheating on quality becomes smaller. For $r=0$ any positive flow of profits would be more than enough to cause a firm to prefer to maintain quality. Viewed differently, any positive flow of profits would be more than enough to compensate an entrant for the finite one-period loss involved in building up the reputation.

Keeping in mind the interpretations of r noted above, the larger is r the more of a gap between p(q) and c(q). The welfare consequences of this will be explored below. See Figure 2.

It is also easy to see how q_0 affects p(q). An increase in q_0 simply shifts the whole schedule down by a fixed amount without affecting the slope. Of course the schedule starts at $q=q_0$, so an increase in q_0 reduces the spectrum of products available in the market. See Figure 3.

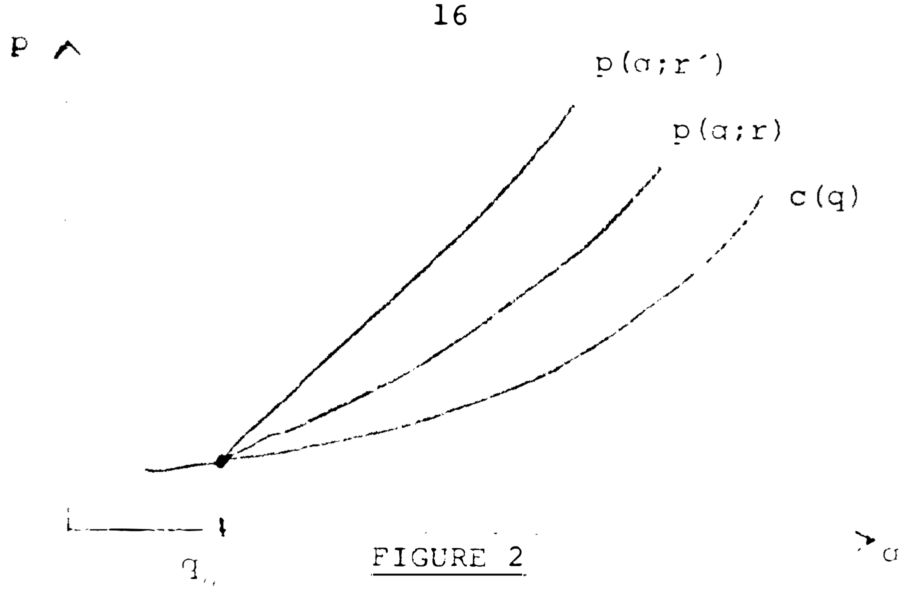

FIGURE 2

Effect of r on p(q): r'>r

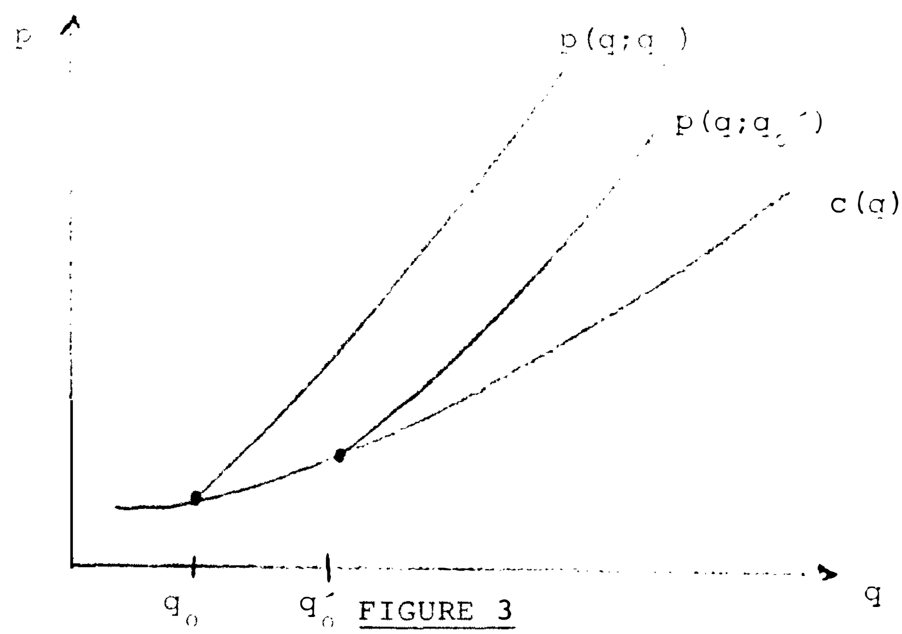

FIGURE 3

Effect of q_0 on p(q): $q_0' > q_0$

Consumers

In this section I look more closely at how various consumers respond to a given p(q) schedule. This is necessary in order to perform a welfare analysis.

As mentioned above, each consumer is described by two parameters, Θ and v. A consumer of type (Θ, v) achieves utility $\Theta q + v - p$ from purchasing one unit of quality q at price p.

Consumers buy either 0 or 1 unit of the good. There is a given distribution of types of consumers f(Θ, v). This distribution is confined to the box $[\underline{\Theta} \times \bar{\Theta}] \times [\underline{v} \times \bar{v}]$ where $\underline{\Theta} > 0$. (Multi-unit demands for the same quality can be treated via the f function).

Consumer (Θ, v), when facing the price-quality schedule p(q), solves the following problem:

$$\max_{q \geq q_0} \Theta q + v - p(q).$$

Differentiating with respect to q we have

(11) $\Theta = p'(q)$

unless $\Theta < p'(q_0)$, in which case $q = q_0$. These describe the choice of q by Θ *if* the product is purchased.

So long as $p''(q) > 0$, which follows from the assumption that $c''(q) > 0$, we know that consumers with a greater taste for quality, higher θ's, consume higher quality items:

$$\frac{dq}{d\Theta} = \frac{1}{p''(q(\Theta))} > 0$$

Substituting our formula (5) for $p(q)$ into (11) we have the quality choice by θ given r, denoted $q(\theta, r)$, defined by

$$(12) \quad \begin{cases} \theta = (1 + r)\, c'(q(\theta, r)) & \text{if } \theta > (1 + r)\, c'(q_0). \\ q(\theta, r) = q_0 & \text{if } \theta \leq (1 + r)\, c'(q_0). \end{cases}$$

It is important to note that q_0 does not affect the slope of $p(q)$ and thus does not affect $q(\theta, r)$ except for those who choose to consume q_0 . It will, however, affect the set of consumers who buy at all.

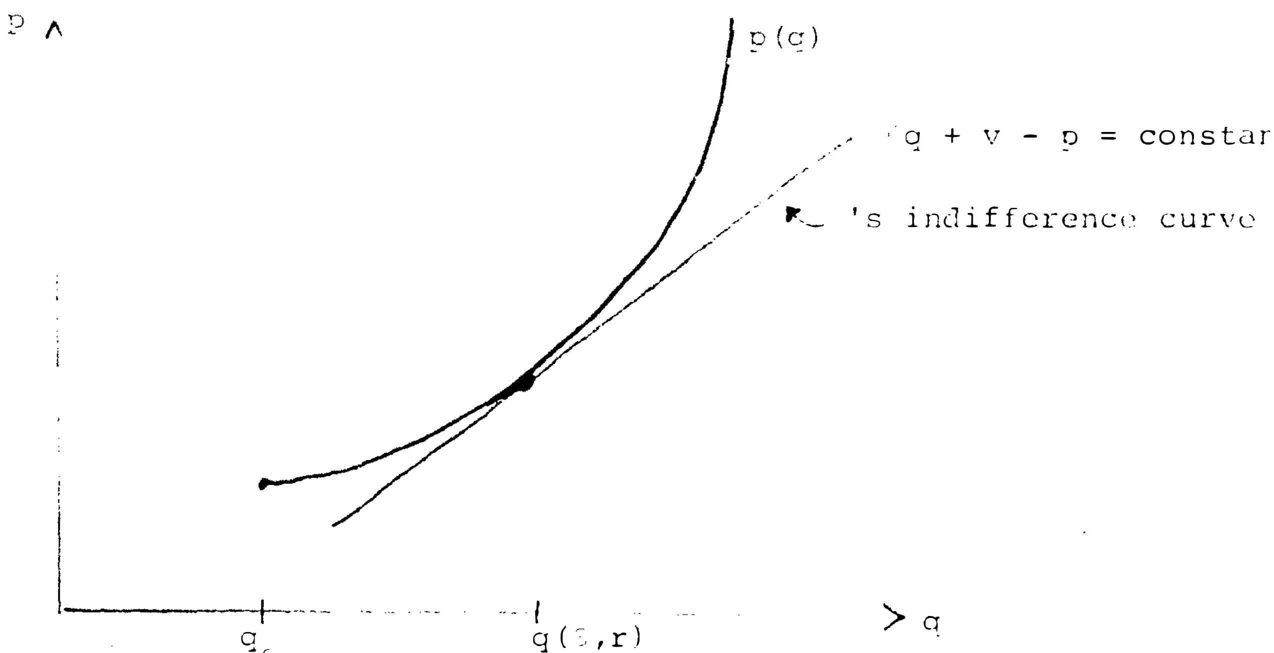

FIGURE 4

Choice of Quality by (θ, v)

Type (Θ, v) will purchase the product if and only if

$\Theta q(\Theta, r) + v - p(q(\Theta, r)) > 0$.

Subtituting for $p(q)$ this becomes

$\Theta q(\Theta, r) + v - [(1 + r)\, c(q(\Theta, r)) - rc(q_0)] \geq 0$

Rewriting, we have:

(Θ, v) purchases the product if and only if

(13)

$v \geq (1 + r)\, c(q(\Theta, r)) - rc(q_0) - \Theta q(\Theta, r)$.

Denote the right-hand side by $v(\Theta; q_0, r)$. Differentiating (13) with respect to q_0 and using (12) we have

$$(14) \quad v_{q_0}(\Theta; q_0, r) = \begin{cases} -rc'(q_0) & \text{if } q(\Theta, r) > q_0 \\ c'(q_0) - \Theta & \text{if } q(\Theta, r) = q_0 \end{cases}$$

This is to be interpreted as follows: when q_0 rises the $p(q)$ schedule shifts down (by $rc'(q_0)$). This causes consumers with high valuations of quality (Θ) to face a more attractive opportunity set and more of them buy. This is represented by region B in Figure 5 ($v(\Theta; q_0, r)$ falls for high Θ's). On the other hand low Θ's $(\Theta < c'(q_0)$ would like to consume products of quality less than q_0, so raising q_0 makes them worse off, and only higher v's will purchase as a result. Those in region D leave the market when q_0 is raised to q_0. For $\Theta\varepsilon(c'(q_0), (1+r)c'(q_0)$, (Θ, v) would like to purchase $q>q_0$ if he only had to pay the cost. He is unwilling to pay the premium as well, however, so purchases q_0. Since q_0 sells at price $c(q_0)$, he prefers q_0 to be raised. This is all summarized in Figure 5 below. (We know $v(\Theta, q_0, r)$ is declining in Θ since higher Θ's derive strictly greater utility from any given quality product, and thus would certainly buy a unit if lower Θ's did.)

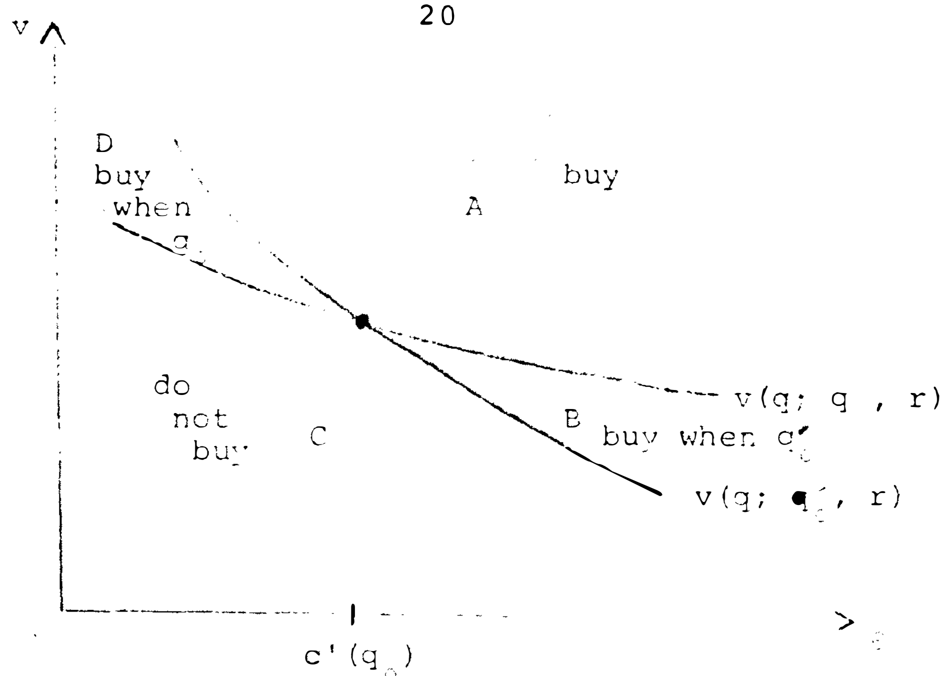

FIGURE 5

$v(\Theta, q_0, r)$: $q_0' \gtrsim q_0$

Having described how (Θ, v)'s response depends on q_0, let me look more closely at the effects of r. When r goes up any consumer who purchases $q > q_0$ finds he must pay more. Consequently, he is worse off; furthermore, since r affects the marginal cost of quality, (1+r) c'(q), it will affect his quality choice, via (12). Differentiating (12) with respect to r we have (for

$\Theta \geq (1+r)\ c'(q_0)$, $0 = (1+r)\ c''(q(\Theta, r))\ \ q_r(\Theta, r) + c(q(\Theta, r))$)

or

$$(15)\quad q_r(\Theta, r) = \frac{-c'(q(\Theta, r)}{(1+r)\ c''(q(\Theta, r))} < 0.$$

As expected, increased r causes a given type of consumer to substitute towards lower quality products (unless Θ was using q_0 already, as would be the case for $\Theta \leq (1+r)\ c'(q_0)$). See Figure 6.

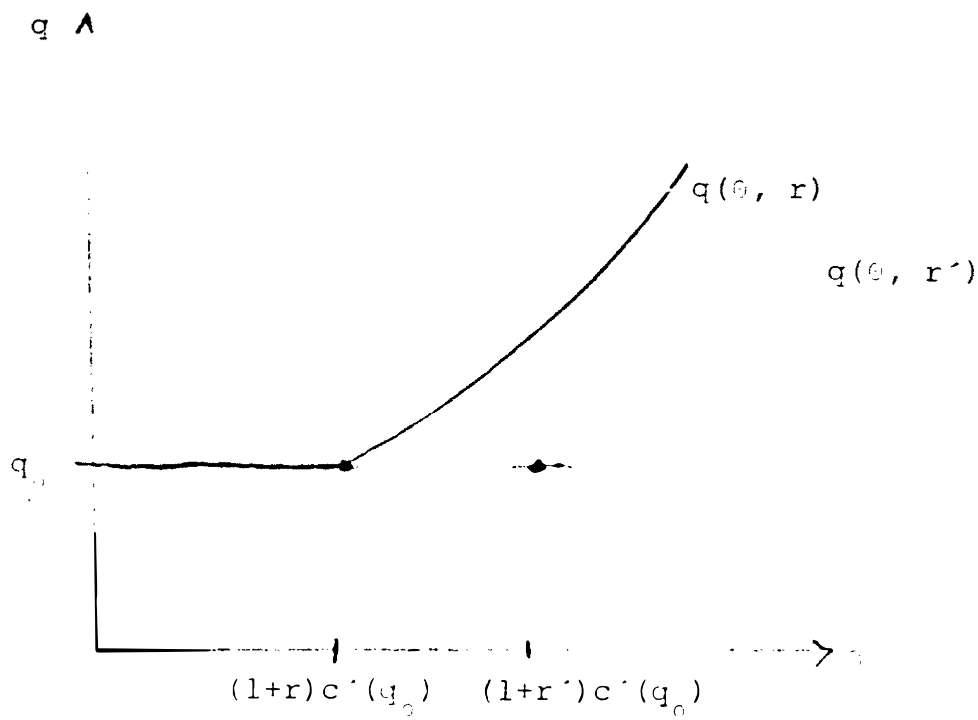

FIGURE 6

q(Θ, r): r' > r

Furthermore, the less favorable p(q) schedule which results from increased r causes fewer consumers of a given Θ-type to consume at all. Differentiate (13) with respect to r to get

$$v_r(\Theta; q_0, r) = (1+r)\ c'(q(\Theta, r))q_r(\Theta, r) + c(q(\Theta, r)) - c(q_0)$$
$$- \Theta q_r(\Theta, r)$$

$$= [\ (1+r)\ c'(q(\Theta, r)) - \Theta]\ q_r(\Theta, r) + c(q(\Theta, r)) - c(q_0)$$

By (12) the first term in brackets is zero for $\Theta > (1+r)\ c'(q_0)$ and so

(16) $v_r(\Theta; q_0, r) = c(q(\Theta, r)) - c(q_0) > 0$

for those Θ's. For $\Theta \leq (1+r)\ c'(q_0)$, $q(\Theta, r) = q_0$, $q_r(\Theta, r) = 0$ and

$v_r(\theta;\ q_0,\ r) = 0$. This is because low θ's continue to use q_0 at price $c(q_0)$, whatever r is. See Figure 7. Those consumers in-between the two curves drop out of the market when r rises to r´.

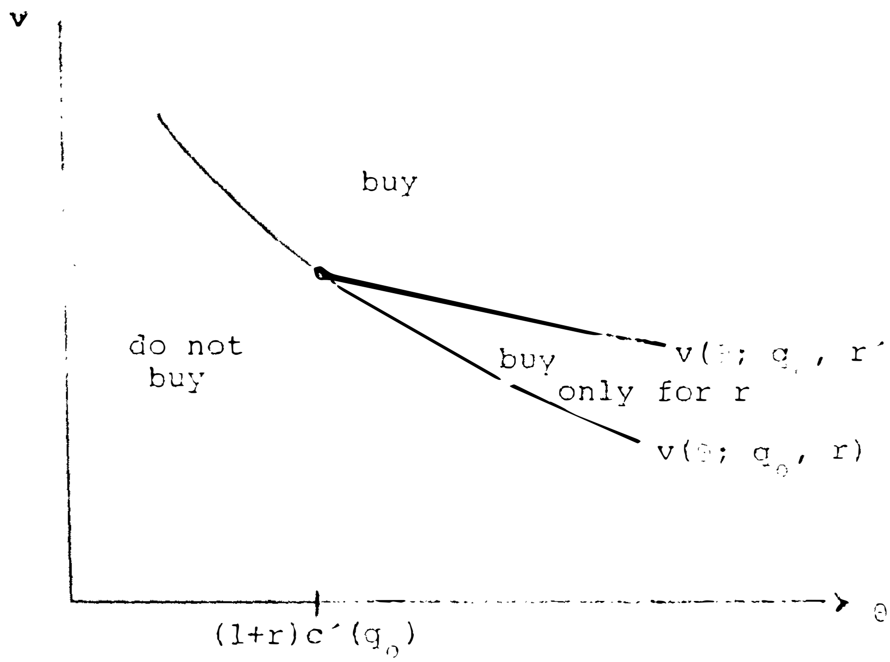

FIGURE 7

$v(\theta;\ q_0,\ r)$: $r' \gtrsim r$

Welfare Analysis of r

Utilizing the analysis of consumer behavior above, we can determine the welfare effects of changing r and q_0. This section studies r; the next will treat q_0. Keep in mind the section on the interpretation of r when considering changing r in this section. Since q_0 will be fixed in this section, it is suppressed in the notation when possible.

The idea behind the welfare theorem in this section is this: as r increases, the wedge between price and cost for high quality products rises. This is like a tax on high quality items. Increases in r lead to increases in the "tax", with associated distortions. Of course, information costs are as "real" as production costs, so this should not be viewed as a market failure so much as a cost due to imperfect information.

The welfare measure used is aggregate consumer surplus plus profits. Equivalently, I will write down expressions for gross utility minus the costs of production. Since producers earn zero profits *ex ante*, we can identify this aggregate welfare measure with consumer surplus. This requires inclusion of the transition period, during which firms take losses to build up reputations, in the welfare analysis. The easiest way to treat this period is to assume[2] that it differs from the steady-state only in the prices charged (all items sell at $c(q_0)$). With this convention there is no difference in social welfare between the transition period and the steady-state, because the same allocation prevails. Consequently, we can identify steady-state aggregate welfare with consumer surplus, by using the zero profit condition.

Look first at the set of all consumers of type θ. For $\theta < (1+r)\, c'(q_0)$ type θ either uses $q=q_0$ or stays our of the market. The aggregate welfare of type θ consumers is

$$W(\theta, r) = \int_{v(\theta,r)}^{\bar{v}} f(\theta, v)[\theta q_0 + v - c(q_0)]dv.$$

For $\theta < (1+r)\, c'(q_0)$, $q(\theta, r) = q_0$ so

$$v(\theta, r) = c(q_0) - \theta q_0 \quad \text{and}$$

$$(17) \quad W(\theta, r) = \int_{c(q_0)-\theta q_0}^{\bar{v}} f(\theta, v)[\theta q_0 + v - c(q_0)]dv.$$

Evidently, r has no influence on these consumers' utility since it neither affects the quality chosen (q_0) nor its price (and hence $v(\theta, r)$).

The situation is very different for $\theta > (1+r)\, c'(q_0)$. Now

$$W(\theta, r) = \int_{v(\theta, r)}^{\bar{v}} f(\theta, r)[\theta q(\theta, r) + v - c(q(\theta, r))]dv$$

Differentiating with respect to r we have

$$W_r(\theta, r) = \int_{v(\theta, r)}^{\bar{v}} f(\theta, v)[\theta q_r - c' q_r]dv$$
$$- v_r f(\theta, v(\theta, r))[\theta q(\theta, r) + v(\theta, r) - c(q(\theta,r)$$

Using (12) to substitute for c', and (13) for $v(\theta, r)$ we have

$$W_r(\theta, r) = \int_{v(\theta, r)}^{\bar{v}} f(\theta, v)\; q_r(\theta - \frac{\theta}{1+r})dv$$
$$- v_r f(\theta, v(\theta, r)) r[c(q(\theta, r)) - c(q_0)]$$

Finally, substituting for q_r and v_r from (15) and (16) we get

$$(18) \quad W_r(\Theta, r) = \frac{r\Theta}{(1+r)^2}\left[\frac{-c'(q(\Theta, r))}{c''(q(\Theta, r))}\right]\int_{v(\Theta,r)}^{\bar{v}} f(\Theta, v)dv$$

$$-f(\Theta, v(\Theta, r))r[c(q(\Theta, r)) - c(q_0)]^2$$

The first term here indicates the welfare loss due to the further distortion in quality choice by type Θ's as r increases. The second term reflects the fact that some type Θ's (namely type $(\Theta, v(\Theta, r)))$ are forced out of the market by the increase in r. There is an unambiguous welfare loss as r increases. The gains from reducing r should be weighed against the costs of any information provision activities which could do so. This welfare analysis is summarized in

Theorem 2. There is a welfare loss as r increases for all consumers who consume qualities above q_0, given r. Increases in r also cause more consumers to leave the market altogether, with additional welfare losses resulting. Changes in r have no effect on consumers who purchase quality q_0. In general consumers substitute to lower quality items as r rises.

Notice that $W_r(\Theta, 0) = 0$; this reflects the fact that there is no loss, to the first order, from imperfect information when we first move away from perfect information (r=0). Notice also that the per-capita welfare losses as r increases tend to be greater for those who value quality the highest (high Θ). Finally, the curvative of the cost function, $\frac{c'}{c''}$, enters into the welfare loss.[3] This is because it determines how severe is the substitution towards lower quality items as a consequence of the premiums for higher quality products.

Welfare Analysis of q_0

In a perfect information world there is no justification for a minimum quality standard. After all, its only effect would be to artificially restrict the range of products offered for sale.

When product quality cannot be observed prior to purchase, however, there may well be justification for such standards. The usual story is that the minimum standard or licensing protects consumers from quacks, frauds, and rip-offs generally. This refers to a disequilibrium situation where consumers may be unpleasantly surprised by the quality of the product they buy.

While such a story is perfectly plausible, it is not the one I am telling in this paper. Rather, I am concerned with the desirability of a minimum quality standard, where the standard influences the equilibrium price-quality schedule. So, even granting that consumers are never surprised (in equilibrium) *i.e.*, that their expectations of quality are fulfilled, it is desirable to impose a minimum standard.

There are, as far as I know, no other formal analyses of minimum quality standards where the supply of products of various qualities is endogenous. The case with exogenous supplies has been treated by Leland [1979].

Since I have already shown how the minimum standard q_0, influences the equilibrium $p(q)$ schedule, (5), and how consumers respond to this, it is relatively easy to do the welfare analysis to determine the optimal minimum quality standard.

Looking at type θ consumers, and taking $r>0$ as fixed, we can write down welfare of type θ's (using the same convention as above to identify consumer surplus and aggregate welfare) as

$$W(\theta, q_0) = \int_{v(\theta,q_0)}^{\bar{v}} f(\theta, v)[\theta q(\theta, r) + v - c(q(\theta, r))]dv.$$

Let me again consider the two classes of θ's separately: first $\theta \leq (1+r)\, c'(q_0)$ and then $\theta > (1+r)\, c'(q_0)$.

For the first group, $q(\theta, r) = q_0$ so $v(\theta,q_0) = c(q_0) - \theta q_0$ and

$$W(\theta,q_0) = \int_{c(q_0)-\theta q_0}^{\bar{v}} f(\theta, v)\ [\theta q_0 + v - c(q_0)]dv$$

And so

$$W_{q_0}(\theta,q_0) = \int_{c(q_0)-\theta q_0}^{\bar{v}} f(\theta, v)[\theta - c'(q_0)]dv$$

$$\text{(19)} \quad W_{q_0}(\theta, q_0) = [\theta - c'(q_0)] \int_{c(q_0)-\theta q_0}^{\bar{v}} f(\theta, v)dv.$$

For $\theta < c'(q_0)$, consumer θ would like to purchase $q<q_0$ and so is hurt by a rise in the standard. For $c'(q_0) < \theta < (1+r)c'(q_0)$, θ is happy to see q_0 raised: so long as he only has to pay the cost of the item and not the premium payment he prefers $q > q_0$ Since minimum quality items sell at cost, these θ's prefer to see the standard raised.

For the second group, $(\theta > (1+r)\ c'(q_0))$, we have

$$W(\theta,\ q_0) = \int_{v(\theta, q_0)}^{\bar{v}} f(\theta,\ v)[\theta q(\theta,\ r) + v - c(q(\theta,\ r))]dv$$

Since q_0 does not influence the quality choice $q(\theta,\ r)$, it has an impact only through the number of consumers who purchase the good.

$$W_{q_0}(\theta,\ q_0) = -\ v_{q_0}(\theta,\ q_0)f(\theta,\ v(\theta,\ q_0))[\theta q(\theta,\ r) + v(\theta,\ q_0) - c(q(\theta.\ r))]$$

From (14) we have $v_{q_0}(\theta,\ q_0) = -rc'(q_0)$ so

$$W_{q_0}(\theta,\ q_0) = rc'(q_0)f(\theta,\ v(\theta,\ q_0)) \qquad [\theta q(\theta,\ r) + v(\theta, q_0) - c(q(\theta,\ r))]$$

Now use (13) to substitute for $v(\theta,\ q_0)$ to get

$$W_{q_0}(\theta,\ q_0) = rc'(q_0)f(\theta,\ v(\theta,\ q_0))r(c(q(\theta,\ r) - c(q_0))$$

Rewriting this we have

$$W_{q_0}(\theta,\ q_0) = r^2c'(q_0)[c(q(\theta,\ r)) - c(q_0)]f(\theta,\ v(\theta,\ q_0))$$

As expected, this is positive. Since $p(q) > c(q)$ for $q > q_0$, some consumers who would purchase the product under perfect information drop out of the market rather than pay the premium. As q_0 increases the premium falls, and some of these consumers, whose valuation of the product exceeds its cost, re-enter the market. This constitutes a welfare gain.

The calculations above can be summarized in

<u>Theorem 3</u>. Given some minimum quality standard q_0, all consumers of type θ such that $\theta > c'(q_0)$ enjoy a welfare gain from raising q_0, while those θ's for which $\theta < c'(q_0)$ suffer as a result.

The calculation of the optimal minimum quality standard is not hard, now that we have computed the welfare achieved in supplying type θ for every θ.

$$W(q_0) = \int_{\underline{\theta}}^{\bar{\theta}} W(\theta, q_0)\,d\theta$$

$$W'(q_0) = \int_{\underline{\theta}}^{\bar{\theta}} W_{q_0}(\theta, q_0)\,d\theta$$

$$= \int_{\underline{\theta}}^{c'(q_0)} W_{q_0}(\theta, q_0)\,d\theta + \int_{c'(q_0)}^{\bar{\theta}} W_{q_0}(\theta, q_0)\,d\theta$$

We know the integrand is always negative in the first integral, and always positive in the second. The optimal q_0, q_0^*, satisfies $W'(q_0^*) = 0$.

For q_0 such that $c'(q_0) \leq \underline{\theta}$ we know $W'(q_0) > 0$. Likewise, for q_0 such that $c'(q_0) \geq \bar{\theta}$, $W'(q_0) < 0$. Consequently

<u>Theorem 4</u>. The optimal minimum quality standard is such that (1) there are some consumers who cannot get as low a quality item as they would prefer under perfect information, and (2) some consumers would prefer a higher standard i.e., would prefer a better product under perfect (or imperfect) information.[4]

In particular, setting a minimum quality standard $\underline{q}$ such that $\underline{\theta} = c'(\underline{q})$, so that no one would want to buy a lower quality than $\underline{q}$, is not optimal.

Continuous Time Model with Quality and Quantity as Controls

In this section I present a model in which firms can choose a sales level, x, as well as a quality, q, at each point in time. When sales are a control variable, the reputation adjustment process must be changed to reflect this fact.[5] I adopt a specification in which the speed of adjustment of reputation depends positively on the sales level.

The resulting model is naturally more complex than the one in which quality is the only control variable. The main reasons for presenting it are two: Firstly, it indicates that the qualitative characterization of the price-quality schedule derived above is not peculiar to a model in which quantity variables are absent. Since many producers of consumer goods can vary their sales over time as well as quality, this is important. Secondly, it allows us to identify an additional welfare loss which is a consequence of imperfect information: there is a production inefficiency induced by the fact that prices for high quality items sell above their minimum average cost. Specifically, active firms operate at above efficient scale. In equilibrium there are too few firms, each producing too much.

In particular, each firm faces an optimal control problem of the following form:

$$\max_{x(t),\, q(t)} \int_0^\infty e^{-rt}[p(R)x - c(x, q)]dt$$

$$\text{s.t.} \quad \dot{R} = sx(q - R)$$

$$R(0) \text{ given.}$$

Here $c(x, q)$ is the cost function in quantity and quality; I assume $c_x > 0$, $c_{xx} > 0$, $c_{qx} > 0$, and that we have U-shaped average cost curves for any q. The parameter s represents the speed of learning by consumers, and $p(R)$ is the price a firm can charge if its reputation is R. Again we have perfect competition, so the firm faces a perfectly elastic demand curve at price $p(R)$.

The current value Hamiltonian for this control problem is

$$H(x, q, \lambda, R) = p(R)x - c(x, q) + \lambda sx(q-R).$$

The necessary conditions for an optimal regime include

(20) $H_x = p(R) - c_x(x, q) + \lambda s(q-R) = 0$

(21) $H_q = c_q(x, q) + \lambda sx = 0$

and

(22) $H_R = p'(R)x - \lambda sx = r\lambda - \dot{\lambda}.$

We can solve for the steady-state conditions by putting $\dot{\lambda}=0$, $q=R$ to get

(23) $p(q) = c_x(x, q)$

(24) $c_q(x, q) = \lambda sx$

(25) $p'(q)x = r\lambda + \lambda sx$

Solve for λ using (24) to get, finally,

(26) $p'(q) = \dfrac{c_q(x, q)}{x} [1 + \dfrac{r}{sx}]$

Notice the similarity between (26) and (6).

The reasoning now parallels the formal derviation of $p(q)$ in the case where x was not a control variable: For an arbitrary $p(q)$ schedule (23) and (26) would imply a unique steady state (x,q) pair, at which all firms would choose to produce (if they settle down at all). This would not satisfy the equilibrium conditions for the same reasons as in the earlier case. But if (23) and (26) are used to <u>define</u> $p(q)$, with the auxilliary variable $x(q)$ as well, then any firm would find it optimal to maintain $q=R$ rather than to deviate.[6]

Before looking more closely at the solution to (23) and (26), it is helpful to define the perfect-information price-quality schedule. It is given by

$$\phi(q) = \min_x \frac{c(x,\ q)}{x}$$

i.e. quality q is supplied at its minimum average cost. The associated scale at which firms operate, $z(q)$, satisfies

$$(27) \quad c_x(z(q),\ q) = \frac{c(z(q),\ q)}{z(q)} .$$

since MC=AC at minimum AC.

Returning to the imperfect information case, we must add the natural boundary condition to solve the differential equation for $p(q)$ given by (26), namely

$$(28) \quad p(q_0) = \phi(q_0) .$$

This is analogous to $p(q_0) = c(q_0)$ in the earlier case.

<u>Theorem 5</u> As $r\to 0$ or $s\to\infty$ the equilibrium $p(q)$ schedule approaches the perfect information schedule $\phi(q)$. For $(\frac{r}{s}) > 0$ $p(q) > \phi(q)$ for all $q>q_0$.

<u>Proof</u>: Let me first show that when $\frac{r}{s} = 0$, $(\phi(q), z(q))$ solves the system given by (23), (26) and (28). Note that for any set of parameters the system has a unique solution. Well, $\phi(q) = \frac{c(z(q), q)}{z(q)}$ by the definition of $\phi(q)$, and that equals $c_x(z(q), q)$ by the definition of $z(q)$ so (23) is satisfied. To verify (26) simply differentiate the equation defining $\phi(q)$, to get

$$\phi'(q) = \frac{z(q)[c_x z' + c_q] - cz'}{z^2}$$

$$= [\frac{c_x}{z} - c]z' + \frac{c_x}{z}$$

But $\frac{c_x(z, q)}{z} = c(z, q)$ so

$$\phi'(q) = \frac{c_q(z(q), q)}{z(q)}$$

which is exactly (26) when $x=z$, $(\frac{r}{s}) = 0$. Now, since the solution to the differential equation is continuous in $\frac{r}{s}$, we have proven the first part of the Theorem. The second part of the Theorem can be shown by a more basic argument. If $p(q) < \phi(q)$ firms selling quality q would be losing money and it could not be optimal to continue doing so. If $p(q) = \phi(q)$ they are breaking even, but they could make positive profits (at least for a little while if $s<\infty$) by running down reputation. Therefore maintaining quality can only be optimal if $p(q) > \phi(q)$.

It is interesting to note how r and s enter only through their ratio. This is very intuitive: for low interest rates or high learning speeds the informational problems are less important.

There is an interesting effect which comes up in this model which could not arise in the earlier model: since $p(q) > \phi(q)$ firms providing quality q operate at above efficient scale.[7]

Theorem 6 For $(\frac{r}{s}) \to 0$, all firms providing non-minimal quality operate at a point above efficient scale. So, in addition to the welfare losses due to imperfect consumer quality matching, and some consumers dropping out of the market, there is a production inefficiency.

Proof: Since $p(q) > \phi(q)$ for $q>q_0$ by Theorem 5, we know that $x(q) > z(q)$ since $c_{xx}>0$ and $x(q)$ is defined by (23). See Figure 8 below. So the number of products of quality q which are sold in equilibrium is not produced in the cost-minimizing manner. There are too few firms, each of which produces too much.

Since average cost is $\frac{c(x(q),q)}{x(q)} > \phi(q)$, some of the premium to high quality items, $p(q) - \phi(q)$ is dissipated by the production cost inefficiency.

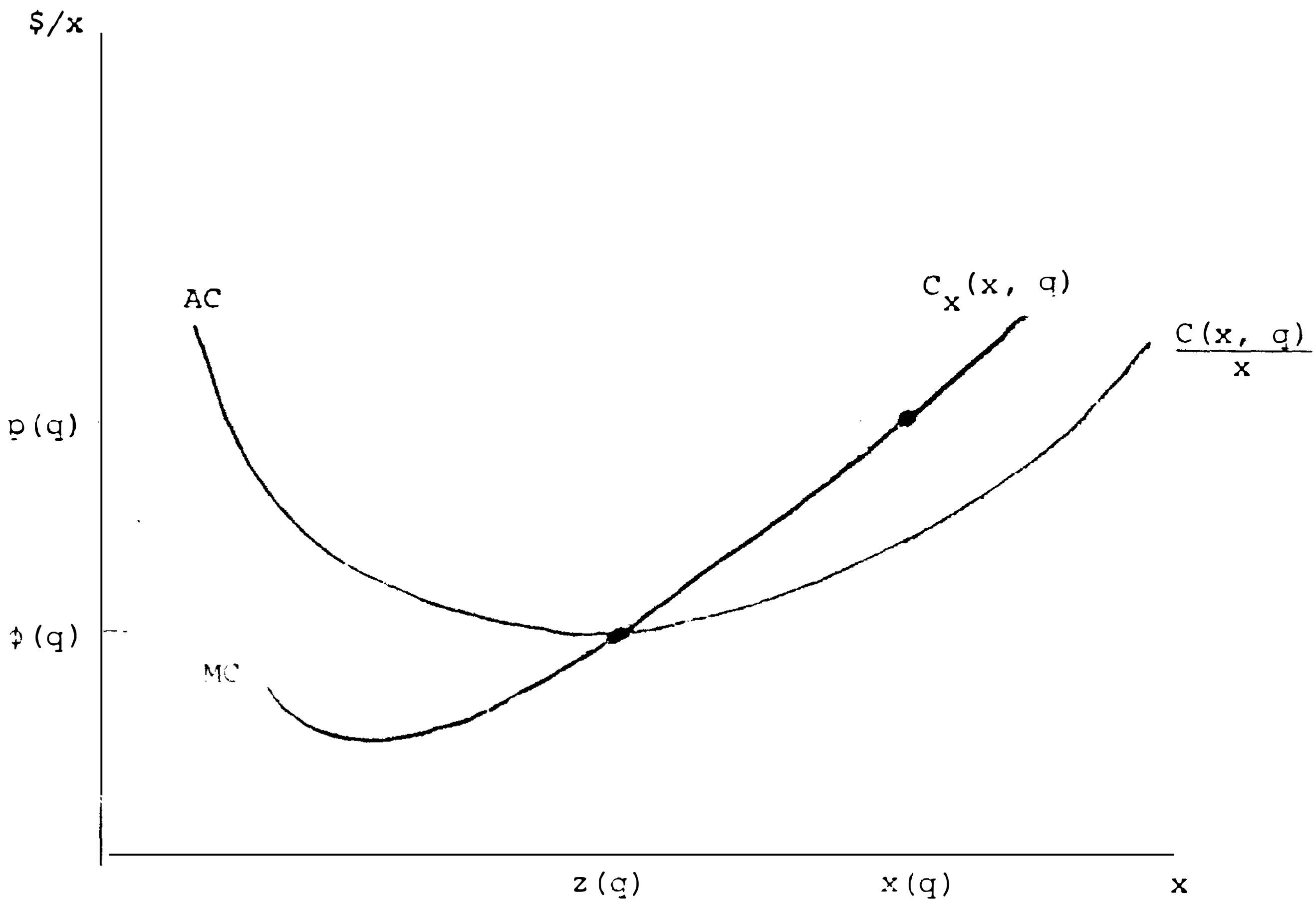

FIGURE 8

Production Inefficienty due to $p(q) > \phi(q)$

Conclusions

This paper has investigated the implications of reputation in a perfectly competitive environment. It has been shown that reputation can operate only imperfectly as a mechanism for assuring quality. High quality items sell for a premium above cost. This premium provides a flow of profits which compensate the seller for the resources expended in building up the reputation.

Several common but informal notions relating to reputations have been challenged by this analysis. First, a good reputation need not confer market power on its owner. Indeed, firms face perfectly elastic demand curves in the model presented above. Second, reputations need not imply a barrier to entry either. It is true that a firm must expand resources initially to build up a reputation, but it is not possible, at least in this model, to earn super-normal profits by virtue of having built up a reputation. In other models, which I hope to explore, it may be the case that there are first-mover advantages in reputation formation, and thus reputation could serve as a barrier. In this first simple model, however, it does not. Finally, care must be taken in evaluating profit data for consumer goods industries. If reputation is not included in the set of assets a firm owns, the calculations of its rate-of-return will exceed the market rate of return. This is misleading, as would be the conclusion based upon it that the firm enjoyed some degree of market power.

Finally, a welfare analysis of information remedies and minimum quality standards is made. There are welfare gains from improving information transmission; these must be balanced against the costs of such a program, of course. Optimal minimum quality standards are also studied. In general it is optimal to exclude from the market items which some consumers would like to purchase, i.e., the standard should be binding. This is because there are welfare gains to consumers who like high quality items which arise from raising the standard. These gains arise because a higher minimum quality standard reduces the premiums for high quality goods.

Notes

1. In fact, a firm would be indifferent to maintaining or deviating, but stability would be provided by any positive adjustment costs to changing quality. such indifference is inevitable in a model in which identical firms choose a variety of actions in equilibrium.
2. The welfare theorems do not depend on this assumption. They only require that a consistent description of what happens during the transition period be maintained throughout the analysis.
3. Recall that is is really the curvative of c(q) relative to utility in q, but q has been scaled such that utility is linear in quality.
4. This Theorem holds no matter what weights are placed on the utilities of different consumers in the welfare measure so long as the weights are positive and finite.
5. It is not plausible that reputation adjustment is independent of sales. Furthermore, if it were, there would be no equilibrium. This is because a firm could build up reputation by selling, say, one good item and then sell a great many bad items when reputation is high. Since this strategy gives more profits from running down reputation than the costs of building it up, firms would never maintain quality. See Shapiro (1979).
6. I have been unable to verify the sufficiency conditions for the optimal control problem when sales levels are a control variable. The maximized Hamiltonian is not concave, but that does not mean the solution is not optimal.
7. This is in contrast to the traditional Chamberlinian result that firms operate below efficient scale in monopolistic competition.

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