MULTI-MARKET STRATEGIES IN A
DOMINANT FIRM INDUSTRY

Steve C. Salop
and
David T. Scheffman

WORKING PAPER NO. 100

April 1984

FTC Bureau of Economics working papers are preliminary materials circulated to stimulate discussion and critical comment. All data contained in them are in the public domain. This includes information obtained by the Commission which has become part of public record. The analyses and conclusions set forth are those of the authors and do not necessarily reflect the views of other members of the Bureau of Economics, other Commission staff, or the Commission itself. Upon request, single copies of the paper will be provided. References in publications to FTC Bureau of Economics working papers by FTC economists (other than acknowledgement by a writer that he has access to such unpublished materials) should be cleared with the author to protect the tentative character of these papers.
MULTI-MARKET STRATEGIES IN A
DOMINANT FIRM INDUSTRY

Steven C. Salop
Georgetown University Law Center

David T. Scheffman
Federal Trade Commission

April 1984
Introduction

Traditional analyses of oligopoly markets have generally been based on models in which a single strategic instrument (price or quantity) is featured. For example, the models of Cournot, Bertrand and Stackleberg assume a single strategic instrument and make different assumptions about the information sets and allowable strategies of the oligopolists. More recent work in oligopoly theory has developed a richer menu of models, many of which feature two or more strategic instruments. Examples include models incorporating product differentiation, advertising, or scarce inputs, where besides strategies governing output or price the firms must make decisions concerning product "quality", advertising, or the purchase of scarce inputs. Another important class of examples is the burgeoning literature on dynamic oligopoly models, where a firm must make decisions which will affect its market at various times in the future (so that e.g., strategies require decisions about price today and price tomorrow).

Because most of the literature concerned with multiple strategies is relatively new, it is just beginning to have an impact on the more applied areas of industrial organization, particularly the economic analysis of competition policy. Until recently, most of the analyses of predation, preemption, exclusion and collusion have used models with a single strategic instrument. For example, most of the literature on predation has focused on predatory price cutting. The preemption literature
has concentrated on attempts to control scarce "inputs". Until recently most of the models of exclusionary behavior have assumed single market strategies such as limit pricing. Finally, much of the literature concerned with collusion has been limited to strategic use of a single instrument - output or price.

Recently, however, there has been a growing recognition that more complex strategies may be powerful methods in effecting an outcome of successful predation, preemption, exclusion or collusion. (See Salop (1981) for a discussion of some of the recent literature). For example, a dominant firm might find that controlling both price and the wage rate can be an effective predatory strategy (Williamson 1968). Similarly, knowing that entrants must match its own stock of advertising to effectively compete, an incumbent monopolist might advertise solely to increase the cost of entry and thereby exclude potential rivals (Rogerson (1982), Salop (1979)). A dominant firm might profitably exclude potential rivals or preempt actual rivals by purchasing a scarce input, even if it must pay a premium to do so (Gilbert 1981). Moreover, it might refuse to sell a scarce input it controls to potential rivals at any price (Ordover-Willig 1981). Finally, control of variables other than price or output may make tacit collusion on price or output more tenable (Plott (1982)).

Although these models are quite different in apparent structure, formally, most of them are simple extensions of the Stackleberg leader - follower model to multiple strategy
environments. Elsewhere, we have sketched a simple model that exploits this common structure. (Salop-Scheffman 1983). In this paper we derive the full implications of a more completely specified version of that model. The model discussed here is general in the sense that predation, preemption, exclusion or coordination are simply different equilibrium outcomes resulting from the optimal strategy of the dominant firm.

The remainder of this paper consists of four sections. In Section II we set out a simple general model of a dominant firm industry in which the dominant firm can use multiple strategies. In Section III we develop a model in which the dominant firm can control both price and a parameter which affects its costs, its rivals costs, and market demand. A specific example of this model is presented in Williamson (1968). We derive various necessary conditions for strategic use of the parameter instrument to be profitable and show that exclusion and coordination are simply two possible outcomes of the dominant firm's optimal strategy. In addition, we show that multiple strategy exclusion may be a profitable strategy in the short run. This is in contrast to the traditional predation analysis in which the dominant firm incurs losses in the short run in order to reap gains in the long run. Thus, the traditional analysis is misleading in characterizing the incentives for predation. Finally, we provide a rigorous welfare analysis of the effects of dominant firm's strategic actions.
In Section IV we develop a model in which one of the strategic instruments under the control of the dominant firm is the price of an input used by the dominant firm and the fringe. This control arises because the dominant firm has monopsony power in this input market and so can affect the price by varying its level of purchase. Necessary and sufficient conditions for the profitable strategic manipulation of the input market are derived. We also show that it may be profitable for the dominant firm to purchase the input at a price above the value of the marginal product of the input. This is in contrast to the conventional monopsonist that would purchase at a price below the value of the marginal product.

Finally in Section V we develop a model of vertical integration. We show that the dominant firm may find the use of vertical integration to be a useful anticompetitive strategic instrument. One important result is that, unlike the standard monopoly analysis, this dominant firm model predicts potential anticompetitive advantages to vertical integration, even for technologies that do not permit input substitutability. (c.f. Bork (1980))

II. The General Model

This paper develops a simple (but general) basic model of a dominant firm-competitive fringe industry. The dominant firm controls two instruments, the product price p and a parameter α which is an argument in industry demand and cost functions. This
parameter could be an input price, advertising expenditure, a regulatory restraint or a product quality parameter. Fringe producers are summarized by a fringe supply function \( S(p,a) \). The fringe is assumed to take prices and \( a \) as given.\(^1\)

**Notation:**

- \( D(p,a) \) industry demand
- \( x \) output of dominant firm
- \( y \) output of the fringe
- \( C(x,a) \) total cost function of the dominant firm
- \( S(p,a) \) supply function of the fringe (i.e., \( y = S(p,a) \))
- \( F(a,p,x,y) > 0 \) inequality summarizing the relationship between \( a \) and other variables of the model

The structure sketched thus far is quite general. For technical simplicity, we will treat \( p \) and \( a \) as scalars; but it should be clear that our results could easily be extended to a more general setting in which all variables and functions are vector-valued.\(^2\) The parameter \( a \) could have a variety of interpretations: the price of an input, expenditures on advertising, a product quality or a regulatory parameter.\(^3\) If \( a \) is the price of an input \( A \), the equation \( [F(a,p,x,y) > 0] \) represents the reduced form equation summarizing the equilibrium in the \( A \) market. Finally, it will sometimes be useful to assume that the fringe supply function \( S(p,a) \) arises from an underlying cost function \( G(y,a) \). In that case, fringe supply is given as follows:
solution of $G_y(y, \alpha) = p$, if $py - G(y, \alpha) > 0$ \hspace{1cm} (1)

$y = 0$, otherwise

Assuming $G_{yy} > 0$, the properties of the supply curve are:

a) $S_p = 1/G_{yy} > 0$  \hspace{1cm} (2)

b) $S_{\alpha} = -G_{ya}/G_{yy} < 0$

Assuming that the fringe takes $p$ and $\alpha$ as given, the dominant firm's profit-maximization problem is given by:

$$\max \; px - C(x; \alpha)$$

subject to: $y = S(p, \alpha)$

$x + y = D(p, \alpha)$

$F(\alpha, p, x, y) \geq 0$

The first order conditions for the general problem do not yield interesting interpretations without imposing more structure on the constraint function $F(\alpha, p, x, y)$. The following sections consider specific versions of this constraint.

III. Direct Control of $\alpha$ by a Dominant Firm

In this section we assume that the dominant firm has complete control over $\alpha$, subject only to the constraint $\alpha \geq \alpha$. Thus, the constraint $F(\alpha, p, x, y) > 0$ can be written as $\alpha \geq \alpha$, where $\alpha = \overline{\alpha}$, corresponds to the dominant firm taking no strategic action. We assume that (i) increases in $\alpha$ raise the fringe's marginal cost ($S_{\alpha} < 0$); (ii) the marginal cost to the dominant firm of
changing $\alpha$ is simply $C_{\alpha};^4$ and (iii) there may be demand effect $D_{\alpha}$.

Several possible interpretations are possible. In Williamson (1968), $\alpha$ is the wage rate set by the large coal mines with the connivance of the union and $D_{\alpha} = 0$. Another interpretation treats $\alpha$ as a regulatory parameter, where the regulatory agency is "captured" by the dominant firm. A third example treats $\alpha$ as a class of R&D or advertising expenditures of the dominant firm that must be matched by the fringe. In this case, it might be that $D_{\alpha} > 0$.

Rewriting the maximization problem in (3), we have

$$
\max\quad p[D(p,\alpha) - S(p,\alpha)] - C(D(p,\alpha) - S(p,\alpha), \alpha)
$$

subject to $\alpha > \bar{\alpha},^5$

The first order conditions are given as follows.

$$
\frac{p - Cx}{p} = 1/\varepsilon^D\quad (5a)
$$

$$
p - Cx = \frac{C_{\alpha}}{D_{\alpha} - S_{\alpha}}, \alpha > \bar{\alpha}\quad (5b)
$$

where $\varepsilon^D$ is the elasticity of demand faced by the dominant firm.$^6$

Equation (5a) is the usual dominant firm price mark-up equation. At an interior equilibrium ($\alpha > \bar{\alpha}$), combining (5a) and (5b), we have

$$
\frac{D_{\alpha} - S_{\alpha}}{D_p - S_p} = C_{\alpha}/x,^7
$$
The interpretation of (6) is as follows:
The left-hand side of (6) is \( \frac{\partial p}{\partial a} \bigg|_x \) where this derivative represents the change in price arising from a combination of the reduction in fringe output and the increase in market demand caused by an increase in \( a \), holding the output of the dominant firm fixed. In other words, the left-hand side of (6) is the derivative representing the vertical shift in the residual demand curve facing the dominant firm. The right-hand side of (6) is the derivative of average cost of the dominant firm (AC) with respect to \( a \) for output of the dominant firm kept fixed. Therefore, an interior solution must satisfy the condition

\[
\frac{\partial p}{\partial a} = \frac{\partial AC}{\partial a}
\]  

(7)

where these derivatives are evaluated at the profit-maximizing point \((a^*, x^*)\).

We now state the following sufficient condition for \( a^* > \overline{a} \).

Theorem 1 Let \( \overline{x} \) be the profit maximizing output for the dominant firm for \( a = \overline{a} \), i.e., \( \overline{x} \) is the solution of (5a) for \( a = \overline{a} \). A sufficient condition for \( a > \overline{a} \) to be profitable is

\[
\frac{\partial p}{\partial a} > \frac{\partial AC}{\partial a}
\]  

(8)

where the derivatives are evaluated at \((\overline{a}, \overline{x})\). That is, the vertical shift in the dominant firm's residual demand curve must exceed the vertical shift in its average cost curve.
Notice that condition (8) requires that either $\left(\frac{-S_a}{S_p}\right) > \left.\frac{\partial AC}{\partial a}\right|_{\bar{a}, \bar{x}}$, or $\left(\frac{-D_a}{D_p}\right) > \left.\frac{\partial AC}{\partial a}\right|_{\bar{a}, \bar{x}}$, i.e., either fringe supply or market demand must shift (vertically) more than does the dominant firm's average cost.

The interpretation of (8) is further clarified by Figure 1, where $D(p, a)$ is the market demand curve and $S(p, \bar{a})$ is the fringe supply curve for $a = \bar{a}$. The residual curve $[D(p, \bar{a})-S(p, \bar{a})]$ is given by $ABC$.

Consider the profit-maximizing price and total output $(\bar{p}, \bar{Q})$ for $a = \bar{a}$ where the output of the dominant firm is given by $\bar{x}$. At this output $\bar{x}$, the marginal revenue for the dominant firm's residual demand curve equals its marginal cost. An increase in $a$ from $\bar{a}$ will shift up the supply curve, $S(p, a)$ and, perhaps, shift up the market demand curve $D(p, a)$ as well, shifting up the residual demand curve. The interpretation of condition (8) is that if the vertical shift of the residual demand curve at output $\bar{x}$ is greater than the vertical shift in the dominant firm's average cost curve at $\bar{x}$, then it is profitable to increase $a$. This follows since the dominant firm's profits are equal to $(p - AC)\bar{x}$.

The properties of the dominant firm's strategy can be further analyzed as follows. Rewriting (6) by substituting $\varepsilon = (-D_p \; p/\bar{D})$ for the market price elasticity or demand, and $\sigma = x/\bar{D}$ for the dominant firm's market share, and combining (5a) and (5b) an interior optimum satisfies
\[
\frac{\epsilon/\sigma}{p - c_x} \left[ 1 - \frac{(C_a/x)/(D_a - S_a)/S_p)}{1} \right]
\] (9)

Similarly the sufficient condition for profitability (8) can be written as

\[
\frac{\epsilon/\sigma}{p - c_x} \left[ 1 - \frac{(C_a/x)/(D_a - S_a)/S_p)}{1} \right]
\] (10)

where all variables are evaluated at \((\bar{a}, \bar{x})\).

These equations can be interpreted as follows:

**Proposition 1.** An equilibrium with \(a > \bar{a}\) is more likely to result,

(i) the larger is the vertical shift in the fringe supply curve resulting from an increase in \((S_a \text{ large})\)

(ii) the more responsive is market demand to an increase in \(a \text{ (D}_a \text{ large)}\)

(iii) the smaller is the impact of an increase in \(a\) on the dominant firm's average costs \((C_a/x \text{ small})\); and

(iv) the less elastic is the market demand curve \((\epsilon \text{ small})\).

Condition (iv) can be seen by considering the case of a perfectly elastic demand curve with \(D_a = 0\). In this case, the equilibrium price cannot be raised, regardless of the magnitude of the fringe supply response. On the other hand, if \(D_a = D_p = 0\), the change in price for fixed \(x\) is exactly the vertical shift in fringe supply.
We now analyze the effects of the dominant firm's strategy on the fringe's price, profits and output, and on welfare.

A. Effect on Price

Denoting by $\pi^D$ the profit of the dominant firm, we have

$$\pi^D = p[D-S] - C(D-S, \alpha)$$

(11)

The profit-maximizing price satisfies the first order condition $\pi_p^D = 0$. Totally differentiating this first order condition, we have

$$\frac{dp}{d\alpha} = -\frac{\pi_p^D}{\pi_{pp}^D}$$

(12)

Since the second-order conditions require $\pi_{pp}^D < 0$,

$$\text{sign } \frac{dp}{d\alpha} = \text{sign } \frac{\pi^D}{\pi_p \alpha}$$

(13)

Partially differentiating $\pi^D$,

$$\pi^D = (D_\alpha - S_\alpha) - C_\alpha(D_p - S_p) + (p - C_x)(D_\alpha - S_p)$$

(14)

The first term on the right hand side of (14) is positive. The second term depends on the sign of $C_\alpha$, the effect of an increase in $\alpha$ on the dominant firm's marginal cost. The sign of the third term depends on the sign of $(D_\alpha - S_p)$, the effect of an increase in $\alpha$ on the slope of the dominant firm's residual
demand curve. Even in the case $D_\alpha = 0$, the sign of $(D_\alpha - S_\alpha)$ is ambiguous. Thus the sign of $dp/d\alpha$ is ambiguous.\textsuperscript{11}

This is a straightforward result. When a monopolist's demand increases, its profit-maximizing price may not rise if demand becomes more elastic. Thus, we have the following result.

**Proposition 2.** Strategic increases in $\alpha$ may result in a decrease in price. A sufficient condition for price to increase is $C_{x\alpha} > 0$ and $(D_\alpha - S_\alpha) > 0$.\textsuperscript{12}

B. **Effect on the Fringe**

Recent literature casts considerable doubt on the viability of successful predatory pricing or limit pricing because those strategies are not credible. In contrast, the multi-market strategies examined here are credible because they are profit maximizing. For example, in the standard predatory pricing analysis, the dominant firm must price below its average cost in order to induce exit by the fringe. If there are no significant re-entry barriers, the fringe will simply shut down and wait until the dominant firm raises price and then re-enter. The dominant firm's threat to lower price is not credible if the fringe remains a viable entrant; given this fact, the dominant firm's predatory strategy is not profit maximizing. In contrast, suppose that the dominant firm can also control a second instrument, $\alpha$. If its profit maximizing choices of $p$ and $\alpha$ induce the
fringe to either reduce its output or exit, then that \((p, a)\) strategy is credible. Therefore, the strategy can be successful because the strategy is profitable, even assuming the fringe remains an actual viable competitor or a entrant.

We will now see that the dominant firm's profit-maximizing strategy has an ambiguous effect on the profits and output of the fringe. That is, the model can capture strategies that might be characterized as either "exclusion" of the fringe or "coordination" with the fringe. Thus our model provides a framework for a general analysis of strategic interaction in which predation, exclusion, preemption, or tacit collusion are all possible equilibrium outcomes.

We now examine the effect of an increase in \(a\) on fringe output. From the fringe supply function, we have

\[
\frac{dy}{da} = S_p \frac{dp}{da} + S_a
\]

(15)

Noting that \(S_a < 0\), if \(\frac{dp}{da} < 0\) then \(\frac{dy}{da} < 0\). However, in some cases it may be profitable for the dominant firm to increase price sufficiently to allow the fringe output to expand. Of course, this depends on the degree to which dominant firm must reduce its own output in order to effect an increase in price. If \(D_a > 0\), the possibility of an increase in fringe output is clear. However, even if \(D_a < 0\), fringe expansion is possible.13 As before, the result depends on the impact on the elasticity of the dominant firm's residual curve. For example, if an increase in \(a\) makes the fringe supply curve much less elastic, the
dominant firm might find it profitable to change from an initial strategy of (exclusionary) limit pricing to a strategy of accommodating moderate entry. Summarizing, we have the following result.

**Proposition 3.** Strategic use of $\alpha$ has an ambiguous effect on fringe output, even if $D_\alpha = 0$. However, if $d\pi/d\alpha < 0$, then $dy/d\alpha < 0$.

Since an increase in $\alpha$ on fringe output has ambiguous effects, it is not surprising that the effect of an increase in $\alpha$ on fringe profits is also ambiguous. For example, suppose that fringe supply is generated by (2) and further entry is blocked so that the fringe, even though behaving competitively, earns inframarginal rents.\(^{14}\)

Franchise profits are given by

$$\pi^F = py - G(y, \alpha)$$

(16)

Differentiating and rearranging terms we have

$$\frac{\partial \pi^F}{\partial \alpha} = y \frac{dp}{d\alpha} - G_\alpha$$

(17)

from (1), we have

$$\frac{dp}{d\alpha} = G_{yy} \frac{dy}{d\alpha} + G_{y\alpha}.$$  

(18)

Substituting into (17), we have
\[
\frac{\partial \Pi^F}{\partial \alpha} = y \left( G_{yy} \frac{dy}{d\alpha} + G_y - \frac{G_a}{y} \right)
\]

Note that \(G_y\) and \(\frac{G_a}{y}\) respectively represent the changes in fringe marginal costs (\(\partial MC^F/\partial \alpha\)) and average costs (\(\partial AC^F/\partial \alpha\)) as \(\alpha\) increases.

In the standard case in which fringe output falls \(\frac{dy}{d\alpha} < 0\), fringe profits fall as long as average costs rise by less than marginal costs. Only if marginal costs rise disproportionately to average costs will fringe profits rise.

Summarizing, we have

**Proposition 4.** There are two alternative sufficient conditions for fringe profits to fall \(\frac{d\Pi^F}{d\alpha} < 0\). Either (i) \(\frac{dp}{d\alpha} < 0\) or (ii) \(\frac{dy}{d\alpha} < 0\) and \((G_y - \frac{G_a}{y}) < 0\).

Sufficient conditions for fringe profits to rise are \(\frac{dy}{d\alpha} > 0\) (which itself requires \(\frac{dp}{d\alpha} > 0\)) and \((G_y - \frac{G_a}{y}) > 0\).

These two conditions are of particular interest because they depend only on the effect on price or on the output of the fringe and its technology.

**C. Effect on Aggregate Welfare**

In this section we analyze the welfare effects of the dominant firm's strategy. We use the conventional welfare indicator, the sum of consumer surplus plus producer surplus and
denote this indicator as $W$. Letting $\bar{p}$ be the equilibrium price for $a=\bar{a}$, we have

$$W(\bar{p}, \bar{a}) = \int_\mathbb{D}(p, \bar{a})dp + \bar{p}\bar{x} - C(\bar{x}, \bar{a}) + \bar{p}S(\bar{p}, \bar{a}) - G(S(\bar{p}, \bar{a}), \bar{a})$$

where $\bar{x} = D(\bar{p}, \bar{a}) - S(\bar{p}, \bar{a})$.

The dominant firm's profits ($\Pi^D$) are the sum of the second and third terms on the right-hand side of (20). By definition of $\bar{p}$, $\Pi^D(\bar{p}, \bar{a}) = 0$. Assuming that increasing $a$ is a profitable strategy, differentiating $W$ with respect to $a$ and evaluating at $(\bar{p}, \bar{a})$, we have

$$\frac{dW}{da} = \int D(p, a)dp - \left[D(\bar{p}, \bar{a}) - S(\bar{p}, \bar{a})\right] \frac{dp}{da}$$

From (22) it should be clear that the sign of $\frac{dW}{da}$ is still ambiguous. For example, if $\frac{dp}{da} < 0$, it would not be surprising that $\frac{dW}{da} > 0$. Furthermore, even if $\frac{dp}{da} > 0$, it could be the case that $\frac{dW}{da} > 0$, because the dominant firm's
profit maximizing strategy may reduce the output of a higher cost rival, increasing producer surplus more than any decrease in consumer surplus.  

Of course, a sufficient increase in price will cause enough reduction in consumers' surplus to result in \( \frac{dW}{d\alpha} < 0 \). By (5b),

\[
\bar{p} - C_X = \frac{D - S}{D_p - S_p}
\]

Rewriting, we have the following sufficient condition for a reduction in welfare:

\[
\frac{dW}{d\alpha} < 0 \text{ if } \frac{dp}{d\alpha} > \frac{S_\alpha}{D_p - S_p} - \frac{C_\alpha + G_\alpha}{D - S}
\]  

As before, the term \( \frac{S_\alpha}{(D_p - S_p)} \) is the price rise due to an increase in \( \alpha \), assuming the dominant firm holds its output fixed. Therefore, (24) holds if the dominant firm does not increase its output with an increase in \( \alpha \). Summarizing, we have the following results:

**Proposition 5.** In general, the effect of a strategic use of \( \alpha \) on aggregate welfare is ambiguous, even if \( D_\alpha = 0 \) and \( \frac{dp}{d\alpha} > 0 \). However, if \( D_\alpha = 0 \) and the dominant firm does not increase its output, aggregate welfare is reduced.

**Proposition 6.** If \( D_\alpha = 0 \) and price rises, consumer welfare is reduced by an increase in \( \alpha \).
D. a an input price

We now place additional structure on the previous model by specifying the way in which α affects demand and costs. We assume that α is the price of an input, A, that is used by both the dominant firm and the fringe. Denoting $\bar{\alpha}$ as the competitive price of A, we assume the dominant firm can raise the input price α above the competitive level. Assuming that $D_\alpha = 0$, the first order conditions for this model are given by (5). Moreover, because α is an input price, additional properties of $C_\alpha$ and $S_\alpha$ can be derived.

By the usual duality properties of cost functions, we have

$$C_\alpha = A^D; G_\alpha = A^F$$

(25)

where $A^D$ and $A^F$ are the (cost minimizing) demands for A by the dominant firm and fringe, respectively.

Assuming that fringe supply is given by (1), then equation (2) and (25), imply that the fringe's (cost minimizing) demand for A satisfies

$$\frac{\partial A^F}{\partial Y} = -S_\alpha / p = G_Y \alpha$$

(26)

Noting that fringe profit-maximization requires $p = G_Y$, and substituting (25) into the first order conditions (9), we have

$$\xi / \alpha = [p - (\alpha A^D / x Y)] / (p - C_x)$$

(27)
where \( \gamma = \left( \frac{aG_y}{G_y} \right) \), denotes the elasticity of fringe marginal cost with respect to the input \( a \). Furthermore, assuming that none of the fringe's inputs are inferior, so that \( \gamma < 1 \), we have the following condition:

**Proposition 7.** If \( a \) is an input price and no fringe inputs are inferior, then a necessary condition for raising the input price to be profitable is given by

\[
\frac{\epsilon}{\sigma} < \frac{[p - (aA^D/x)]}{(p - C_x)}
\]

(28)

where all variables are evaluated at the (non-strategic) equilibrium \((\bar{a}, \bar{x})\).

Equation (28) is useful because it requires no data concerning the fringe, beyond the knowledge that inputs are not inferior. Moreover, equation (28) has some useful corollaries. For example, if the dominant firm's average input cost of \( A \) (given by \( (aA^D/x) \)), exceeds its marginal cost \( C_x \), (28) is only satisfied if market demand is inelastic.\(^{19}\)

**E. \( a \) as promotional expenditures**

The parameter \( a \) could be interpreted as promotional expenditures incurred by the dominant firm. Again, \( S_a < 0 \) is a plausible assumption, since an increase in \( a \) would increase the dominant firm's market share, ceteris paribus, and the presumed reaction of the fringe would raise fringe costs. Denoting by \( \Delta(p,a) = [D(p,a)] - S(p,a) \), the residual demand curve facing the
dominant firm, then this simple advertising model is technically identical to the usual simple model of the monopolist who chooses both price and advertising. In that model the effect of advertising on equilibrium price depends on how advertising affects the elasticity of demand. In the model presented here this effect is complicated because advertising expenditure affects fringe supply as well as the elasticity of market demand.

IV. Indirect Control of \( \alpha \) By The Dominant Firm

In this section we place additional structure on the model. We assume that the input \( A \) is supplied by a competitive industry according to the supply curve \( A(\alpha) \). Thus, in order to raise the input price, the dominant firm must purchase additional quantities of the input. As a result, its marginal factor cost of increasing \( \alpha \) exceeds \( C_\alpha \) (where \( C(x, \alpha) \) is the minimized cost of buying inputs to produce \( x \) at input price \( \alpha \)).

The equilibrium condition in the input market corresponding to the constraint \( F(\alpha, p, x, y) > 0 \) in (1) is now given by

\[
A(\alpha) - A^F(\alpha, p) - A^D = 0
\]  

(29)

where \( A^F(\alpha, p) \) is the fringe demand function for input \( A \) and \( A^D \) is the quantity of \( A \) purchased by the dominant firm. We make the standard assumptions that \( A'' > 0, A^F_\alpha < 0, A^F_p > 0 \). Rewriting (29), we define

\[
-21-
\]
\[ A^D = A^D(\alpha, p) = A(\alpha) - A^F(\alpha, p), \quad \text{where} \quad A^D > 0, \quad A^D < 0 \quad (30) \]

Let \( z = (z^1, \ldots, z^m) \) denote the quantities of other inputs, \( r = (r^1, \ldots, r^m) \) denote their prices, and \( f(z^D, A^D) \) denote the dominant firm's production function. The modified cost function of the dominant firm, \( \tilde{C}(x; \alpha, p) \), may be defined as follows:

\[ \tilde{C}(x; \alpha, p) = \min \{ z^D \} \quad \text{subject to} \quad f(z^D, A^D(\alpha, p)) = x \quad 20 \]

By the Envelope Theorem, we have

\[ \tilde{C}_\alpha = A^D + (\alpha - \theta f_\alpha)A^D_\alpha \quad 21 \]

\[ \tilde{C}_p = (\alpha - \theta f_\alpha)A^D_p \quad 22 \]

where \( \theta = \partial C/\partial x \), the marginal cost of the dominant firm.

Now, as in (4), the equilibrium is given by the solution of

\[ \max p[D(p) - S(p, \alpha)] - \tilde{C}[D(p) - S(p, \alpha), \alpha, p] \quad 33 \]

The first order conditions can now be written as

\[ \frac{p - \tilde{C}_x}{p} = (1 - \tilde{C}_p/x)/\epsilon^D \quad 34a \]

\[ p - \tilde{C}_x = -\tilde{C}_\alpha/S_\alpha \quad 34b \]

The equations in (34) can be combined (as in (6)), yielding

\[ \frac{S_\alpha}{D_\alpha - S_\alpha} = \tilde{C}_\alpha/x + (\tilde{C}_p/x) \frac{S_\alpha}{D_\alpha - S_\alpha} \quad 35 \]

\[ -22- \]
which is analogous to equation (7). As before, \( \frac{(S_a)}{(D_p - S_p)} = \frac{\partial p}{\partial \alpha} \) and the right hand side of (33) is 
\[ \frac{\partial AC}{\partial \alpha} = \left[ \frac{\partial AC}{\partial \alpha} \right. + \left( \frac{\partial AC}{\partial p} \right) \left( \frac{\partial p}{\partial \alpha} \right) \], evaluated at \((\bar{a}, \bar{x})\).

Because of the added complexity of this model, simple sufficient conditions such as (8) do not obtain. We no longer have the convenient benchmark equilibrium of \( \alpha = \bar{a} \). However, notice that (35) still requires that the vertical shift in the fringe marginal cost curve \((-S_a/S_p)\) to exceed the vertical shift in the dominant firm's average cost curve.

In the previous model, in principle, it was straightforward to determine whether the dominant firm was acting strategically with respect to the fringe - a necessary and sufficient condition was \( \alpha > \bar{a} \). In the present model, however, all purchases of \( A \) automatically increase \( \alpha \), so that as long as the dominant firm actually uses its purchase of \( A \) to produce output, we do not have a simple benchmark.

However, a useful benchmark does arise by comparing (34) to the case in which the dominant firm behaves as a simple monopolist-monopsonist facing residual demand curve \( \Delta(p, \alpha) \) and residual supply curve \( A^D(\alpha, p) \) - i.e., ignores the effect of \( \alpha \) on \( \Delta \) and ignores the effect of \( p \) on \( A^D \). For such behavior, the optimal choice of \( A \) would require \( \alpha - \bar{f}_A < 0 \), i.e., its marginal revenue product greater than factor price \( \alpha \). In terms of the model of equation (32), the first order conditions for output would be \( \bar{C}_\alpha = 0 \). However, in the model at hand in which the
dominant firm realizes and acts on the knowledge that $a$ affects $\Delta$ and $p$ affects $A^D$, by (34b), $\bar{c}_a > 0$, i.e.,

$$\left( a - \theta f_A \right) A^D_a + A^D > 0$$

(36)

Relative to the case in which the dominant firm behaves non-strategically, the strategic firm in this model purchases more $A$. It is in this sense that the strategic dominant firm overpurchases $A$. This is illustrated in Figure II, in which the usual simple monopsony diagram is depicted. The curve $\theta f_A$ is the marginal revenue product of $A$, drawn on the assumption that $\theta$ is fixed. The curve $A^D(a)$ is the net supply of $A$ facing the dominant firm, assuming $p$ is constant. Treating the curve $A^D(a)$ as the average factor cost curve, the curve labeled $MC^D_A$ is then the marginal factor cost of $A^D$, assuming $p$ is constant, i.e.,

$$MC^D_A = a + \frac{A^D}{a}.$$  

As depicted, the simple monopsony equilibrium is depicted at point M, where $MC^D_A = \theta f_A$. In contrast, equilibrium in our model requires $MC^D_A - \theta f_A > 0$, depicted at point S. The intuition is straightforward. The dominant firm, recognizing the effect of an increase in $a$ on fringe supply finds it in its interest to purchase relatively more $A$ than if it were a simple monopsonist. It is in this sense that the dominant firm "overpurchases" the input. This is one variant of a 'vertical squeeze' strategy, (except as noted above, such a strategy, may in fact, benefit the fringe).
**Proposition 8.** A necessary and sufficient condition for a profitable strategic use of \( a \) is \( MC_A^D - \theta f_A > 0 \), i.e., the marginal factor cost of \( A \) exceeds the dominant firm's marginal revenue product of \( A \).

In principle, \( MC_A^D \) and \( \theta f_A \) could be quantified, so that an empirical test of the incidence of a strategic control of \( a \) exists. A somewhat easier test arises from the fact that the dominant firm may find it in its interest to overpurchase \( A \) to the extent that \( a - \theta f_A > 0 \), i.e., it may purchase \( A \) at a price exceeding the marginal revenue product of \( A \). In fact, it may profit the dominant firm to purchase \( A \) at a price \( a \) that exceeds the value of the marginal product of \( A \) \((pf_A)\).\(^{24}\)

**Proposition 9.** It may be profitable for the dominant firm to purchase \( A \) at a price exceeding the marginal revenue product or the value of the marginal product of \( A \).

**The Effects of the Strategic Control of \( a \)**

Because of the lack of the convenient benchmark of \( \bar{a} \), it is not possible to conduct a detailed analysis here of the effects of a strategic use of \( a \) on the fringe and on welfare as was done earlier in section III.C. As an alternative we might consider as a benchmark an equilibrium in which the dominant firm does not act strategically with respect to the fringe - i.e., where the dominant firm acts as non-strategically with respect to his
residual demand curve \([D(p, a)] - S(p, a)\) and his residual factor supply curve \(A^D(a, p)\). Mathematically, this would mean that the dominant firm maximizes profits over choice variables \((p, a)\), ignoring the effect of \(a\) on \([D(p, a)] - S(p, a)\) and ignoring the effect of \(p\) of \(A^D(a, p)\). Unfortunately, this model is too complicated to allow a comparison of the benchmark and strategic equilibria. Intuition suggests that the strategic equilibrium would have a higher \(a\) and lower \(p\) than the benchmark equilibrium.\(^{25}\) However, the results derived in section III suggest that there are not likely to be any general conclusions possible.

V. Vertical Integration

One further extension is of interest. Consider a situation in which the dominant firm can also produce input \(A\), i.e., the dominant firm is vertically integrated into the production of \(A\). This possibility can easily be incorporated into the previous model as follows.

Interpreting \(A^D(a, p)\) as net purchases of input \(A\) by the dominant firm (where \(A^D(a, p) < 0\) means that dominant firm is a net seller of \(A\)), let \(A^D\) be the quantity of input \(A\) produced by the dominant firm and let \(c(A^D)\) be its cost of production. Then the modified cost function for the dominant firm is given by
\[
\hat{c}(x; \alpha, p) = \min \{ z_i^D + \alpha A^D(\alpha, p) + c(A^D) \} \quad (37)
\]
subject to \( f(z^x, \hat{A}^D + A^D(\alpha, p)) = x \)

Minimizing (37) with respect to \( \hat{A}^D \) requires

\[
c'(\hat{A}^D) - \theta f_A = 0 \quad (38)
\]

This condition implies that the dominant firm always produces input A efficiently, i.e., at the level at which its marginal cost of production is equal to its marginal revenue product.

The other equilibrium conditions are the same as (34). Therefore, since \((a - \theta f_A) > 0\) may be characteristic of an equilibrium, \((a - c'(A)) > 0\) may also obtain. In short, it may even be profitable for the dominant firm to purchase the input at a price exceeding its own marginal cost of producing the input.\textsuperscript{26} This is because purchases of the input raise the costs of the fringe and the reduction in fringe supply may more than compensate for the dominant firm's increased input cost.

Recall from Section II that when \( D_\alpha = 0 \), fringe marginal cost must increase more than the dominant firm's average cost in order to fulfill the sufficient condition (8) for profitability an increase in \( \alpha \) (for \( D_{\alpha} = 0 \)). In a model with asymmetrically vertically integrated producers, such an asymmetry that will enhance the likelihood that increasing \( \alpha \) is profitable, even in a model with no substitutability in inputs.

-28-
For example, consider a simple model in which the dominant firm and the fringe each require one unit of the input to produce one unit of output. Assume that only the dominant firm is vertically integrated into input production. In this case, an increase in $\alpha$ of $\Delta \alpha$ then increases the dominant firm's average costs directly by $B\Delta \alpha$ where $B$ is the proportion of the total amount of that input used by the dominant firm which is purchased (rather than produced internally). Thus the direct effect on the dominant firm's average cost, $B\Delta \alpha$ is smaller than the direct effect on fringe marginal cost $A\Delta \alpha$ (recall that one unit of input produces one unit of output). Of course, there is also an indirect effect on the dominant firm's average cost of $\varepsilon_A(\Delta \alpha)A(\alpha)$ arising from the increased purchases required to increase $\alpha$ by $\Delta \alpha$, where $\varepsilon_A$ is the price elasticity of supply of A. Nonetheless, the asymmetry between the vertically integrated dominant firm and the unIntegrated fringe is likely to make an increase in $\alpha$ profitable for the dominant firm, even if there is no input substitutability. This is summarized in the following result.

Proposition 10. There can be strategic advantages from vertical integration, even with a technology that permits no input substitutability.

-29-
Proposition 10 shows that a fixed coefficient technology is not a sufficient condition for the absence of anticompetitive impact of a vertical merger. (c.f. Bork (1980)) Only if the upstream firm is a monopolist (as opposed to it having "incomplete" market power as analyzed here) and the upstream product is used in fixed proportions downstream is it necessarily the case that a vertical merger can have no adverse competitive impact.

VI. Summary and Conclusions

In this paper we developed a general model of a dominant firm industry in which the dominant firm can use multiple strategies. Our results suggest that non-price strategies can be an important anticompetitive instrument. Such strategies can include preemption in upstream input markets and vertical integration, use of the regulatory process, advertising and product differentiation. We believe that we have made a useful contribution to the beginning of a general theory of non-price strategies. Much remains to be done. In particular, an attempt to create a general oligopoly model with non-price strategies is the obvious next step.
FOOTNOTES

1. The fringe supply function can be given a long run interpretation in order to encompass potential entry by price-taking firms.

2. Under one interpretation then the model could be intertemporal.

3. Our model is obviously somewhat limited by the assumption that the fringe does not act strategically. We have begun to develop a more general strategic model elsewhere.

4. This model assumes that all the dominant firm's costs of increasing \( \alpha \) are captured completely by \( C(x, \alpha) \). This would not obtain, for example, if \( \alpha \) were the price of an input \( A \), over which the dominant firm had monopsony power. In this case, purchases of \( A \) by the dominant firm and the fringe would also affect cost. See Section IV below.

5. If the fringe has a constant returns-to-scale technology the problem becomes:

\[
\begin{align*}
\text{max} & \quad P[D(p, \alpha) - y] - C[D(p, \alpha) - y, \alpha] \\
\text{subject to} & \quad \alpha > \bar{\alpha}, \ p - ACF(\alpha) < 0
\end{align*}
\]

where \( y \) is output of the fringe and \( ACF(\alpha) \) is the average cost of the fringe.
6. \( \epsilon^D = -\left(\frac{\partial(D - S)}{\partial p}\right)\left(\frac{p}{(D - S)}\right) = -\frac{p(S_p - D_p)}{(D-S)} \)

7. It can easily be shown that the condition corresponding to (6) if the fringe has constant returns-to-scale technology (see fn. 1) is:

\[
1 - \frac{p - C_x}{p} \epsilon \frac{D - C_x}{D_a - C_a/D} = \frac{-1}{ACF'(a)}
\]

with \( \epsilon \) the price elasticity of market demand and \( ACF'(a) \) is the derivative of the fringe's average cost with respect to \( a \).

The first order conditions for \( y \) require

\[
p - C_x > 0, \ (p - C_x)y = 0
\]

8. Derived from \( x = D(p,a) - S(p,a) \) for fixed \( x \).

9. The proof follows from (5) and (7).

10. This can be derived from (5a) and (5b) and

\[
\epsilon^D = [\epsilon + (1-a)S_p/S]/\sigma
\]

11. An example in which \( dp/da < 0 \) is as follows. Suppose market demand is \( Q = 1-p \), and that initially fringe supply is inelastic at one unit. Suppose further that the dominant firm's cost are zero and that an increase in \( a \) changes fringe supply to the function: \( y = p^2/100, \ p < 10; \ y = 1, \ for \ p > 10 \). It is easy to
see $S_a < 0$. The initial equilibrium price is $p^* = 5$, while the equilibrium price after the increase in $a$ is $p^* = 4.7$, so that the price falls with an increase in $a$. The key to this example is that although $S_a < 0$, the fringe supply curve becomes much more elastic with an increase in $a$, making the dominant firm's residual demand curve more elastic.

12. This should clear from the preceding discussion since $(D_{p,a} - S_{p,a}) > 0$ is a sufficient condition for $\alpha E D/\beta a < 0$.

13. An example in which $dy/da > 0$ is as follows. Suppose market demand is $Q = 11 - p$ and the dominant firm's costs are zero. Suppose further that for $a = 0$, fringe supply curve is perfectly elastic at a price of one, but for $a > 0$ the fringe supply curve becomes $y = (p - 1)/a$ for $0 < a < 1$, $y = p - 1$ for $a > 1$. Then for $a = 0$, the equilibrium price $p^* = 1$ and output of the fringe is zero. However, if the dominant firm can set $a$, it will set $a^* = 1$ leading to a new price of $p^* = 5/2$ with fringe output rising to $3/2$. Of course the key to this example is that the increase in $a$ makes the fringe supply curve much less elastic.

14. If we give the example of footnote 8 this interpretation, the strategic use of $a$ increases fringe profits.

15. $\bar{p}$ is the solution of (5a) for $a = \bar{a}$. 
16. Suppose demand is $Q = a - bp$ and the dominant firm's costs are zero. Suppose further that for $a=0$, fringe supply is $y = (a-4b) + bp$ and for $a=1$ fringe supply is $y = (a - 6b) + 2bp$. It can easily be shown that the equilibrium price is $p = 1$ for $a=0$ or $a=1$, so that consumers' surplus is the same in each equilibrium. However, producer surplus is larger in the case $a=1$ because a greater proportion of the output is produced by the lower cost dominant firm (which has zero costs).

17. This specification of this model is similar to that of Williamson (1968). However, Williamson makes some restrictive assumptions (fixed coefficients), constant-returns-to-scale technology for the dominant firm and the fringe), which, we will see, greatly limited the range of possible equilibrium outcomes.

18. By (26), $G_y = \partial A_F / \partial a$. For example, if the fringe also uses another input, $B$, with price $\beta$, then $p = a \partial A_F / \partial y + \beta \partial B_F / \partial y$, so that $y < 1$ if $\partial A_F / \partial y$, $\partial B_F / \partial y > 0$.

19. The interpretation of (27) and (28) can be clarified by considering the one input case. In that case, $C(x, a) = A_D(x)$, $G(y; a) = A_F(y)$. Then $a A_D/x = A_C$, the average cost of the dominant firm, and $0 \equiv 1$. Therefore, in the one input case, (28) becomes: $\epsilon/\sigma = (p - A_C)/(p - M_C)$, where $M_C = C_x$ is the marginal cost of the dominant firm. Since $\sigma < 1$, for an interior equilibrium ($a > \bar{a}$) in the one input case, then if market demand
case, then if market demand is elastic, the dominant firm must have increasing average costs ($MC^D > AC^D$). On the other hand, if the dominant firm has decreasing average costs ($MC^D < AC^D$), interior equilibrium requires that demand be inelastic.

To see why these conditions are necessary consider a market with a dominant firm with constant average costs and an elastic demand curve. According to the necessary conditions $\alpha > \bar{\alpha}$ will never be a profitable strategy in this case. To see this, consider strategy with $\alpha > \bar{\alpha}$. Now let $\alpha$ be cut by $\delta$ percent. Because of constant average costs, $AC^D$ then falls by $\delta$ percent (one input case). Next let the dominant firm increase output so that $p$ falls by $\delta$ percent. Since $p$ and $\alpha$ both fall by $\delta$ percent, output of fringe remains unchanged. Therefore, in order to maintain the fall in price the dominant firm must increase output $\delta \epsilon / \sigma$ percent which is greater than $\delta$ percent since $\sigma < 1$ and $\epsilon > 1$. But then the dominant firm's profits ($(p-AC^D)x$) must go up since $(p-AC^D)$ fell by $\delta$ percent but $x$ increased by more than $\delta$ percent. Thus $\alpha > \bar{\alpha}$ could not have been an optimal strategy.

20. Notice that the cost function would not be of this form if the dominant firm were a perfect competitor or a simple monopsonist in the A-market; in either case $\bar{C}$ would not be a function of $p$. $\bar{C}$ is a function of $p$ because the dominant firm has market power in the A-market and in the output market.
21. If the dominant firm were a perfect competitor in the A market then $\dot{C}_a = A^D$ (the usual duality relationship). If the dominant firm were a monopsonist in the A-market, the first order conditions for $a$ would require $\dot{C}_a = 0$.

22. $\dot{C}_p$ is the change in the dominant firm's costs arising from a change in $p$, resulting from the fact that a change in $p$ changes the fringe demand for $A$ and therefore the net supply of $A$ to the dominant firm.

23. The first order conditions require $p - \dot{C}_x > 0$. To see this rewrite (31) as

$$\max \quad \{x, p, a\} \quad \text{subject to} \quad D(p) - S(p, a) - x = 0.$$ 

Then the first order conditions for $x$ require $p - \dot{C}_x > 0$.

24. Suppose that the fringe requires one unit of $A$ to produce one unit of output. Then, assuming that the fringe has increasing costs in other inputs, a plausible fringe supply function is $S(p, a) = (p - a)$, and fringe demand for $A$ is $A^F(a, p) = (p - a)$, for $p - a > 0$. Suppose that $A$ is supplied inelastically at $A$, but that the dominant firm does not use $A$ in production. This assumption is made because then $\theta e_{\lambda} = 0$, so that the marginal revenue product and the value of the marginal product of $A$ are zero. Then, an equilibrium with $a > 0, A^D > 0$ proves the result. Assume that the dominant firm has constant
average cost of production of \( c \). Finally, assume that industry demand is linear, \( D(p) = a - bp \). It is easily shown that the equilibrium in this model is \( p^* = (a + bc - \overline{A})/2b \),
\[ \alpha^* = \frac{a-(b+1)\overline{A}}{2b}, \quad A^D* = \frac{\overline{A} - c}{2}, \quad \text{if} \quad a-(b+1)\overline{A} > 0. \]
If this condition holds \( \alpha^* > 0 \) and \( A^D* > 0 \), even though the marginal productivity of \( A \) for the dominant firm is zero. Thus the dominant firm purchases \( A \) at a price above its marginal revenue product and above the value of the marginal product of \( A \) (which in this case are both zero).

25. A simple example along the lines of the one in footnote 24 can be constructed which bears out this intuition. Let market demand be \( a - bp \). Suppose both the fringe and the dominant firm use one unit of \( A \) to produce one unit of output, and that \( A^F(\alpha) = (p - \alpha) \). Suppose further that \( A \) is supplied according to the supply function \( A(\alpha) = k\alpha \). Finally, assume that the dominant firm's only cost of production is the cost of \( A \). Then it can be shown that the strategic equilibrium has a lower \( p \) and higher \( \alpha \) than the equilibrium in which the dominant firm acts as a simple monopolist-monopsonist.

26. An example in which \( (\alpha - c'(A^D)) > 0 \) can be easily constructed along the lines of the example of footnote 16. Assume now that the dominant firm must also use one unit of \( A \) to produce one unit of output and that it can produce \( A \) at a constant average cost of \( c \). Assuming the demand curve and
technology of the fringe is the same as in footnote 16, the equilibrium in this model is also $p^* = (a+bc-A)/2b$, $a^* = [a - (b+1)A]/2b$, $A^* = \bar{A}-c)/2$, $x^* = (a+1(l-b(c))/2$, and $A^* = [a + (2-b)c -A]/2$, where $A^*$ is the amount of $A$ produced by the dominant firm (assuming $(a+(2-b)c-A) > 0)$. Since $a^*$ doesn't depend on $c$, it is clearly possible to have $a^* - c > 0$.

27. See the example in footnote 25.
REFERENCES


