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A MODEL OF NON-COMPETITIVE INTERDEPENDENCE AND ANTITRUST LAWS*

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ABSTRACT

The received, Cournot-Stackelberg class of non-competitive interdependence models, and its modern counterpart, the Von Neumann-Morgenstern-Nash-Selten class of n-person, non-cooperative games, assume no strategic communication. No player can affect the current strategies of others because no player can pre-communicate his strategy to the other players. The present paper constructs a general model of non-competitive interdependence under perfect strategic communication. In such a model, between any pair among n interacting monopolists, one exhibits a prior reaction function while the other simply picks a point on the function. We derive rational reaction functions and characterize the resulting solutions for both unconstrained interaction and interaction constrained by antitrust laws. Results of preliminary tests of each of these models are very encouraging.

INTRODUCTION*

The received, Cournot-Stackelberg class of non-competitive interdependence models, and its modern counterpart, the Von Neumann-Morgenstern-Nash-Selten class of n-person, non-cooperative games, assume no strategic communication. No player can affect the strategies of others because no player can communicate his strategy to the other players before the others select their own strategies. Yet the repeated interactions between large firms in most industries in the real world make it extremely implausible that such firms do not find ways to communicate how they will respond to the actions of their rivals. While numerous dynamic models of learning and strategic interaction have been developed, their lack of simplicity and generality have left economists with no common, static-equilibrium framework with which to view modern oligopolistic industries. In this paper, we attempt to remedy the situation by developing a static equilibrium model from an assumption of <u>perfect</u> strategic communication, where <u>strategic education is complete</u> and occurs before ordinary production decisions are made.

In Section I, we first specify our general model and show that a <u>necessary condition</u> for the existence of solutions to all monopoly interaction problems featuring perfect strategic

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communication is a hierarchy of firms containing a recursive set of reaction functions.

A sharp contrast emerges between the resulting static model and conventional economic theory. Our firms need not simply pick prices or outputs. They may also pick their price or output reactions to the prices or outputs of other firms. In other words, they may each pick their own, committed, reaction functions. In contrast, the modern approach to problems of sophisticated, oligopolistic interaction has been to supply highly involved, but still highly unrealistic, dynamic descriptions in which price and output choices at each point in time maximize the present values of prospective profits. (The recent F.T.C. volume on predation and entry deterrence edited by Salop and the recent J.E.T. collection of papers on reputation investment by Kreps, Milgrom, Roberts, and Wilson testify to the popularity of the approach.)

We object to this narrow rationality approach to non-competitive price-quantity choices first because of the large number of equally-plausible dynamical solutions with radically different conclusions (Aumann, Dixit, Friedman, Rubinstein) and second because the real world does not--even theoretically--follow the simple Alchian-rule of selecting for success and long-term survival those firms that will choose future prices or outputs that will maximize the then-expected present values of their then-prospective profits. Rather, it selects for survival those organizations that exhibit initially profit-maximizing strategies, such strategies generally employing price or output responses to others that will not maximize the expected present values of their then-prospective profits given the chosen behavior of others. Correspondingly, surviving, successful, real-world oligopolies have evolved separations of ownership from short-run control and thereby greatly facilitated their ability to respond to the near-future actions of others in ways that do not maximize the firm's value of its then-prospective profits given these actions. Observed, surviving, oligopolies thus consistently reject the current-wealth-maximizing calculations of economists and the like in favor of short-run decision rules of more principled, sometimes war-like, sometimes benevolent, professionally trained managers.

Once our basic theory is developed in Sections I A and B (and the Appendix), the exact form of the survival-determining decision rules and the characteristics of the resulting solutions become the main concern of the paper.

The solutions to unconstrained interaction are shown in Section I C to have a predatory, robber-barron characteristic. In Section II, we introduce government antitrust policy. Given a simple, first approximation of the reaction constraints implied by observed policy, we derive the form of the new, constrained, optimal reaction functions of the interacting firms. We use this result to construct a long-run equilibrium in a linear special case and derive its equilibrium size distribution of firms, concentration ratios, and mark-ups. Firms are each twice as large as the next smallest firm, except possibly for the largest firm, which may be a bit larger in order to deter subsequent entry into the industry. Most important is the approximate efficiency of the resulting oligopoly solutions. For example, the equilibrium mark-up with at least 6 firms is shown to be no more than about 3 percent of the pure monopoly mark-up.

In Section III, we note the rough empirical accuracy of our theories of monopoly for the U.S. experience before, during, and after the establishment of our federal antitrust statutes. The central empirical result of this analysis is that U.S. antitrust laws have altered the form of rational reaction functions so as to convert a world dominated by highly inefficient monopolies into one whose typical outputs are probably very close to purely competitive levels.

Section IV provides a much more realistic, second approximation of the effects of U.S. antitrust laws on rational firm strategies, both for long-run, capital decisions and short-run, output decisions. By exposing the rational, individualistic, basis of actual producer "agreements" and "cooperation," our theory provides us with a new, simplified, view of the operation of these laws. While the positive and normative conclusions of our first approximation largely remain, several unfortunate judicial interpretations of these laws appear to have made the recent costs of these laws substantially higher than is necessary.

The analysis up to the Appendix of the paper follows our basic theory of social organization (1980, 1981) in assuming that the identity of a firm able to establish a prior reaction function over other firms is determined prior to its commitment to a particular reaction function. The Appendix, however, introduces an explicit, competitive process determining the identity of this firm and, simultaneously, its rational reaction function. After demonstrating the existence of a general solution, we show that the solution characteristics of the simpler model in the text are not substantially altered by this change in the model as long as the number of firms is not very small. (An empirical application to a small numbers case is noted in footnote 17).

The features distinguishing our oligopoly, or interdependent monopoly, allocation problem from a general allocation problem with perfect strategic communication in a private property system (Thompson-Faith, 1980, 81) are that: (1) Buyers are noncooperating price takers and (2) sellers are only horizontal rivals and, correspondingly, do not make transfer payments to other sellers. These commonly employed, simplifying restrictions shall remain in force throughout our formal discussion.

I. THE GENERAL MODEL

A. The Environment

We consider an economy containing n commodities and m firms, $(n, m) \geqslant (2, 2)$. The economy's output allocation set is denoted:

$$x = (x_1, x_2, \ldots, x_m), \; x_f \geqslant 0, \text{ for all } f = 1, \ldots, m, \tag{1}$$

where x_f is the f^{th} firm's n-dimensional output vector, or "output". $x_f \in X_f$, the f^{th} firm's feasible output set. The set of output vectors of all firms except firm f is denoted:

$$x_{-f} = (x_1, x_2, \ldots, x_{f-1}, x_{f+1}, \ldots, x_m). \tag{2}$$

Each firm, f, can produce all commodities and has a profit function,

$$\pi_f(x) = \pi_f(x_f; x_{-f}), \; f = 1, \ldots, m. \tag{3}$$

We think of this function as one summarizing firm f's technology, factor costs, and output demand conditions in a private property system. Hence, $\pi_f(0; x_{-f}) = 0$, and the profit functions of the various firms are not generally identical.

B. Firm Interaction Under Perfect Information Regarding Others' Strategies

We now diverge from anything we have ever seen in formal economic theory by having some firms choose, not an x_f, but a reaction function to its competitors. The rationale is quite simple: Our firms make no errors and always correctly evaluate the reaction functions of others; so these other firms, recognizing that their reactions are being taken into account, merely select reaction functions, thereby making the output choices of other firms determine their own outputs. From a game-theoretic perspective, we are introducing an assumption of perfect information with respect to the strategies of others,[1] so the strategies must be selected in sequence rather than simultaneously because any given strategy selector must observe the strategies of the other strategy selectors. In particular, for perfect information with respect to the strategies of others, the first strategy selector, firm 1, must announce his strategy, or reaction function, to the others:

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[1] Standard game theory does not cover this level of informational perfection in that even "perfect information" assumes zero information with respect to the strategies of others, thereby motivating the conventional Nash noncooperative solution concept. The possibility of perfect information with respect to strategies was noted by Von Neumann-Morgenstern (p. 84), but as it created no new existence problems, it was passed over. Our interest is in optimality, where, we shall see, it makes a great deal of difference whether there is perfect information with respect to others' strategies or just conventional "perfect information," i.e., perfect information with respect to others' outputs. The conventional "perfect information" solution is simply an m-firm Stackelberg solution. We shall elaborate on this in Section II.

The "metagame" model of Howard does permit the communication of strategies but inappropriately employs a conventional Nash solution concept rather than a Von Neumann-Morgenstern perfect information solution concept to the resulting normal form. The result is a solution set which is uninterestingly large in that it still contains ordinary Cournot-Nash solution points. This is elaborated elsewhere (Thompson-Faith 1981).

$x_1 = x_1(x_2, x_3, \ldots, x_m)$,

while firm 2, the second strategy selector, exhibits the reaction function,

$x_2 = x_2(x_3, \ldots, x_m)$,

and the third firm exhibits

$x_3 = x_3(x_4, \ldots, x_m)$,

and so on up to the $m-1^{st}$ firm's simple reaction function. A solution output allocation set is easily constructed by having the m^{th} firm, viewing these reaction functions, selecting an output. This output determines the output of the $m-1^{st}$ firm given its established reaction function, which in return gives the two outputs necessary for the $m-2^{nd}$ firm to determine its output, and so on until the first strategy selector's output is determined.[2]

The m^{th} firm's output is assumed to maximize its profit in view of the various solutions which would result from his various

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[2] In general, any strategy selector selects more than a reaction function; he announces his reactions to reaction functions which subsequent strategy selectors might present to him. But it is easy to see that such strategies by subsequent strategy selectors can never benefit them and can be ignored. For example, if the m^{th} strategy selector selects, rather than a simple action, a reaction function to present to prior strategy selectors, he is giving the choice of x consistent with the above, prior, m-1 reaction functions to others and therefore can be no better-off than by choosing x himself. Therefore, m's choice of a reaction function is rational only when it is redundant in that it yields him the same return as a simple output choice.

output selections. But before we can derive the profit maximizing choices, the reaction functions, of the other firms, we must specify the cost of becoming a prior strategy selector. We must also determine the order of priority in strategy selection. One possibility is to arbitrarily assign an order of strategy selection (corresponding, say, to the order of birth of the various firms.) A solution is then obtained by first allowing the m-1st firm to select a reaction function that maximizes its profit given the m-2 prior reaction functions and in view of the various rational output choices of the mth firm for the various possible reaction functions of the m-1st firm. Then the m-2nd firm selects a reaction function that maximizes its profit given the prior m-3 reaction functions and the various rational outputs of firms m-1 and m which result from its various reaction functions. The process continues until the m-1 rational reaction functions have been formed.

Another possibility is to have the firms engage in costly competition to determine the order of strategy selection. A model of such competition is specified in the Appendix, where we demonstrate the existence of solutions to this new kind of competitive process, and compare the solutions resulting from these two possible methods of assigning priority of strategy selection. The chief result is that there is no substantial difference between the solutions resulting from the alternative assignment processes when the number of interacting monopolists is not very small. This result will hold both in the case of unconstrained interaction and when an antitrust law restricts the set of feasible reaction functions.

So, for the formal text below, we assume that there is a costless assignment of hierarchial position determined, say, by the historical sequence of entry into the economy.

To characterize the resulting solutions, it will be instructive to consider first the case of duopoly, where the 2nd firm simply chooses a set of outputs, x_2^*, that maximizes $\pi_2(x_2;\ x_1(x_2))$. The resulting solution determines a dependency of x_2^* on the functional, $x_1(x_2)$, a dependency which we write as

$$x_2^* = x_2^*\{x_1(x_2)\}. \tag{4}$$

This dependency is not a reaction function; it merely shows how a subsequently selected strategy depends upon a prior strategy. In view of firm 2's rational response given by (4), firm 1 chooses a reaction function $x_1^*(x_2)$ which maximizes $\pi_1(x_1(x_2);\ x_2^*\{x_1(x_2)\})$. This duopoly solution concept is equivalent in game theoretic structure to an implicit solution concept used by Schelling (Ch. 5) to solve two-person bargaining and prisoner's dilemma problems (see Thompson-Faith, 1981).

To guarantee the existence of such solutions, we can add that X_f is finite and $\Pi_f(\cdot)$ is real-valued. Sufficient conditions for the existence of a general, n-firm, solution to this variational maximization problem when X_f is infinite are given in the Appendix.

C. Characterizing the Solution

An extremely simple characterization of the above solution is achieved by adding the following "punishability" condition: For any positive $x_2 \varepsilon X_2$, there exists a "punishment output," an $x_1^p(x_2) \varepsilon X_1$ such that $\pi_2(x_2; x_1^p(x_2)) < 0$. Then, letting x^1 be an economy-wide output allocation that maximizes firm 1's profits over all feasible x subject to $\pi_2(x_2; x_1) \geqslant 0$, a rational reaction function of firm 1 is given by

$$
\begin{aligned}
x_1^*(x_2^1) &= x_1^1 \\
x_1^*(x_2) &= x_1^p \text{ for } x_2 \neq x_2^1 .
\end{aligned}
\tag{5}
$$

Facing this, firm 2 chooses x_2^1, thus yielding x^1 as a solution. It is readily shown (See Appendix) that this dictatorial solution also holds for $m > 2$.

It is useful to consider the case of a Cournot technology, wherein $\pi_f = x_f[a-b(x_1 + x_2)-c]$, $(a,b,c,) > 0$, $a > c$, and $x \varepsilon R_+^2$. The condition on the existence of a set of punishment outputs obviously holds in such an environment as firm 1 may, for any output of firm 2, merely set $x_1 = \frac{a-c}{\_}$ in order to make $\pi_2(x_2,x_1) < 0$ for all $x_2 > 0$. As the industry's simple monopoly output, $\frac{a-c}{2b}$, for firm 1 and a zero output for firm 2 is obviously the most profitable allocation to firm 1, its reaction function is given by

$$x_1^*(0) = \frac{a - c}{2b}$$

$$x_1^*(x_2) = \frac{a - c}{b} \text{ if } x_2 > 0.$$

Firm 2 rationally picks a zero output, and a pure monopoly output results.

The reactions described above are extremely predatory. Such a high degree of predatory behavior does not appear to characterize the typical real-world relationship between currently interacting firms. This may be due to the failure of the assumption of perfect information regarding others' strategies to guide real-world relationships. Indeed, as our general solution describes the complete behavior of a cartel with no side payments,[3] Professor Stigler's well-known analysis of the informational problems of cartels applies. In particular, the enforcement of the cartel behavior on a given set of firms requires that the enforcers--in our model, the firms with prior reaction functions--observe or infer the actual outputs of the obviously reluctant firms. Since the total industry information cost relative to the potential industry profit increases with the number of firms in the industry, the <u>initial</u> number of firms may easily be so large that observations on the behavior of other firms become impractical. In such a case, each firm will rationally assume that the outputs of the others are unaffected by his

[3] When inter-firm payments are allowed, our general solution becomes a "perfect" cartel, one which achieves a joint profit maximum for all the producers (Thompson-Faith, 1980, '81).

own output and a familiar Cournot-Nash solution becomes appropriate. This apparently corresponds to industries such as agriculture, mining, furniture and garment manufacturing, industries with a large number of locally established firms prior to the mid-nineteenth century (Chandler).

In contrast, most national industries born during the transportation-communication revolution of 1860-1940 quickly became dominated by a few large firms. These firms, who have, by and large, remained dominant to this day, have clearly not interacted in a wholely Cournot-Nash fashion. Firms in these industries can typically observe or infer one another's recent actions. Yet the extremely predatory reaction functions described above do not appear to have characterized most of these industries in modern times. A plausible hypothesis to explain this is simply that such reactions are forbidden by law. The following section of the paper formalizes this hypothesis.

II. THE EFFECT OF ANTITRUST POLICY

A. Anti-Monopoly Laws

A seemingly first-best antitrust law would simply: (1) allow monopolies to form at will in every industry; (2) subsidize firm outputs in order to prevent the usual monopolistic under-productions arising under imperfect price discrimination, and (3) tax away monopoly profits lump-sum (say by having the government sell the monopoly by auctioning-off the right to the first reaction function, as described in the Appendix) in order to prevent a costly devotion of real resources to obtaining the monopoly profit. However, this first-best efficiency is only apparent. Any such antitrust law would admit extremely large efficiency losses by tempting firms to employ discriminatory price systems (Thompson, 1983). To prevent these losses, an efficient government may adopt a policy of splitting up monopolistic firms so as to create at least two rival producers for each kind of output. With some, justifiable exceptions,[4] such a policy is, in fact, observed in the U.S. The legislative basis of the policy is provided by section 2 of the Sherman Act and sections 7 and 8 of the Clayton Act. We might think of these

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[4] Monopolies are typically permitted when they provide: (1) a relatively inexpensive way to reward recent innovations (U.S. vs. E.I. duPont, 118 F. Supp. 41, D. Del. (1953)), (2) a way of redistributing away from other countries (the Webb-Pomerene Act), and (3) a way of internalizing the effect of one supplier's quality change on the perceived qualities of the given outputs of other suppliers (antitrust-exempt sports leagues and standard-setting professional associations).

structure-determining laws as the "anti-monopoly" portion of our antitrust laws.

The remaining policy problem is to make the resulting, two or more firms act as independent competitors. By outlawing "conspiracies in restraint of trade," section 1 of the Sherman Act clearly reveals a legislative attempt to restrict inter-firm reaction functions. How such conduct-determining laws, which can be thought of as the "anti-cartel" portion of our antitrust laws, actually restrict these reaction functions and generate observed oligopoly outcomes is the subject of the remaining text of this paper.

B. The Effect of Anti-Cartel Laws on Reaction Functions--A First Approximation

On the basis of existing anti-cartel laws, it is not unreasonable to assume, for a first approximation, that if any firm expands its output in reaction to increases in the outputs of its competitors--either existing or entering firms--that firm would be subject to prosecution under both the Sherman Act and section 5 of the FTC Act for its "predatory practices."[5] Thus, whenever a firm increase its output for a given level of industry

[5] This interpretation assumes that the courts can distinguish between increases in output due to efficiency reasons from increases for predatory or punishment reasons. Several Sherman Act cases, such as the U.S. Steel Case of 1921, lend support to the assumption that courts at least attempt to make such a distinction. We shall discuss later the recent trend away from this view of what constitutes illegal predation.

demand and cost, we shall assume that government anti-cartel policy prohibits another firm from increasing its output. This restriction upon the output reactions of firms precludes the use of punishment strategies.

In the absence of punishment strategies, a firm may also try to induce the production of some desired industry output by "rewarding" other firms for their outputs, that is, decreasing its output if the other firms decrease their outputs down to the desired levels. We shall assume, for our first approximation, that such behavior will be viewed as "collusion" by the government policy-makers, and likewise be prohibited.

The effect of our first approximation of actual anti-cartel policy is, therefore, to limit each firm's choice of reaction function to a choice among the members of the class of non-increasing functions. Thus, firm i faces only two kinds of alternatives given a change in the output choice of the firm i+k, where k=1,...,m-i. i may exhibit a zero or a negative reaction to i+k's change in output. Comparing the results of these two alternatives, firm i rationally decides upon the zero reaction function.

Our demonstration is as follows: Suppose the i^{th} strategy selector adopted a negative reaction function. Then, if firm i+k increased its output, say from x^{o}_{i+k} to x'_{i+k}, and firms i+1,...,i+k-1, i+k+1,...,m did not contract as much as i+k expanded so that there were a net expansion in industry output, or $\sum_{k=1}^{m-i}(x'_{i+k}-x^{o}_{i+k}) > 0$, the result of i's contracting its output in response would clearly be to encourage the output expansion of firm i+k and thus the aggregate output of his competitors. Inducing an increased output by its competitors could only benefit firm i if it ultimately permitted i a different solution output. But x_i for any $(x_{i+1},\ldots,x_m)$ may be changed independently of i's reactions to the output changes of these firms! Firm i need only change its reaction function from $x_i(x_{i+1},\ldots,x_m)$ to $x_i(x_{i+1},\ldots,x_m) + \delta$. So with firm i able to alter its solution output independently of its responses to the output changes of later strategy selectors, its negative reaction function would be dominated by a particular, constant output, reaction function. (Similarly, if i+k reduced its output and the induced output changes by the rest of the firms resulted in a net decrease in industry output, firm i would not rationally respond with an increase in its output and thereby discourage i+k from decreasing its output in the first place. For firm i could produce the desired, higher output by simply committing itself to produce the desired output as a constant regardless of i+k's reaction and thereby induce i+k to produce a lower output than he would if i presented i+k with a negative reaction.) Finally, if, when firm i+k increases (decreases) its output, the rest of the firms other than i decreased (increased) their aggregate output so that the net output of the industry were reduced, (increased) firm i+k would continually expand (contract) its output until this were no longer the case. So there would be no equilibrium under this final possibility. Thus, in equilibrium, each firm's rational reaction function is a constant-output reaction function.

C. Equilibrium

Using this result we now derive an equilibrium with m firms producing a homogeneous output in which each firm's hierarchical position is exogenously given. This amounts to a generalization of the Stackelberg duopoly model.[6] We again assume a fixed hierarchy of strategy selectors, or "strategy makers," with firm 1 being the "primary maker," firm 2 the "secondary maker," and firm m the "pure taker."[7] Also, we again adopt a Cournot technology. Thus, industry demand is assumed to be linear and of the form:

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[6] In the Stackelberg-on-output model of duopoly (see, for example, Intriligator), one firm, called the "follower," assumes the other will exhibit a constant output and makes his rational output choice on this assumption. The other firm, "the leader," selects his output subject to the follower's rational response function. The result is a Stackelberg equilibrium. In our antitrust-constrained monopoly model, the second strategy selector acts as a Stackelberg follower by choosing his output subject to constant-output reaction functions. The first strategy selector behaves as a Stackelberg leader since he chooses his reaction function (fixed output) subject only to the profit-maximizing behavior of the other firms. Firms 2, ..., m-1 introduce into the model additional relationships not described in previous models of which these authors are aware. Nevertheless, our controlled monopoly model with its constrained reaction functions and added relationships generates what can be interpreted as a generalized Stackelberg model. For, as derived above, adding more firms to our model merely creates a hierarchy of partial Stackelberg leaders.

[7] We introduce these new firm descriptions to remind readers that the firms are interacting under constraints not present in our basic model developed in Section I. As Stackelberg's "leader-follower" terminology suggests certain pricing relations that are not relevant to a single-output model, we avoid it here. It will, however, reappear in a fixed-price, multiple-output discussion of Section IV.

$$p = a-b\sum_{i=1}^{m} x_i, \tag{6}$$

where p is the price of the industry's output and, again, a and b are positive constants. And marginal costs, c, are assumed to be constant and identical for each firm so that firm f's profits can be expressed as

$$\pi_f = x_f(a-b\sum_{i=1}^{m} x_i-c), \tag{7}$$

where, to assure positive outputs, $c < a$. The condition for profit maximization for each firm is

$$p - x_f b \sum_{i=1}^{m} \frac{dx_i}{dx_f} - c = 0, \text{ or} \tag{8}$$

$$x_f = \frac{a-c-b\sum_{i\neq f} x_i}{b\left(2+\sum_{i\neq f}\frac{dx_i}{dx_f}\right)} = \frac{p - c}{b\left(1+\sum_{i\neq f}\frac{dx_i}{dx_f}\right)} . \tag{9}$$

Since all reaction functions are constant output reaction functions, for each firm j,

$$\frac{dx_i}{dx_j} = 0 \qquad \text{for all } i < j. \tag{10}$$

This yields a profit-maximizing expression for m of:

$$x_m^* = \frac{a-c-b\sum_{i=1}^{m-1} x_i}{2b} = \frac{p-c}{b} . \tag{11}$$

Since firm m-1 is a taker with respect to 1,..., m-2, and the latter exhibit constant output reaction functions,

$$\sum_{i=1}^{m-2} \frac{dx_i}{dx_{m-1}} = 0 .$$

And from (11), we know $\frac{dx_m^*}{dx_{m-1}} = -\frac{1}{2}$. Thus, using (9) and (10),

$$x_{m-1}^* = \frac{a-c-b\sum_{i=1}^{m-2} x_i}{2b} = \frac{p-c}{b/2} \tag{12}$$

To obtain m-2's profit-maximizing condition, we have to calculate m-1's and m's rational responses to a change in x_{m-2}. From (1) we know that

$$\frac{dx_{m-1}^*}{dx_{m-2}} = -1/2 . \tag{13}$$

And from (10),

$$\frac{dx_m^*}{dx_{m-2}} = \frac{\partial x_m^*}{\partial x_{m-2}} + \frac{\partial x_m^*}{\partial x_{m-1}} \cdot \frac{dx_{m-1}^*}{dx_{m-2}} = -1/4. \tag{14}$$

Hence, again using (9),

$$x_{m-2}^* = \frac{a-c-b\sum_{i=1}^{m-3} x_i}{2b} = \frac{p-c}{b/4} . \tag{15}$$

Similarly, we find that

$$x^*_{m-3} = \frac{a-c-b \sum_{i=1}^{m-4} x_i}{2b} = \frac{p-c}{b/8} , \tag{16}$$

$$x^*_{m-4} = \frac{a-c-b \sum_{i=1}^{m-5} x_i}{2b} = \frac{p-c}{b/16} ,$$

$$\vdots \qquad\qquad \vdots$$

$$x^*_1 = \frac{a-c}{2b} = \frac{p-c}{b/2^{m-1}} \tag{17}$$

The resulting size distribution of firms is obviously

$$x^*_{m-i} = x^*_m \, 2^j , \; j = 0, 1, \ldots, m-1. \tag{18}$$

D. Corollaries

It is interesting to note from (17) that the first firm always produces the simple monopoly output. Its solution output, and correspondingly the solution outputs of subsequent makers, is therefore unaffected by the addition of new firms to the industry.

There is also a corollary concerning the "concentration ratio" of our industries. It is that the t-firm concentration ratio, the share of the top t firms in the industry, decreases as the number of firms in the industry increases. From the above theorem, the total output of the top t firms in the industry can be written:

$$K \sum_{j=m-t}^{m-1} 2^j = K2^{m-1-t} \sum_{j=1}^{t} 2^j = K2^{m-t}(2^t-1), \tag{19}$$

where $t \leqslant m-1$ and K is some positive number. The total output of the m firms is

$$K \sum_{j=0}^{m-1} 2^j = K(2^m-1). \tag{20}$$

Hence, the output share of the top t firms in the industry, $t \leqslant m - 1$, is given by:

$$S_t = \frac{2^{m-t}(2^t-1)}{2^m - 1} = \frac{2^t - 1}{2^t - \frac{1}{2^{m-t}}} . \tag{21}$$

Thus we see that as the number of firms in the industry expands and thus the output becomes more competitive, the concentration ratio, S_t, for any t decreases.

This decrease, however, is very slight once the number of firms in the industry becomes at all significant. For example, if $m \geqslant 8$, then the percentage error in using $\frac{2^t - 1}{2^t}$ as an estimate of S_t is always less than one half of one percent. Observed large-firm concentration ratios should therefore be roughly the same across oligopolistic industries with several firms and no threat of entry.[8]

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8 Using the above analysis and the results of Part II of the Appendix, it can be shown that this same asymptote is approached, and the same approximation result holds, when competitive bidding for hierarchial position is allowed. The only difference which arises when such competition is allowed is that the concentration ratio increases, rather than decreases, to its asymptotic level as $m \to \infty$.

A third corollary regards the proximity of our generalized Stackelberg equilibrium to a competitive equilibrium. From (17), the equilibrium mark-up in our model is $\frac{a-c}{2^m}$. Under pure monopoly, the mark-up would be (a-c)/2. (This is the same as the uncontrolled monopoly mark-up since the rational maker in this industry model could not do better than he could by producing an output such that $p < c$ whenever any other firm produced a positive output.) Given the distribution of output among firms in our antitrust-constrained solution, the equilibrium mark-up relative to the pure monopoly mark-up is therefore given by

$$\frac{\frac{a-c}{2^m}}{\frac{a-c}{2}} = \frac{1}{2^{m-1}} . \tag{22}$$

So with, say, 6 firms in the industry, the equilibrium mark-up is only about 3 percent of the pure monopoly mark-up.

This insignificant mark-up not only assures essentially competitive outputs from industries with as few as a half of a dozen firms. It reduces to near-insignificance the resources which the firms devote, say in the form of an over-building of factories ahead of demand, to establish the priority of their output commitments. These results speak for the powerful efficiency of the simple antitrust policy outlined above.

E. Entry Threatened Industries

We have been ignoring the possibility of entry and, correspondingly, have not considered the overhead costs of firm operation. Clearly, if the variable profit to an $(m + 1)^{st}$ firm in an $(m + 1)$ - firm solution to the above model were below these overhead costs to potential competitors, there would be no threat of entry. But if the solution profits to an $(m + 1)^{st}$ firm in an $(m + 1)$ - firm interaction exceeded the firm's overhead cost, then either entry would occur or existing firms would alter their strategies to forestall this undesirable prospect. It is easy to show that the latter will always occur in our simple model. Any firm in the industry would obviously be better-off producing the outputs that would be produced by equal and constant-variable-cost, would-be entrants. The first maker has the first opportunity. If he doesn't take it, a subsequent established firm will, and, in any case, the same, Sylos-Labini-type, entry-forestalling, industry output emerges. Our solution thus has the first maker producing its normal, constrained-monopoly output plus the outputs of the potential entrants and then the rest of the firms producing their normal, constrained monopoly outputs. To verify the uniqueness of this 11:4:2:1 distribution, where 3 is the total output of the would-be entrants, simply note that the marginal revenue of the first maker beyond an output equal to 11 is way below its cost since it only produced such a large output because it was profitable to deter entry. For the same reason, subsequent selectors also have marginal revenues significantly below costs at larger outputs. And at lower outputs, all but the m^{th} firm would merely be giving up the opportunity to replace the potential competitor to a subsequent selector. A lower output by the m^{th} firm would induce entry and would therefore, as in Sylos-Labini, be unprofitable. (This no-entry result is substantially generalized in Schwartz-Thompson.)

Such a solution would, of course, be even more efficient than our nearly efficient, non-entry-constrained, Stackelberg-on-output solution.

The top firm would obviously be even better-off if it could establish a commitment to predate against the new entrant by expanding its output in response to the entry. But, as we have been assuming, such conduct is illegal.

This interpretation of existing antipredation law has been recently challenged by Williamson, who has argued in favor of <u>strengthening</u> existing U.S. antipredation law to the exact constraint on reactions to entry that our above, first approximation uses to describe the <u>existing</u> law regarding reactions to rival output changes. Our second approximation of existing law, which was written subsequent to Williamson's paper in order to explain in greater detail why our first approximation is a reasonable approximation of existing law regarding reactions to existing rivals, also shows how a policy change extending our reaction constraint against existing rivals to new rivals would represent a <u>weakening</u> of current U.S. antitrust constraints on reactions to new entry.

F. A Policy Equivalent

The legal restrictions on output reactions generating our above, first approximation is equivalent, for firms producing a homogeneous output, to a prohibition of output reactions that are necessarily "irrational in the narrow sense," i.e., necessarily precommitted, whatever the demand and cost conditions facing the industry. To see this, first note that when a firm expands its output for a given state of industry demand and costs, any uncommitted, narrowly rational, set of reactions by its substitute-producing rivals implies that they contract their subsequently chosen outputs. (See Section IV B.1 for a description of the multiple-output models permitting us to extend this simple, Stackelberg-on-output, solution characteristic to more realistic environments.) The reason, of course, for this solution characteristic is that the given output expansion lowers the joint demand curve facing the rivals. Similarly, when a firm contracts its output, narrowly rational rivals will always want to expand their subsequently selected outputs. A contraction response to a rival contraction, like an expansion response to a rival expansion, would therefore be narrowly irrational, carried out only because of a prior commitment to do so. In effect then, the law eliminates responses that are unambiguously precommitted, narrowly irrational, reactions regardless of the nature of industry demand and costs.

An immediate corollary is that positive reactions to rival expansions imply that the reactors have expanded to where they are selling at prices below their marginal opportunity costs, costs which include foregone, monopoly-type, internalized price-effects. This provides a simple economic interpretation of the classic, "selling-below-cost," legal description of economic predation, an interpretation quite contrary to the numerous economic interpretations that have recently appeared. (See, e.g., Telser, Areeda-Turner, Koller, Williamson, Joskow-Klevorick, or Ordover-Willig.) These authors view "predation," which is explicitly outlawed in the Sherman Act, as restricted to narrowly rational, monopolizing investment while we view predation as including the narrowly irrational responses necessary to maintain a previously established monopoly position. More generally, they view observed predation as a rational expenditure on aggression to acquire future property while we view it as including a narrowly irrational expenditure on war necessary to execute a previous commitment to protect one's previously acquired property. Correspondingly, while we have been arguing that effectively outlawing such narrowly irrational, war-like, predatory actions is an essential part of our anti-cartel laws, these other authors have argued--erroneously--that irrational predation is not worth considering because the real world will always select against individuals exhibiting such behavior even in the absence of legal sanctions.

These authors point out that "predation" (in their narrow sense) is rarely profitable in the real world, but do not consider the possibility that the main reason such predation is rarely profitable is that existing U.S. antitrust laws--including the laws effectively eliminating predation in the broad sense--have largely eliminated the profit to predation in the narrow sense.

G. Extension to More General Environments

The approximate efficiency of our now-generalized interpretation of observed anti-cartel law (wherein all responses to rival behavior that are narrowly irrational are outlawed) easily extends to worlds admitting the possibility of direct, lump-sum, transfers between the firms. As we have already pointed out, the unconstrained-interaction solution is characterized by perfect, joint-profit-maximizing, cartelization once such transfers are allowed (Thompson-Faith, 1980). The extra-dimension responsible for the "perfection" is that firms can be rewarded for output cut-backs by direct, lump-sum payments rather than less direct, generally less internally efficient, matching cutbacks in output. The interaction can thus be viewed as including exchanges of ownership claims for output reductions. Now, the last person to deliver in any exchange, including exchanges of ownership claims for rival output reductions, choose to deliver only because of a prior commitment to do so. Final delivery is always a narrowly irrational act. The delivery is therefore precluded by our generalized anti-cartel law, which outlaws all responses to rival behavior that are unambiguously narrowly irrational. Thus, for them, the outlawing of such responses is sufficient to achieve approximate social optimality even when the environment is generalized to include lump-sum transfers.

However, as we shall see in Section IV, the approximate efficiency of outlawing narrowly irrational responses to rival behavior does not extend to short-run, nominal-price-setting behavior. That is, it is not optimal to outlaw all forms of collusion, predation and, more generally, irrational responses to rival nominal price behavior. Moreover, we shall finally conclude that in failing to recognize this exception, our legislature has allowed our judges, lawyers, and economists to recently evolve a perverse system in which nominal price-fixing has become a per se violation of our antitrust laws while sophisticated short-run quantity interactions have become almost completely immune to antitrust prosecution, thereby creating the recent trend toward substantially increased litigation without any apparent lessening in the degree short-run cartalization.

III. EVIDENCE

The rational-reaction-function, perfect-information approach to non-competitive interdependence can be tested by attempting to verify empirically the implications developed above. We use the U.S. experience since we are somewhat less ignorant of it than of the experiences of other countries.

A. Prior to Antitrust Laws

According to most accounts, no substantial monopolies other than government-granted and small, local monopolies appeared before the Civil War. After that war, the communications-transportation revolution and the emergence of the corporate form of organization apparently opened up unprecedented opportunities for large-scale private organizations and thus private monopolies operating in nationwide markets. In this environment, industrial giants grew in several new industries, each coming to dominate his industry by using unprofitable price-cutting as a weapon against smaller firms in order to keep them "in line." These "robber barons" were, in our terms, simply prior strategy selectors, and their "cutthroat competition" was merely their application of punishment outputs to deviant firms.

The view that these "robber barons" even existed has been challenged in the economic literature by Professor McGee. Regarding theory, McGee argues that a punishment strategy is irrational, which indeed it is if one accepts conventional Cournot-Nash assumptions. But we are allowing one individual's strategy to affect the strategy selections of others. So, as we have seen, a punishment strategy may be rational. Regarding the real world, McGee argues that the Standard Oil Company, widely considered a company which regularly adopted punative reaction functions (Tarbell), did not, in fact, adopt such strategies. His evidence for this unusual claim was of two kinds. First, he argued that most of the refineries that were acquired by Standard in developing its monopoly were purchased at about book value, or slightly higher, from sellers who did not complain about "predatory pricing." But since most of the refineries were purchased during an era of unexpected crude oil discoveries and booming demand for refined products, book-values were probably a significant underestimate of competitive values throughout the period. (Indeed, what examples appear in McGee (pp. 151-2) and Tarbell (p. 33) indicate that discounted earnings streams from the purchased refineries were far in excess of book values or purchase prices.) And, while the refiners selling during Standard's growth to near-monopoly status (1871-1886) frequently complained that their high expected transport cost relative to Standard's was the main reason for selling to Standard, McGee did not regard Standard's acquisition of a pipeline monopoly and control over railroad rebate policies as a source of "predatory pricing." That is, McGee did not regard over-purchasing in a competitor's input market to be equivalent, from the standpoint of predation, to overselling in the competitor's output market.

(While McGee points out that railroad rebates were often given Standard's competitors during the period of Standard's growth into a monopoly, he fails to point out that the rebates typically took on a narrowly irrational form rather than a narrowly rational, sliding scale reflecting overhead transport costs: Frequently no rebate was given to large refiners simply because their volume was below that of Standard; more frequently, Standard's agreement with the railroads was that higher transport charges were to apply to its competitors; and several key contracts were also signed in which the railroads were obligated to pay Standard a large fee for each barrel of oil shipped for independent refiners (Tarbell, esp. Chs. 2 and 11).)

Standard's effect on independent refiners' transport costs was obviously one that punished these competitors for staying independent. While these punishments were not harsh enough to prevent all entry, the entry that did occur may well have provided Standard with refineries, market experiments, or collections of refinery equipment at a lower cost than if they had built the plants themselves for their booming new industry. In our terms, Standard's dictatorial output set had other firms initially producing substantial outputs (in order to establish new refining patterns) and, after a while, zero outputs (as Standard took over). Standard's policy of purchasing successful refineries at just slightly over book-values is certainly consistent with this interpretation.

The second argument of McGee is that while predatory pricing behavior may have existed on the retail level, it did not result in any significant retailing monopoly for Standard. The problem here is that Standard's use of predatory retail pricing was used not to acquire a redundant retailing monopoly but rather to aid their near-monopoly in refining by punishing retailers in certain locations for buying from competing refiners (Tarbell, Ch. 10).[9]

B. <u>During the Development of Our Basic Antitrust Law</u>

Under the above interpretation of our anti-cartel laws, the immediate effect of this constraining of reaction functions to

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[9] McGee's subsidiary claim that a series of nonpredatory buy-outs to form a monopoly is cheaper than predation is also incorrect. Commitments to predate against rivals refusing "reasonable" prices are of obvious value in reducing buy-out prices. Moreover, since, in the absence of such predatory commitments, all firms would demand at least their profit as the last-acquired firm and since it always pays to acquire an equally informed competitor, a McGee-type buy-out solution would not generally exist. For example, in our antitrust-constrained, linear model of Section II, it is easy to show that if the industry started with four or more firms, it would never be profitable for any firm to buy-out <u>all</u> of the others even though they could be obtained for only their opportunity value, that obtained by remaining an independent follower in a Stackelberg-on-output duopoly. And since it always pays to buy out a single firm to achieve a joint output restriction, there is no pure-strategy solution to this interaction. For pure strategy buy-out solutions without information differences, committed reaction strategies are generally required.

non-collusive, non-predatory forms would be to induce consolidations, or multi-firm mergers.[10] In fact, a record number of consolidations--by far the largest in our history[11]--immediately followed the landmark U.S. Court decisions validating Section 1, the anti-cartel portion, of the Sherman Act. (U.S. vs. Addyston Pipe and Steel Co., et al., 85 FED 271 (6th Circuit, 1898), affirmed 175 U.S. 211 (1899)).

In sharp contrast, the usual, Stiglerian view of cartels leaves us with no explanation of this merger wave. Since the numerous pre-1898 cartels were--according to this popular theory--largely ineffective, outlawing the cartels should have had no significant effect on the popularity of substitute forms of monopolistic organization. Moreover, even if the effectiveness of the pre-1898 cartels were, somehow, admitted, the standard, Stiglerian view would still imply an absence of monopolizing consolidations because it would always pay an individual, competitive firm more to stay on the outside rather than to join

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10 A consolidated group of firms differs from a group of unconstrained, independently operated, interactors only in that the former can freely make lump-sum transfers between the interactors and thereby achieve joint productive efficiency in the absence of transactions costs (Thompson-Faith, 1980). It is therefore likely to be at most a minor inconvenience for previously independent operators to merge in order to retain their monopolistic output restrictions.

11 Brozen points out that the 1898-1902 merger wave, representing about 53 percent or the book value of all manufacturing and mining corporations, was more than five times larger than any other merger wave in our history. And Nelson tells us that "75 percent of the 1895-1904 firm disappearances took place by consolidations of five or more firms."

an output-restricting, profit-sharing consolidation (Mackay). So even if the pre-1898 cartels were effective, the post-1898 consolidation wave should never have occurred under the conventional theory.

An implicit assumption of the conventional argument is that insiders cannot pre-commit to predatory reactions to non-joining members of the industry. While the large numbers involved in both the typical pre-1898 cartel and the typical post-1898 consolidation strongly suggest the presence of contractual pre-commitments to punish such "free-riding", our only direct evidence concerns the huge consolidation forming the U.S. Steel monopoly in early 1901. The detailed facts leading up to this consolidation (Hendrick) reveal the thoroughness of the predation involved: Andrew Carnegie, who had, by the late 1890's, amassed a fortune through his intelligence as a competitive raw steel producer and optimism as a investor, decided to retire from the rigors of business by selling his company. His offer of 320 million dollars in March 1900 to various fabricators and horizontal competitors through J. P. Morgan and various other stock promoters attracted little interest. Then Carnegie announced a plan to build his own, giant, steel fabricating plants. His extremely optimistic president, C.W. Schwab, went to work acquiring the property and drawings sufficient to build the plants and made the announcement of the reaction function to the initially incredulous Morgan, who represented several steel fabricators: Either the fabricators would join in buying-out Carnegie and help form a giant, integrated, steel consolidation or Carnegie would flood their markets. Morgan's response was simply to have Carnegie name his price; the biggest single deal in U.S. history was struck without a bit of haggling. Morgan's group wound up paying almost $600 million to the Carnegie Company just months after rejecting the offer of $320 million and Morgan would later admit to Carnegie that they would have readily paid a price that was $100 million higher. Schwab's fabricating plants were never built. What explains this approximate doubling in stock value in less than a year, when other, much smaller, acquired firms barely maintained their market values, is the formation of the above, predatory reaction function. Morgan, being the only middleman who could be trusted to evaluate the Carnegie reaction function for the fabricators (with whom he shared a substantial equity interest) reaped a net promotional profit of over 60 million dollars. The non-fabricating firms in the merger forming U.S. Steel were also threatened with a narrowly irrational reaction commitment. The other large integrated raw steel producer, Federal Steel, was facing the prospect of Carnegie's exercising his contract with Rockefeller assuring Carnegie of essentially all the iron ore he could use over the next fifty years at about half its market price. If Federal paid enough for part of Carnegie Steel, the latter would combine with the Rockefeller interest to prevent this predatory over-exploitation the iron ore reserve. Federal jumped at the chance and the merger forming U.S. Steel was achieved.

Also in line with our general model, the great consolidation wave was brought to an abrupt halt in 1904 by a Supreme Court decision (Northern Securities Co. v. U.S., 193 U.S. 197) establishing the applicability of section 2 of the Sherman Act to monopolizing mergers. This decision completed the basic form of existing U.S. antitrust law by adding an anti-monopoly component to the anti-cartel component established in 1898.

C. The Modern Era: A Test Of Our First Approximation

Our first-approximation model of the effect of existing antitrust law implies, as demonstrated in Section II B, a hierarchy of makers in which each of the makers presents the industry with a fixed long-run output for a given level of market demand and industry costs. Empirically, this means that the larger firms in an industry can be expected to commit themselves in sequence to announced shares of the expected, solution, market output and retain the fixed outputs necessary to generate these shares regardless of the peculiar economics of individual firms. That large firms in the U.S. have typically come to determine their outputs in this way rather than computing their own demand and supply curves has, in effect, been claimed by numerous modern business historians (e.g., Chandler).

Evidence for the corresponding size distribution of firms derived in Section II C was obtained from observations on relative firm sizes within selected U.S. industries. Our hypothesis, from equation (18), implies that

$$\log x_{m-j} = K_1 + b_1 \cdot j, \qquad j = 0, 1, \ldots, m-1, \tag{23}$$

where $b_1 = \log 2$ for each industry. The hypothesis relating firm outputs to rank dominant in the literature (Simon, and Simon and Bonini) has been:

$$\log x_{m-j+1} = K_2 + b_2 \log j+1, \quad b_2 < 1 \tag{24}$$

This hypothesis, which has little theoretical rationale, is clearly contrary to ours in that ours, as represented in equation (23), has firm size increasing more than in proportion to a firm's ordinal size in the industry, j, while (24) has firm size increasing less than in proportion to the firm's ordinal size. We obtained our initial data from Standard and Poor's "Compustat" tape for 1971, which has data on all of the relatively large U.S. companies within industries disaggregated to the four-digit industry level.[12] This data was used to generate least-squares

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[12] We only included an industry when it (1) included 4 or more companies (for statistical reasons), (2) had a firm producing over 50 million dollars of sales (to avoid the exclusion of large producers due to their being a subsidiary of a diversified firm), (3) sold its product in a national market (to avoid local monopoly effects and interactions with firms in foreign markets), (4) sold its product to economic agents which are not substantially larger than itself (to avoid including industries in which some of the outputs are produced by vertically integrated firms, would not be counted as part of the industry), (5) and marketed a relatively homogeneous commodity. This is a highly subjective selection of industries, but we know of no better way to provide a fair test of the hypothesis with so much of the data obviously irrelevant.

fits of the two hypotheses.[13] The regressions for equation (24) produced coefficients less than unity in only three industries. In each of these industries (cement, roof and wallboard, and savings and loans) it appeared that we had erred in considering the markets for their product a national rather than a local market. It is not surprising that equation (24) fits better than (23) for local industries as it is well known (cf. Simon) that city sizes follow a distribution such as (24).

For the remaining thirty-eight industries, the fit of equation (23) was better (higher R^2) than (24) in over 90 percent of the cases. The regression estimates of equation (23) are described in Table 1. The average estimated ratio of each firm's size over the next smallest firm's size (the average of the antilogs of the esti-mated coefficients in (23)) was 1.86, which was less than 1/4 of the standard deviation of the estimates from the theoretical value of 2.00. Nevertheless, a little more than half of the industries had b-coefficients significantly different than the theoretical value of .69 at the 5 percent level. Furthermore, an analysis of covariance reveals significant differences among these coefficients at the 5 percent level, leading us to

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[13] We had data on both current sales and assets as measures of size, assets being perhaps better than current sales as a measure of future sales. We ran regressions for both measures of size and chose the measure for each hypothesis that yielded Durbin-Watson statistics closest to 2. The rationale here is that we wanted to be as generous as we could to each hypothesis regarding which measure of size would conform the best to the curvature assumptions of the hypothesis.

Table 1: Fit of Equation (23): $\log x_{m-2-i} = K_1 + b_1 i$
$i=1,...,m-3$

Industry	b_1	s_{b_1}	e^{b_1}	D.W.	R^2	m-3
gold mining	.68	.1	1.98	1.57	.85	6
coal	1.13	.21	3.10	1.66	.90	5
housing construction	.39	.04	1.48	2.37	.93	7
packaged foods	.26	.03	1.30	1.13	.88	11
dairies	.64	.13	1.90	1.78	.83	7
canned foods	.33	.02	1.39	1.55	.96	12
animal foods	1.14	.16	3.13	1.70	.95	5
biscuits	1.22	.67	3.39	3.00	.80	3
confectionary	.69	.09	2.00	2.06	.91	6
brewers	.27	.01	1.31	1.81	.97	15
distillers	.51	.03	1.67	2.53	.98	7
soft drinks	.69	.09	2.00	2.06	.91	7
tobacco	.47	.10	1.60	1.73	.83	7
forest products	.49	.04	1.63	1.67	.95	11
mobile homes	.33	.04	1.39	2.11	.90	11
home furnishings	.27	.02	1.31	1.85	.94	15
paper	.35	.04	1.42	1.28	.84	14
books	.49	.05	1.63	1.75	.95	7
drugs-ethical	.22	.02	1.25	1.21	.93	17
drugs-proprietary	.57	.07	1.77	.70	.91	9
medical & hospital supply	.38	.03	1.46	2.08	.94	10
soap	.99	.18	2.69	1.57	.91	5
cosmetics	.30	.01	1.35	2.45	.99	16
paint	.60	.11	1.83	1.74	.86	7
tires & rubber goods	.31	.02	1.36	.89	.92	17
plastics	.33	.04	1.39	1.77	.91	8
shoes	.38	.02	1.46	1.81	.96	13
concrete gypsum and plaster	.37	.08	1.45	1.98	.83	7
aluminum	.60	.21	1.83	2.29	.80	4
motor vehicles	1.01	.22	2.75	2.14	.91	4
photographic	1.07	.07	2.92	2.38	.98	7
watches	.65	.08	1.82	1.93	.96	5
musical instruments, parts	.97	.23	2.64	2.27	.86	6
games	.41	.05	1.51	1.85	.94	6

Table 1--Continued

Industry	b_1	s_{b_1}	e^{b_1}	D.W.	R^2	m-3
radio-TV broadcasters	.34	.03	1.40	2.91	.95	11
wholesale foods	.52	.06	1.68	1.78	.93	9
retail lumber yards	.96	.23	2.61	2.37	.90	4
motion pictures	.55	.06	1.73	1.37	.91	9
relevant averages	.58		1.86			
relevant standard deviations	.28		.60			

reject the hypothesis that the true value of b_1 is the same for each industry. On the other hand we did not really expect this hypothesis to be true. Deviations from our linearity assumption produce deviations in the theoretical b_1 coefficients. But we did expect the deviations to be unsystematic. That is, we expected the average of these coefficients to be not significantly different than .69. Indeed, the actual average of the b_1 coefficients was .58, less than half of a standard deviation away from its expected value.

Encouraged, we duplicated the above pair of regressions on an FTC sample of 697 5-digit industries from a 1950 survey of the 1,000 largest firms in the country. Besides the advantage of a higher level of disaggregation, the sample was based on estimates by the firms of their plant capacities in the various industries and therefore did not have to be adjusted for differences in product mixes among the sampled firms (as in footnote 12). Moreover, as the sample contained large as well as small numbers of firms in each industry, a far superior contrast between industry-types was provided. The results of these numerous fits of equation (23) are summarized in Table 2.

Table 2. Average e^{b_1} Coefficients Using the FTC Sample

Number of Firms in Industry	Below Median Industry Size	Above Median Industry Size
$n < 20$	2.47	1.95
	($\bar{n}$=7)	($\bar{n}$=11)
$n \geq 20$	1.30	1.23
	($\bar{n}$=24)	($\bar{n}$=23)

The oligopolistic industries (those with $n < 20$) followed the same theoretically predicted pattern as in the above sample, while the competitive (those with $n \geq 20$) industries displayed e^{b_1} coefficients that were consistently way below theoretical oligopoly values.

Although the R^2's for the oligopolistic industries, averaging .91, were slightly better than the R^2's for the corresponding Pareto-distribution regressions described in equation (24), which averaged .89, we expected our form to show a greater degree of superiority. However, subsequent work on more recent FTC, plant-by-plant, data has revealed a large-firm size distribution that is very significantly more skewed (i.e., more in our direction) than Simon's Pareto-distribution (Kwoka).

Also somewhat encouraging is the recent confirmation of our semilogarithmic form provided by a 1980 survey of 200 large corporations producing in 1,218 markets (Buzzell). With an average reported share of their market's total output equal to 33 percent for the top firm, 19 percent for the second-largest firm, 12 percent for the third-largest firm, and 7 percent for the fourth-largest firm, our constancy of the ratio of successive firm sizes is clearly in evidence. While the median value of e^{b_1}, slightly less than 1.7, was significantly less than its theoretical level of 2.0 the discrepancy can be explained, we shall argue in the next subsection, through a generalization of our simple model encompassing short-run influences. Note that the 1.7 estimate is our only estimate based solely on current output data, capacity data being entirely absent from the sample producing this relatively low estimate.

D. Other Evidence

Further, less direct, econometric evidence for our general model is the similarity of oligopolistic size distributions across regions with greatly differing market sizes (Pryor, Scherer, pp. 70-74) and the near-normality of the profit rates characterizing most oligopolistic industries, with more concentrated industries doing only slightly better than the less concentrated ones (Scherer, p. 92).

A direct test determining whether deviations of b_1 about its theoretical value are due solely to non-linearities rather our legal constraints is possible by complementing the above test with a direct fit of the underlying reaction functions, as specified in equations (10), (13), (14), etc., (c.f., Thompson, Faith, and Rooney). It is important to note that, due to the recursive nature of our model, our reaction functions are extremely easy to fit statistically while the opposite is true of received theory, which allows every firm's output to depend upon every other firm's output, thereby presenting an immense simultaneity problem (Iwata).

IV. THE EFFECT OF ANTI-CARTEL LAWS--A SECOND APPROXIMATION

A. Capital Reaction Functions

The above, first approximation of our anti-cartel laws--being based on a constant-cost technology and a single choice variable--is suited only to applications concerning long-run, single-point choices. Thus, in our historical examples, we had Standard Oil and Carnegie Steel committed to predatory reactions to the one-shot, capital decisions of certain rivals.

Laws eliminating one-shot, predatory capital reactions, and thereby creating a capital size distribution approximating our simple 4:2:1 solution, need not directly outlaw predation. The private cost of predatory reactions to the capital choices of one's rivals is substantially lower when the predator can absorb rival capital stocks. Therefore, outlawing large-firm acquisitions of rival capital stocks many, as a by-product, eliminate most committed, predatory respones to rival capital choices. Thus, section 2 of the Sherman Act and section 7 of the Clayton Act, by effectively eliminating large-firm acquisitions of rival capital stocks, may well have simultaneously eliminated most cases of long-run, capital predation and left us with a simple, sequential, Stackelberg-type, capital interaction and hence our approximate 4:2:1 capital size distribution in most oligopolistic industries. The corresponding organizational adjustment would be the observed, rapid evolution away from the personality-dominated long-run decision structure of closed, family-run organizations and toward the rationality-dominated long-run decision structure imposed by the stockholder representatives of modern, widely-held corporations (Chandler). The ability of our model to predict the observed change to the typical, modern capital size distribution and the corresponding organizational adjustment should not be surprising to believers in the effectiveness of section 2 of the Sherman Act and section 7 of the Clayton Act. It should also not be surprising that these two sections of our antitrust laws, serving both anti-monopoly and long-run anti-cartel functions, have been so frequently employed.

The above argument suggests that the 4:2:1 capital size distribution provides a structural test for the absence of capital predation. A top firm that is much more than twice as large as the second-largest firm in its industy is suspect.[14] When the industry also reveals monopoly performance characteristics and the top firm exhibits notoriously aggressive, matching-type capital reactions despite the legal constraints on its ability to buy-out its victims, then the top firm should be prosecuted for monopolization under Section 2 of the Sherman Act.[15]

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[14] Size distribution evidence is not sufficient. The first strategy maker in a high-profit, entry-threatened industry would, as shown in Section IIE, expand to where the potential entrants faced a non-positive profit rate. This would create a size distribution in which the largest firm was somewhat larger than twice the size of the second-largest firm, although the remaining firms would retain their 4:2:1 capital size distribution.

[15] IBM comes to mind. (See, e.g., Telex Corp. v. IBM, 367 F Supp. 258 (N.D. Okla., 1973) and Memorex Corp. v. IBM, 458 F

(footnote continued)

If we could assume technologically constant capital-output ratios, our analysis would be finished. But we should recognize in this more realistic, second approximation, that output may vary substantially in the short-run and further that firms cannot typically observe the short-run output or variable input levels required to apply our basic model.

B. Short Run Interaction

1. Competitive-type Interactions

Several kinds of short-run interaction exist, each defined by its unique technology. First, when the sold output is durable

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(footnote continues)

Supp. 423 (N.D. Cal. 1978).) While IBM eventually escaped prosecution, the financial cost to IBM of "winning" their cases, as most economists know, were extremely high. IBM's legal victories were largely the result of the above-discussed, unfortunate trend in legal thought--based no doubt on an overly literal reading of elementary economics texts--in which predation has come to mean "selling below ordinary production costs." It is likely that these decisions would have gone the other way if the "costs" referred to were full opportunity costs rather than ordinary production costs, in which case "selling below costs" merely means selling an irrationally large output (using the term "irrational" in the narrow sense.) As pointed out in IIF above, our simple, powerful, anti-cartel law of Section IIB, which prohibits all positive output responses to rival output increases, equivalently prohibits all unambiguously "irrational" output responses to rival output increases. We believe that previous generations implicitly had our definition in mind. Using the more recent, overly narrow, definition of predation, where a predatory firm is one that rationally attempts to lower prices sufficiently to induce competitors to exit the industry so it can increase prices in the more distant future, modern authors have ignored the possibly large profit to establishing a reaction function commitment to perform a narrowly irrational action in case a subsequent maker deviates from the primary maker's announced plan.

and homogeneous with the stocks of used outputs--primary comodities such as primary metals are an example--it is natural to assume competitive price-taking behavior on the part of current producers, whose short-run decisions have no significant influence on these prices.

Similar, essentially competitive, price outcomes generally arise in the absence of strategic communication even when current outputs are not close substitutes for existing stocks of marketed durable goods. In particular, when rival prices as well as out-outputs are not observable, then a Bertrand assumption applies so that each firm will shade its rivals's expected price to marginal, informed buyers down to where these prices are once again equal to marginal costs. And when rival prices are observable, the ability of uncommitted, later price-setters to shade the effective price of the first price setter--the conventional Stackelberg price leader--and thereby sell all they desire at these readily observable prices means that the Stackelberg price leader must absorb the entire Walrasian excess supply at its chosen price. As the resulting, "dominant firm" solution has all firms but one behaving as pure, price-taking competitors while the largest firm, the price leader because it receives the greatest gain from being the leader (see Appendix), faces a demand elasticity well over twice the market demand elasticity, the industry output is again close to perfectly competitive. Because the largest firm is cut-back relative to its smaller, price-taking rivals, the equilibrium size distribution of outputs becomes less skewed than the equilibrium size distribution of capital stocks, a theoretical result which may well explain the corresponding empirical differences noted above in Section IIIC.

2. Monopoly-type Interaction

A highly inefficient, monopoly-type, short-run interaction is also possible. This occurs when firms can precommit themselves to short-run, reaction functions of the type characterizing our basic model of Section I, although the reaction functions of our basic model should then be reinterpreted to admit price-and-quality, rather than simple output, reaction functions because nominal prices and qualities are typically more easily observable than rival outputs in the short-run.

These precommitted, price-and-quality, reaction functions imply contract-like, matching reaction commitments of the form: "I'll raise my price or lower my quality by $1 if you raise your price or lower your quality by $1." Such functions obviously yield monopoly-type solutions and are the appropriate target of our anti-cartel laws. But this effective price cooperation is dramatically different, from a welfare standpoint, than nominal price cooperation, where quality is not part of the reaction function.

(a) Simple, Nominal, Price Reaction Functions. Simple "price-fixing" or "tacit price collusion", where the reaction functions concern only nominal prices and not qualities, merely set a nominal price for the industry. Quality competition then proceeds to create a noncooperative, competitive-type equilibrium. The important, largely unrecognized, value of such nominal price cooperation is that it enhances the ability of sellers to make firm price commitments. Without such price commitments, aggressively shopping buyers could easily create a "ruinously competitive" buying environment, wherein sellers inefficiently provide no buyer-specific, pre-contract services and buyers over-devote resources to shopping and negotiating in order to buy the good at its general, incremental, production cost. (Thompson, 1983a.) In contrast, cooperatively enforced, nominal prices would enable sellers to collect for their buyer-specific, pre-contract, services and permit the achievement of an efficient competitive equilibrium with personalized services (Makowski, V. Thompson).

For example, suppose that the general cost of producing a unit of output of optimal quality is $1.00 and the cost of producing the corresponding, optimal level of transaction-specific, pre-contract services,--i.e., the optimal specific selling cost--is 25¢. A fixed nominal price of $1.25 per unit would induce Bertrand-competing sellers to provide the optimal qualities of both the general and specific outputs. (Any simple quality reduction would obviously leave the seller with no business while any equal-cost substitution of one form of quality for the other would make buyers worse off and thereby also leave the seller with no customers.) And this zero-profit equilibrium is clearly superior to the laissez-faire competitive equilibrium, wherein the one-dollar price covers only general unit cost of production, buyer-specific investments by sellers are non-existent, and buyers are induced to aggressively over-shop in order to obtain the one-dollar price. The optimal, $1.25, nominal price level may easily be the level achieved by price (but not quality) colluding firms. Raising the collusive price to a monopoly level without augmenting this with corresponding quality reaction functions would merely raise the Bertrand-competitive qualities to new, higher, zero-profit levels. (To see why the collusive nominal price solution approximates a social optimum, note that the choice is, in effect, of a quality that maximizes the competitive rents of the owners of existing capital. Assuming demand and cost functions that are additive in quality and quantity, quality is rationally expanded whenever the demand-price increase exceeds the marginal cost increase, i.e., whenever the social value of the quality increase exceeds its social cost.)

When the firms in an industry: (1) Incur substantial specific selling costs, (2) transact with large, infrequent buyers so that price commitments are very difficult to establish, and (3) produce outputs that are homogeneous in production, then we would expect the above, efficient form of price-fixing to emerge. In fact, the two industries where price-fixing has been most frequently found, heavy electrical equipment and wholesale paper products, are both excellent examples of such industries. While it is theoretically possible that there has also been complete quality cooperation in these industries, the large absolute size and large specific service component of the transactions involved should serve to preclude rivals from forming committed reaction functions to the buyer-specific qualities. (Cooperation restricting the non-specific quality provided by the sellers at a given nominal price on these goods would merely increase the quality of the specific benefits delivered to the buyer.) So it is a priori unlikely that there has been any monopolizing cartelization in these heavily prosecuted industries. In fact, a careful study of Sultan's thorough history of our heavy electrical equipment industry fails to turn up a single example of monopolistic structure, conduct, or performance by this industry (we're sparing the reader the details of our critical study of this lengthy work; we'll send a summary on request). Reinforcing this conclusion, the classic study of price-fixing agreements of Hay and Kelly tells us that most of the usual, reasonable predictors of cartelizing collusion are unsuccessful in forecasting the presence of detected, prosecuted price-fixing. Rather, consistent with our theory, the main determinants of observed, prosecuted, price-fixing agreements (besides small numbers) are, (1) the homogeneity of the basic product, and (2) the infrequency and lumpiness of the transactions. Furthermore, the study of Phillips shows us that price-fixing has only a minor effect on long-run, accounting profit rates.

We conclude from all this that anti-cartel law is probably working poorly with respect to price-fixing. Price-fixing should not be a per se violation of the antitrust laws. Our Supreme Court probably erred in U.S. v. Socony-Vacuum Oil Co. 310 U.S. 150, 224-26, n. 59 (1940). A rule of reason should probably be applied to prosecute only those price-fixers also involved in a restraint of trade. This would halt the wholesale outlawing of a potentially very useful economic institution.

Some "price fixing" cases have included charges of market-splitting, where reaction functions cover readily observable, locational outputs and thereby have produced genuinely monopolistic solutions of the type described in Section I and the Appendix. Many of the early anti-cartel cases, including the landmark, Addyston Pipe case, were of this variety. The relative ease of governmental detection of most market-splitting solutions has apparently turned them into a rarity and provided a measure of success for existing anti-cartel law to weigh against the apparent failure of the price-fixing portion of the law. (The appearance of some market-splitting in recent times (Koller) is probably what explains the slight positivity of Phillips' measured effect of "price fixing" activities on long-run accounting profits.)

(b) Price and Quality Reaction Functions. The only case involving genuinely monopolizing, price-and-quality reaction functions that we have found--the landmark, American Tobacco Co. V. United States, 328 U.S. 781 (1946)--occurs in the area of informal, or "tacit", collusion. The amorphous state of the theory describing this "consciously parallel" form of collusion is evidence for the lack of a coherent, pre-existing theory of short run, non-competitive interdependence. It also gives us an opportunity to illustrate the workings of our own, hopefully coherent, theory in a short-run setting.

There were two key economic observations leading to the 1946 tobacco decision. First, the big-three cigarette manufacturers substantially raised their effective prices in 1931, after both demand and cost had fallen. This observation suggests either an increase in the cigarette manufacturers' degree of monopoly in 1931 or a pricing error that the leader of a cartel could plausibly make but independent competitors could not. In either case, monopoly performance was revealed. Second, a fixed, sequential pattern of equal nominal price changes evolved by the early 1920's following the formal break-up of the original Tobacco Trust in 1911. Reynolds Tobacco would introduce a change in the price of Camels; then American Tobacco would follow with an equal price change for Lucky Strikes; and finally the smaller Liggett and Myers Co. would go along by changing the price of Chesterfield by an equal amount. The usual interpretation of this sequence--the one used by the Court to "explain" the price increase of 1931 and convict the Tobacco companies--is that the price leader simply leads the narrowly rational remainder of the group to a monopoly-type soluton.

But such a price leader is only a dominant firm and, as we have already noted, will not lead narrowly rational followers to a monopoly solution as its rivals will slightly shade its effective price, say with quality or advertising increases, so as to force the leader to absorb the entire Walrasian excess supply himself as he moves to supracompetitive prices.[16] This difficulty with the usual "conscious paralellism" argument has apparently never been cleared up. Applying our model, one of the firms would have to have a committed, narrowly irrational, reaction function to the others in order to generate a monopoly-type solution. Indeed, Nicholls' detailed history of pricing in the Tobacco industry reveals (p. 49) that American had evolved a dominating leader, George Washington Hill, with a price-and-quality reaction function he personally described as one in which he set "a little higher price on Luckies than on Camels.... because of....national advertising...we needed a little more money to spend per thousand....than they required." Thus, if Reynolds raised the price of Camels, Hill would raise the price of Luckies by the same amount <u>and</u> simultaneously maintain the same advertising (quality) differential. Liggett would go along too, presumably because its advertising program had long been a

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16 Regarding the goods as non-homogeneous, differentiated, substitutes, the point is that subsequent, narrowly rational, price selectors would change their prices less than the first price selector and thereby pick up some extra business at the expense of the initiating, Stackelberg-leader. The resulting, 3-firm Stackelberg solution generally yields prices that are much lower than monopoly prices.

low-key appeal to the "discriminating" segment of the market, claiming that it had a superior cigarette but was civilized enough not to charge a higher price for it; price discounts and premiums, together with large variations in advertising, were therefore inconsistent with its marketing strategy. The resulting interaction yielded a pure monopoly solution. When Reynolds raised the price of Camels, its rivals would match the increase, without any serious attempt to shade the effective price increase to expand its share of the market. And the peculiar price increase of 1931 can be directly understood by applying this theory. It was based on an error by Reynolds, which raised the price of Camels in mid-1931 thinking that its new humidor (waterproof cellophane) pack would allow it to command a relative price premium. In fact, the market was not impressed with the pack and the relative demand for Camels did not substantially increase. Hill, seeing only a higher price for Camels and no relative demand shift, followed his firmly committed reaction function and raised the price of Luckies; and Chesterfield prices had to follow suit to protect its advertised image (Nicholls, pp. 85-86.) That the group had overdone it was soon apparent as a flood of tiny competitors expanded beyond the fringe in 1932. (As our model suggests, this fringe expansion led American Tobacco, the only firm with a narrowly irrational reaction function, to reduce its prices in early 1933 to where it under-cut both the fringe and its usual "competitors", expanding its own output and selling Luckies at a loss (in the conventional sense) until 1934, when most of the fringe firms had fallen back to their traditional, insignificant places in the industry (Nicholls, pp. 114-122)).

Despite a widespread belief that the Tobacco decision, which did not offer an explicit remedy, had little effect on the industry in that price leadership remained, the industry profit rate after the decision quickly fell from its historic level of about twice the average manufacturing profit rate to right around that average and stayed there despite the rapidly growing demand during the first half of the Post World War II period (Tennant). This observation can also be explained within our model. The model implies that the industry would become an ordinary, competitive-type, Stackelberg industry if American, the only firm with the narrowly irrational reaction function, became the price leader. This would mean that only narrowly rational responses would follow a change by the leader so that a noncooperative Stackelberg solution would emerge. This is, in fact, what happened: American, showing an understanding of the source of the monopoly far superior to that of its prosecutors, became the price leader immediately after the 1946 decision (Nicholls, p. 164, Tennant). As a result, cigarette price increases were small and greater fringe growth was permitted (especially by Phillip Morris).[17]

C. Brief Summary

A brief summary of our complex, second approximation is that U.S. antitrust laws simulate our first approximation despite an almost complete lack of understanding by our lawyers, judges and economists of what comprises monopolistic conduct. This is done by: (1) Harshly applying our anti-merger laws to substantially reduce the return to capital predation and (2) prosecuting industries exhibiting monopoly performance characteristics in responding to short-run shocks even though the legal argument purporting to outlaw monopolistic conduct typically represents such a gross violation of rational economic thought that it also outlaws, at substantial legal cost, some highly efficient, non-monopolistic, arrangements between horizontal competitors.

[17] A final structural anomaly was that American and Reynolds had about the same average market share during the "golden," inter-war years. Why didn't American choose a reaction function giving it a dominant, robber-baron position in the industry? The answer, we believe, is that the companies, being formed at the same time, initially engaged in a direct competition to be the primary maker. American, therefore, could reduce the cost of becoming maker by offering a kinder reaction function (i.e., organizational form) to its rivals. As the model of the Appendix shows, with only two serious competitors to be maker, the resulting solution reaction function yields a simple, joint-profit-maximizing set of outputs. The observed, matching behavior is sufficient to achieve such a solution as long as the relevant, short-run, cost curves are identical as well as increasing. We have, however, been unable to find sufficiently detailed descriptions of the 1911 dissolution negotiations to directly test this hypothesis.

APPENDIX

The Competitive Determination of Priorities in Strategy Selection

For our model in which strategy selection priorities are determined in a competitive fashion, a firm, in order to exhibit a reaction function, must establish a commitment through its "manager", whose services, which have no alternative value, are obtained via competitive bidding by the firms. We shall give one individual, called the "top manager", the ability to enforce a firm's commitment prior to all other firms' managers. For the top manager, each firm submits m-1 bids, each bid representing the amount the firm is willing to pay to be the first strategy selector in place of a specified, alternative firm. A winning bidder is a firm whose bid against his least preferred, alternative, first strategy selector is no less than the maximum of the bids against him. The reason a winning bidder must bid as if the worst possible alternative is the actual alternative is that the manager is free to choose the bidder's alternative and will rationally choose an alternative which will maximize the bid of the winning bidder. The winning bidder, however, does not generally pay his bid to the manager; he matches the second highest bid. Subsequent positions in the m-1 firm hierarchy of reaction functions are determined in a similar fashion. Our auction is unusual in that the bidders have different payoffs, and therefore different bids, depending on who would otherwise win the auction and on what he would do as the winning bidder.

Since the reaction function chosen by the manager depends on the incentive systems he is given by his firm, if a firm presents an incentive system which leads to a relatively generous reaction to the firm which is the closest competitor for the top manager, it will face a lower competing bid for the manager and thereby obtain the manager for a lower salary.

We have been careful not to give to our managers too much influence on the bidding process. It would be unrealistic, for example to allow a manager to encourage bidders to exhibit reaction functions that raise the winning bid. It would be similarly unrealistic to allow a manager to require payments from losing bidders for not selecting an even worse maker from their points of view.

Overall joint profit maximization is not a general solution only because side payments are disallowed. That overall joint profit maximization (and Pareto optimality under perfect price discrimination) results when there are side payments and an absence of transaction costs in all possible transactions is shown in Thompson-Faith, 1980.

Part I of the following discussion contains, for the unconstrained monopoly case, a specification of the cost of being a prior strategy selector, a derivation of the identity of a prior strategy selector, a characterization of general equilibrium solutions, and a proof of the existence of equilibrium solutions under some additional restrictions. Part II contains a similar analysis when the producers are constrained by our anti-trust laws. In both cases we show that the quantity solutions are close to the solutions in the text when the number of firms is not very small.

I. EQUILIBRIUM WITH UNCONSTRAINED MONOPOLY

A. The Existence of Punishment Outputs

We shall assume that for each firm, there exists a "punishment set of outputs." More formally,

(a.1) For each i, there exists an x_i, say $x_i^{'}$, such that $\pi_j(x_j;x_1 \ldots x_{j-1},x_{j+1},\ldots x_i^{'},\ldots x_m) < 0$ for all x_{-i} with $x_j > 0$ and all $j \neq i$.

If firm i is the first strategy selector, it can make a commitment which will induce each firm to produce its specified output. Faced with firm i's commitment, each of the remaining m-1 firms will rationally choose to produce their respective profit-maximizing outputs, the outputs specified by firm i.

More formally, let x_i^i and x_{-i}^i be solution values to the problem,

$$\max_x \; [\pi_i(x) - C_i(x)] \text{ subject to } \pi_f \geq 0 \text{ for all } f \neq i, \tag{25}$$

where $x_f = 0$ if $\pi_f(x_f) < 0$ for all $x_f > 0$, and where C_i is the cost to i of becoming the first strategy selector, or the "strategy maker." Thus, x_f^i is the output of the f^{th} firm which maximizes the net maker profit of firm i subject to the non-negativity of profits of each of the other firms, who are "strategy takers." The rational reaction function for firm i is then:

$$x_i(x_{-i}) \Big\} \begin{array}{l} = x_i^i \text{ when } x_{-i} = x_{-i}^i \\ = x_i^{'} \text{ otherwise.} \end{array} \tag{26}$$

The commitment made by i guarantees that i will produce x_i' when firm f deviates from producing x_f^i even if it implies lower profits to i than some alternative values of x_i given $x_f \neq x_f^i$. Such apparently irrational behavior by firm i is rational by virtue of our assumption of profit-maximizing behavior of all firms, which implies that firm f will produce x_f^i in equilibrium rather than an alternative output.

A strategy-taker, any firm $j \neq i$, faces the problem:

max $\pi_j(x_j;x_{-j})$ subject to

$$x_{-j} \begin{cases} = x_{-j}^i \text{ if } x_j = x_j^i, \text{ and } x_k = x_k^i, \text{ all } k \neq j,i \\ = (x_1,\ldots, x_i',\ldots,x_m) \text{ otherwise.} \end{cases} \qquad (27)$$

This leads the j^{th} firm, knowing the rational responses of the other takers, to choose $x_j = x_j^i$. We have assumed this holds even if $\pi_j(x_j^i) = 0$ for $x_j^i > 0$. That is, the taker will choose to produce the maker's optimal output choice even though his profits there are zero and he has the equally profitable possibility of quitting business.

Given our assumption on the existence of a punishment output, the problem of the existence of an unconstrained monopoly equilibrium when there is an arbitrary determination of the strategy maker (and thus when $C_i(x) \equiv 0$) thus reduces to a problem of the existence of an x which maximizes firm i's profit. This existence follows immediately from the minor, additional assumptions that X_f is a non-empty, compact set for each f and π_i (x) is a continuous function. The existence of an uncontrolled monopoly equilibrium under our competitive bidding process will be established in subsection E below, after we have specified the nature of the $C_i(x)$ function under competitive bidding and examined the solution characteristics of the two models in both small and large numbers cases.

Note that disregarding the cost of becoming a maker, no firm is ever worse-off by being the strategy-maker as opposed to being a strategy-taker. This is because an individual firm can always do as well by choosing its own output as having it chosen by another. Hence, each firm will have non-negative bids for the top manager's services regardless of whom he is bidding against.

B. The Two-Firm Case

Consider two firms, i and j. The amount firm j is willing to offer to the top manager equals the difference between j's profit as a maker and j's profit as a taker. Since j's profit as a taker depends on i's choice of outputs as a maker, the cost to i of being the maker, which is the cost of just beating j's bid, is a function of the x that i would choose as maker. Hence, we can write:

$$C_i(x) = \pi_j(x^j) - \pi_j(x), \tag{28}$$

and, using (5), describe firm i's <u>maximum maker profit</u> as:

$$\pi_i^M = \max_x \; [\pi_i(x) - (\pi_j(x^j) - \pi_j(x))] \text{ subject to } \pi_j(x) \geq 0, \tag{29}$$

where $\pi_j(x^j)$ is the value of π_j implied by the solution to:

$$\max_{x} \; [\pi_j(x) - (\pi_i(x^i) - \pi_i(x))] \text{ subject to } \pi_i(x) \geq 0, \qquad (30)$$

where $\pi_i(x^i)$ is the solution value of π_i implied by (9).

Solutions to (29) and (30), if they exist, yield explicit values of $\pi_i(x^i)$, $\pi_j(x^j)$, $\pi_i(x^j)$, and $\pi_j(x^i)$ from which we obtain the value of each firm's bid. These values are interpreted as i's and j's operating profit as a maker, and i's and j's operating profit as a taker, respectively.

Noting that x^i is independent of $\pi_j(x^j)$, we see from (29) that firm i is maximizing its joint-profits with firm j. Similarly, from (30), firm j is maximizing its joint-profits with firm i. If we assume that the joint-profit maximizing output is unique, then the same output vector will be chosen regardless of which firm is the strategy-maker. Hence, each firm's bid for the rights to be maker would equal zero since its profit as a maker is the same as its profit as a taker. In this case, the final determination of the strategy maker is arbitrary.

If the joint-profit maximizing output is non-unique, the two firms' bids will still be equal, but they may then be positive. For example, at x^i, let $\pi_i(x^i) = 50$, $\pi_j(x^i) = 40$, and at x^j, let $\pi_j(x^j) = 60$, $\pi_i(x^j) = 30$. Notice that both i's and j's bid will equal 20. Since the bids are equal, the selection of strategy maker is still arbitrary.

The joint-profit-maximizing solution when there are two firms contrasts sharply with the solution when there is no competition to determine a maker. In the latter case, the arbitrarily selected maker simply determines a set of outputs which maximizes his own profit and applies his punishment if the takers do not oblige him. This is generally far from a joint-profit-maximum.

Given any number of firms which produce a single output, an arbitrarily selected maker produces a simple monopoly output, flooding the market with an output which would enforce negative profits on all other active firms in the industry if any of them produced a positive output. This predatory, "robber-baron" strategy holds regardless of the nature of production costs and demand, and regardless of the number of takers, giving us a single-active-monopoly solution among any group of interacting firms selling a homogeneous product. We shall soon see, however, that this robber-baron solution is also approached in the case of competitive bidding for hierarchial position as the number of firms increases beyond two.

C. Competitive Bids to be Maker in the m-Firm Case

With m firms, $m \geq 3$, although there are m-1 competing bids with which a prospective strategy maker must contend, any prospective maker need only be concerned with the highest of his rivals' bids. This highest rival bid is the explicit cost to i of becoming the maker. Thus, (28) becomes:

$$C_i(x) = \max_{f \neq i} (\pi_f(x^f) - \pi_f(x)). \tag{31}$$

The m-1 opposing bidders are each rationally assuming that firm i will be the strategy maker if they are not. The resulting bid of each firm then measures how much a firm is willing to pay to be maker instead of being a taker of i's reaction function. We can now describe firm i's maximum maker profit, π_i^M, as:

$$\pi_i^M = \max_x \; [\pi_i(x) - \max_{f \neq i} \; (\pi_f(x^f) - \pi_f(x))] \text{ subject to } \pi_f \geq 0, \qquad (32)$$

where $\pi_f(x^f)$ is the operating profit to firm f when f is solving for its maximum maker profit.

Firm i's alternative maker is that firm which will be the maker if i is not. Firm i's bid when j is his alternative maker, the difference between i's profit as maker and i's profit as taker of j, is

$$\pi_i(x^i) - \pi_i(x^j) = B_{ij}(x^j), \qquad (33)$$

where j is i's alternative maker.

By computing maximum maker operating profit for all m firms, if these profits exist, and taker profit in a similar fashion, we can compute each firm's bids from the explicit values of maker and taker profits.

D. Characterizing an m-Firm Equilibrium

Distinguishing features of the m-firm case ($m \geq 3$) under competitive bidding to be maker are that at a solution there is more than one highest-bidding taker, that the solution is not a joint-profit maximum and that the solution approaches the arbitrary maker solution as m increases.

At any choice of output allocation set of the maker, i, there is either a distinct firm determining i's managerial cost--i.e., an unique f, solving (31), or there is a tie bid between some of the takers. Suppose there is an unique maximum in (31). Since the maker is responsive only to changes in the bid of the single highest-bidding taker, say, j, the maker and this taker will adopt a joint-profit maximizing relationship as in the two-firm case. Therefore, if the output choice of i is, in fact, a solution, it also corresponds to a joint-profit maximum between i and j. If the joint-profit maximizing output is unique, j's bid against i is zero. Since the remaining bids are non-negative, such an output choice is unattainable because the alternative maker's zero bid is then not higher than the other takers.

If the joint-profit maximizing output of i and j is non-unique, the same result obtains. Suppose that joint-profits between i and j are maximum and therefore equal at both x^i and x^j. Then the difference between i's maker profit at x^i and x^j,

$$\pi_i(x^i) - [\pi_j(x^j) - \pi_j(x^i)] - \pi_i(x^j) + [\pi_j(x^j) - \pi_j(x^j)]$$
$$= \pi_i(x^i) + \pi_j(x^i) - [\pi_j(x^j) + \pi_i(x^j)] = 0.$$

Thus, i is indifferent between x^i and x^j which implies that i's bid is zero. Again, an inconsistency results as the other bids are non-negative. Thus, the equilibrium solution is inconsistent with the existence of an unique maximum in the alternative bids.

Therefore the optimal output choice of the strategy-maker occurs where there exists a tie in the maximum bids of some of the takers.

A solution occurring at this point does not correspond to a joint-profit maximum. Although the maker is responsive to a change in the bid of any one of his several maximum-bidding-takers, he is not concerned with the sum of their bids, which i requires for joint-profit maximization. Thus, the solution is not a joint-profit maximum.[1]

The greater the number of firms, the "closer" the solution is to the arbitrary maker solution in the following sense: With a greater number of firms, there is a greater number of takers whose bids are equal to or a greater number whose bids are less than the maximum bid of the takers. If there is an expansion of those whose bids are less than the maximum, there are more firms whose non-negative profit variations are of no concern to the maker and thus more firms whose output is determined just as it is in the case of an arbitrarily selected maker. If there is an expansion of takers whose bids are maximal, then the maker internalizes less of the variation in the total profits to these

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[1] An m-firm joint-profit maximum would mean that the maker's marginal profit (assuming differentiability) equals the sum of the other firms' marginal profits. In our case, the marginal profit of the maker is equal to each firm's marginal profit. Let there be an m-firm tie where m = 3. If marginal profit to i, the maker, equals $1 (thus, marginal profit equals $1 a piece to the takers), then marginal joint-profit equals minus $1, rather than 0.

takers, moving toward the extreme in which he is arbitrarily selected and therefore internalizes none of this variation.

In both the two and m-firm cases, since the solution maker need only match the highest-bidding rival firm(s), the amount going to the manager will equal the value of the second highest bid over all firms. But, whereas the manager's fee is always zero in the two-firm case when the joint profit maximizing output set is unique, it may be positive in the m-firm case.

E. A Theorem on the Existence of Equilibrium

We will be working in Euclidean space, R^y; the dimensionality y of the space equals the number of commodities (n) times the number of firms (m), or nm.

Let X, a subset of R^y, equal the feasible output set. An element, x, of this feasible output set is a y-dimensional vector of outputs of each commodity by each firm.

From the above discussion, for each firm there is a profit function, $\pi_f(x)$, f=1,...,m; defined on X. Similarly defined on X is:

(d.1) the i^{th} firm's maker profit,

$$\pi_i(x) - \max_f \,[\pi_f - \pi_f(x)] \quad f=1,\ldots,m;\ f \neq i\ , \tag{34}$$

where π_f is f's operating profit as a maker, a given number to i; and

(d.2) the i^{th} firm's bid function, given that f, who dictates output x, is the alternative maker,

$$B_{if}(x) = [\pi_i(x^i) - \pi_i(x)], \quad f=1,\ldots,m;\ f \neq 1 . \qquad (35)$$

An <u>equilibrium</u> is (a) a set of output allocation vectors, $\underline{x}^1$, $x^2, \ldots,$ x_m^m, $x^f \varepsilon X^f$, such that each $\underline{x}^i$ maximizes the maker profit of firm i given $\pi_f = \pi_f(\underline{x}^f)$, all $f \neq i$, and (b) a winning bidder, a firm, i, such that $\max_f B_{if}(\underline{x}^f) \geq \max_k B_{ki}(\underline{x}^i)$.

We now make the following assumptions:

(a.2) X is a non-empty, compact, convex set.

(a.3) $\pi_f(x)$ is a continuous, real-valued function, $f=1,\ldots,m$.

(a.4) For any f and any given $(\pi_1,\ldots,\pi_{f-1},\pi_{f+1},\ldots,\pi_m)$, there is at most one value of $\underline{x}^f$. (This is slightly weaker than the strict convexity of the set, $X_j = \{x_i: \pi(x_i) \leq \pi\}$ for all π.)

<u>Theorem</u>: Given assumptions (a.1) - (a.4), there exists an equilibrium.

The proof will consist of two parts. Part 1 will prove that there exists a set of outputs $\underline{x}^1,\underline{x}^2,\ldots,\underline{x}^m$. That is, for any i, there exists maximum maker profits, π_i^M, with consistent values of π_f, for all $f \neq i$. Part 2 will prove that there is always at least one firm which is a winning bidder, i.e., one firm whose maximum bid against his alternative makers is no less than the maximum of the bids against him.

Proof:

Part 1.

First we show that for given values of maker profit of other firms, firm i has a maximum maker profit. To do this we will employ the wellknown theorem in analysis that a continuous, real-valued function defined over a closed and bounded set attains a maximum at some point in the set. Let

$$g_i(x) = \max_f \; [\pi_f \qquad - \pi_f(x)] \quad f=1,\ldots,m;\; f \neq i, \quad i=1,\ldots,m. \tag{36}$$

That is, $g_i(x)$ is the function describing the maximum bids against i for each point in X selected by i. Since $\pi_i(x)$ is continuous by (a.2) and the sum of two continuous functions is continuous, firm i's maker profit in (34) is continuous if $g_i(x)$ is continuous.

Lemma: The function $g_i(x)$ is continuous.

At any point in X, and any i, there is either (a) an unique maximum bid in (36), or (b) there is an equality between the highest two, or more bids in (36).

(a) If there is an unique maximum in (36) at some point in X, then since each bid function, $B_{fi}(x)$, is continuous (the difference of two continous functions is continuous), (36) is continuous at such points in X.

(b) Let x_s be a point in X where $B_{ji}(x_s) = B_{ki}(x_s) = g_i(x_s)$, $j \neq k$. Suppose, for any $\delta > 0$, there is an $\varepsilon < \delta$, $\varepsilon > 0$, such that

$$g_i(x_s-\varepsilon) = B_{ji}(x_s-\varepsilon) > B_{ki}(x_s-\varepsilon) \text{ , and} \qquad (37)$$

$$g_i(x_s+\varepsilon) = B_{ji}(x_s+\varepsilon) \leq B_{ki}(x_s+\varepsilon) \text{ , } j,\ k=1,\ldots,m;$$

$$j\neq 1,\ j,k\neq i.$$

It is obvious that since each bid function is continuous and equal at x_s, $g_i(x_s)$ is continuous at x_s.

In the case where the second relation in (37) does not hold, the bid of j is a maximal bid over the entire δ-neighborhood so the continuity of $g(x_s)$ follows from the continuity of $B_{ji}(x_s)$.

In the only remaining case, where only the first relation in (37) does not hold, $B_{ji}(x) \equiv B_{ki}(x)$ about an δ-neighborhood of x_s, then either bid is maximal in that neighborhood. Since the bid functions are continuous at all points in X, then $g_i(x)$, $i=1,\ldots,m$, is continuous over all of X.

It follows from the lemma and the well-known theorem in analysis stated above that π_i^M and $\underline{x}^i$ exist for any set of values $\{\pi_f\}$ and thus for any set of vectors $\{x^f\}$, $f \neq i$.

Now one firm's optimal maker output vector depends upon the optimal maker output vectors of other firms. This leads to the question of whether the optimal output vectors of the various firms are mutually consistent. Proving this establishes the existence of a set of output vectors, $(\underline{x}^1,\ldots,\underline{x}^m)$, such that, for each f, $\underline{x}^f$ yields maximum maker profit to firm f for the $\underline{x}^i$ of all $i \neq f$, $i,f = 1,\ldots,m$. Consider m feasible sets of nm outputs, each representing an output allocation vector arbitrarily selected by each f, or $(x^{10},\ldots,x^{m0})$. Given the values of

$(x^{20},\ldots,x^{m0})$, the value of x maximizing firm 1's maker profit, x^{11}, is calculated. Using x^{11} and $x^{30},\ldots,x^{m0}$, the value of x maximizing firm 2's maximum maker profit, x^{21}, is calculated. Continuing in this manner, the output set, $(x^{11},\ldots,x^{m1})$ is attained.

The resulting transformation,

$$(x^{10},\ldots,x^{m0}) \to (x^{11},x^{21},\ldots,x^{m1}), \tag{38}$$

then is a transformation from a set of output sets, X^m, into itself. To show that there exists a consistent set of maximum maker profit over all m firms, it is sufficient to show that there exists a set of outputs, $(\underline{x}^1,\ldots,\underline{x}^m)$, which remains unchanged over the transformation (18). By the Kakutani fixed point theorem, the set, $(\underline{x}^1,\ldots,\underline{x}^m)$ exists of X^m is a compact, convex set, and the complete transformation (38) is continuous.

By assumption (a.1)--and the fact that the Cartesian product of closed, bounded, and convex sets is itself closed, bounded, and convex--we know that X^m is closed, bounded, and convex. Since each transformation in (38) is a calculation of some firm's profits, it is sufficient to show the continuity of (38) by showing the continuity of $\underline{x}^i$ as a function of

$$x^{-i} = (x^1,\ldots,x^{i-1},x^{i+1},\ldots,x^m) \tag{39}$$

for any i, $i=1,\ldots,m$. Suppose the function,

$$\underline{x}^i = \underline{x}^i(x^{-1}), \tag{40}$$

is not continuous at some $\overline{x}^{-i}$. This implies that there is an infinite sequence, $\{x^{-i}\}$, approaching $\overline{x}^{-i}$ such that

$$\underline{x}^{-i} = \underline{x}^i(\bar{x}^{-i}) \neq \lim_{x^{-i} \to \bar{x}^{-i}} \{\underline{x}^i(x^{-i})\}.$$

(The existence of this limit is implied by the boundedness assumption in (a.2) and the Weierstrass Theorem.) Since each firm's profit is a continuous function of $(x^1,\ldots,x^m)$, there is also an infinite sequence,

$$\{\pi_{-i}\} = \{\pi_1,\ldots,\pi_{i-1},\pi_{i+1},\ldots,\pi_m\}, \tag{41}$$

which approaches $\bar{\pi}_{-i}$ such that

$$\underline{x}^{-i} = \underline{x}^i(\bar{\pi}_{-i}) \neq \lim_{\pi_{-i} \to \bar{\pi}_{-i}} \{\underline{x}^i(\pi_{-i})\}.$$

Now the uniqueness of $\underline{x}^i(\bar{\pi}_{-i})$ expressed in (a.4) implies that there is a $\delta(\varepsilon), \delta > 0$, such that for any $x^i \varepsilon X$ not in an ε-neighborhood of $\underline{x}^{-i}$,

$$\pi^i(\underline{x}^{-i}, \bar{\pi}_{-i}) - \pi^i(x^i, \bar{\pi}_{-i}) > \delta(\varepsilon), \tag{42}$$

where π^i is firm i's maker profit. Then, from the linear manner in which π_{-i} enters i's maker profit function (34), there is an $\omega > 0$ such that for all π_{-i} satisfying $|\pi_{-i} - \pi_{-i}| < \omega$, and for all $x^i \varepsilon X$ <u>not</u> in an ε-neighborhood of $\underline{x}^{-i}$,

$$\pi^i(\underline{x}^{-i}, \pi_{-i}) - \pi^i(x^i, \pi_{-i}) > \delta(\varepsilon) \tag{43}$$

Consider the ε-neighborhood of

$$\lim_{\pi_{-i} \to \bar{\pi}_{-i}} \underline{x}^i,$$

and select an ε sufficiently small that the intersection of this neighborhood and the ε-neighborhood of $\underline{x}^{-i}$ is empty. If the $\underline{x}^i$ in the former neighborhood are indeed profit maximizing, for all $\underline{x}^i$ in that neighborhood,

$$\pi^i(\underline{x}^{-i},\pi_{-i}^M) - \pi^i(\underline{x}^i,\pi_{-i}^M) < 0, \tag{44}$$

for all $\pi_{-i}^M \to \bar{\pi}_{-i}^M$ generating the ε-neighborhood of

$$\lim_{\pi_{-i}^M \to \bar{\pi}_{-i}^M} x^i.$$

This is a direct contradiction of the immediately preceding inequality (43).

Hence, $\underline{x}^i$ is a continuous function of x^{-i} and likewise the transformation (38) is continuous. This is sufficient for the Kakutani fixed point theorem to apply; and therefore, the set $(\underline{x}^1,\ldots,\underline{x}^m)$ exists.

Part 2.

We shall now prove that--given the array of maximum maker profits in (34), and therefore an array of bids against all alternative makers described in (35)--a winning bidder exists. Consider the matrix B, representing the bids of each firm against the others, with zeros along the main diagonal:

$$B = \begin{bmatrix} 0 & B_{12} & B_{13} & \cdots\cdot & B_{1m} \\ B_{21} & 0 & B_{23} & \cdots\cdot & B_{2m} \\ \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ B_{m1} & B_{m2} & B_{m3} & & 0 \end{bmatrix}. \tag{45}$$

From the definition of a solution maker, i is a solution maker if $\max_j B_{ij} \geq \max_k B_{ki}$, that is, if the maximum bid by i exceeds the maximum of the bids against i. In B, i is a solution maker if the maximum of the elements in the i^{th} row exceeds the maximum of the elements in the i^{th} column. Let B_{qr} be a maximal element of B. Then, $B_{qr} \geq \max_k B_{kq}$ so that q is a maker. Hence, there is always a winning bidder.

II. EQUILIBRIUM WITH ANTITRUST POLICY

This part of the Appendix shows that the introduction of competition in making commitments, and thus of competitive bidding for each position in the hierarchy, makes the model of equilibrium with antitrust policy in the text decidedly more complex but does not substantially alter its conclusions as long as the number of firms is not very small.

Referring to the model with antitrust policy in the text, while competitive bidding for hierarchial position may increase the costs to all firms, it does not, of course, increase the costs of the pure taker. Since all firms have the same variable cost functions, the variable maker profit of each firm is, in equilibrium, equal to the simple variable profit of the pure taker. It also follows from the equality of variable costs between our firms that every firm is indifferent to any position in the hierarchy so that all bids for each position in the hierarchy are identical. One's position in the hierarchy is determined by the priority of his commitment. Hence, the first auction is for the position of primary maker, the second for the secondary maker, etc. In the first auction, there are m bidders, in the second there are m-1, and so on until, finally in the m-1st auction, there are only two bidders. Let us see how these auctions alter the rationally chosen outputs from those selected in the model with no competition for hierarchal position.

The pure taker obviously has the same output choice function as in the model with no competitive bidding for hierarchal positions, so again

$$x_m^* = \frac{a-c-b\sum_{i=1}^{m-1} x_i}{2b} = \frac{p-c}{b}, \tag{46}$$

where X_i^* is the solution output of firm i in the present model. But the output of the m-1st firm is now sensitive to a bid of the m^{th} firm for his hierarchal position. The bid of m for position m-1 equals

$$B_{m,m-1}(x_{m-1}) = \max[0,\ \pi_{m,m-1} - \pi_m(x_{m-1})],$$

where $\pi_{m,m-1}$ is the operating profit m would make if he were the $m-1^{st}$ maker. Firm m-1 therefore selects an x_{m-1} which maximizes

$$\pi^M_{m-1} = \pi_{m-1} - \max[0,\ \pi_{m,m-1} - \pi_m(x_{m-1})] \text{ subject to (36) and the given values, } x_1,\ldots,x_{m-2}. \tag{47}$$

If the solution x_{m-1} were such that $B_{m,m-1}(x_{m-1}) > 0$, then the output that satisfies (47) would be a joint profit maximum subject to (46) and $x_1,\ldots,x_{m-2}$. This joint profit is $(p-c)(x_{m-1} + x_m)$ which, as above, reaches its maximum at $x_{m-1}+x_m = \frac{p-c}{b}$. But then, from (46), x^*_{m-1} would equal zero. Since variable profits exceed zero at some positive outputs, firm m-1 would do better as the pure taker. Hence, $B_{m,m-1}(x^*_{m-1})=0$. But if so, the variable profits are equal for both firms m and m-1. Therefore.

$$x^*_{m-1} = x^*_m, \tag{48}$$

and using (36),

$$x^*_{m-1} + x^*_m = \frac{a-c-b \sum_{i=1}^{m-2} x_i}{3b/2} = 2(\frac{p-c}{b}). \quad (49)$$

This solution may be constructed by starting firm m-1 at its output in the previous model, an output which maximizes its operating profit and exceeds x_m, and then making it pay m's bid to be maker to a manager. It is then obvious from (47) that it pays m-1 to reduce his output in order to reduce m's bid against him. This occurs until $x_{m-1} = x_m$, at which point the bids become zero. It then no longer pays m-1 to reduce his output for there is no further reduction in m's bid that is possible. To compute m-2's optimal output, we need the profit of firms m and m-1 as a function of x_{m-2}. Using (49),

$$\pi_{m-1} = \pi_m = \left[a-c-b \sum_{i=1}^{m-2} x_i - b \left(\frac{a-c-b \sum_{i=1}^{m-2} x_i}{3b/2}\right)\right]\left(\frac{a-c-b \sum_{i=1}^{m-2} x_i}{3b}\right) = \left(\frac{a-c-b \sum_{i=1}^{m-2} x_i}{9b}\right)^2. \quad (50)$$

Firm m-2's maker profits can now be written, using (49),

$$\pi^M_{m-2} = \left[a-c-b \sum_{i=1}^{m-2} x_i - b \left(\frac{a-c-b \sum_{i=1}^{m-2} x_i}{3b/2}\right)\right] x_{m-2} - \max[0, \pi_{m,m-2} - \pi_m(x_{m-2})]. \quad (51)$$

Assume that $\pi_{m,m-2} - \pi_m(x^*_{m-2}) \geq 0$. Then, using (50) and (51),

$$\pi^M_{m-2} = \left(\frac{a-c-b \sum_{i=1}^{m-2} x_i}{3}\right) x_{m-2} + \left(\frac{a-c-b \sum_{i=1}^{m-2} x_i}{9b}\right)^2 - \pi_{m,m-2}.$$

Maximizing this profit, we find

$$x^*_{m-2} = \frac{a-c-b \sum_{i=1}^{m-3} x_i}{4b} \text{ , and} \tag{52}$$

$$\pi_{m-2}(x^*_{m-2}) = \left(\frac{a-c-b \sum_{i=1}^{m-3} x_i}{16b}\right)^2 \tag{53}$$

Substituting x^*_{m-2} into (49),

$$x^*_{m-2} = x^*_{m-1} = x^*_m \tag{54}$$

The assumption that $\pi_{m,m-2} - \pi_m(x^*_{m-2}) \geq 0$ is satisfied, for at x^*_{m-2} the bid of m (and of m-1) is zero.

Similarly, if the output solution to the problem,

$$\max_{x_i} \ [(p-c)x_i - \max_j(\pi_{ji} - \pi_j(x_i))], \ i < m, \tag{55}$$

exceeds x_m, then, because bids are positive in such a solution, this output solution is also the solution to the general problem:

$$\max_{x_i} \ [(p-c)x_i - \max_j \ [0, \ \pi_{ji} - \pi_j(x_i)]. \tag{56}$$

Performing the maximization in (55) for firm m-3, using (52), (53) and (54), we maximize

$$\left(a-c-b \sum_{i=1}^{m-3} x_i\right) x_{m-3} - 3/4 \ \left(a-c-b \sum_{i=1}^{m-3} x_i\right) \ x_{m-3} + \left(\frac{a-c-b \sum_{i=1}^{m-3} x_i}{16b}\right)^2$$

with respect to x_{m-3}. The solution output can be written:

$$x^*_{m-3} = \frac{a-c-b \sum_{i=1}^{m-4} x_i}{3b} = \frac{a-c-b \sum_{i=1}^{m-3} x_i}{2b}. \tag{57}$$

This output is twice the output of firm m-2 and thus is also a solution to (56). We also find, using (52), (54) and (57) that variable profits for each firm are:

$$\pi^*_m = \pi^*_{m-1} = \pi^*_{m-2} = \pi^{M*}_{m-3} = \frac{(a-c-b \sum_{i=1}^{m-4} x_i)^2}{36b}. \tag{58}$$

Thus variable maker profit to the m-4th firm is

$$\pi^M_{m-4}(x_{m-4}) = \left[a-c-b \sum_{i=1}^{m-4} x_i - \frac{5}{6}\left(a-c-b \sum_{i=1}^{m-4} x_i\right)\right]x_{m-4} + \left(\frac{a-c-b \sum_{i=1}^{m-4} x_i}{36b}\right)^2$$

Maximizing this with respect to x_{m-4}, we find

$$x^*_{m-4} = \frac{a-c-b \sum_{i=1}^{m-5} x_i}{5b/2} = \frac{a-c-b \sum_{i=1}^{m-4} x_i}{3b/2}. \tag{59}$$

This is twice the output of firm m-3 and four times the outputs of firms m, m-1, and m-2. We also find that

$$\pi^*_m = \pi^*_{m-1} = \pi^*_{m-2} = \pi^{M*}_{m-3} = \pi^{M*}_{m-4} = \frac{(a-c-b \sum_{i=1}^{m-5} x_i)^2}{100b}.$$

These profits form the bids for the m-5th position in the hierarchy, and the procedure continues until we reach the top position. The distribution of outputs, moving on to m-5, m-6 and m-7 and again indexing the output of the mth firm to unity, is easily seen to be

1, 1, 1, 2, 4, 8, 16, 32.

The obvious generalization is that

$$x^*_m = x^*_{m=1} \text{ and } x^*_{m-2-i} = x^*_m 2^i,\ i = 0,1,\ldots,\ m-3. \qquad (60)$$

To prove that this generalization is, in fact, the solution distribution of firms, we provide an inductive proof. In particular, we shall prove that if the hypothesized distribution holds for $i = r$, i.e., if $x_{m-2-i}/x_{m-1-i} = 2$ for any i such that $0 \le i \le r$, then it holds for $i = r + 1$, i.e., $x_{m-3-r}/x_{m-2-r} = 2$. To do this, we first note that variable profit to the $m-3-r^{th}$ firm is

$$\pi_{m-3-r} = \left(a-c-b \sum_{i=1}^{m} x_i\right) x_{m-3-r} + \left(\frac{a-c-b \sum_{i=1}^{m} x_i}{b}\right)^2.$$

By hypothesis, for the firms from m to m-2-r we have:

$$x_{m-2-r} = \frac{a-c-b \sum_{i=1}^{m-3-r} x_i}{b(2^r+1)/2^{r-1}} \qquad (61)$$

and

$$\sum_{i=m-2-r}^{m} x_i = \frac{a-c-b \sum_{i=1}^{m-3-r} x_i}{b} \left(\frac{2^{r+1}+1}{2^{r+1}+2}\right), \quad r \ge 0 \qquad (62)$$

Using (52),

$$\pi_{m-3-r} = \left(a-c-b \sum_{i=1}^{m-3-r} x_i - b \sum_{i=m-2-r}^{m} x_i\right) x_{m-3-r} + \left(\frac{a-c-b \sum_{i=1}^{m-3-r} x_i - b \sum_{i=m-2-r}^{m} x_i}{b}\right)^2$$

$$= \frac{1}{2^{r+1}+2} \left[\left(a-c-b \sum_{i=1}^{m-3-r} x_i\right) x_{m-3-r} + \frac{\left(a-c-b \sum_{i=1}^{m-3-r} x_i\right)^2}{(2^{r+1}+2)^2 b}\right]$$

Maximizing this expression with respect to x_{m-3-r},

$$0 = \frac{1}{2^{r+1}+2}\left(a-c-b\sum_{i=1}^{m-3-r} x_i\right) - \frac{bx_{m-3-r}}{2^{r+1}+2} - 2\frac{\left(a-c-b\sum_{i=1}^{m-3-r} x_i\right)}{(2^{r+1}+2)^2}$$

$$= 2^{r+1}\frac{\left(a-c-b\sum_{i=1}^{m-3-r} x_i\right)}{(2^{r+1}+2)^2} - \frac{bx_{m-3-r}}{2^{r+1}+2}$$

$$x_{m-3-r} = \frac{a-c-b\sum_{i=1}^{m-3-r} x_i}{b\left(\frac{2^r+1}{2^r}\right)}$$

Using (51), we see that

$$\frac{x_{m-3-r}}{x_{m-2-r}} = \frac{(2^r+1)/2^{r-1}}{(2^r+1)/2^r} = 2.$$

This establishes the theorem.

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