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INVESTMENT IN HEALTH: LIFE CYCLE CONSUMPTION OF HAZARDOUS GOODS

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1. Introduction

In recent years considerable attention has been directed towards the identification of goods which are hazardous to human health. Scientific studies regularly reveal hazards associated with products previously considered safe. Consumption items such as cigarettes, eggs (cholesterol), saccharin and certain types of radial tires are typical of products currently recognized as hazardous. Industrial pollutants, such as asbestos and kepone, represent another large class of goods which are under scrutiny for their detrimental effects on human health.

Economic theory suggests that once a good is identified as hazardous, consumption of the good should be reduced. The utility associated with consumption of the good is traded for an increase in expected life or a reduction in the likelihood of illness.[1] Thus, an individual's decision to restrict the consumption of a hazardous good can be regarded as one aspect of his overall decision to invest in health.

Investment in health has been dealt with by many authors. Grossman (1972), for instance, has formulated a model in which individuals view health as a capital stock which depreciates with age but can be increased by investment. Cropper (1977) has extended the Grossman analysis by allowing the relationship between health capital and illness to be random rather than deterministic. In both models, the age of death is assumed either to be predetermined or to occur when the health stock falls below some predetermined value.

Implicit in these general models are a variety of issues related to health. The choice of occupation, the decision to smoke, drink alcohol or exercise, and even the choice of entertainment are all elements of overall health investment. However, because these models are concerned with the broad issue of health investment, they do not provide much insight into the behavior of individuals relative to single consumption issues associated with health.

The purpose of this paper is to study this more specialized form of investment in health. Specifically a model is developed to analyze the optimal life cycle consumption of a hazardous good, that is, of a good that increases the probability of death either immediately or in the future. A key feature of the model is that death from "other causes" is not assumed to be predetermined. In fact, the uncertainty of life plays a key role in the consumption decision.

A second point of departure from the previous literature is that our analysis does not necessarily assume that individuals have the relevant health information when they first begin to consume the hazardous good. In fact, a primary concern of this paper is the rational reaction to new information concerning a hazard. This is an important issue in this context since individuals are often made aware of a hazard only after they have been consuming the good for some time. By investigating rational consumer reaction to new health information, the analysis yields predictions for the movement of aggregate consumption statistics following the announcement of a hazard.

The simplest type of hazardous good is considered in Section 3. In this case, each unit of consumption is assumed to be fatal with a specified (constant) probability, and death, if it occurs, is instantaneous. Under minimal assumptions on the probability of death from "other causes," the optimal lifetime consumption path for such a hazard is shown to be monotonically increasing. Further, if consumption had been assumed to be safe and information is made available regarding the hazard, the long term adjustment should be achieved immediately.

In Section 4, this model is generalized to cumulative hazards, that is, to hazards for which the likelihood of death per unit consumed depends on past cumulative consumption. In particular, we focus on the case of an increasingly hazardous good, that is, of a good where the first units consumed are not very dangerous but those consumed after years of consumption are. In this case, the results are dramatically different. The shape of the lifetime consumption path depends on the age at which the hazard information is received: a U-shaped consumption path being possible if the information is received early enough; a monotonically increasing path being optimal otherwise. Further, periods of abstinence are usually optimal only upon first hearing of the hazard; that is, surprisingly, it is not usually optimal to consume the relatively riskless portion of the hazard when young, abstain during the middle years of life, and if alive, resume consumption when old.

Finally, in contrast with the constant hazard case, aggregate consumption statistics should not achieve long-term equilibrium immediately upon announcement of the hazard. For increasing hazards, a rational reaction would cause aggregate consumption rates to drop on announcement of the hazard, but this reduction should erode over time, stabilizing in the long run at some point below the preannouncement rate. Further, the consumption rates for young age groups should stabilize more quickly than those for older age groups.

In Section 5, we expand the model still further by allowing a delay between consumption of the good and the eventual death caused by it. This model is motivated by the fact that many cancer-causing agents are known to have a lengthy incubation period. In this case, consumption paths may again be U-shaped or monotonically increasing and are affected by age at the time of the announcement. However, the reaction will not be as strong as for instantaneous hazards, and consumption rates will rise to preannouncement rates sooner than for their instantaneous counterparts. Further, even constant hazards will require time to achieve long-term adjustment if there is a delayed effect.

Finally, we note that the choice of a hazardous occupation was analyzed by Cropper (1977). Her model can be represented in our terminology as an increasing hazard where the age of death from "other causes" is known with certainty.

2. Assumptions of the Analysis

Let us suppose that an individual receives a constant rate of income over his entire life which he divides among expenditures on n consumption goods. Suppose that the utility derived from this consumption is specified by a well-defined concave utility function which is constant over the individual's adult life. Finally, suppose that the individual faces constant prices and an instantaneous budget constraint.

These assumptions are not critical to the analysis which follows but serve merely as a convenient mechanism to isolate the relationship between age and the consumption of a hazardous good. In particular, by abstracting from price and income effects and age-related changes in the underlying utility function, we can conclude that prior to any hazard announcement, consumption of the hazardous good (and all other goods) is constant over life.

The term "hazardous good" is used in this analysis to refer to goods which increase the probability of death. That is, it is assumed that individuals restrict consumption of the good to avoid premature death and do not consider the disutility of illness which might precede death. This assumption makes the model more applicable to goods which cause fatal accidents, heart attacks, strokes or cancer than to, say, diabetes, which might eventually cause death but whose major impact is one of "discomfort" prior to death.[2]

For simplicity, let us also suppose there is a single hazardous good in the consumer's bundle of n goods. Ignoring the hazardous aspects of the good for the moment, we define

$U(x)$ = Instantaneous utility from consumption of x units of the hazardous good, given that the consumer rearranges his remaining bundle of (nonhazardous) goods so as to maximize his utility subject to his budget constraint.

It follows directly from the concavity of the underlying utility function that $U = U(x)$ is a concave function of x and that its maximum is achieved when $x = \bar{x}$, the pre-announcement consumption rate.[3] We assume further that U is twice continuously differentiable on $(0,\bar{x})$.

At the start of adult life $\bar{s}$, the consumer is assumed to know the probability that he will be alive at all future ages assuming no hazard to consumption. More specifically we define

$p(t)$ = Probability that the consumer will be alive at age t given that he is alive at age $\bar{s}$, $\bar{s} < t < T$, and T is the maximum attainable human age.

$p(t)$ is assumed to be continuously differentiable (the time derivative is denoted by .) and satisfies

$$0 < p(t) < 1 \quad p(\bar{s}) = 1 \quad p(T) = 0 \quad \dot{p}(t) < 0 \qquad (2.1)$$

Note that this formulation of the uncertainty of life is equivalent to Yaari's (1965) model where the age of death is viewed as a random variable.

We will also assume that the conditional probability of dying is an increasing function of age; that is,

$$-\dot{p}(t)/p(t) \text{ increases with } t \qquad (2.2)$$

This assumption is not critical to all the results which follow but is consistent with mortality statistics over the adult span of life in the U.S.

As the individual lives past age $\bar{s}$ to age s, it is assumed that he updates his probability of living function; that is, at age s he considers his probability of living to age t to be p(t)/p(s), for t $\geq$ s. However, for reasons of tractability, we will assume that he receives no other information which induces him to update his probability of living function. Thus, we are explicitly ruling out the situation where the individual develops "early warning signs" and cuts back his consumption of the hazardous good as a result.

Finally, we will assume no subjective discounting of the future, that is, future utility is discounted only to the extent that there is uncertainty of survival. Inclusion of a subjective discount rate would alter the analysis in the usual way but would not change the major results.

3. Life Cycle Consumption of a Constant Instantaneous Hazard

A. Basic Model - To illustrate our model and to provide a benchmark against which to measure the more complex analysis which follows, we begin by considering the simplest type of hazardous good, namely a good which leads to instantaneous death with probability q for each incremental amount consumed.[4] The critical feature of this hazard is that the probability q does not depend on past consumption. Admittedly this class of hazards does not include many of the goods of concern to consumers. However, foods which have a probability of being tainted, airline travel, or driving without a seatbelt might be considered examples of hazards in this class.

If a rational consumer has been ignorant of the hazard and information regarding its ill effects is made available to him, other things being equal, he will adjust his consumption pattern over life, taking this new information into account. In this section, we will formulate this constant hazard problem and investigate the characteristics of the consumer's optimal consumption path over life.

Suppose that at the time of the announcement, the consumer is s years of age and has consumed Y units of the hazardous good to date.[5] If he stops consuming the hazardous good, his probability of being alive at age t, $t > s$, is $p(t)/p(s)$. Each unit of the hazardous good that he consumes between ages s and t decreases the likelihood that he will be alive at t and beyond, and thus decreases his chance of reaping the satisfaction from consumption of this and all other goods in future years. To begin our analysis, we define

$x(t)$ = Planned consumption rate of the hazardous good at age t, given the information available at age s, $s < t < T$.

$X(t)$ = Cumulative consumption of the hazardous good at age t given consumption at the rate $x(t')$ between the ages of s and t ($s < t' < t$) and cumulative consumption of Y at age s.

The consumer's optimal strategy at age s is to find a consumption rate x(t) for each future age t which maximizes his expected lifetime utility.[6] Given the simple nature of a constant hazard, the conditional probability of being alive at age t if consumption is X(t) by that point is $k^{X(t)-Y}$ p(t)/p(s) where $k = e^{-q}$.[7] Therefore, the consumer's problem is to find the consumption path which maximizes the functional

$$J_s(X,x;Y) = \int_s^T k^{X(t)-Y} \frac{p(t)}{p(s)} U(x(t))dt \tag{3.1}$$

subject to

$$\dot{X}(t) = x(t) \tag{3.2}$$

$$X(s) = Y. \tag{3.3}$$

Before proceeding to solve (3.1) - (3.3), note that the simple change of variable Z(t) = X(t) - Y transforms the consumer's problem at any age s into the equivalent problem:

$$J_s(Z,x) = \int_s^T k^{Z(t)} \frac{p(t)}{p(s)} U(x(t))dt \tag{3.4}$$

subject to

$$\dot{Z}(t) = x(t) \tag{3.5}$$

$$Z(s) = 0 \tag{3.6}$$

From this formulation of the problem, it is clear that the consumer's decision as viewed from age s is independent of past consumption Y. If the consumer has not been killed by past consumption and if he does not consume any more of the good, his life expectancy is equal to that of a person who has never consumed the good. Further, the harm associated with consumption of each additional unit of the good is not affected by past consumption. Consumption planned for the future, however, affects the probability of being alive at older ages.

To find the consumer's optimal consumption path x(t) for $s \leq t \leq T$, we note that (3.1) - (3.3) is a non-autonomous optimal control problem[8] over a fixed time period. The corresponding Hamiltonian function is

$$H(t,X,x,\lambda) = k^{X-Y} \frac{p(t)}{p(s)} U(x) + \lambda x$$

Here $\lambda(t)$ is an auxiliary function corresponding to X and x chosen to satisfy

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial X} = qk^{X-Y} \frac{p(t)}{p(s)} U(x)$$

Applying the non-autonomous version of the Pontryagin Maximum Principle, we conclude that the optimal consumption path must satisfy

$$k^{X(t)-Y} \frac{p(t)}{p(s)} U'(x(t)) + \lambda(t) \leq 0$$

with equality holding unless x(t) = 0. Further, the following transversality condition is satisfied at T:

$$\lambda(T) = 0$$

Computing $\lambda(t)$ from (3.8) and (3.10), it is optimal for a consumer to plan either no consumption of the hazardous good at age t (x(t) = 0) or a positive consumption rate x(t) which satisfies

$$k^{X(t)-Y}\frac{p(t)}{p(s)}U'(x(t)) = q \int_t^T k^{X(t')-Y} \frac{p(t')}{p(s)}U(x(t'))dt' \quad (3.11)$$

Condition (3.11) simply indicates that any positive consumption planned for a given age t must be chosen so that the expected marginal utility of consumption at age t (left side of (3.11)) is equal to the expected marginal cost in the future, the shadow price $\lambda(t)$ (right side of (3.11)). This future cost is due to the decrease in the probability of living at each point in the future if consumption of the hazardous good is increased at t.

B. Characteristics of Life Cycle Consumption

To analyze the nature of the optimal consumption path over life, note that by (3.9) consumption satisfies

$$U'(x(t)) \leq q\int_t^T k^{X(t')-X(t)}\frac{p(t')}{p(t)}\,U(x(t'))dt' \qquad (3.12)$$

at every age t, with equality holding unless $x(t) = 0$. Thus if an individual lives to old age (t near T), he will consume a positive amount and eventually his consumption rate will approach the pre-announcement rate $\bar{x}$, since by (3.12)

$$U'(x(T)) = 0 \qquad (3.13)$$

To analyze the properties of the optimal consumption path further, we differentiate equation (3.9) with respect to t and use (3.8) to establish the fact that whenever positive consumption is optimal, the following condition must hold:

$$U''(x)\dot{x} = -q[U(x) - xU'(x)] - \frac{\dot{p}}{p}U'(x) \qquad (3.14)$$

Suppose now that it is optimal for an individual to reduce his consumption rate over some period of life; that is, that $\dot{x} < 0$ at some age t^0. For ages $t > t^0$ where $x \equiv x(t) < x(t^0) \equiv x^0$, the convexity of U and the increasing conditional probability of death from other causes imply that

$$0 < U''(x')\dot{x}(t') = -q[U(x') - x'U'(x')] - \frac{\dot{p}(t')}{p(t')}U'(x')$$

$$< -q[U(x) - xU'(x)] - \frac{\dot{p}(t)}{p(t)} U'(x)$$

$$= U''(x)\dot{x}(t)$$

Hence it must be true that

$$\dot{x}(t) < 0 \text{ whenever } t > t' \text{ and } 0 < x < x'. \qquad (3.15)$$

Since we know that the optimal consumption rate must eventually rise to $\bar{x}$ and must be continuous (since U was assumed twice differentiable), this is impossible.

Thus, for a constant hazard, it can never be optimal for an individual to reduce his consumption of the hazardous good over any portion of life. When combined with (3.13), this result allows us to assert that an individual consuming a constant hazard may find it optimal to abstain completely in the early part of life, but at some point he should begin to consume the hazardous good and should do so at a monotonically increasing rate. In fact, if he lives to be old enough, his consumption should rise to the pre-announcement rate.

Figure 3.1 depicts the possible forms of the lifetime consumption paths for this type of hazard. This behavior is intuitively appealing. Since the probability of death associated with consumption of the hazard is fixed at q, the only factor affecting the consumption decision is the cost should death occur. As an individual gets older, this cost decreases--termed the age effect--since the expected number of years remaining in life is falling. Also, as expected, the likelihood of dying from other causes is relatively high for an old person so that the marginal effects of consuming the hazard become negligible, causing the consumption rate to rise to the pre-announcement level.

Finally, we note that once the information is available to consumers, the optimal consumption path $x(t)$ does not depend on the individual's age at the time of the announcement. That is, since past consumption does not affect the cost of future consumption, all individuals (with the same income and utility function) who receive information regarding the hazard will adjust their consumption rates so that they consume at the same rate at the same age. Thus, once the information is available the observed consumption path $x(t)$ for an individual over time should conform to a cross-section depiction of the consumption rates for like individuals as a function of age. Both the instantaneous nature of the long-term adjustment and the consistency of the lifetime and cross-section consumption rates depend critically on the constant hazard assumption.

4. Life Cycle Consumption of A Cumulative Hazard

A. Cumulative Hazards

Most health hazards cannot be adequately characterized as constant hazards. A more plausible model is one where the probability of death per unit consumed is not fixed but depends on the level of previous consumption. Intuitively, for instance, many hazards are viewed as having a degenerative effect on the body so that the first units consumed are not likely to cause death but units consumed after the body has been weakened may. This type of increasingly hazardous good might be typified by the connection between diet (salt or cholesterol intake) and stroke or by the dangers of excessive consumption of alcohol.

Alternatively, some hazards are most dangerous when first consumed as in cases where the body develops a tolerance for the hazard (e.g. the dangerous side effects of some drugs) or when learning is required to use a product safely. Similarly, hazards which only affect a portion of the population can be modeled as a decreasing hazard if there is no ex ante method of determining susceptibility; nonfatal past consumption serves as an indication that the person is in the immune group.

Pollutants might also fall into the class of cumulative hazards. These are usually viewed as being gradually absorbed by individuals in proportion to exposure, accumulating in the body until some critical level is reached and death occurs. If an individual's threshold is assumed to be unknown but distributed according to a specified density function, then the level of past consumption serves to determine the probability of death associated with future consumption.[9] The case of an industrial pollutant which is an increasing hazard was considered by Cropper (1977) under the assumption that the age of death from other causes is certain and the threshold is fixed over life.

B. The Model

In the case of a cumulative hazard, the rate at which the hazard is consumed is often an important determinant of the danger associated with further consumption. (One bottle of wine per week for one year is not equivalent to 51 weeks of abstinence followed by 52 bottles of wine in one week!) In order to incorporate this feature in our model, we begin by redefining one of the previous variables, namely, the stock variable X(t) associated with a given consumption path x(t). In particular, define

X(t) = Net cumulative consumption of the hazardous good at age t resulting from consumption at the rate x(t'), $s \leq t' \leq t$, given that net cumulative consumption was Y at age s and cumulative consumption decays at a rate δ.

Note that if there is no decay, X(t) again represents cumulative consumption. The decay rate δ can be interpreted as the rate at which the body can compensate for the cumulative effects of the hazard.

The essential characteristic of a cumulative hazard is that the harm associated with marginal consumption depends on the current level of net cumulative consumption; that is, on the level of past consumption from which the body has not recuperated. Thus, we define

f(X) = Conditional probability density of death due to consumption of the hazard given a net cumulative consumption of X.

Thus, f(X) can be interpreted loosely as the probability that an individual will die from further consumption of the hazardous good given that his current net cumulative consumption is X.[10] As for the constant hazard model of Section 3, we will assume that there are no lagged health effects to consumption (these are treated in Section 5); if consumption of a unit of the hazardous good does not kill an individual instantly, its only future effect is to make him more (or less) susceptible to further consumption of the good. Thus, if he abstains from further consumption, his life expectancy is assumed to be the same as for someone who has never consumed the good. Further, we assume that the rate of consumption does not influence the hazard of consumption beyond its effect on net cumulative consumption. Though admittedly an idealized model of a hazard's effects,[11] this formulation does capture the essential nature of a cumulative hazard. For a smoker, for instance, we are assuming that if his consumption of cigarettes has not caused a fatal heart attack to date and he abstains from further smoking, he will not suffer a heart attack in the future due to his past behavior. However, each additional cigarette that he smokes is assumed to have an increased probability of giving him a fatal heart attack.

To formalize these concepts, for every consumption path x and associated net cumulative consumption path X, we define

$P(t)$ = Probability that an individual is not dead from consumption of the hazard by age t if he is alive at age s, has a net cumulative consumption of Y by that age, and plans to consume at the rate $x(t')$ between ages s and t, $s \leq t' \leq t$.

Note that P(t) must satisfy [12]

$$\dot{P} = -P\, f(X)x$$
$$P(s) = 1 \qquad (4.1)$$

Recalling the definition of $p(t)$ from Section 2, the conditional probability that the individual is alive at age t is simply the product of the probabilities that he does not die from consumption of the hazardous good and that he does not die from other causes, thus, $P(t)p(t)/p(s)$.

As with the constant hazard model, the consumer at age s must determine the consumption rate $x(t)$ $(s < t < T)$ over life which maximizes his expected lifetime utility. However, in this case, past consumption has an effect on the current decision.

Thus the consumer's problem is to maximize the functional

$$J_s(X,P,x;Y) = \int_s^T \frac{P(t)p(t)}{P(s)p(s)} U(x(t))dt \tag{4.2}$$

where

$$\dot{X} = x - \delta X \tag{4.3}$$

$$\dot{P} = -P\, f(X)x \tag{4.4}$$

$$X(s) = Y \tag{4.5}$$

$$P(s) = 1 \tag{4.6}$$

As in the constant hazard case, the consumer's problem can be viewed as a non-autonomous optimal control problem. The corresponding Hamiltonian is

$$H(t,X,P,x,\lambda,\gamma) = \frac{P(t)p(t)}{P(s)p(s)}U(x) + \lambda(x-\delta X) - \gamma P f(X)x \tag{4.7}$$

where λ and γ are auxilliary functions corresponding to x, X and P chosen to satisfy

$$\dot{\lambda} = -\frac{\partial H}{\partial X} = \delta\lambda + \gamma P x f'(X)$$

$$\dot{\gamma} = -\frac{\partial H}{\partial P} = -\frac{p(t)}{p(s)} U(x) + \gamma f(X)x$$

Thus we can again apply the Pontryagin Maximum Principle to conclude that either it is optimal for the consumer to plan to abstain completely at a given age t ($x(t)=0$) or he should plan his consumption to satisfy

$$\frac{P(t)p(t)}{P(s)p(s)}U'(x(t)) = -\lambda(t) + \gamma(t)P(t)f(X(t)) \quad (4.10)$$

In the cases where zero consumption is optimal, inequality ($\leq$) will hold in (4.10). Further, the following transversality conditions must hold

$$\lambda(T) = 0 \qquad \gamma(T) = 0 \quad (4.11)$$

It is straightforward to verify that (4.8) - (4.9) and (4.11) are satisfied by λ and γ given by:

$$\gamma(t) = \int_t^T \frac{P(t')p(t')}{P(t)\ p(s)} U(x(t'))dt' \quad (4.12)$$

$$\lambda(t) = -e^{\delta t}\int_t^T e^{-\delta t'} x(t')P(t')f'(X(t'))\gamma(t')dt'$$

Note that using the notation of (4.2), (4.12) can be written as

$$\gamma(t) = \frac{p(t)}{p(s)} J_t(X,P,x;X(t)) \quad (4.14)$$

and using (4.10) in (4.9) that

$$\dot{\gamma} < 0. \quad (4.15)$$

Note also from (4.8), that

$$\dot{\lambda} - \delta\lambda = \gamma P x f'(X) > 0$$

and hence that

$$\dot{\lambda} > \delta\lambda$$

As in the constant hazard case, the intuitive explanation for (4.10) is simple. Positive consumption will not be planned for age t unless the expected marginal benefits from consumption at t (LHS of (4.10)) can be balanced against expected marginal costs. These marginal costs can be decomposed into two distinct costs: the expected loss due to death at age t (the second term on the RHS of (4.10)) plus the expected loss (or gain)[13] in the future due to the cumulative effects of having increased consumption at age t (the first term on the RHS of (4.10)).

Finally note that as in the constant hazard case, if the individual lives to be old enough, he will consume a positive amount of the hazardous good and that amount will rise to the pre-announcement rate $\bar{X}$ as his age approaches T (let t=s and then let $s \to T$ in (4.10)).

In order to analyze the optimal consumption path in more detail, we will initially restrict our study to the case of an increasingly hazardous good. Decreasingly hazardous goods will be considered below.

C. <u>Consumption of an Increasingly Hazardous Good</u>

An increasingly hazardous good is defined as one for which the conditional probability density of dying is an increasing function of net consumption X, that is,

$$f'(X) > 0. \qquad (4.16)$$

Clearly (4.16) implies that each unit consumed is more hazardous than the previous one and the rate at which consumption becomes dangerous depends on the derivative of f(X).

To analyze the consumption path for an increasing hazard, we differentiate (4.10) and use (4.3), (4.4) and (4.8)-(4.10) to conclude that whenever positive consumption is optimal, the following condition must hold:

$$U''(x)\dot{x} = -f(X)[U(x) - xU'(x)] + (\delta-\frac{\dot{p}}{p})U'(x) - \delta\frac{p(s)}{p}\gamma[f(X) + f'(X)X] \qquad (4.17)$$

Since U is assumed to be a concave function, the first and third terms on the RHS of (4.17) are negative and the second term is positive. The sign of $\dot{x}$ is thus determined by the magnitudes involved. In particular, for an increasingly hazardous good which is not very harmful initially (that is, f(X) is small for X small), the sign of (4.17) is unambiguously positive at an early age (s near $\bar{s}$ - the start of adult life). Relating this to the previously determined behavior, we can conclude that for such an increasingly hazardous good, the consumption rate should initially fall but at some point will rise again to the pre-announcement rate $\bar{x}$. Thus a U-shaped lifetime consumption path is consistent with rational behavior for this type of increasing hazard.

This behavior is strikingly different from that found in the constant hazard case of Section 3 where the individual would never find it optimal to decrease his consumption rate over time. In the current case, the shape of the planned consumption path is seen to depend critically on the nature of the hazard and on past consumption Y and hence in our model on the age at which the consumer receives the information regarding the hazard.

The previous argument is not sufficient to allow us to conclude that we should ever expect to see a U-shaped lifetime consumption path since we have only considered planned consumption $x(t)$, $s \leq t \leq T$. However, the planned optimal consumption path in this case coincides with the optimal consumption path an individual will actually follow if he lives. Thus we need not make a distinction between the two.[14]

Given our analysis to this point, it seems plausible that for increasing hazards we might expect to see rational individuals consume a hazardous good when young and again when old but abstain from consumption in their middle years. However, in most cases, we can show that such a period of abstinence in midlife is not optimal. Consider first the case where the decay rate δ is very large. In this case the hazard is approximately a constant hazard since any stock of past consumption decays rapidly and can be approximated by $f(X) = f(0)$. Thus a period of abstinence is not optimal by the argument of Section 3. Alternatively if the decay rate is zero (or small enough) the right-hand side of (4.17) is increasing over any period of abstinence. Thus, since U is concave and (4.17) must hold at the start t_1 and the end t_2 of the period of abstinence, it must be true that $x^-(t_1) > x^+(t_2)$ where the plus and minus indicate the right and left hand derivatives respectively. But since the optimal control must be continuous (given our smoothness assumptions), it must be true that $x^-(t_1) \leq 0 \leq x^+(t_2)$, which is a contradiction. Thus no such period of abstinence can exist in this case.

For decay rates between these two extremes, the situation is not entirely clear. If the optimal consumption path is characterized by the fact that expected lifetime utility $J_t(X,P,x;X(t))$ at age t is monotonically decreasing over life[15] and the rate of increase of the hazard is not too sharply decreasing (that is, if $f(X) + Xf'(X)$ is increasing in X), then we can again establish the nonoptimality of a period of abstinence. To see this note that by (4.14), γ/p is thus assumed decreasing. Then, as above, we can show that the right-hand side of (4.17) is increasing during a period of abstinence and thus gives us a contradiction.

The only cases remaining, then, for which a period of abstinence is even a possibility are those where the rate of increase of the hazard is sharply diminishing or where the optimal path is characterized by a period in which expected lifetime utility J_t is increasing with age.

Cropper (1977) also proved the nonoptimality of a period of abstinence for her model of hazardous employment. However, in the Cropper model, this result depends on the fact that the model is a fixed threshold model (see Footnote 9) and hence that it is always optimal to consume at a rate greater than the decay rate; that is, $x > \delta X$, since this rate of consumption is harmless. In our model, the increasing marginal probability of death from other causes is enough to guarantee the result in most cases even though there is a hazard to any positive level of consumption. Thus, the fact that a hazard increases the probability of death is not usually, in itself, enough to induce an individual to take "recuperative breaks" in consumption <u>except</u> upon first hearing of the hazard.

Finally, it should be noted that the increasing marginal probability of death from other causes plays a significant role in this analysis. This illustrates the sensitivity of life cycle models to this assumption and underscores the fact that models which use a constant discount rate as a proxy for uncertain lifetimes may lead to qualitatively different results.

Figure 4.1 depicts the alternative lifetime consumption paths $x(t)$, $\bar{s} < t < T$, for an individual aged s at the time the hazard information is made known. In all cases the consumption rate is $\bar{x}$ before the hazard announcement is made. If the individual is young enough at the time of the announcement (s small) then his cumulative consumption Y will be small and his consumption path may take the form of either (a) or (b); if he is older when the announcement is made, his consumption path will be monotonically increasing as in either (a) or (c).

The intuitive explanation for the behavior depicted in Figure 4.1 is quite simple. There are two opposing forces associated with increasing hazards which influence the individual's consumption decision. The first, termed the age effect, is the same force that dictated the consumption behavior for the constant hazard; namely, the expected cost of dying (the loss of expected future utility) is decreasing as an individual ages. This factor induces him to increase his consumption rate over time.

Acting in the opposite direction is what we will term the discount effect. As each unit is consumed, the risk associated with further consumption increases. Since the likelihood of dying from other causes is increasing with time, the consumer tends to discount future consumption of the hazardous good in favor of current consumption. That is, it is optimal for him to consume the relatively riskless quantity of the hazardous good before the risk of dying from other causes becomes significant.[16]

The Relationship Between Age and New Information

The age at which the individual receives the information regarding the hazard clearly plays a significant role in the model developed here. To examine this factor further we will restrict our attention to the case where the cumulative effects do not decay ($\delta=0$). In this case consider the phase diagram depicted in Figure 4.2. Here the cumulative consumption X is graphed as a function of age t. Until the consumer receives the information, his cumulative consumption X rises along the line $X = \bar{x}(t-\bar{s})$. Once he receives the information, his consumption rate is reduced, causing his cumulative consumption to rise less rapidly. Note that in all cases, if the individual lives to age T, his consumption rate (as reflected by the slope of the curve) will have risen to the preannouncement rate $\bar{x}$. Further if he receives the information later in life than another individual, his cumulative consumption at each age t will never be less than that of the other person.[17]

The rate of consumption x over life is perhaps of more interest than cumulative consumption X. To analyze the effect of age at the time of the announcement on x consider two optimal consumption plans x (and corresponding X) and x' (and its corresponding X') over the same age span $s < t < T$ which differ only because of different net cumulative consumptions at s. In particular, suppose that the first individual has accumulated consumption of Y at age s and the second has accumulated Y' at age s with $Y < Y'$. Elementary properties of the optimal consumption path dictate that $X \leq X'$ for every age t. Further, $X < X'$ at age t, unless $X = X'$ for the remainder of life.

In Appendix A it is shown that under these conditions the lifetime consumption rates for the two individuals will also consistently reflect their relative initial positions, that is, if $Y < Y'$ at age s, $x > x'$ at every age t, $s < t < T$ unless $x = x'$ over the last part of life.

Figure 4.3 depicts the lifetime consumption rates for like individuals who receive the hazard information at various ages. Note that the consumption rate at any age is never larger for a person who received the information later in life. This follows directly from the analysis in Appendix A since his cumulative consumption at any age s will be larger than that of the person who received the information earlier. Intuitively this result is attributed to the fact that because of the increasing nature of the hazard, the individual who receives the information later in life faces a higher price for marginal consumption at every age. As a result, he consumes less than his earlier informed counterpart.

This relationship between consumption rates and age at the time of the announcement can have a significant effect on aggregate statistics. For simplicity, suppose the age profile of the population is constant over time. Then at the time of the hazard announcement, individuals would adjust their consumption as a function of age. This initial cross-section profile for individuals with the same utility function and income is depicted in Figure 4.3 by the dashed curve connecting the initial points of the individual consumption paths.

Figure 4.4 represents the changes that will take place in this cross-section profile as time passes and more of the population has known about the dangers of consumption from an earlier age. Note that in the long run (after $T - \bar{s}$ years) the cross-section profile will coincide with the lifetime behavior of an individual who knows of the hazard from the start of adult life. However, until $T - \bar{s}$ years have passed, the cross-section profile will be continuously changing as the individuals who have consumed nonoptimally for the longest period die. After $s - \bar{s}$ years, the cross-section profile will coincide with the long run profile for individuals aged less than s but will deviate for individuals older than s (see Figure 4.4.a).

If the age profile of the population is not changing over time and utility functions and income are constant, these results have a well-defined effect on aggregate statistics. Once the information has been received by individuals, there should be an initial drop in the average consumption rate. As time passes, the average consumption rate should rise until reaching an equilibrium (after $T - \bar{s}$ years) which is less than the pre-announcement rate $\bar{x}$. This erosion of the initial reduction simply reflects the smaller average reaction required to compensate for past nonoptimal behavior.

The same phenomenon should be observed for each age group. That is, if the consumption rate is measured each year for individuals aged s_i, the initial drop in the consumption rate should tend to erode over time until reaching an equilibrium after $s_i - \bar{s}$ years. Thus if individuals are correctly perceiving the hazard information, cross-section statistics should quickly become stable for young adults but remain "transitory" for older adults for a longer period of time.

These results have all assumed that the age profile of the population is constant over time. If the profile is changing (as it is in the U.S.), even in the long run aggregate statistics should reflect movement attributable to the differences in consumption rates which are optimal for individuals of different ages. The direction of the aggregate movement depends on both the nature of the population shift and the parameters of the particular hazard considered.

Finally, it should be noted that while it is true that lifetime consumption rates for an increasing hazard should never be monotonically decreasing over life, as an empirical matter they may be in the relevant range. Since a controlling parameter in the analysis is the <u>maximum</u> attainable age T, the rise in consumption rates may occur after the age at which statistics are normally reported. In this case, both individual lifetime consumption rates and cross-section consumption rates may be decreasing as a function of age in the reported range.

Consumption of a Decreasingly Hazardous Good

In this section, we will briefly consider hazards where the probability of death from consumption decreases with past consumption. Since the analysis required for this case is quite similar to that for the previous two cases, it will not be repeated. Instead, the basic results will be outlined and the source of differences indicated.

As in the case of the constant hazard, decreasing hazards should be characterized by a lifetime consumption path that is monotonically increasing. Since further consumption is always less riskly than past consumption, this force acts in the same direction as the "age effect" (rather than in the opposite direction as in the case of an increasing hazard.) Thus both forces cause consumption to increase with age.

The decreasing hazard differs from the constant hazard, however, in that the long run effect is not achieved immediately. Because of the dependency on past consumption, the individual's consumption path varies with his age at the time of the announcement. As in the case of the increasing hazard, aggregate consumption statistics will change over time until the full adjustment has taken place (after $T - \bar{s}$ years). However, in contrast with the increasing case, these aggregate statistics will be falling over time, reflecting the fact that past nonoptimal consumption has put individuals (who have survived!) in a more favorable current position. As new members enter the population, they will consider the hazards associated with the initial consumption and tend to reduce average consumption rates.

5. Hazardous Goods With a Delayed Effect

In the previous two sections it was assumed that the effects of hazardous consumption were instantaneous. This assumption is admittedy a strong one for many of the health hazards of current concern. Goods which have been linked to cancer, for instance, are typically characterized as having a rather long incubation period. Recent disclosures on the hazards of asbestos and nuclear radiation indicate incubation periods in excess of 20 years.

In this section we will modify the previous model to consider the case where the hazard's effects are felt only after a specified incubation period of length θ. The delayed effect of the hazard significantly changes rational consumption decisions. In particular, we will show that the delay in the hazard's effects reduces the reaction to new information about a hazard. During the period between consumption of the hazard and its effects, the consumer may die from other causes or from the effects of past consumption. If consumption is fatal, the stream of utility which is lost is θ years shorter than in the instantaneous case. Both of these factors reduce the expected costs faced by the consumer and hence cause him to increase current consumption when compared with the instantaneous case. In particular, this effect causes consumption to rise to the pre-announcement level much earlier in the life cycle.

Since there is a delay of θ years between consumption of the hazard and its possible effects, the probability $P(t)$ of not having died from consumption at age t, given that the individual is alive at age s, is given by

$$\dot{P}(t) = P(t)f(X(t-\theta))x(t-\theta) \tag{5.1}$$

$$P(s) = 1 \tag{5.2}$$

Note that it is consumption θ years ago (at age t-θ) that affects the probability of death at t (compare with (4.1)). Thus, the intuitive notion embodied here is typified by a model of the hazards of cigarette smoking where consumption today is viewed as triggering cell changes that will result in lung cancer 20 years from now.

For notational simplicity, we define the lagged probability variable

$$Q(t) = P(t + \theta).$$

At age s, the consumer's problem is to maximize lifetime utility

$$J_s(X,Q,x;Y(t),s-\theta<t<s) = \int_s^T \frac{Q(t-\theta)p(t)}{Q(s-\theta)p(s)}U(x(t))dt \tag{5.3}$$

where

$$\dot{X}(t) = x(t) - \delta X(t) \tag{5.4}$$

$$\dot{Q}(t) = - Q(t)f(X(t))x(t) \tag{5.5}$$

$$X(t) = Y(t) \qquad s-\theta < t < s \tag{5.6}$$

$$Q(s-\theta) = 1 \tag{5.7}$$

The history of consumption given by (5.6) determines the value of Q for θ years; in particular,

$$Q(t) = \exp \{-\int_{s-\theta}^{t} f(X(t))x(t)dt\} \qquad s-\theta < t < s \tag{5.8}$$

Thus, the consumer's history of consumption for the past θ years enters into his current consumption decision. This is the critical feature of the model. While the consumer can alter his consumption in the next θ years, he cannot change his probability of death in those years. This has already been determined by his past consumption of the hazard. Thus, the gains from reducing consumption today will be realized only *if* the individual lives more than θ years into the future.

Equations (5.3) - (5.6) and (5.8) constitute a well-defined delayed control problem (see Pontryagin, or Kharatishvili). The related Hamiltonian function[18] is

$$H(t,X(t),Q(t),Q(t-\theta),x(t),\lambda(t),\gamma(t)) = \frac{Q(t-\theta)p(t)}{Q(s-\theta)p(s)}U(x) + \lambda(x-\delta X) - \gamma Q(t)f(X)x \tag{5.9}$$

where γ and λ are auxilliary functions chosen to satisfy

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial X} = \delta\lambda + \gamma Q f'(X)x \quad \text{for } s < t < T \tag{5.10}$$

$$\dot{\gamma}(t) = -\frac{\partial H}{\partial Q} - \frac{\partial H}{\partial Q(t-\theta)} = -\frac{p(t+\theta)}{p(s)}U(x(t+\theta)) + \gamma f(X)x \quad \text{for } s < t < T-\theta \tag{5.11}$$

$$\dot{\gamma}(t) = -\frac{\partial H}{\partial Q} = \gamma f(X)x \qquad \text{for } T-\theta < t < T \tag{5.12}$$

$$\lambda(T) = 0 \tag{5.13}$$

$$\gamma(T) = 0 \tag{5.14}$$

The Maximum Principle for delayed problems insures that the optimal consumption path will be characterized by

$$\frac{Q(t-\theta)p(t)}{Q(s-\theta)p(s)}U'(x) = -\lambda + \gamma Q f(X) \tag{5.15}$$

except where $x = 0$ and inequality ($<$) holds in (5.15).

To analyze the optimal consumption path, we show first that near the end of life, the hazards of consumption should be ignored, that is, that the shadow price of consumption is zero. To establish this fact, we demonstrate first that the direct mortality component of the shadow price is zero during the last θ years of possible life, that is,

$$\gamma(t) = 0 \quad \text{for } T - \theta < t < T \tag{5.16}$$

Suppose alternatively that $\gamma(t) > 0$ near T. Then by (5.12), $\dot{\gamma}(t) > 0$ near T. But this contradicts (5.14). Similarly if $\gamma(t) < 0$ near T, then $\dot{\gamma}(t) < 0$ near T which also contradicts (5.14). Thus $\gamma(t) = 0$ in some neighborhood of T. The argument can be repeated until (5.16) is established in the entire range for $T - \theta < t < T$.

Using (5.16) in (5.10), we have $\dot{\lambda} = \delta\lambda$ for $T - \theta < t < T$. But then using the same type of argument as above, we have

$$\lambda(t) = 0 \qquad \text{for } T - \theta < t < T. \tag{5.17}$$

Using (5.16) and (5.17) in (5.15), it is clear that $U'(x) = 0$ for $T - \theta < t < T$ and hence that

$$x(t) = \bar{x} \qquad \text{for } T - \theta < t < T \tag{5.18}$$

Thus in contrast with the instantaneous models, consumption rates for the delayed hazards should rise to the pre-announcement rate $\bar{x}$ much sooner. Hazards which take θ years to have an effect become harmless if there are fewer than θ possible years remaining in life.

Using these results, it is easy to verify that the two cost components of consumption are

$$\gamma(t) = \int_{t+\theta}^{T} \frac{P(t')p(t')}{P(t+\theta)p(s)} U(x(t'))dt' \qquad \text{for } s < t < T-\theta \qquad (5.19)$$

$$\lambda(t) = -e^{\delta t}\int_{t}^{T} e^{-\delta t'} x(t')P(t'+\theta)f'(X(t'))\gamma(t')dt'$$

$$\text{for } s < t < T-\theta \qquad (5.20)$$

$$\gamma(t) = \lambda(t) = 0 \qquad \text{for } T-\theta < t < T \qquad (5.21)$$

Note that as in the instantaneous case, γ represents the direct mortality cost of marginal consumption, and λ represents the cost of the increased dangerousness of all future consumption (for increasing hazards) caused by marginal consumption. As is evident from a comparison of equation (5.19) with equation (4.12), the costs of consumption for a delayed hazard are substantially lower than for an instantaneous one. Two factors are responsible for this reduction: first, if consumption is fatal, the number of years lost is θ years less for the delayed hazard (note the limits of integration in (5.19)), and second, the likelihood of incurring this loss is reduced by the probability that the individual will die before age $t + \theta$ for other reasons (either unconnected with the hazard or due to the hazard's lagged effects.) These two factors substantially reduce the cost of hazardous consumption and hence reduce the response to hazard information.

Differentiating (5.15) and using (5.10) and (5.11) we can establish the fact that when positive consumption is optimal, it must satisfy

$$U''(x)\dot{x} = (\delta - \frac{\dot{p}}{p} - \frac{\dot{P}}{P})U'(x) - \frac{\delta\gamma p(s)Q}{p(t)Q(t-\theta)}(f(X) + Xf'(X)) \qquad (5.22)$$

$$- \frac{Q(t)p(t+\theta)}{Q(t-\theta)p(t)} f(x)U(x(t+\theta)) \qquad \text{for } s < t < T - \theta$$

At young ages (s near $\bar{s}$), there are no lagged effects altering the consumption decision so that $Q(t-\theta) = 1$, $x(t-\theta) = 0$ initially. Thus

$$U''(x)\dot{x} = (\delta - \frac{\dot{p}}{p})U'(x) - \delta\gamma\frac{p(s)}{p(t)}Q(f(X) + Xf'(X)) \tag{5.23}$$

$$- \frac{Qp(t+\theta)}{p(t)}f(X)U(x(t+\theta)) \quad \text{for } \bar{s} < t < \bar{s} + \theta$$

As in the case where the hazard has an instantaneous effect, delayed increasing hazards which are not very dangerous initially will be characterized by a consumption path that decreases in early years of consumption. This follows directly from the fact that when $f(X)$ and X are small, $U''(X)\dot{x}$ is positive by (5.23) and hence $\dot{x} < 0$. Thus, the delay in the hazard's effects does not alter the U-shaped character of the optimal consumption path for an increasing hazard.

The previous results regarding the movement of aggregate consumption statistics are altered, however, by the delayed effect. In particular, it is no longer true that the adjustment should be immediate in the case of constant hazards ($f(X) = k$). Since the effects of past consumption will be felt for θ years <u>after</u> receipt of the information, individuals should be expected to consume more upon hearing of the hazard than they would have if they had been consuming at the optimal (lower) rate.[19] Thus as the proportion of individuals who have consumed non-optimally becomes smaller, aggregate statistics should fall after the announcement reaching long term equilibrium after $T - \bar{s} - \theta$ years.

The effect of the delay on aggregate consumption statistics, then, is to introduce a negative factor into the forces we have previously identified. This does not affect the qualitative results for decreasing hazards - aggregate consumption statistics should fall over time. However, for increasing hazards it introduces an ambiguity. If the hazard is increasing fast enough or the length of the delay is short enough, then the aggregate statistics should fall upon announcement of the hazard but then increase until reaching equilibrium in $T - \bar{s} - \theta$ years. On the other hand, if the hazard does not increase rapidly or the delay is long enough, the aggregate statistics should have a discrete drop upon announcement of the hazard followed by a U-shaped or monotonically decreasing movement for the next $T - \bar{s} - \theta$ years.

6. Conclusion

This paper has presented a model of the individual's decision to consume a hazardous good. The individual is viewed as receiving a certain utility from consumption of the good which costs him the sum of the purchase price and the expected life cost. A primary focus of the paper has been to emphasize several factors which affect the expected life cost of consumption and to analyze how these factors should change consumption behavior over time.

One such factor is the nature of the hazard. In particular, the timing of a hazard's effects is shown to be an important determinant of the reaction to information about hazards. A safety hazard, such as failure to use a seat belt or occupation as a construction worker, is essentially different from smoking or exposure to asbestos -- the effects of the safety hazard are immediate, the cancer-causing agents take years to have an effect. The rational reaction to the seat belt hazard is shown to be substantially greater than to an equivalent cancerous agent (that is, one with the same probability of death).

The cumulative nature of the hazard is another important factor in predicting the reaction to hazard information. The failure to use a seat belt might be regarded as a constant hazard (there is a fixed probability of being killed per mile), but dietary hazards may be better characterized as increasing hazards (e.g. each additional gram of salt consumed has an increasing probability of causing a fatal heart condition.) The optimal consumption behavior in each case is again different. This paper has focused on the cumulative effects of a hazard--whether it is increasingly/decreasingly dangerous or not--and on the timing of the effects--whether death is immediate or after an incubation period. Many other characterizations are possible, but the general point is clear: the nature of the hazard may play a critical role in determining a rational response to information regarding the hazards of consumption.

Another primary factor in the consumption decision is age. The cost of consumption clearly depend on the expected number of years remaining in life. This factor (in isolation) causes consumption to rise with age so that if an individual lives to be old enough, his consumption rate should approach what it would have been if there were no hazard to consumption.

The expected age of death also plays a critical role in determining rational consumption of hazards. An individual who expects to die earlier than normal (for genetic reasons, say) faces a lower cost for hazardous consumption and should consume more of every hazardous good. Thus, rational behavior would tend to exaggerate any natural differences in expected life. This natural selection bias has implications for attempts[20] to measure the value of life from observed consumer behavior. It is widely recognized that isolation of a single risk-taking activity may serve only to measure risk-taking behavior in general. However, the result presented here illustrates that even if we could control for all risk-taking activity, rational behavior by consumers would exaggerate the cost of hazardous consumption. Those who expect to live longer would cut consumption of all hazardous goods and vice versa. Unless the issue of expected life is addressed directly, the estimated cost of hazardous consumption will be high because it will, in part, reflect this genetic difference. The corresponding value of life estimated from consumption activity will also be somewhat low.[21]

Another important feature of this analysis is that the age of death is not treated as an exogenous variable. The uncertainty of life itself acts to increase consumption of hazardous goods and to change the pattern of life-cycle consumption. If the age of death from "other causes" were known with certainty, consumption of even an increasingly hazardous good would rise monotonically over life.[22] However, when life is uncertain and the conditional probability of death is increasing with age, consumption is higher in early years and is characterized by a period of decrease before rising again later in life. Thus, when confronted with the fact that younger consumers are often less concerned with dietary hazards or smoking only to "come to their senses" in a few years, our analysis would not necessarily attribute this to irrational behavior; rather it is consistent with perfectly rational consumption.

While life cycle consumption of hazards is interesting in its own right, probably a more pertinent question in this era of ongoing health research is how should consumers react to new health information regarding hazards. It is often the case that consumers are not aware of the hazards of consumption until after they have been consuming the hazard for a number of years. For the simplest type of safety hazard--a constant and instantaneous hazard like failure to use a seat belt--this past ignorance should not inhibit an immediate move to a new equilibrium. That is, if consumers are rational and if consumers accurately perceive the hazard involved, they should quickly adjust their consumption of the hazard to achieve a new long-term equilibrium.

For other hazards, the situation is not as simple. Individuals who in ignorance have been consuming an increasing hazard in too large a quantity will find themselves in a relatively disadvantageous position upon announcement of the hazard. As a result, our analysis shows that rational consumption behavior will require them to reduce their consumption far more than if they had been consuming optimally since the start of life. In contrast, new individuals entering the population will consume optimally from the start of life and will therefore be at a better relative position at every age. Thus for increasing hazards which do not have much of a delay in their effect, aggregate rational consumption[23] should be characterized by a drop in the consumption rate upon announcement of the hazard followed by a gradual increase until stabilizing once the population has been completely replaced. Further, statistics for young age groups should stabilize much sooner than for older age groups.

The reaction to new information regarding cancer-causing hazards is more difficult to predict. If hazards like smoking can be characterized as increasing hazards with a delayed effect, then there are two opposing forces affecting aggregate consumption statistics. The phenomenon described above should cause statistics to fall upon announcement of the hazard and then rise somewhat before stabilizing. However, the delay in the development of cancer has the opposite effect. Those who have been consuming an excessive amount of the hazard before the announcement know that there is a probability that they have already triggered the development of cancer in the next (say) 20 years. Since their chance of dying in these years is therefore higher than otherwise, their cost of consuming the hazard is less. This factor causes these individuals to consume more of the hazard than otherwise. As these individuals are replaced in the population by those who have consumed optimally from the start of life, consumption rates have a tendency to fall. Which of these forces dominates is therefore an empirical matter, but in general, a rational reaction to information regarding cancer-causing goods should be characterized by an immediate drop in consumption followed by additional movement (up if the delay is short and the hazard is an increasing one or down if the delay is long) until the population has been totally replaced. However, rational behavior should again be characterized by the fact that statistics for younger age groups should stabilize sooner than for older age groups.

To be sure, this analysis has ignored many of the factors which affect consumption decisions in the light of new information on hazards. We have considered only the mortality costs of such consumption and we have not dealt with any addictive or habit characteristics of consumption. Nevertheless, by simplifying the analysis, this model of consumption gives an initial understanding of some of the qualitative characteristics of rational consumption behavior and thus provides a necessary first step in the understanding required to test the rationality of consumer response to health information and ultimately assess the value of public sponsored programs to provide such information.

Footnotes

[1] This view of the consumption decision is closely related to the Schelling (1968) and Mishan (1971) arguments that the relevant benefit measure of an activity which reduces the probability of death is reflected in the sum of individuals' willingness to pay for such a reduction.

[2] One approach to this problem is to adopt the device used by Cropper (1977) where illness is modeled as a disruption in the individual's utility stream. In this case, a lifetime illness becomes equivalent to death and our analysis would carry over with the probability of death replaced by the probability of illness. However, this device seems extreme except for serious illnesses.

[3] That is, the rate of consumption before the hazard is known.

[4] More formally, a constant hazard is defined as one for which the conditional probability density function is constant. Loosely speaking, this can be interpreted as a constant probability of death if consumption is increased marginally, given that the individual is currently alive. In a discrete model, the equivalent assumption would be that each unit of consumption has a constant probability of causing death.

[5] Note that our previous assumptions imply that consumption prior to the announcement would be constant at $\bar{x}$.

[6] Work by Luce and Krantz (1971) and Jones-Lee (1974) has shown that the individual acts as an expected utility maximizer in circumstances like ours.

[7] The probability of not having died from consumption of the hazardous good at $t + \Delta t$ depends on two independent events: that the individual has not died from consumption by t and that he does not die from consumption during $(t, t+\Delta t)$. Thus,

$$P(t+\Delta t) \sim P(t)[1 - qx\Delta t]$$

or

$$\frac{P(t+\Delta t) - P(t)}{\Delta t} \sim -qP(t)x.$$

Taking the limit as $\Delta t \to 0$ and solving for $P(t)$ yields the stated form.

[8] See, for instance, Pontryagin (1962) or Hadley and Kemp (1971).

[9] Formulation of a threshold model involves the resolution of a technical matter which deserves mention. If an individual is assumed to have a fixed threshold over life, then the accumulated stock of the hazard should never be allowed to decay since this stock level has been proven to be safe for the individual. Thus consumption at the decay rate $x = \delta X$ represents no hazard to life.

An alternative formulation of the threshold model assumes that the threshold is random and subject to a specific density function at each instant t. This model would correspond, for instance, to circumstances in which random outside events (illness, weakness, recent good diet, etc.) make the individual more or less susceptible to the hazard by changing his threshold accordingly. A level of accumulation of the hazard which is not fatal today may prove to be fatal in the future. Thus, any consumption of the hazardous good has an associated probability of death; that is, there is no "totally safe" level of consumption (unless that level has always been safe.) It is only the degree of the hazard that is affected by the stock X.

The approach adopted here will focus on the second formulation since it is the more complex. The Cropper (1977) model of occupational choice was of the first type: a fixed threshold model. Only slight changes in the arguments presented here will be required to deal with the fixed threshold case and we will point them out as appropriate.

The appropriateness of each model depends on the nature of the hazard. If genetic factors play a significant role in the determination of susceptibility, then the fixed threshold model may be more suitable; the "unknown" here is whether one inherited the susceptibility or not and this is fixed for the individual. However, if there is significant uncertainty regarding the other factors which affect the individual's susceptibility, then a random threshold model may be more appropriate. An individual's threshold for alcohol, for instance, may depend on his weight, chemical balance, state of mind, physical condition and many other factors which vary substantially over life.

[10] Note that in a discrete model f(X) would represent the probability of death from consumption of an additional unit of the hazardous good if net cumulative consumption was X. However, in our continuous framework, f(X) corresponds to a marginal increase in consumption.

[11] For instance, we are ignoring the presence of other hazards, connections between hazards and other such complicating factors.

[12] The probability that the individual will not have been killed by the hazardous good at $t + \Delta t$ is equal to the product of the probabilities of two independent events: that he has not been killed by age t and that he is not killed in the intervening period. So

$$P(t + \Delta t) \approx P(t)\,[1 - f(X(t))x(t)\Delta t].$$

Taking the limit as $\Delta t \to 0$, we get (4.1).

In the case of a fixed threshold model (see Footnote 9), (4.1) becomes

$$\dot{P} = -Pf(X)\dot{X} \qquad (4.1')$$

where it can be demonstrated that $\dot{X} > 0$.

[13] Note that in the case of a decreasing hazard, if the individual does not die at age t, the marginal consumption will produce the beneficial effect of decreasing the hazard to further consumption throughout life. However, if the hazard is an increasing one (the case we consider in most detail), the hazard to further consumption will be increased.

[14] Recall that we have explicitly ruled out a subjective discounting function as well as any new information which would cause the individual to adjust his probability of living at a given age (beyond the factors considered here.) This allows us to avoid the problem of consistency (See, for instance, Strotz (1957) or Phlips (1974)).

[15] The potential for an increasing lifetime utility arises from the possibility that X(t) decreases significantly.

[16] Note that a subjective discount rate would simply augment this factor.

[17] This assertion follows from the simple argument that if the two cumulative consumption paths ever crossed, the piecewise optimality principle applied at that point would insure coincidence for the remainder of life.

[18] Note that whenever the argument is not explicitly stated, t is implied.

[19] Past behavior has increased the probability of dying in the next θ years and thus reduced the cost of additional consumption.

[20] See, for instance, Blomquist (1978) and Thaler and Rosen (1975).

[21] To be sure, there are many other potential biases affecting these estimates in either direction.

[22] Here we ignore the fact that the cumulative effects of the hazard may decay with time and alter this result somewhat.

[23] In this discussion, we assume the population age profile is not changing since this would have a corresponding effect on average consumption figures.

Appendix A

Proof That Lifetime Consumption Rates Consistently Reflect Initial Positions

Consider two like individuals of the same age s who have accumulated consumption of Y and Y' with $Y < Y'$. Our aim is to demonstrate that their consumption rates over the remainder of life will consistently reflect this fact; that is, that the first individual will find it optimal to consume more at every age, so that

$$x > x' \qquad \text{for } s \leq t < T$$

unless $x = x'$ over the last part of life.

From previous results, we know that $X < X'$ unless $X = X'$ over the latter part of life. Suppose that it is optimal for the second individual to consume at a greater rate than the first individual near the end of life, that is, assume

$$x < x' \quad \text{for } t \text{ near } T.$$

Then at these ages $X < X'$ and

$$\begin{aligned} U''(x)\dot{x} &= - f(X)[U(x)-xU'(x))] - \frac{\dot{p}}{p}U'(x) \\ &> - f(X')[U(x') - x'U'(x')] - \frac{\dot{p}}{p}U'(x') \\ &= U''(x')\dot{x}' \end{aligned} \tag{A.1}$$

It follows directly that

$$\dot{x} \leq \dot{x}' \quad \text{at } T. \tag{A.2}$$

But since $x < x'$ near T and $x(T) = x'(T)$, we must have

$$\dot{x} = \dot{x}' \quad \text{at } T.$$

But in this case (A.1) implies that $f(X) = f(X')$ at T and since the hazard is an increasing one, that $X = X'$ at T. This is a contradiction of our assumption that $x < x'$ near T, since $X < X'$ near T and $X = X'$ at T implies that $\dot{X} > \dot{X}'$ for t near T and therefore $x > x'$ near T. This establishes the proposition that

$$x \geq x' \quad \text{near } T. \tag{A.3}$$

Suppose now that $x = x'$ at some age $t < T$. Either the consumption paths are tangent at this point or they cross. For the tangent case, (A.1) will hold with equality and therefore $X = X'$ at t. In this case the paths would coincide from t to T. In the non-tangent case, (A.1) will hold strictly and the concavity of U implies

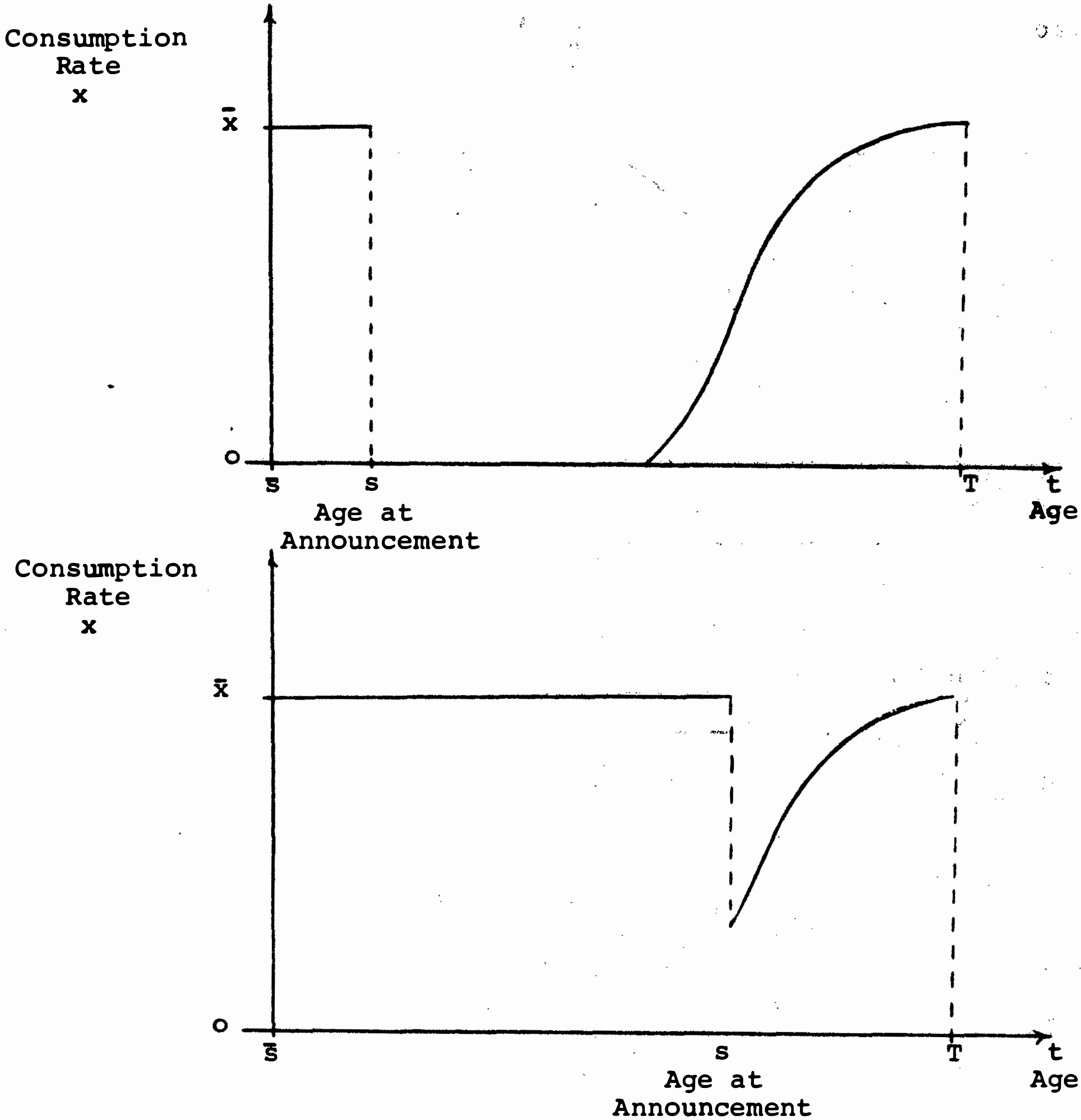
$$\dot{x} < \dot{x}' \quad \text{at } t \tag{A.4}$$

This contradicts the assumption that the curves cross.

Thus, we have demonstrated that

$$x > x' \quad \text{at every age } t, \text{ for } s < t < T$$

unless $x = x'$ over the last part of life.



<u>Figure 3.1</u> Lifetime Consumption Path For A Constant Instantaneous Hazard

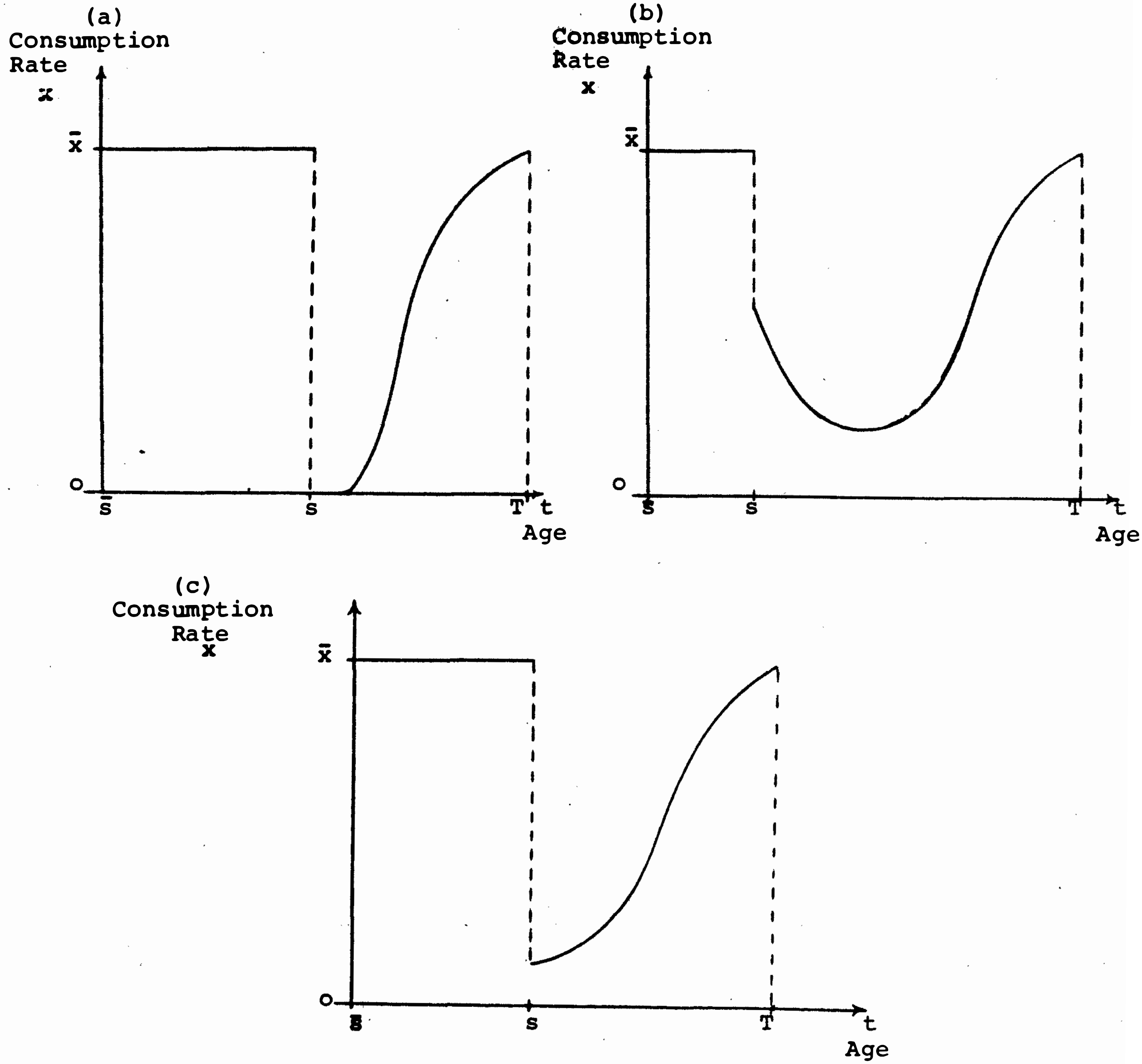


Figure 4.1 Lifetime Consumption Path for an Increasingly Hazardous Good -- Announcement at Age s.

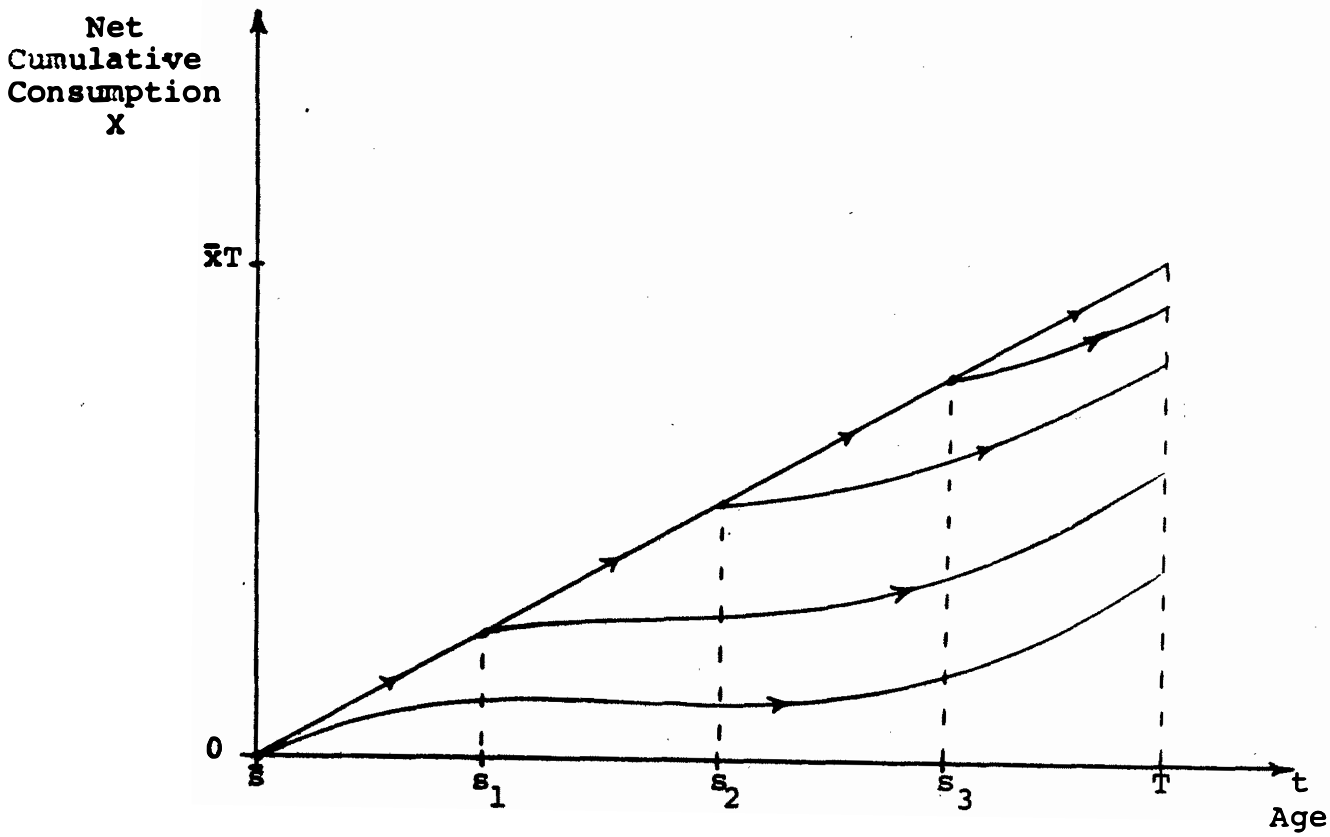
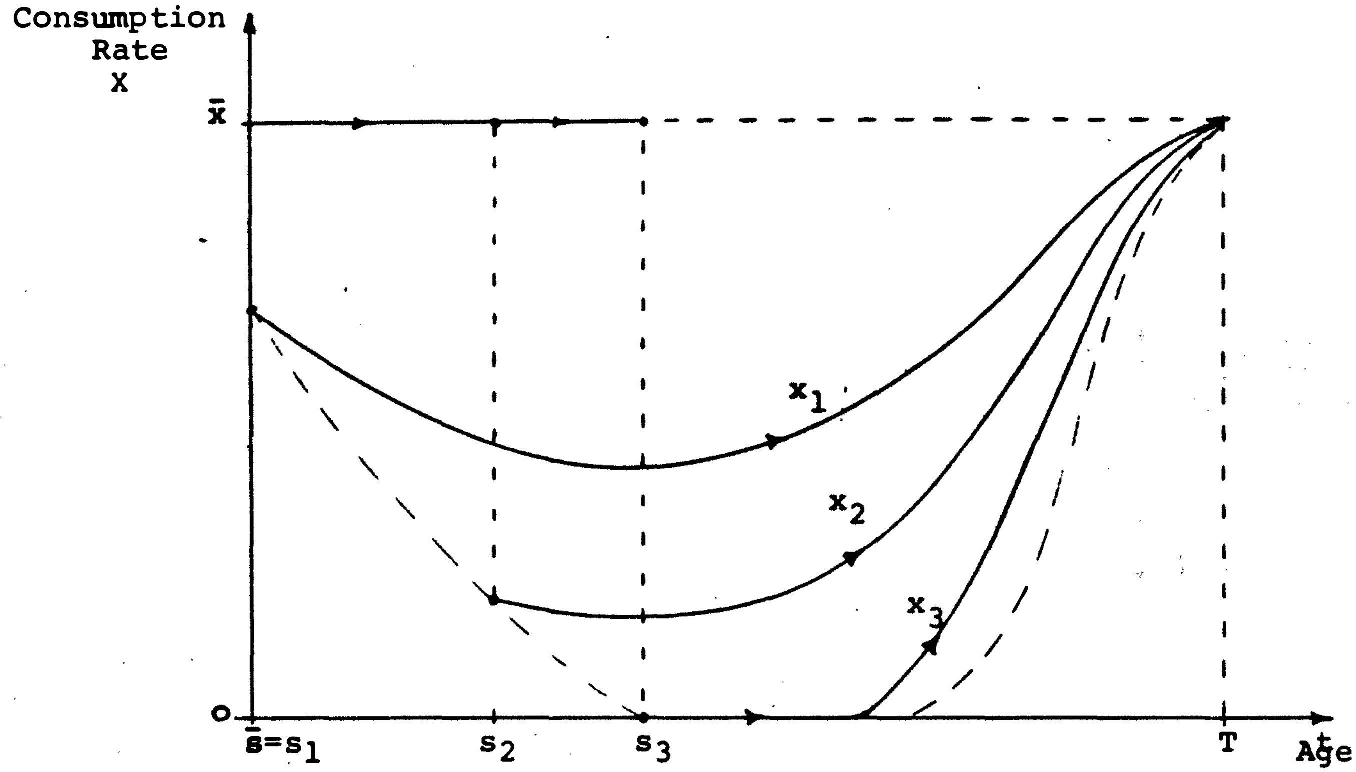


Figure 4.2 Phase Diagram Depicting Growth of Net Cumulative Consumption X: Linear path before announcement of hazard; Curved path after announcement depends on age s_i at time of announcement.



<u>Figure 4.3</u> Lifecycle Consumption Rates for Individuals Receiving Hazard Information At Age s_i.

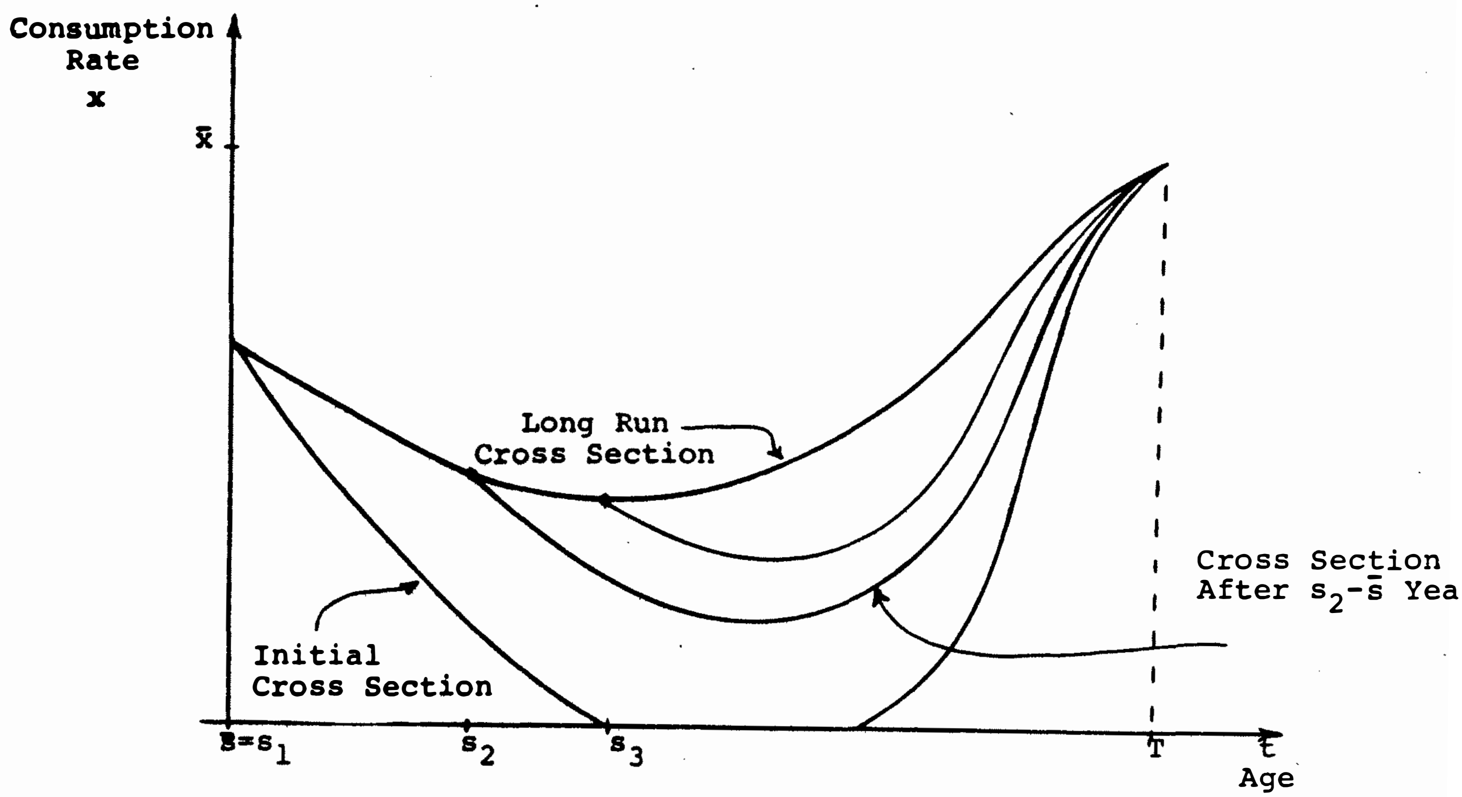


Figure 4.4 Cross-Section Consumption Rates After $s_i - \bar{s}$ Years.

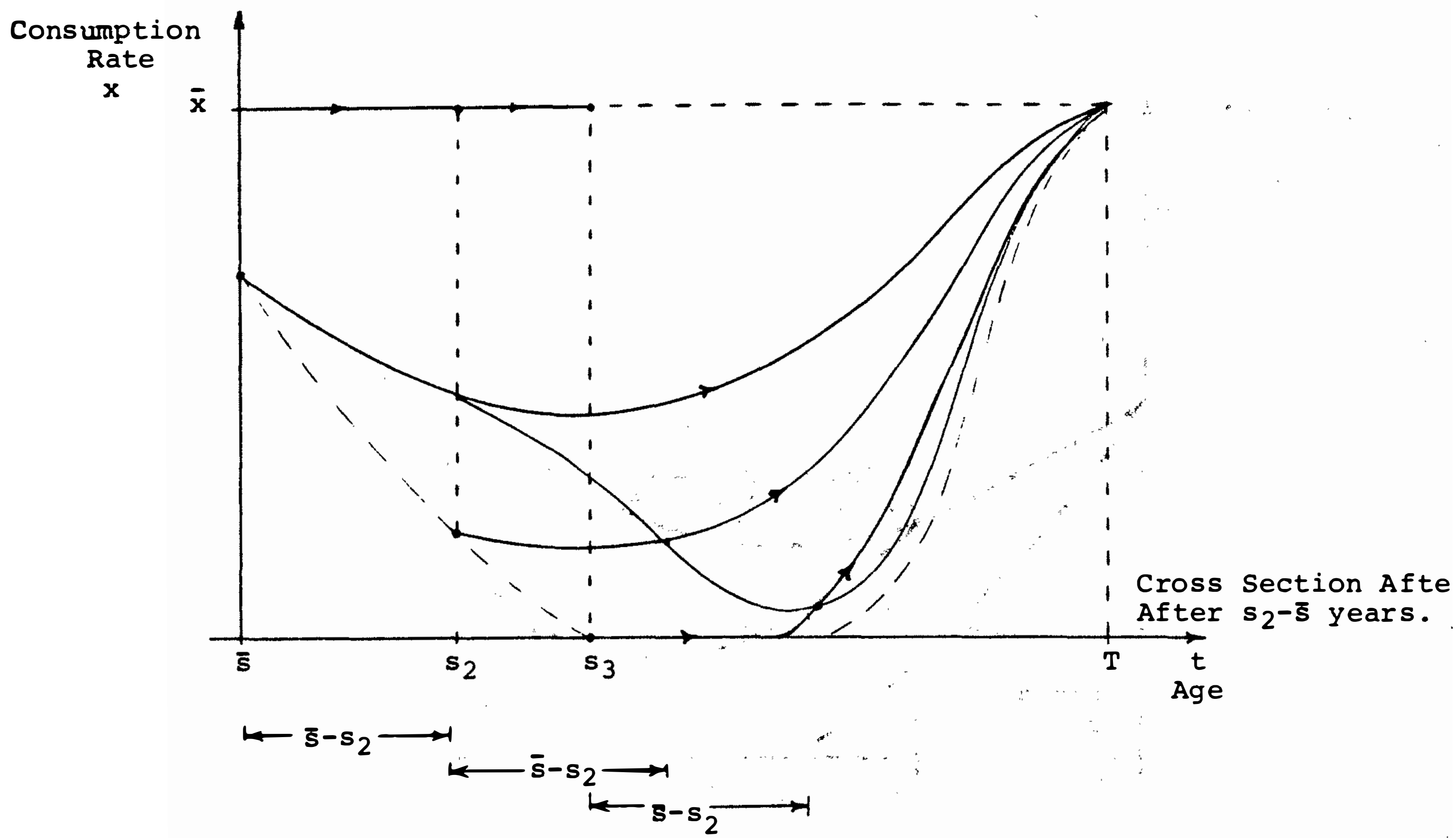


Figure 4.4 (a) Derivation of Cross Section Curves From Life Cycle Profiles.

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