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INTRA-FIRM SUBSIDIZATION AND REGULATION:

DO PROFITS COVER LOSSES, OR DO LOSSES JUSTIFY PROFITS, AND DOES IT MATTER?

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Introduction.

Cross-subsidization within regulated or public enterprises and private firms persists as a policy issue dispite yeoman efforts to eliminate it (McGee, 1958 ; Bailey, 1973). The best efforts erase all but the topic's Cheshire-like grin. This paper investigates the second part of the classic Averch-Johnson (1962) paper. In that section, A-J advance the unproven conjecture that a regulated firm may enter non-regulated markets and sell there at output levels where price in that market falls short of covering marginal costs incurred in that market. This paper provides a demonstration of the validity of this conjecture under both symmetric (mark-up over cost) and assymetric (rate-of-return) regulation regimes. The demonstration proceeds at the level of the firm by both analytical means and by producing examples of profit maximizing sub-marginal cost production. The paper also has some implications for the evolution of regulation. Regulation at the level of the firm can be thwarted by entry into new markets. As a result, regulatory bodies either must pass rules strictly limiting the regulated firm's product domain, or the regulators must become intimately involved in product line specific matters including product line marginal costs and demand elasticities. Recent development in telecommunications regulation (Brock, 1980) indicates how practically difficult such regulation of multi-product firms may actually become.

Analytically, following Baumol & Bradford's (1970) reformulation of Ramsey's (1927) tax problem, cross-subsidy arises in the context of regulated monopoly displaying scale economies when regulated prices fail to systemmatically reflect demand elasticity differences in the marginal cost-of-service.[1] The natural monopoly setting traditionally conducive to regulatory treatment requires either a direct subsidy, generated presumptively through the political process in some ongoing fashion, if prices are set equal to marginal cost, or with some set of also politically determined allocations of overhead to marginal cost prices. Though political in nature (Posner, 1971; Stigler, 1964, 1971, 1972; Peltzman, 1976; Jerrell, 1977; Lee, 1980) these mark-ups may still satisfy Ramsey conditions, though there is no reason to suppose they will.

Analyses of private cross-subsidization (deep pocketing

1. Unregulated, private multi-product firms also adjust output prices to reflect elasticity based differences in marginal costs, but for at least some product lines the elasticity mark-ups are too large. Ramsey pricing requires the elasticity mark-ups to be based on an explicit consumers' surplus maximization. This immediately involves the regulatory body in discussions of product demands and appropriate marginal costs. The Baumol-Bradford paper has been extended by Lee (1980) to explicitly include regulatory costs in the context of collusion prone industries with higher private policing costs than those incurred by a public industry oversight body.

predation) typically share one characteristic with public or regulatory analyses. Profits earned, or expected to accrue after successful predation, justify or cover current, short-run losses arising out of predatory behavior in some market(s).[2] Public cross-subsidization differs only in that it is on-going rather than short-run since the ultimate objective is something other than profit maximization. The sub-title of this paper is designed to provoke some dissussion of this approach to public cross-subsidization. We believe Averch & Johnson's conjecture is correct and that, at least expositionally, the modelling should proceed as if the problem were one of internalizing an externality. The externality, under simple regulation at the level of the firm, is simply that entry into new market, entailing expenditures on productive factors that enter the overall regulatory constraint, permits an initially effectively regulated firm to regain

2. Salop (1981) contains a recent treatment of private predation. Public and private models apparently differ on information assumptions. Public cross-subsidization need not imply imperfect knowledge. The whole point, in fact, is to find prices for output(s) that reward political or regulatory allies and covers any losses on such sales by finding other prices that recover more than cost from other, less favored, groups of buyers. Private predation, if it occurs, seems *ab initio* to *require* imperfect knowledge. Potential predators must believe predation can succeed, and their victims - to have entered or remained in the contested territory - must believe predation will not be tried or that it will fail.

some of the net quasi-rents taken away by public spirited control. The model does not hinge upon price discrimination, and, as explicitly noted, the firm maximizes profits, rather than revenues or sales.[3] Showing that entry into new markets tends to circumvent simple regulation and may lead to sub-marginal cost pricing has some moment for current (de)regulation debates. Faulhaber's (1975) excellent examination of the generalizability of natural monopoly demonstrated that the multi-product 'natural monopolist' may need extra-market protection from non-innovative, welfare reducing entry. This possibility, however, together with the initial regulatory circumventing possibility of new market entry, creates an incentive for firms to mimic conditions where natural monopoly is not sustainable without protection from entry. Even if such aping is not exact, the multi-product firm using technologies displaying common costs may thwart simple regulatory attempts to make regulation effective by limiting or scrutinizing new market entry.

Recent Department of Justice-AT&T consent negotiations provides one example of efforts partially designed to reduce purportedly inefficient cross-subsidization by injecting some market competition.[4] Attempts to deregulate transport (Railroad Revitalization and Regulatory Reform Act, 1976)[5] may abet new competitive struggles between integrated, multi-mode,

3. Kafoglis (1969) showed that revenue or sales maximizing single product regulated firms product beyond p=mc.
4. Contrasting views regarding the efficacy of antitrust and regulation in telecommunications are given by Brock (1980) and Phillips (1982).
5. At this point it is not clear that transport regulatory reform has cut public control over rates and routes much at all.

the stand alone, single-mode transport companies long fostered under regulation. The issue of cross-subsidization may dim, but it will not disappear so long as some firms or industry operations remain subject to regulation and firms provide multiple services from common capacity. Recently, for example, the railroads newly won rights to adjust coal rates ran into strong opposition from a coalition of regulated coal-burning utilities, public service commissions, and the Department of Energy. These groups contended that energy (coal) was improperly singled out for rate increases in order to subsidize the transportation system by bearing an 'unfair' overhead allocation. Both of these examples involve highly integrated, partially regulated, multi-product firms operating in at least partly competitive (contestable) markets.

This paper suggests that sales below marginal cost by profit maximizing firms in a partially deregulated environment may more commonly occur than is commonly supposed. This result can be found whether or not the regulated multi-product firm employs a technology with joint or common costs. This untoward possibility demands neither more and better regulation <u>or</u> complete deregulation and with it an influx of possibly welfare cutting antitrust monitored competition. As hard as it may be at this stage of our understanding of industrial processes and market forces where regulation already exists, the immediate solution is some more or less amicable blend of public regulation (deregulation) and antitrust oversight. Industrial organization specialists understand the validity of contestability propositions. Products may be produced under scale and scope economies without material entry barriers (Buchan & Siegfried, 1978). Yet under some conditions (Willig, 1979, 1980), deregulation and complete reliance on antitrust may eliminate desireable natural monopolies. As yet we do not know how commonly satisfied the assumptions underpinning these analyses are in practice. The territory remains nearly empirically virgin if not theoretically so.

The remainder of the paper investigates regulation-induced sub-marginal cost pricing under, first, symmetric mark-up-over-cost control and then under rate-of-return regulation. The last section of the paper contains two numerical examples.

Most commonly cited regulatory models simplify firms by asserting single product production.[6] This formulation very effectively restricts attention to factor combination and input distortion issues and questions of pure 'gold-plating at a given level and scope of output. Such models tend to overstate the changes in output and factor mix regulation creates.[7] This is nothing more than an application of the famous LeChatelier-Samuelson principle.

6. Cowing & Stevenson (1980) provides a notable exception.
7. Empirically, Baron & Taggert (1977) found that regulated firms are under-capitalized. They explain this result by adding a financial market feed-back loop to simple rate-of-return regulation. However, entry into new product markets also tends to reduce the predicted A-J input bias.

Symmetric regulatory contraints.

The possibility for sub-marginal cost pricing for some products by a regulated firm does not depend upon profit regulation based asymmetrically upon one or a subset of a firms inputs or costs of operation. This section shows that sub-marginal cost pricing accompanying inefficient entry may arise under pure mark-up over cost regulation.

Let the unregulated monopoly choose a product vector, $y=(y_1,\ldots,y_n)$, from the profit maximization condition:

$$(1) \qquad \underset{y_n}{\pi\max}\ (y_1,\ldots,y_n) \geqslant \underset{y_k}{\pi\max}\ (y_1,\ldots,y_k) \qquad k=1,2,\ldots,n,n+1,\ldots$$

The firm's management selects product lines in which to operate and markets in which to sell, in general, based upon demand interactions, production interrelationships (scope economies), and marketing and managerial (decision-making) costs related to the number of product lines offerred. The initial model in simplest form assumes the following about non-monopoly products and markets: (a) there are no production interrelationships, i.e., no (dis) economies of scope; (b) non-monopoly products and markets are competitive; and (c) entry into each competitive market requires payment of a lump-sum fee related, possibly, to managerial cost burdens associated with gearing-up to make decisions and process information for a more complex firm. The latter assumption is necessary in this model to explain (describe) why the firm operates in some, but not all, markets. Without this assumption the phrase 'America, Inc.' takes on a new possibility.

Given (1), the unregulated firm solves:

$$(2)\quad \underset{y_n}{\text{Max}}\ (y_n' \cdot g_n(y) - C(y_n);\ y_n \geq 0_n)$$

where g_n represents a vector of inverse demand functions defined over y_n. The natural set of first order Kuhn-Tucker conditions:

$$(3.1)\quad g_n(y) + \nabla_y g(y) \cdot y - C_y \leq 0,\ y_n \geq 0_n$$

$$(3.2)\quad \text{diag}(g_n(y) + \nabla_y g(y) \cdot y - C_y) \cdot y = 0$$

where diag($\cdot$) is a diagonal n x n matrix containing on the main diagonal the elements of the column vector within the brackets. Usually the vector product $\nabla_y g_n(y) \cdot y_n \leq 0$ in equilibrium (Diewert, 1980).

Imposing regulation mandates that the firm must earn revenues equal to or less than some regulatory constant multiplied by total cost. The firm now solves:

$$(4)\quad \underset{y_n}{\text{Max}}\ (y_n' \cdot g_n(y) - C(y_n);\ \text{s.t. } d \cdot C(y_n) - y_n' \cdot g_n \geq 0;\ y_n \geq 0_n)$$

The new first order conditions are:

$$(5.1)\quad (g(y) + g(y) \cdot y_n)(1-h) - C_y \cdot (1-d \cdot h) \leq 0,\ y \geq 0_n$$

$$(5.2)\quad d \cdot C(y) - y_n' \cdot g_n(y) \geq 0,\ h \geq 0,\ h \cdot (d \cdot C(y) - y_n' \cdot g_n(y)) = 0$$

EQ(5.1 & 5.2) are complete if we assume an interior solution for y_n.

We can show, following Bailey (1973, p. 31), that $0 \leq h \leq d^{-1}$.

By the same technique, the firm will not engage in wasteful acts if $g(y) + g(y) \cdot y_n > 0_n$.[10] Comparing (3.1) and (5.1) shows that the regulated firm produces more of every product than does its unregulated counterpart, when

10. Policies aimed at detection and penalization of wasteful behavior can induce regulated firms to produce where demand is inelastic. The ability to detect blatant waste and disallow it, implying a lower feasible value for d, has implications for the extent to which a regulated firm will choose to enter new markets even though doing so implies sales at less than marginal cost.

the service mix is held fixed.

Simplifying, assume regulated firms produce one service prior to regulation's imposition. Entry into any new, and competitive, market entails payment of a fee. For simple regulation to be effective some device, like the hypothesized entry fee, is necessary. Without the fee, the regulated firm could always attain the price vector, $p = (p_m; p_1^c, p_2^c, \ldots, p_k^c)$, where p_m is the profit maximizing monopoly price and p_j^c is the established competitive price in market j, by repetitive costless entry at minimum efficient scale into an arbitrary number, k, of markets. The entry fee that limits free entry may flow in real life from managerial costs tied to enhanced decision-making complexity, legal limitations on market entry, or some institutionally fixed combination. Our purpose is not at this stage to adequately explain how and why firms, in specific cases, select or reject markets and products. That topic is important. However we merely want to point out its importance for both effective simple regulation and for simple modelling of sub-marginal cost profit maximizing pricing.

Assume the regulated firm pays a fee, f(n), related monotonically to the number of product lines, n, offerred. Assume for the single product firm that f(0)=0=f(1). For simplicity assume f(3) is prohibitively high. The firm chooses exit, the monopoly product cluster, or the monopoly and first competitive product clusters. Having paid the fee, the competitive product, q_c, is produced by the monopolist with the standard technology, represented by an independent cost structure, $C(q_c)$, with $C_q > 0$ and, eventually, $C_{qq} > 0$.

Compare the simple single product monopoly and the two product monopolist entering the competitive market at the point of quasi-short-run efficient production. Free entry by any single product competitor, $f_c(1)=0$, implies the monopolist faces the following conditions:

$$(6.1)\ P_c = C_q = \min C(q)/q$$

$$(6.2)\ \Pi^c_{dir} = -f(2)$$

where Π^c_{dir} represents direct profits earned in the competitive market line when production is doubly efficient: production takes place on the operating cost frontier, $C(q)$; and $R^c_q = C^c_q$. From a quasi long-run view, however, $dR/dn < dC/dn$ resulting in persistent losses, $-f(2)$, in the competitive sector.

However, simple regulation of the firm's overall revenue or net earnings requirement creates or induces an externality between the original monopoly and the newly entered competitive sectors which the firm internalizes. Implicitly, total competitive market line revenue, $p_c \cdot q^*_c$, where for now q^*_c represents minimum efficient scale production, requires costs equalling $(1/d) \cdot C(q^*_c)$. Using an accounting convention of this sort, the competitive product line has a reported net operating earnings figure of $p_c \cdot q^*_c \cdot (1-1/d)$. The remaining costs, $(1-1/d) \cdot C(q^*_c) + f(2)$, justify under simple regulation additional enterprise profits equal to:

$$(7.1) \quad \Delta\tilde{\pi} = (d-1)\cdot(1-1/d)\cdot C(q_c^*) + (d-1)\cdot f(2)$$

If regulation of the single-product firm was successful in initially inducing expanded output and lower price, then potential profits are available for recapture.

Furthermore, only f(2) has not already been paid for (covered) by receipts from sales in the competitive market. This poses an interesting situation. Heretofor, single product regulatory models have been used to dominstrate how excessive regulatory zeal can perversely raise regulated industry costs. If regulators tighten the screws excessively by lowering d too much then the efficient firm operating along its cost frontier would produce where product demand is inelastic. The profit maximizing firm will respond by incurring unnecessary costs - waste - holding output to the level consistent with e_d=-1. However, when entry into new markets is possible wasteful expenditure fails as an optimal response to excessively tight regulation.

When $R_q \leq 0$ any added spending on productive inputs raises cost and lowers revenue. Spending wastefully, say $1, raises only costs. Indeed, since costs have risen, higher revenues may be justified. So waste is better than productive expenditure. But pure waste involves a net out-of-pocket payment. Suppose the firm could find a way to justify higher revenues in the monopoly market line without a net out-of-pocket payment. Certainly profit would rise compared to wasteful buying used to attain the same revenue point. If the firm can enter new markets and essentially shift costs - costs as in our example that have already been paid for, as $(1-1/d)\cdot C(q_c^*)$, by the sale of that other market's output - monopoly market revenues can be raised at essentially no opportunity cost to the multi-product firm. Any regulated firm that can shift its effective regulatory constraint at no cost to itself will always pick that route in preference to one that requires an uncompensated expenditure.

Provisionally, the firm maximizes in the regulated market:

$$(8)\quad \underset{y}{\mathrm{Max}}(y\cdot g(y) - C(y);\ d\cdot C(y) + d\cdot(1-1/d)\cdot C(q_c^*) + d\cdot f(2) - y\cdot g(y) \geq 0;\ y \geq 0)$$

Geometrically, given $(1-1/d)\cdot C(q_c^*) + f(2)$, the constrained demand curve in the monopoly market has as one segment part of a rectangular hyperbola. For example, let the monopoly product's cost structure display constant costs - at least in the relevant range. From the constraint inequality the price in the monopoly market must satisfy:

$$(9)\quad g(y) = p_m \leq d\cdot v + H(q_c^*;\ d,\ f)/y$$

where $H(q_c^*;\ d,\ f) = d\cdot(1-1/d)\cdot C(q_c^*) + d\cdot f(2) =$ constant. Figure 1 illustrates the model. The heavily shaded lines represent the regulation induced (constrained) average and marginal revenue functions. Clearly:

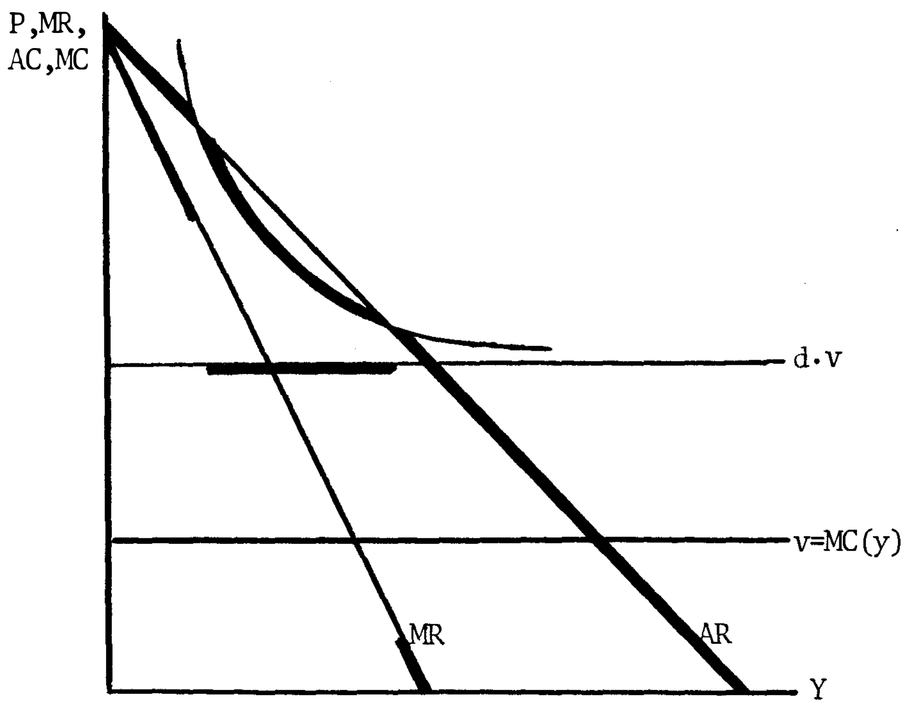


Figure 1

The more elastic the monopoly demand, the smaller the implicit cost (revenue) transfer from the competitive (monopoly) market needed to justify full monopoly pricing.

To this point the firm presumtively selected the quasi-short-run loss minimizing output level for the competitive product and then maximized in the monopoly market with the constrained average revenue function parameterized on q_c^* via the function $H = H(q_c^*; d, f)$. If such quasi-efficient competitive production justifies full monopoly pricing, then:

$$(10) \quad -p_m^*\left\{\frac{e^*(d-1)+d}{e^*}\right\} \lesseqgtr (d-1)C(q_c^*) + d \cdot f(2)$$

where the right-hand side represents the total revenues in the monopoly sector justified by costs incurred in the competitive market, p_m^* and e^* are, respectively, the unrestrained monopoly price and the elasticity of monopoly product demand corresponding to unrestrained monopoly pricing. The left-hand side can be shown without difficulty to equal that part of total revenue generated at pure monopoly pricing that is unjustified by the costs incurred in the monopoly market in production of y^*.

The partial maximization, conditional on $q_c = q_c^*$, illustrates how costs tied to competitive market production enter the restrained profit function for the monopoly product when the constraint binds. From the perspective of the firm, marginal revenue in the competitive sector - equal to the competitively fixed price - is only part of the firm's earned marginal revenue attributable to entering and producing the competitive product.

Generally, the firm maximizes the following:

$$(11)\ \underset{y,q_c}{\text{Max}}\ (y\cdot g^*(y;q_c) - C^m(y) + p_c\cdot q_c - C^c(q_c) - f(2);\ d\cdot C^m(y) + d\cdot C^c(q_c) + d\cdot f(2)$$
$$- y\cdot g^*(y;q_c) - p_c\cdot q_c \geq 0;\ (y,q_c) \geq 0_2)$$

The first order conditions, omitting the complementary K-T conditions for brevity, become:

$$(12.1)\quad y\cdot g^*_{q_c}(y;q_c)(1-h) + p_c(1-h) - C^c_{q_c}(1-h\cdot d) \leq 0;\ q_c \geq 0$$

$$(12.2)\quad g^*(y;q_c) + y\cdot g^*_y(y;q_c)(1-h) + C^c_y(1-h\cdot d) \leq 0;\ y \geq 0$$

$$(12.3)\quad d\cdot(C^m + C^c + f(2)) - y\cdot g^* - p_c\cdot q_c \geq 0;\ h \geq 0$$

Focus first on equation (12.1). The first term reflects the firm's internalization of the regulatory induced externality. Production of the competitively marketed product shifts the regulation restrained monopoly demand curve. Suppose the regulatory constraint does not bind at some solution vector (y',q_c'). This implies from (12.1) that $p_c = C^c_{q_c}$ because $h=0$. Given the initial set of assumptions, if regulation fails to bind at some solution vector, that vector must equal (y^*,q_c^*) and the firm's profits equal the unrestrained monopoly level minus the competitive market entry fee, f(2). Efficient short-run competitive entry justifies or supports the unrestrained monopoly market demand curve, $g^*(y;q_c) = g(y)$.

When the constraint binds, however, the non-negative character of this first term implies that $p_c < C^c_{q_c}\cdot(1-h\cdot d)/(1-h) = C^c_{q_c} + C^c_{q_c}\cdot(1-d) < C^c_{q_c}$ since $d > 1$. Selecting profit maximizing values for y and q_c implies that $d\Pi = \Pi_y dy + \Pi_{q_c} dq_c = 0$ for all (small) directional vectors (dy,dq_c).

The first order conditions then imply:

$$(13)\quad p^m\cdot(1+1/e^m)/C^m_y = (p^c+p^m_{q_c}\cdot y)/C^c_{q_c}$$

The restrained optimum requires the ratio of revenue increments to cost increments consequent to a change in the production of any good or service must be constant across all product lines. From the private perspective, competitive sales below marginal stand alone cost are not irrational since stand alone revenues are not the only ones a decision-maker considers. The right-hand side term, $p^m_{q_c}\cdot y$, accounts for the added monopoly market revenues justified by competitive market production.

This point may help clear away some semantic confusion. Cross-subsidization carries with it, now, unsavory connotations. First, private market cross-subsidization (predation) suggests big, powerful and rich firms beating up on smaller, weaker competitors. Predation implies aggression. Past profits or anticipated profits permit or induce competition killing behavior. Second, economists may disapprove of cross-subsidization because it is inefficient. Some customers pay less than the cost of serving them. The model outlined above indicates two potential sources of confusion in the context of regulatory proceedings. First, the regulated firm modelled above has no particular incentive to use already earned profits to support costs incurred in the competitive market in order to necessarily drive competitors out. The title of this paper was intended to capture this nuance. The monopolist incurs competitive market costs in order to justify or recapture profits in the monopoly sector. Second, the monopolist's private view is that the sales to competitive market customers do pay their full freight. The total revenues do not come entirely from competitive market sales, however. This is the consequence of the regulatory induced externality internalized by the firm when it is subject to simple firm level regulation.[11]

Common Costs, Economies of Scope and Regulation-Induced R&D.

Simple regulation at the level of the firm was chosen as the most effective vehicle for discussing the possibility of sub-marginal cost pricing by a regulated firm for several reasons. First and most obvious, if regulation can be so effectively designed that each and every product line can be optimally controlled a la Ramsey pricing, for example, then sub-marginal cost pricing cannot occur. Naturally, given the extreme assumption used above that the monopoly and competitive product are produced by technologically separate processes, an automatic incentive exists for regulators to

11. The monopolist's actions, however, may result in displacement of competitive market firms. This is especially true when the entry fee, $f(\cdot)$, is a function only of the number of entered markets and not a function of the number of optimally sized plants.

begin to intrude and try to control product selection. However, as noted at the outset, if regulations or legal restrictions on product or service bundle selection is inexact, that is the legal rulings do not precisely confine the firm to its original monopoly market cluster, then the sub-marginal cost possibility remains. The rule in additional to some specified mark-up over cost scheme results in effective regulation. But the imprecision of the rule together with the cost-based control induces the firm to enter as many competitive markets as the rule allows. Once these market niches are filled, the rule acts to generate a condition analogous to our assumed $f(3)=\infty$ condition.

But another possibility also must be considered. Schmookler's (1966) classic work on innovation strongly suggests that empirically market opportunities, and not *ex ante* underlying scientific or technical base, are most intimately associated with technological and product innovation and invention. This may be explained if we accept that at any time the underlying knowledge base is compatible with a larger number of technical possibilities than actually materialize. Market demand (economic opportunity) guides the process that selects which possibilities become operative.

If regulation of the simple kind creates an incentive to enter new markets in order to evade the regulation's full profit cutting thrust, then further regulation designed to thwart simple product extension may lead to other, less simple, forms of evasion. This provides another way of examining the current discussions involving cross-subsidization, sustainability of monopoly, and economies of scope. If regulators can (do) detect simple waste or gold-plating, they can also notice firms expanding beyond their customary market boundaries. Surely if, for example, AT&T began growing wheat in Nebraska or North Dakota, Federal and state regulators would question the wisdom of permitting wheat growing costs to be rolled in with telecommunication costs in setting rates.

But suppose AT&T, for example, uncovers a technology that yields, as common products, both wheat and telecommunications services. Suppose the total costs of the new technology has the following bounds:

$$(14)\quad C^m(y) + (1/d)\cdot C^c(q_c^*) \leq C(y,q) < C^m(y) + C^c(q_c^*)$$

where q_c^* as before is the output level tied to efficient stand-alone production of the competitive product. The firm can reduce production in the monopoly market line, maintain production in the competitive line at q_c^*, and raise earnings from what they would have been in the absence of the transition to a joint or common cost technology.[12]

12. This form of regulation is based on constraining revenues by costs. Suppose, as an alternative, Baumol-Bradford-Ramsey pricing is suggested where the firm is constrained to have some level of earnings equal to E*. This tactic removes the firm's incentive to enter competitive markets or to wastefully employ resources. The earnings constraint is not weakened by either activity. But, as Takayama first suggested in the A-J model when market and allowed returns are equal, the firm's choices are no longer strongly guided by profit-maximizing behavior. The firm has, potentially, an infinite number of paths leading to E*. The regulators must then decide if the chosen path is optimal.

Assume the following about C^c, C^m, C, C^c_q, C^m_y, and C_q:

(15.1) $C^c(q) = M + F(q)$, where $F(\cdot)$ is homothetic;

(15.2) $C^m(y) = K + G(y)$, where $G(\cdot)$ is CRS;

(15.3) $C(y,q)$ is linear in y and separable in y & q, and $C_{qq} \lesseqgtr 0$.

(15.4) $K+G(y) + (1/d)\cdot(M+F(q)) \leq C(y,q) < K+G(y) + M+F(q)$.

EQ(15.4) guarantees that the multi-product firm can earn more than the single-product monopolist (left-hand inequality), and that $C(y,q)$ displays economies of scope and sub-addativity relative to two single-product line producers. This condition also implies that the new technology embodied in $C(\cdot,\cdot)$ has overcome the original entry fee barrier, f(2).

One simple way to write $C(y,q)$ embodying (15.3) and (15.4) is:

(15.5) $C(y,q) = D_0 + G(y) + W(q)$, defined over R^2_+

where $W_{qq} \lesseqgtr 0$. Suppose that $W(q) = ((F(q^*_c)+M)/q^*_c)\cdot q + I^q_0 z(\theta)d\theta$, where $I^q_0(\cdot)$ represents the integral of $(\cdot)$ from 0 to q. Compare C with the right-hand-side of (15.4). For (15.5) to satisfy (15.4, rhs),[13] $D_0+W(q) < K+M+F(q)$. Substituting for $W(q)$, this implies:

(15.6) $I^q_0 z(\theta)d\theta/q < AC(q) - \min AC(q) + (K - D_0)/q$

where AC(q)=average cost under the stand-alone structure C^c and minAC(q)= the minimum total average cost achieved by all efficient stand-alone competitive producers. If $K > D_0$, then the right-hand side of (15.6) is positive. This condition is important because it is necessary if there is to exist a $z(\theta) > 0, \forall\theta$. When this condition is met, then

13. Satisfying the LHS of EQ(15.4) for all q requires only:
$$AC(q) - \min AC(q) + (K - D_0)/q - \left(\frac{d-1}{d}\right)\cdot AC(q) \leq I^q_0 z(\theta)d\theta/q$$

by construction we have $W_q > F(q_c^*)/q_c^* + M/q_c^* = p^c, \forall q$. By the left-hand side property of (15.4) we know that common cost production is more profitable, potentially, than is stand-alone production.

Certainly there are other ways to show that common cost technologies may simultaneously enhance firm profits and yield a profit maximum restrained by regulation where p_c falls below marginal cost. One way to heuristically see this is to assume that production of multiple-products with the stand-alone structures (15.1) and (15.2) is so profitable under simple regulation that the firm can recover all its monopoly quasi-rents by efficient production in the competitive market. Now simply lower M, or in the case of common cost production (EQ(15.5)), lower D_o. The private inefficiency is spread across both product lines to equate marginal profit losses.

This conclusion, in either the simple case of stand-alone techniques, or the more complex case of regulatory induced common cost structures, warrants more careful attention. Heretofor, most regulatory modelling has implicitly assumed that symmetric regulation, such as simple mark-up over cost, is - if not pushed too far, i.e., $R_q > 0$ - internally costless. This modelling suggests that in complex worlds where effective regulation cannot pretend to penetrate to the level of product lines, but must for information assymetries and decision-making limitations remain at the level of the enterprise, symmetric regulation is not a welfare enhancing free lunch.

A-J Regulation, Once Again

As is well-known, rate-of-return (assymetric) regulation induces the firm to over-capitalize at every chosen output level. Bailey (1973, pp. 104-109) attacks the problem by positing a regulated marginal cost which differs over some part of the domain of y - the monopoly output - from efficient, unregulated marginal cost. In her modelling, A-J (regulated) marginal cost is a continuous function corresponding identically with efficient (frontier) marginal cost at every output level less than y^* - the unfettered monopoly product level. At y^* Bailey's regulated marginal cost begins to diverge continuously from frontier marginal cost. This depiction seems curious because it apparently implies - erroneously - that if the firm happened to select the monopoly output level under A-J regulation it would produce that output without any distortion in its selected factor combination.

Normally, marginal cost is defined with given and invariant perceived factor prices. If this is taken as applying in the A-J case, then as is well-known the regulation can be modelled as creating a wedge between social (market) rates-of-return and the privately perceived rate. This wedge is invariant to input useage. In the simplest case assume $\dot{y}$ is produced under (local) CRS conditions. Also assume that the demand facing the firm for y is also given by, say, parameter(s) θ.

Then:

$$(16)\quad y = f(k,l) = y\cdot f(a(r/w;\theta),b(r/w;\theta))$$

where $a=k/y$, $b=l/y$. In regulated equilibrium a & b depend upon the market return to capital, r, the price of the variable input(s), w, and the demand structure θ. By strict convexity of isoquants (strict quasi-convexity of f) for given θ, $a_r < 0$, $a_w > 0$ and the reverse for b. Simple regulation alters the argument, r/w, to (r-e)/w, where $e=g(d) \gtrless 0$ and d is the allowed rate-of-return. The firm selects y and incurs costs given by $C(y) = r\cdot a((r-e)/w)\cdot y + w\cdot b((r-e)/w)\cdot y$ for θ held fixed. Marginal cost becomes:

$$(17)\quad C_y^e = r\cdot a((r-e)/w) + w\cdot b((r-e)/w)$$

By strict convexity, $C_y^e > C_y$, $e > 0$, for all y.

Given the CRS assumption, the regulated firm reacts to an allowed demand function of the form:

$$(18)\quad p_m = d\cdot a((r-e)/w) + w\cdot((r-e)/w) \lesssim p_m^*$$

The right-hand side is a constant.

Let the firm enter a new product market where competition prevails. Assume to simplify that the firm can do this in a socially efficient fashion, i.e., f(2)=0. Let the firm pick the efficient capital and labor combination, k_c, l_c, at each output level. Given factor prices and the output level, write the capital demand function in the competitive market as $k_c = k_c(q_c,r,w)$. We emphasize that this demand for capital is contingent, and only temporarily so, on the assumption that the firm initially selects efficient input points for the competitive product, q.

Given the quasi-profit maximizing choice, q_c^*, capital used is $k_c^*=k_c(q_c^*)$ where r and w are suppressed. The firm can, now, effectively shift $(1-r/e)\cdot k_c^*$ into the monopoly market. Given the rate-of-return constraint, the firm requires only the fraction $s=r/d \quad 1$ of the competitive line capital stock to justify the operating revenues earned there.

Extended Footnote: Optimal Choice of Product Line Rates-of-Return.

An alternative modelling may provide added insights. The firm, constrained by an overall restriction based on d, can freely choose product line prices and input combinations as well as individual product line rates-of-return, $d_1, d_2, \ldots, d_n$ subject to a new constraint:

$$(19) \quad d = \sum_1^n d_i(k_i/k)$$

where $k = \sum_1^n k_i$. The firm engages in a sort of double maximization. First, a set of d_i is picked satisfying (19). This can be done by picking the d_i, maximizing the equation set below, and then checking to see of (19) is satisfied as an equality. Equation set (20) sets out the initial maximization:

$$
(20)\quad
\begin{array}{l}
\max_{k,l} \; R_1 - C_1 \;\; \text{s.t.} \;\; d_1 \cdot k_1 - R_1 + w_1 \cdot l_1 \geq 0 \\
\vdots \\
\max_{k_n, l_n} \; R_n - C_n \;\; \text{s.t.} \;\; d_n \cdot k_n - R_n + w_n \cdot l_n \geq 0
\end{array}
$$

The second stage of the maximization requires picking the set of d_i which produces the maximal profit level from among all the maximum profit levels associated with each set of d_is. This modelling, while cumbersome, illustrates that, unlike the original A-J model, here the allowed d_i are not independent of the choices for k_i. The d_i, k_i, and l_i are interdependent along the constraint frontier defined by (1). Illustrating the problem is the following maximization:

$$
(21)\quad \max_{k_1, k_2, l_1, l_2, d_1, d_2} \; \Pi = \Pi^1 + \Pi^2 \;\; \text{s.t.} \;\; d_i \cdot k_i - R^i - w^i \cdot l_i \geq 0, \; i = 1,2
$$
$$
d_o - d_i \cdot k_1/k - k_2 \cdot k_2/k \geq 0
$$

which yields the following:

$$
(22)\quad
\begin{array}{ll}
\text{a} & \Pi^i_{l_i}(1-h_i) = 0 \\
\text{b} & h_1 = h_2, \; h_i \geq 0 \\
\text{c} & k = k_1 + k_2 \\
\text{d} & \Pi^i_{k_i} + h_i(d_o - R^i_{k_i}) = 0 \\
\text{e} & d_i \cdot k_i - R^i + w^i \cdot l_i > 0, \; h_i \cdot (d_i \cdot k_i - R^i + w^i \cdot l_i) = 0 \\
\text{f} & d_o - d_1 \cdot k_1/k - d_2 \cdot k_2/k > 0, \; u \cdot (d_o - d_1 \cdot k_1/k - d_2 \cdot k_2/k) = 0
\end{array}
$$

The equations (b) are important because they confirm our intuition that either all the constraints bind--lagrangeans h_1, h_2, u non-zero--or none of them do. Equation set (a) indicates that, given the choices of k_i and d_i, the l_i are chosen so that the marginal profit product of labor in each line is zero. Equation set (d) illustrates that the marginal profit product of each capital input, when regulation binds, is negative.

Sequential Maximization.

Let the firm enter a new product market where competition prevails. Assume the firm can do this efficiently by choosing the socially optimal capital labor combination, k_c, l_c, at each output level. Given factor prices and the output level, we can write the capital demand function in the competitive market as: $k_c = k_c(q_c, r, w)$. We must emphasize that this demand for capital is contingent--and only temporarily so--on the presumption that the firm selects efficient production points. This assumption is soon to be relaxed.

Given the quasi-profit maximizing choice q_c^*, the capital used becomes $k_c(q_c^*) = k_c^*$ where r and w are suppressed for simplicity. Now, the firm can, effectively shift $(1-r/d)k_c^*$ into the monopoly market. Given the rate-of-return constraint the firm requires only the fraction $s=r/d<1$ of the competitive line capital stock to justify the operating revenues earned there. The firm then selects an input configuration in the monopoly market by maximizing:

$$(23)\ \max_{k,L} (y\cdot g(y) - w.L - r.k_m;\ d\cdot k_m + (d-r)k_c^* - y\cdot g(y) + w\cdot L \geq 0;\ k,L \geq 0_2)$$ [14]

14. y is functionally related to k and L through a prior maximization:

$$y(k,L) = \max_y (y: (y,k,L) \varepsilon T)$$

where T is the production possibilities set facing the firm.

Once k_m and L are chosen for any level of y, the price that can be charged becomes:

$$(24) \qquad p'_m \leq (d\cdot k_m + w\cdot L/y + (d-r)k^*_c/y$$

Given the CRS assumption, the first term on the right can be rewritten as:

$$(25) \qquad d\cdot a((r-e')/w) + w\cdot b((r-e')/w)$$

where $e'=h(d;k^*_c)\leq e$. Hence, by permitting the firm entry into a competitive market the input distortion diminishes in the monopoly market. If $k^*_c(d-r)$ is sufficiently large, $p'_m = p^*_m$ and the firm engages in monopoly pricing.

Assume the two products are technologically separate. Let the firm select the optimal input configuration for the competitively marketed product yielding an output-input vector in that sector of (q^*_c, k^*_c, l^*_c). To satisfy the rate-of-return constraint, the firm must apply $(r/d)k^*_c$ of the competitive product line capital input against the net operating revenues, $p_c\cdot q_c(k^*_c,l^*_c)-w\cdot l^*_c$, generated on competitive product sales. The remainder, $(1-r/d)k^*_c$, can be applied against earnings from the monopolized market. Given $(1-r/d)k^*_c$ as a parameter, the firm selects the vector (y,k_m,l_m) by

$$(26) \quad \max\ (y\cdot g(y) - w\cdot l_m - r\cdot k_m; dk_m + (d-r)k^*_c - y\cdot g(y) + w.l_m = 0;$$

$$k_m, l_m\ k_m, l_m = 0_2;\ y(k_m,l_m) = \max_y((y,k_m,l_m)\ T)$$

This leads to the following first order conditions:

$$(27.1)\ [\nabla g_y\cdot y + g(y)]y_k(1-\lambda) - r(1-\lambda) + \lambda\varepsilon = 0$$

$$(27.2)\ [[\nabla g_y\cdot y + g(y)]y_L - w](1-\lambda) = 0$$

$$(27.3)\ d.k_m + (d-r)k_c - y\cdot g(y) + w\cdot L_m \geq 0; \quad \lambda \geq 0$$

$$(27.4)\ \lambda[d\cdot k_m + (d-r)k_L - y\cdot g(y) + w.L_m] = 0$$

If the conditions for a regular (interior) maximum are satisfied, we can apply the implicit function theorem and solve for k_m, l_m, in terms of the parameter k_c^*.

This solution provides the following sensitivity conditions when the solution functions, $k_m=k_m(k_c^*)$, $l_m=l_m(k_c^*)$, $h=h(k_c^*)$, are inserted into the first order equations (3.1)-(3.3) converting them into identities:

$$\partial k_m/\partial k_c^* = \frac{\begin{vmatrix} -h\cdot g_{k,k_c} & \pi_{kl}+h^*g_{kl} & g_k \\ -h\cdot g_{l,k_c} & \pi_{ll}+h^*g_{ll} & g_l \\ -g_{k_c} & g_l & 0 \end{vmatrix}}{H}$$

(28) $$\partial l_m/\partial k_c^* = \frac{\begin{vmatrix} \pi_{kk}+h^*g_{kk} & -h\cdot g_{k,k_c} & g_k \\ \pi_{lk}+h^*g_{lk} & -h\cdot g_{l,k_c} & g_l \\ g_k & -g_{k_c} & 0 \end{vmatrix}}{H}$$

$$\partial h^*/\partial k_c^* = \frac{\begin{vmatrix} \pi_{kk}+h^*g_{kk} & \pi_{kl}+h^*g_{kl} & -h\cdot g_{k,k_c} \\ \pi_{lk}+h^*g_{lk} & \pi_{ll}+h^*g_{ll} & -h\cdot g_{l,k_c} \\ g_k & g_l & -g_{k_c} \end{vmatrix}}{H}$$

Where H is the bordered Hessian of the first order conditions, π_{ij} is the second order partial of the profit function, g_{ij} is the second order partial of the constraint, $g=d\cdot k_m+(d-r)k_c^*-y\cdot g(y)+w\cdot l_m$, h is the lagrangean multiplier, $h^*=h(k_c^*)$. Now in equilibrium, $g_1=0=\nabla_y g(y)\cdot y+g(y)-w$, while $g_k>0$ from the first order condition and the fact that $d>r$. Furthermore, $-\pi_{ij}=g_{ij}$. As a result, the first equation can be shown to be non-positive, for it reduces to: $\partial k_m/\partial k_c^* = (d-r)(g_k)(1-h)\pi_{11}$. Since $g_k= d - (g(y)+\nabla_y g(y)\cdot y_k \geq 0$, $1 > h \geq 0$, $(d-r) \geq 0$, and the usual assumptions leading to $\pi_{11}<0$ the case is proved. This result is comforting for it substantiates the common-sense intuition that shifting capital from one product line to another allows the firm to reduce its monopoly line capitalization toward the efficient factor combination level.

Naturally, the second and third terms are less easily defined. However, using a variation of the envelope theorem (Silberberg, pp. 168-71) we know that $d\pi^*/dk_c = \pi_{k_c} + h\cdot g_{k_c} = 0 + h\cdot(d-r)\geq 0$. Hence, as $k_c(d-r)$ rises, so does the constrained profit, π^*.

Since constrained monopoly profit rises as k_c increases from 0 to (discretely) k_c^*, the issue depends upon whether or not $h>0$ in the neighborhood about $(k_c^* + dk_c)$. If the constraint, constructed as above from the original monopoly constraint, still binds, then the issue of sub-marginal-cost pricing can be resolved by investigating the cheapest method to expand k_c by some small, finite amount dk_c in the competitive sector. This is a natural step, since we have begun in the competitive sector where stand-alone profits have been maximized.

By expanding capital by some amount, dk_c, we change profits (incur losses) in the competitive sector at the margin of:

(29) $$d\Pi_c = p_c \cdot q_c(k_c^*+dk_c, l_c^*+dl_c) - r\cdot(k_c^*+dk_c) - w\cdot(l_c^*+dl_c) - p_c\cdot q_c(k_c^*, l_c^*) + r\cdot k_c^* + w\cdot l_c^* < 0$$

Assuming that q_c can be represented adequately by a Taylor Series in which all terms higher than the second order are truncated, we have:

(30) $$d\Pi_c = p_c\cdot(q_k dk + q_l dl + q_{kk}\cdot(dk)^2/2 + q_{kl}\cdot(dk)(dl) + q_{ll}\cdot(dl)^2/2) - r\cdot dk - w.dl$$

Now, we have already specified the length of dk. As a result, we can write $dl = a\cdot dk$, where a is some real number which can be positive or negative. Replacing dl by $a\cdot dk$ in the above and minimizing with respect to a yields:

(31) $$a = (1/q_{ll}\cdot dk)(w/p_c - q_l - q_{kl}\cdot dk)$$

Now, if the first order conditions apply, as they must since we chose k_c^*, l_c^* to reflect a competitive maximum and q_l in the Taylor expansion was evaluated at that point, then the term $w/p_c - q_l = 0$. Thus, $a = q_{ll}^{-1}\cdot q_{kl}$ since the $dk>0$ cancel. Now we generally assume $q_{ll}<0$. Thus, if $q_{kl} = 0$, the inputs are independent in production, it always pays, irrespective of the value of dk that is chosen, to expand production in the competitive sector from $q_c(k_c^*, l_c)$ to $q_c(k_c^* + dk, l_c^*)$. The competitive sector not only uses the 'wrong' factor combination, but it also produces more than would be produced by a stand-alone firm. On a stand-alone basis, the competitive sector is being driven beyond the point where price equals marginal cost. When k and l are complements, $q_{kl} > 0$, then $a > 0$ and the same result obtains. The possibility of raising dk and simultaneously lowering l so that production remains unchanged--in which case regulated marginal cost has a jump discontinuity at q_c^*--requires k and l to be substitutes.[15]

The remainder of the paper contains some numerical examples where entry into initially competitive is induced by regulation at the level of the firm. Again, it must be stressed that if regulators can confine firms to their original product niches by regulatory rule-making, then regulation can be both effective and minimally efficient. On the other hand, if regulatory rules are a potential source of inefficiencies because they may absolutely eliminate incentives by regulated firms to search, globally, for more effective production technologies - technologies which may also expand the product offerring from common facilities - then the regulatory process becomes much more information and decision intensive. Weeding out inefficient entry into new product lines, or inefficient non-Ramsey Ramsey pricing, demands that regulators know things about marginal costs, total costs, scope and scale economies, and demand elasticities which, at best, a known - or knowable - only imperfectly even to industry or firm insiders.

15. If labor is a better substitute for labor than is capital, then $-1 < a$.

Numerical Examples:

A. Cost Based Regulation.

Let the firm face the following demand and cost conditions in its monopoly product, y, market and a competitive product, q, market:

Demand: $P(y) = (10 - y/2)\cdot y$

$p_c = 1.34$

Cost: $C(y) = 2\cdot y$

$C(q) = q\cdot(q-2)^2 + 1.34\cdot q$

Without regulation, the firm would be indifferent between production at y=8, q=0 and y=8, q=2. If the firm faces a regulatory constraint restricting overall revenues as, $R < 2.5\cdot C$, then the single product firm would produce y=10 units for a 25% increase in output. With the mark-up, d=2.5, the firm maximizes constrained profit where MR(y)=0 and earns a profit level, $\pi(y,reg)=30$. If the firm efficiently enters the competitive market, selling q=2, it will sell y=9.1183 units in the monopoly market and earn profits, $\pi(y,q;reg)=31.4062$. However, if the firm maximizes in both markets together, it sells y=8.74, q=2.36 and earns an overall profit, $\pi^*(y,q;reg)=31.4766$. Since the unconstrained profit maximum generated net earnings $\pi^*(y)=32$, the firm by entering another market and producing inefficiently there has recovered 73.8% of the profits it would have lost had it operated efficiently in one market only.

The model can be altered to illustrate the effect of tighter regulation. Let the allowed mark-up fall to d=1.5. The single product monopoly firm would select an output level $y^{reg}=14$ and earn $\pi(y,reg)=14$. If the firm elected to engage in wasteful expenditures, it would produce y=10, buy waste =13.333, and earn $\pi(y,w;reg)=16.667$. However, the firm has a superior alternative.

Let it enter the competitive product market at efficient scale, $q=2$, continue to produce the revenue maximizing output $y=10$, and buy waste=12.44 and earn $\Pi(y,q,w;reg)=17.56$. However, the best alternative involves no waste at all. Instead, the firm produces $y=9.783$ and $q=3.784$. The firm then earns $\Pi^*(y,q;reg)=18.384$. Twenty-three percent of lost profits have been returned.

As noted earlier, waste is a sub-optimal choice. Consider, if regulation would push the efficient single line firm beyond $MR(y)=0$, then by buying waste=∂W the firm can justify added revenues equal to $d \cdot dW$, leaving a residual profit of $(d-1)\cdot\partial W$. However, if the firm enters another market line and spends $C=\partial W+p_c\cdot\partial q$, it adds ∂W to the firms' unpaid for expenses. But as a result, the firm can shift $\partial W+(1-1/d)p_c\cdot\partial q$ to the monopoly line and justify added revenues $\partial R=d\cdot\partial W+(d-1)p_c\cdot\partial q>d\cdot\partial W$ if $d>1$. Thus, for the same 'unpaid' addition to cost, the firm can justify more monopoly-line revenues.

The key in this model to the extent to which below marginal cost pricing is pushed, depends upon the elasticity of cost in the competitive market line. Uinsg a technique outlined above, the multi-product firm will use the following rule in fixing the competiive product line output level:

$$0 = \frac{\partial\Pi\ \text{firm}}{\partial q_c} = \lambda d\ (C'-(1/d)p_c) + P-C' = \frac{\partial\Pi_m^*}{\partial q_c} + \frac{\partial Vc}{\partial q_c}$$

or

$$C' = \frac{(1-\lambda)}{(1-d\cdot\lambda)}\cdot P_c > P_c \qquad \text{if } \lambda > 0,\ d > 1$$

B1. Asymmetric (A-J) regulation.

Suppose the firm, first, has access to two markets with the following demand and production functions:

$$\text{Demand: } P(y) = 10 - y/2$$

$$P(q) = 2$$

$$\text{Production: } y = (k_m \cdot l_m)^{\frac{1}{2}}$$

$$q = (k_c \cdot l_c)^{1/3}$$

The first demand and production functions are completely compatible with the monopoly demand and cost functions in the first example. The second demand function is identical with the demand function in the first example, but for simplicity in solution the second production function differs from one required to generate the cost function used earlier. The following Table provides solution values for various key values of k_c.

All the columns, except those headed "d_c" and "d_m", are self-explanatory. The d_c and d_m columns provide an alternative way of viewing the choice problem facing the regulated monopolist. Regulation, as posited here, merely requires observing an overall rate of return ceiling - here set at 2.5. The firm may choose to operate its various product lines and report accounting rates of return for the separate lines which differ from the allowed firm-level figure. In the example as worked out here this leads to reported rates of return which are highest in the monopoly market line and lowest in the competitive market line. This result obtains because of the assumption that the firm is actually <u>exporting</u> the capital costs above the amount required to justify competitive line revenues into the monopoly market. With an alternative arrangement, the firm could just as easily report the higher rate-of-return in the competitive market line.

	k_c	L_c	q_c	V_c	k_m	L_m	y	V_m	V_f	d_c	d_m	Conditions
1	0	0	0	0	8	8	8	32	32	-	5	unregulated monopoly
2	0	0	0	0	18	4.5	9	27	27	-	2.5	regulated monopoly
3	8/27	8/27	4/9	8/27	17.954	4.509	8.997	27.034	27.3303	2.0	2.5057	monopoly plus efficient competitive product line
4	2.85	.91894	1.3784	-1.0121	15.6143	5.0326	8.8645	28.708	27.6964	.6448	2.8386	approximate two product regulated maximum
5	1	1	1	0	17.3410	4.6357	8.9659	27.488	27.4885	1.0	2.5852	zero profit competitive operation with efficient input ratio.
6	1.18461	.61539	.9	0	17.2120	4.6630	8.9590	27.583	27.5831	1.0	2.6026	zero profit competitive operation with biased factor ratio.

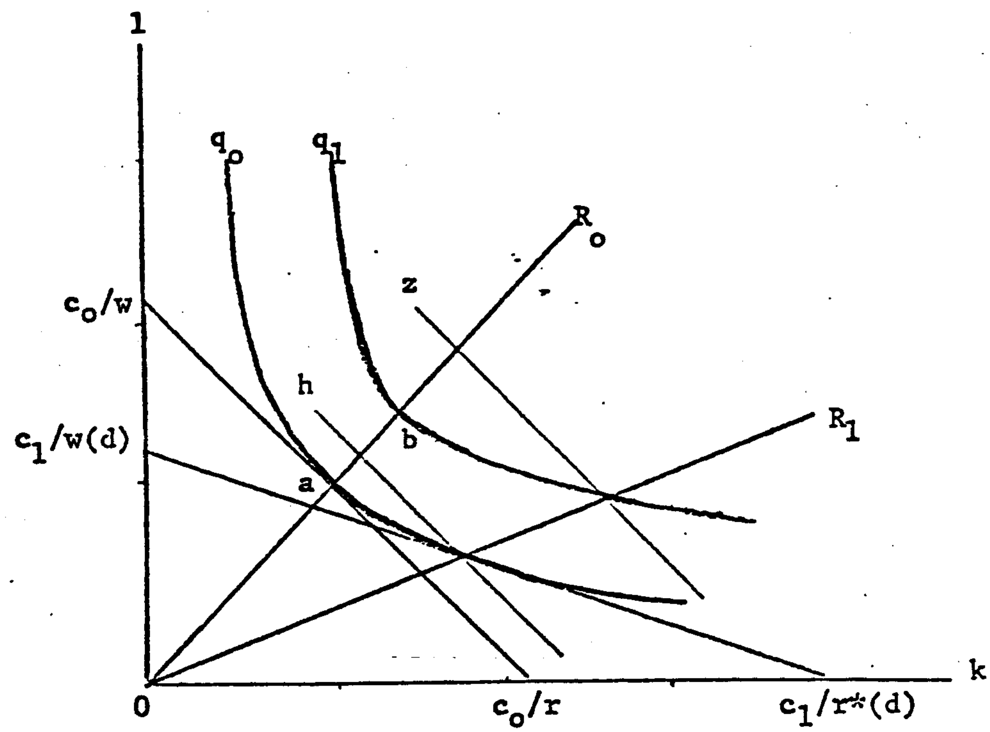
The Table illustrates how gently rising the constrained profit function is in a neighborhood of the maximum. This example illustrates how entry by the regulated firm into competitive production may have important efficiency consequences. Taken by itself, the monopoly-run competitive plant uses 9.619 times the capital and 3.101 times the labor employed by an efficient single-line profit maximizing competitor and produces 3.101 times the single-line producers output. Taken together, the joint product firm uses .60032 more labor units than needed by two efficient disintegrated producers to yield the same output. The integrated firm uses .18376 fewer capital units than would be needed by a single-line regulated monopoly and approximately 3.101 efficient single-line competitors. Given the assumption that $w=r=p_k=1$, the joint firm uses a greater input value to produce its output level than does the separate regulated monopoly and the competitive, efficient, producers. The multi-product firm, if its input acquisition has general equilibrium consequences, tends to raise the price of labor relative to capital compared to the situation prevailing under disintegrated regulated production and competitive single-line operation. This result has some important consequences for empirical work on factor distortion bias, e.g., Baron & Taggart (1977).

Suppose, for example, that the firm actually enters a competitive market line that is, in the heirarchy of production, prior to the main monopoly line. By setting appropriate transfer prices between the competitive selling subsidiary and the monopoly buying subsidiary, the firm could transfer revenues from the latter to the former. In this case, the competitive market line would appear quite profitable - possibly much more so than a stand-alone competitive firm of efficient size. This sort of possibility is important for public policy because it might lead, erroneously, to the conclusion that strong technological complementarities existed between the competitive and monopoly market lines. This example was designed to preclude any possibility that technical (cost) interactions would emerge justifying the expansion of the firm.

B2. Marginal Cost.

The simple mark-up over cost model provided no new problems in computing appropriate marginal cost. The rate-of-return sort of model does. Traditionally, marginal cost is defined as that minimum increment to cost which is required to increase output by one properly defined unit. A-J regulation, as noted earler, creates a bias in factor usage away from the cost-minimizing combinations. This means that added output could be brought forth at negative added cost. Thus traditionally defined minimum marginal cost at a point of production like the one given by row 6 in the Table would always be less than price, $p_c > 0$. We have chosen to define marginal cost by analogy with the expansion path properties associated with traditional MC definitions. In the present case, we will describe the changes in factor usage needed to generate one additional unit of production based on the expansion path passing through the profit-maximizing competitive-line factor combination. This factor increment will be valued based on the market factor prices, r and w.

Using this approach, the Averch-Johnson marginal cost at the production combination given in line 4 is 8.7641, which exceeds the price of the competitive product, $p_c=2$. Thus in the example we have given, not only is average stand-alone competitive market line cost above price, but so is marginal cost as we have defined it. The following figure illustrates, for a homothetic production function of the sort used in the example, the issue.



With the homotheticity assumption and the assumption that factor combinations at the margin are computed along R_1 it is easy to prove that marginal cost computed between the rays z and h exceeds marginal cost computed efficiently between points a and b. In the example given above, marginal cost is elevated for two reasons if we assume normal factors of production. First, production occurs off the efficient expansion path. Second, the level of product has been pushed beyond the efficient single-plant level by 210.1 percent.

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