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I. Introduction

Most of the production theory literature considers the capital input to the production process as, in some sense, a physically measurable entity. The vector of machines and structures is taken to be representative of the firm's fixed plant with the corresponding market prices entering the cost function. The problem with this method is that a change in the specific machines or structures used by the firm is difficult to measure in terms of changing capital intensiveness, and what combinations represent the "same" amount of capital are difficult to identify. The production specification is not severely affected by these ambiguities, but the economic factors as embodied in the cost function are significantly undermined. How does one evaluate a change in the price of a specific machine in the firm's decision process on general capital inputs and investment? How can the effect of this change be observed through factor substitution?

One solution is to consider an hedonic pricing model for capital. Such a model takes the view that the capital input decision of the firm is made at the plant level. That is, the firm's engineers present alternative plant designs and plant level cost estimates to the firm's management for selection based on the initial investment required and the ensuing short run cost and output characteristics of each design. The decision to invest is made on the basis of what the plant can do and the associated costs of undertaking a production plan. The initial cost of acquiring the plant is the "price" of capital to the firm, expressed as a function of the plant's characteristics.

The use of hedonic pricing for production inputs in the economics literature has primarily been in the modeling of steam electric power generation. The boiler-turbine-generator (b-t-g) complex that is the heart of electric power production is well suited to hedonic pricing in that labor is relatively unimportant in the generating process and the *ex post* characteristics of the technology are fairly rigid and well known. Hence Stewart (1980) and Cowing (1974) use plant capacity and fuel efficiency, or heat rate, as the main elements in determining the purchase price of capital at the plant level. Since electricity exists only as a flow, plant capacity is defined as the technologically inflexible rate at which electric power is produced by a given b-t-g unit. The amount of fuel, in BTU equivalents, necessary to produce a kilowatt of electricity with a given b-t-g complex is also fairly rigid and well known by the firm's engineers. The ratio of BTUs of fuel to kilowatts of electricity then serves as an efficiency parameter associated with any given b-t-g unit. The cost of obtaining a steam electric generating plant is then determined by its capacity and fuel efficiency.[1]

In applying this approach to industrial production generally there are two significant problems: (a) how to operationally define the concept of capacity, and (b) how to choose and observe an appropriate efficiency parameter in various production settings. The first of these can be considered in a familiar economic context, while the second requires a sojourn into the practices of engineering economy. We will show that the hedonic pricing of capital is related to the "economic balance" concept used by engineers. It also implicitly restricts the substitutability of subsets of the inputs to the underlying production technology. When this restriction is imposed, the expenditure functions associated with hedonic pricing models yield factor demand relationships that are equivalent to those of neoclassical duality theory.

Section II discusses the general issues of plant capacity and efficiency, while section III relates hedonic capital pricing models to dual cost and production function models. Section IV summarizes and concludes.

II. Plant Capacity and Efficiency Measures

The idea of capacity is related to the ability of a firm to use its existing physical plant to produce goods. There are several ways to define the quantity of output that corresponds to the capacity output of a plant. The usual practice in economics is to define capacity output as that quantity that minimizes short run average cost, or as the firm's planned output over the expected life of a plant. These two methods are, however, not the same. The first would have us choose the quantity corresponding to the minimum point of the short run average cost curve as the capacity output of the plant. The second method yields the quantity corresponding to the tangency of the short and long run average cost curves.

It can be argued that the minimum point of short run average cost corresponds to the engineer's concept of designed capacity, since for an individual machine the design capacity is the rate of output at which the machine produces a "unit" at lowest average cost in material inputs. Nevertheless, for collections of machines, such as an entire factory, the design capacity is the smallest common multiple of the machines' design capacities, hence changing a plant's design capacity corresponds to moving along the long run average cost curve. For firms making investment decisions we would expect the long run concept to be decisive, since it is more closely tied to the specific plant decision.

In the flow-fund cost and production model developed by Georgescu-Roegen (1970, 1971, 1972) and Klein (1980, 1983), the rate of production variable q is a ready correlate to plant design capacity.[2] Thus, we take the planned instantaneous rate of output of a plant as an indicator of the capacity of that plant. This capacity rate q_o along with the planned time utilization of the plant, t, corresponds to the firm's planned daily output $Q = tq_0$. q_0 is the quantity produced by a plant running continuously for a fixed period of time and t is the proportion of the time period that the plant actually operates. Q is then the actual output observed at the end of the period. Hence we use the capacity rate q_o to represent the "size" of plant the firm desires as well as the capital input the firm buys. A casual reading of a week's Wall Street Journal makes it clear that firm's have a definite idea of the capacity, or planned output per time period,[3] of the new plants or additions they decide to build. Given the firm's entire production plan including time utilization, the instantaneous rate of output q_o could be calculated.

Of course we can imagine several designs that could yield the same rate of output q_o and, when the engineers are given the task of producing Q in a day such that they must choose t and q combinations, the design process should produce plant designs with unique collections of characteristics. This leads us to the question of how the engineers decide on the best design capacity. A common technique used by engineers for this purpose rests on the concept of economic balance. From long experience with many types of design problems engineers have discovered that there is typically a single design variable that reduces materials cost and increases investment or capital cost simultaneously.

Vernon Smith (1961) uses the following diagram to illustrate the use of economic balance. Figure 1 shows the cost of producing a given quantity of "product" in a given period of time as a single design variable is "increased." Variable, or direct, costs include raw material and energy costs, and decline as the design variable rises. Conversely the capital or investment

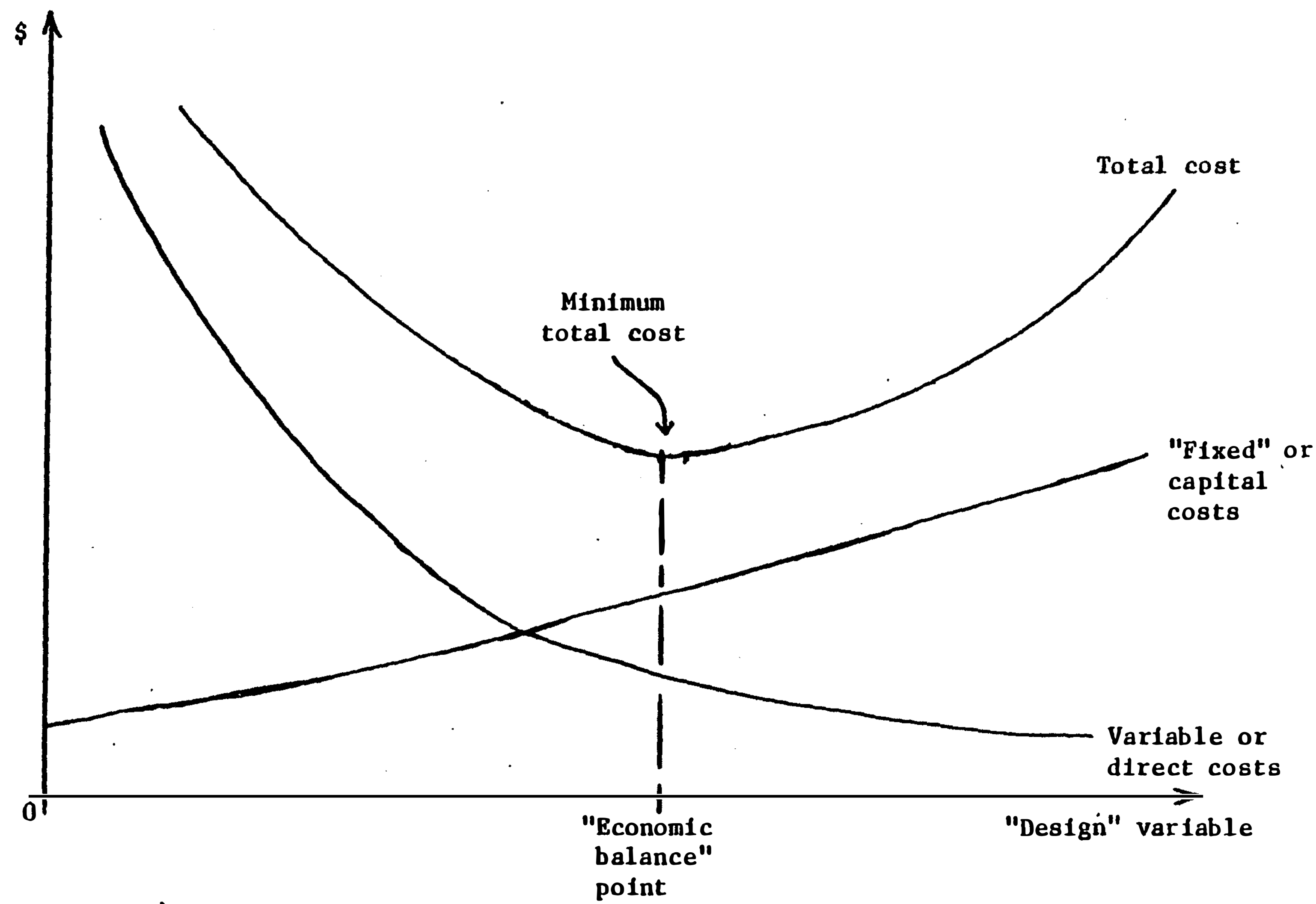

$
Total cost
Minimum
total cost
"Fixed" or
capital
costs
Variable or
direct costs
0
"Economic
balance"
point
"Design" variable

FIGURE 1

costs rise with increases in the design variable. A clear optimum exists at the point where the reduction in variable cost just offsets the increase in capital cost, and this corresponds to the minimum point of total cost. The change in variable cost "balances" the change in capital cost at the balance point: the slope of the variable cost curve is the negative of the slope of the capital cost curve. In this way the engineering optimization mirrors the familiar economic marginal conditions.

Smith suggests insulation thickness, conductor size, pipe size, number of evaporators, pump capacity, and the like for the design variable. Electric power generation is an excellent example. For a given capacity b-t-g unit (in kilowatts) increasing thermal efficiency, or declining heat rate BTU/kw, causes the fuel requirements per kw to fall and so reduces fuel costs. Simultaneously, a lower heat rate requires higher temperatures and pressures in the boiler-turbine complex which in turn require stronger and more heat resistant construction. This tends to increase the purchase price of the b-t-g unit. Thus heat rate, or thermal efficiency, serves as the design variable that determines the economic balance point and the optimal design.

Ammonia production is a similar case.[4] The first stage involves mixing natural gas with steam and injecting the mixture into tubes filled with a catalyst inside a reforming furnace. This produces hydrogen at which time air is added to produce more hydrogen and nitrogen for later use in the synthesis of ammonia. The heat and pressure in the furnace tubes is crucial since higher pressure gives better heat transfer but also reduces the yield per pass. Hence reforming tube pressure may serve as the ammonia industry's equivalent of heat rate in electric power.

These processes do not employ labor directly in the production process and use fairly homogeneous raw materials to produce a chemically well defined product. These factors contribute significantly to the ease with which an efficiency parameter can be identified. Industries such as steel where the input proportions determine the character of the finished metal present a more preplexing problem. Steel production does exhibit relatively fixed labor requirements *ex post*, so further search for a physical efficiency ratio of input and output may be fruitful.

Assembly operations such as automobile production are even more complex in that an efficiency index of tons of inputs per ton of output may not be very meaningful. Short run or *ex post* labor requirements in assembly operations may also be subject to change or responsible for minor alterations in the use of the fixed plant. Engineers typically figure capital requirements and material inputs by the economic balance method, but assign labor using textbook recommendations. The use of labor is then studied after the plant begins operation and altered as seems fit. This indicates the process that economists tend to lump into the concept of the learning curve. At any rate, when labor is a signifcant participant in the direct production of the output this learning effect could bias the attempt to observe an efficiency parameter *ex post*. We will continue to base our hedonic price on a generalized efficiency index with the cautionary note that the particular measure chosen for specific cases should be derived from study of the individual process to be modeled.

We would like, then, to discover the precise properties of a production technology that make it suitable for hedonic pricing of the capital input. We also desire to illuminate the relationship between the cost function derived from an hedonic pricing approach and the cost function for the specific capital vector method. As we will see, when certain conditions are satisfied these two functions yield identical values and the investigator is free to choose the most empirically productive technique.

III. Duality Theory and Hedonic Pricing of Capital

Suppose we are given an hedonic price function for plant capacity that gives us the price per unit of capacity as a function of the total capacity q and an efficiency parameter ε : $P(q, \varepsilon)$. The function $P(q, \varepsilon)$ reflects the nature of costs in the plant producing industry such that the purchase price of a plant with characteristics q and ε depends on those characteristics alone. We define $\varepsilon \equiv x/q$, assuming only one flow input x and that x and q can be measured in common units. The economic balance problem can then be stated as[5]

$$(1) \quad \min_{\varepsilon} \{q \cdot P(q, \varepsilon) + t \cdot \varepsilon \cdot q \cdot p_x : \text{given } t, q \text{ where } t \cdot q = Q\}$$

where the optimal ε, ε^*, is the "economic balance point" in Figure 1.

Both Stewart and Cowing assert a production technology composed of a rate function in terms of capital and the flow inputs

$$(2) \qquad q = G(K, x)$$

and a labor requirement of the form

$$(3) \qquad H = H^*(q)$$

where cummulative output is Q = tq, q is the output produced by running the plant at a constant instantaneous rate for the entire period, and t is the fraction of the period that the plant operates. The flow-fund production function in this case is

$$(4) \qquad q = f(K, H, x) = \begin{cases} G(K, x) & \text{if } H > H^*(G(K, x)) \\ 0 & \text{if } H < H^*(G(K, x)) \end{cases} .$$

Labor requirements are determined by the rate of production alone, and do not depend on the amounts of the other factors used. This is compatible with the engineering practice of setting labor requirements for a particular design by predetermined standards. If the labor requirement is met, the rate of production is determined by the capital employed and material input flow. If the labor present is not adequate, no output is produced.

Klein (1983) shows that a cost function

$$C(q,P) = \min_{K,H,x} \{p_K K + p_h(t)H + tp_x x : f(K,H,x) \geq q,\ tq = Q\}$$

exists for f continuous from above. P is the modified price vector $P \equiv (p_K, p_h(t), tp_x)$ which takes into account the variation in payments to labor and material inputs as t changes. C has all the properties associated with dual cost functions.[6]

Equation (4) requires the production function f to display no substitutability between the input groups (K,x) and (H). The dual cost function must mirror this property of f with respect to the corresponding groups of input prices, such that the factor demand equations have the form

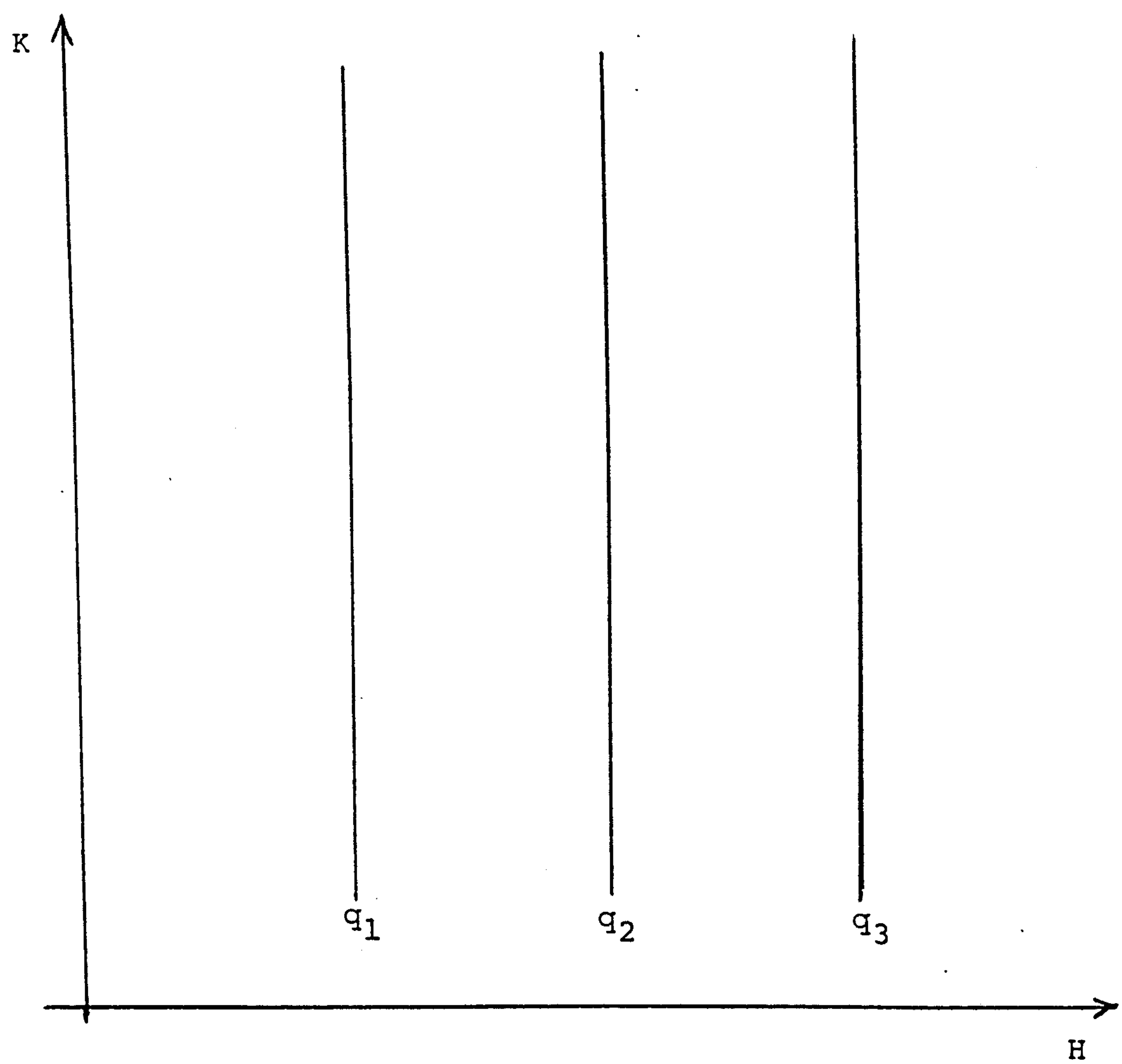
$$(5) \qquad C_K = K(q;\ p_K,\ tp_x)\ ,\ C_H = H(q, p_h(t))\ ,\ C_x = x(q;\ p_K,\ tp_x)$$

The lack of substitutability requires the elasticity of substitution between H and the other factors to be zero.[7] This in turn requires, for example, that $C_{HK} = 0$. Thus the price of capital does not enter the labor demand equation. In fact, the isoquants in K and H space have the shapes shown in Figure 2.

To construct the equivalent of (1) from (4), capital must be determined by q and ε. We can find such a function for capital, if G is invertible, by using the definition of ε such that $x = \varepsilon \cdot q$. This yields

$$(6) \qquad K = g(q,\ \varepsilon) = K(q,\ p_K,\ tp_x).$$

FIGURE 2



For the cost minimization problem, the firm will never choose $H > H^*(q)$. This fact with (6) allows us to write the economic balance problem as

$$(7) \quad \min_{\varepsilon} \{P_K g(q, \varepsilon) + P_h(t)H^*(q) + tp_x \varepsilon q : t, q \text{ given}\} .$$

Since (1) and (7) are equivalent, we must have

$$(8) \quad P_K g(q, \varepsilon) = qP(q, \varepsilon) \rightarrow P(q, \varepsilon) = P_K g(q, \varepsilon)/q .$$

Furthermore, the optimal inputs implied by (7) must be equal to the dual input demand functions in (5), such that

$$P(q, \varepsilon^*) = p_K g(q, \varepsilon^*)/q = p_K K^*(q; p_K, tp_x)/q$$

$$(9) \quad H^*(q) = H(q, p_h(t))$$

$$\varepsilon^* q = x^* = x^*(q; p_K, tp_x)$$

The solution to the economic balance problem in (1) leads to capital and material input demands identical to those from a cost function dual to the restricted flow-fund production function f in (4).

Furthermore, our earlier discussion implies the following properties of the hedonic price function for plant capacity:

$$(8) \quad \partial P(0, \varepsilon)/\partial q < 0; \ \partial P(\infty, \varepsilon)/\partial q > 0;$$

that is, $P(q, \varepsilon)$ is U-shaped depending on economies of plant scale and

$$(10) \quad \partial P(q, \varepsilon)/\partial \varepsilon < 0.$$

In order to insure the existence of solutions to the cost minimization problem we may require that $P(q, \varepsilon)$ satisfy

$$(11) \quad \partial^2 P(q, \varepsilon)/\partial q^2 > 0 \ ; \ \partial^2 P(q, \varepsilon)/\partial \varepsilon^2 > 0.$$

This condition is sufficient for convexity of P, but not necessary.[8]

IV. Conclusion

We conclude that the "economic balance/engineering production function" method using hedonic pricing of capital is consistent with a production technology of the type defined in (4); and an expenditure function derived from (1) will yield factor demand equations equivalent to those from the cost function dual to (4). It is easy to see how the approach can be generalized to the case of more than three inputs. The efficiency parameter ε becomes a vector such that some subset of the inputs can be determined by the ouput vector q and efficiency vector ε. The remaining non-capital inputs must satisfy restrictions of the type specified for labor in (4).

Although the production technology in (4) is highly restrictive, our discussion suggests that there may be several production processes that meet that restriction. Zudak's (1970) investigation of labor demand in a steel plant suggests just this sort of restriction, as does Oi (1983). Kopp and Smith (1980) have recently confirmed this property for electric power generation. When the technology can be described by (4), and when the observation of capital or its price is difficult, the hedonic pricing approach can be the answer to a problematic investigation.

FOOTNOTES

1 Cowing lets total capital costs be a direct function of these characteristics, while Stewart posits a price function for units of capacity that depends on b-t-g capacity and heat rate. The two are formally equivalent since Stewart's price function is conceptually identical to the average capacity cost function used by Cowing for his empirical work.

2 Georgescu-Roegen suggests a production function of the form

$$q = f(K,H,x)$$

where q corresponds to an instantaneous rate of production sustained for a 24 hour period; K is the number of each type of machine present during the "day"; H is the number of each type of worker present while the plant operates; and x is the flow of material inputs required to produce q.

Then, if t is the proportion of the day that the plant actually operates, the observed total production for the day is given by

$$Q = tq = tf(K,H,x).$$

Klein has shown that a cost function dual to this technology exists.

3 Either per year, per day, or per hour depending on the sort of process and the magnitude of the units involved.

[4] See Levin (1977) for a discussion of this and other industries from a process specific point of view.

[5] This is also identical to Stewart's (1980) method.

[6] These properties are listed in Diewert (1982) and in Klein (1980). Note that the interest component of the capital price has been suppressed for simplicity. The inclusion of an interest factor would not change the results in any fundamental way.

[7] The elasticity of substitution can be defined as

$$\sigma_{KH} = C \cdot C_{KH} / C_K \cdot C_H = 0.$$

Since total cost, capital demand and labor demand are presumed positive for positive q, $\sigma_{KH} = 0$ requires $C_{KH} = 0$.

[8] Diewert (1982) shows that any non-linear price function is acceptable, as long as it can be linearized in the vicinity of the current price.

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