DISCLOSURE OF PRODUCT QUALITY UNDER IMPERFECT INFORMATION

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Disclosure of Product Quality Under Imperfect Information

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This paper examines the incentives for firms to disclose information on product quality. I show that within the context of a simple model when firms have reliable information on the quality of their products, they will always disclose the information. This result was also shown by Grossman (1982) in the context of product quality. Stiglitz (1975) had previously used the same argument for the labor market. Next, I examine the incentives for firms to disclose product quality when they do not always have reliable information about product quality. I show that within this model, firms do not disclose all the information they possess and as a result social welfare is lower. I then explore the incentives for firms to test for product quality when testing is costly. When testing is costly, there are many equilibrium levels of testing. Firms never test too little and may test too much. Next, I explore the incentives for firms to improve product quality when information is imperfect and testing costly. In this model, a lemons market occurs: average quality is too low. However, the lemons market inefficiency can be alleviated by mandating testing. Whether mandated testing raises or lowers welfare depends on the gain from improved quality and the cost of testing. Finally, I suggest how my model may be
applicable to a wider range of problems.

It is normal for the producer to have better information on a product's attributes or quality than the consumer. Disclosure by the producer is a common method for this information to be passed on to the consumer. In Section 1, I discuss Grossman's result that firms will always disclose information about product quality. There are many necessary conditions for this result. One crucial assumption is that producers always know product quality.

In Section 2, I show that in a model without this assumption producers withhold information whenever it reveals that product quality is well below the market average. The level of quality below which information is withheld by firms is defined as a trigger level. Information about quality less than this trigger is bad information and is not released to consumers. Information about quality greater than the trigger is good information that firms freely disclose. In Section 3, I discuss the incentive for firms to test their products in order to discover their quality. I show that there are multiple stable equilibria and that firms often test more than is socially optimal. In Section 4, I demonstrate that with imperfect information, average product quality is lower than socially optimal. In Section 5, I show that mandating testing may either raise or lower welfare. When testing is mandated, firms have the
incentive to improve quality. The gain from increased quality must be weighed against the cost of testing.

In Section 6, I discuss general policy implications and discuss other areas to which this model can be extended. Perhaps, the most important implication of this paper is that results from these models are quite sensitive to the assumptions. I explore how changes in certain assumptions change the results. There are many more possible extensions of the model. Disclosures are not always believable and firms sometimes deceive consumers. Also, consumers are not always as rational as I assume. Therefore, my principal conclusion is that it is important to understand how realistic a set of assumptions are for a particular problem before making definitive policy recommendations.

1. Disclosure with Perfect Information

Disclosure is an effective method by which information about product quality can reach the market. Grossman (1982) has argued that all verifiable information will be disclosed in the market. Thus, information about low-quality as well as high-quality products is disclosed. He argues that, if consumers have prior beliefs about the range of product quality in the market, firms selling products at the top of this range will disclose their product's quality. Otherwise, their products will be thought to be just average and will bring a lower price. Products with quality just
below the top of the range will then also be disclosed as such, because consumers would otherwise believe them to be of lower quality than they actually are. This argument can be repeated until the quality of all but the lowest quality product is disclosed, and its quality is known by default.

Grossman's argument can be formalized in a model which will also be useful for later analysis.

1.1. The Model

Consumers:

(1) Consumers have unitary demand for the product, i.e., they purchase at most one unit of product, regardless of quality. (See Figure 1.)

(2) All consumers have identical expected utility, $V$. $V$ is linear in expected quality, $E(q)$, and in risk, $\text{var}(q)$, and separable in income: where $r$ is the degree of risk aversion:

$$V = E(q) - r \text{var}(q) + \text{Income}.$$

(3) Consumers maximize utility: they purchase the product that maximizes consumer surplus.

Producers:

(4) Each firm produces exactly one unit.

(5) The production process is the same for all firms, but quality is stochastic and varies between minimum quality, $q_{\min}$, and maximum quality, $q_{\max}$. The production process is characterized by a uniform distribution between $q_{\min}$ and $q_{\max}$. For simplicity, it is assumed that $q_{\max} = q_{\min} + 1$. Thus, the uniform distribution has a height of one. (See Figure 2.)

(6) The cost of engaging in production is $c$ per unit produced.
Figure 1

Consumer Demand

Price-$

$E(q) - r var(q)$

$price(e(q), var(q))$

quantity
Figure 2

Distribution of Quality

1

$q_{\text{min}}$ $q_{\text{max}} = q_{\text{min}} + 1$

quality
Structure of Information:

(7) Firms know their product quality and can release it in a believable and verifiable form.

(8) Consumers know quality if disclosed. Consumers know that a product of undisclosed quality is undisclosed because a firm is maximizing profits by not disclosing. Consumers also know the production process as described above.

Market Equilibrium:

(9) There is free entry into the market, and consequently expected profits are equal to zero. Prices are set competitively.

I now derive the equilibrium price function, i.e. a product's price as a function of its expected quality and the variance of its quality. By assumptions (2), (3), and (9), all consumers must achieve the same consumer surplus, $k$, in equilibrium. Otherwise, consumers with lower consumer surplus would bid more for products where consumer surplus is higher. Therefore:

$$E(q) - r \text{ var}(q) - \text{price}[E(q), \text{var}(q)] = k,$$

and

$$\text{price}[E(q), \text{var}(q)] = E(q) - r \text{ var}(q) - k.$$

Thus, a product of known quality, $q'$ sells for $q'-k$. A product with expected quality, $q'$, and a variance of $1/12$ (the variance of the uniform distribution) sells for $q'-r/12-k$.

$k$ is the consumer surplus derived from purchasing the product. (See Figure 1.) $k$ is determined by assumption (9), the free entry condition:
\[ E(\text{price}) = E(q) - r \, \text{var}(q) - k = c, \]

or

\[ k = E(q) - r \, \text{var}(q) - c. \]

\( k \) also depends on the equilibrium disclosure policy of firms. If all information is disclosed then \( \text{var}(q) = 0 \), and

\[ k = E(q) - c = q_{\text{min}} + 1/2 - c, \]

and thus,

\[ \text{price}(q) = q - (q_{\text{min}} + 1/2) + c. \]

If no information is disclosed then \( \text{var}(q) = 1/12 \) and

\[ k = q_{\text{min}} + 1/2 - r/12 - c, \]

and thus,

\[ \text{price}(q) = c. \]

Therefore, the price is the same for all products. When no information is disclosed consumer surplus is lower by \( r/12 \) than when all information is disclosed.

The used car market is an example of a market like the one described in the model. Five-year-old Chevies vary in quality depending on the care given them by previous owners and other random factors. If used car dealers know the car's quality, they must decide whether or not to release their information to consumers, who otherwise have no good way of determining quality. The model may also characterize other goods. Firms may begin with the same basic technology for producing widgets. After production, however, not all widgets have the same quality, because of various uncontrollable shocks, such as unperceived differences among
firms in worker or manager quality, or differences in humidity during production. Firms are aware of the quality of their product and must decide whether or not to disclose it. (The incentives for firms to test their products when testing is costly are explored in Section 3.)

A summary of the model may be given as follows:

(1) \( \text{price}(q) = E(q) - r \text{var}(q) - k \) price equation

(2) \( f(q) = 1 \) \( q_{\text{min}} < q < q_{\text{min}} + 1 \) production process

(3) Firms know their products quality

(4) Consumers know what firms disclose about quality and that if firms do not disclose quality that the firms are maximizing profits by not disclosing. Consumers also know the production process.

1.2. **Proof of Grossman's Result**

In this section, I show that within the context of the above model firms always disclose quality. Assume for now that consumers are risk neutral, i.e. that \( r=0 \). The proof goes through equally well if consumers are risk averse, but it is not as tractable.

Suppose no firms disclose quality. The price received by all firms is \( q_{\text{min}} + 1/2 - k \). All firms with quality \( q > q_{\text{min}} + 1/2 \) have an incentive to disclose their product's quality, since they would then receive price \( p(q) = q - k > q_{\text{min}} + 1/2 - k \). All remaining firms not disclosing now receive the \( q_{\text{min}} + 1/4 - k \), since \( q_{\text{min}} + 1/4 \) is the average
quality of undisclosed products. Thus all firms with product quality $q > q_{\text{min}} + 1/4$ disclose, lowering the price for non-disclosing firms to $q_{\text{min}} + 1/8 - k$. This line of reasoning continues with all firms with product quality $q > q_{\text{min}} + (1/2)^n$ disclosing in the $n$'th iteration. Since the limit of $(1/2)^n$ as $n$ goes to infinity is zero, all firms with $q > q_{\text{min}}$ disclose their product's quality. The result is complete disclosure.\(^3\)

Not only is complete disclosure an equilibrium, but it can be shown that complete disclosure is the only equilibrium. The proof is a proof by contradiction. Assume that there exists another equilibrium where products of quality, $q \in V$ are not disclosed. Also, assume that $V$ is closed and of positive measure. Let $b$ equal the maximum $q$ over the set $V$. Thus,

$$b > E[q|q \in V] \text{ as } b \text{ is the maximum value of } q \text{ and the expectation gives positive weight to } q < b.$$  

If the quality is undisclosed,

$$\text{Price}[q \in V|q \text{ undisclosed}] > \text{Price}[q \in V|q \text{ disclosed}],$$

or else firms would prefer to disclose quality. Also, the price of an undisclosed product is $E[q|q \in V]$. However, price of a product of quality $b$ is $E[b|q \text{ disclosed}]$, which equals $b$. Thus, combining the above equations yields,

$$E[q \in V|q \text{ undisclosed}] > b.$$  

This contradicts the definition of $b$. Thus, there cannot
exist a set $V$ where quality is not disclosed. Therefore, the only equilibrium is complete disclosure.

2. Disclosure with Imperfect Information

In Section 1, I assume that the firm always knows product quality exactly. The used car dealer knows the quality of his car and the widget manufacturer knows the quality of his widget. While this may not be an extreme assumption for some industries, it seems implausible that a used car dealer has perfect knowledge about all attributes of the car. Furthermore, even with proper design and testing for most products, some information about an attribute will most likely not be known to the producer. This unknown information is not necessarily bad information. A manufacturer may produce an extremely durable product, but not discover it until years later.

Assume that a firm tests the product after production, but that testing is inconclusive with probability $p$ and that firms cannot retest. For example, a new design can be tested for durability by subjecting it to intensified use. The product being tested may break after 100 uses indicating poor quality, or it may break after 1000 uses indicating high quality. Alternatively the product might malfunction on the first use, regardless of the actual quality, because the testing procedure was not applicable for this new design. Thus, in this latter case, the test can said to
have been inconclusive.

Therefore, the firm knows the true quality, \( q \), with probability \( (1-p) \). When testing is inconclusive, the after-test information about the product is the same as the pre-test information about the product; that is, quality \( q \) is uniformly distributed between \( q_{\text{min}} \) and \( q_{\text{min}}+1 \). In this case, the firm cannot disclose product quality since the firm does not know the quality. However, it may disclose that it has tested the product and the test was inconclusive, by labeling the product "quality unknown" ("QU"). A product labeled "QU" may be thought of as a product labeled "run of the mill." That is, the firm makes no claims about the quality of the product.

When the test is conclusive, the firm may choose not to reveal the information, but instead to also label the product "quality unknown." It is possible, but unlikely that consumers or the government could detect that the firm is lying in this situation. The firm can always maintain that it never performed the test. (A firm that claims that its product has a quality superior to its actual quality will usually be detected. However, a firm that says it does not know the quality when it actually does will rarely be detected.)

2.1. The Decision to Disclose Information

Given that with probability \( p \) a firm does not know its own quality and that it can with impunity declare "quality
unknown" even when the quality is known, the firm must decide what information to disclose and what information to conceal. A firm will disclose the quality of its product whenever the price for the disclosed quality is greater than the price of a product labeled "quality unknown." This proposition follows directly from the assumption that firms maximize profits. Since a firm sells only one unit of the good, maximizing profits corresponds to maximizing the price by deciding whether to disclose quality.

A disclosure policy can be characterized by the decision to disclose quality whenever quality is greater than a trigger, $t$. Quality greater than the trigger ($q > t$) is good information and is disclosed, and quality less than the trigger ($q < t$) is bad information and is concealed. Thus, $t \in (q_{\min}, q_{\max})$ is the trigger value and the disclosure policy can be denoted by $t$. (Also, define $T$ as the amount the trigger exceeds $q_{\min}$, that is, $T = t - q_{\min}$.)

From Section 1, the price of a product with disclosed quality, $q$, is $q - k$. The price of a product labeled "QU" is $E(\epsilon|"QU", t) - k$. Define, $E(\epsilon|"QU", t) - k = a(t)$. Thus, a firm will prefer disclosing quality if $q - k > a(t) - k$ or $q > a(t)$. Therefore, $a(t)$ is also the trigger in equilibrium, that is, $a(t) = t$ is an equilibrium condition.

To calculate $a(t)$ Bayes Theorem is used. When a firm announces "quality unknown", either it knows $q$ and $q < t$ or
else it truly does not know quality and \( q \in [q_{\text{min}}, q_{\text{max}}] \).

Below are the probabilities that a firm labels the product "QU" given that it does and does not know quality.

\[
\begin{align*}
\text{Prob}(\text{"QU"}\mid \text{Firm does not know}) & = 1 \\
\text{Prob}(\text{"QU"}\mid \text{Firm knows}) & = T.
\end{align*}
\]

The latter probability is equal to \( T \) because \( \text{ex ante} \) \( q \) is uniform between \( q_{\text{min}} \) and \( q_{\text{min}} + 1 \), i.e., \( \text{Prob}(q < t) = T \). From before,

\[
\begin{align*}
\text{Prob}(\text{Firm does not know}) & = p \\
\text{Prob}(\text{Firm knows}) & = 1 - p.
\end{align*}
\]

Using Bayes Theorem:

\[
\begin{align*}
\text{Prob}(\text{"QU"}) & = \\
& = \frac{\text{Prob}(\text{"QU"}\mid \text{Firm does not know})\text{Prob}(\text{Firm does not know}) + \text{Prob}(\text{"QU"}\mid \text{Firm knows})\text{Prob}(\text{Firm knows})}{p + T(1-p)}.
\end{align*}
\]

\[
\begin{align*}
\text{Prob}(\text{Firm Knows}\mid \text{"QU"}) & = \\
& = \frac{\text{Prob}(\text{"QU"}\mid \text{Firm knows})\text{Prob}(\text{Firm knows})}{\text{Prob}(\text{"QU"})} \\
& = \frac{T(1-p)}{p + T(1-p)}.
\end{align*}
\]

\[
\begin{align*}
\text{Prob}(\text{Firm does not know}\mid \text{"QU"}) & = \\
& = \frac{\text{Prob}(\text{"QU"}\mid \text{Firm does not know})\text{Prob}(\text{Firm does not know})}{\text{Prob}(\text{"QU"})} \\
& = \frac{p}{p + T(1-p)}.
\end{align*}
\]

The expected value of \( q \) given a firm announces "QU" is:

\[
\begin{align*}
a(t) & = \text{Prob}(\text{Firm knows}\mid \text{"QU"})E[q\mid \text{Firm knows}] + \\
& \quad \text{Prob}(\text{Firm does not know}\mid \text{"QU"})E[q\mid \text{Firm does not know}].
\end{align*}
\]
I

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Solving \( a(t) = t \), yields the equilibrium \( t \), of

\[
\frac{T(1-p)}{[p+T(l-p)]} \left( q_{\text{min}} + \frac{T}{2} \right) + \frac{p}{[p+T(l-p)]} \left( q_{\text{min}} + \frac{1}{2} \right) = q_{\text{min}} + \frac{T^2(1-p)}{2(p+T(l-p))}.
\]

Solving \( a(t) = t \), yields the equilibrium \( t \), of

\[
t = \frac{p^5}{[1+p^5]}.
\]

Figure 3 shows the distribution of quality for products labeled "QU".

Remembering that \( p \) is the probability of an inconclusive test, and \( t \) is the trigger above which information on quality is disclosed, the equation for the equilibrium trigger yields some expected results. If \( p=0 \), \( t = q_{\text{min}} \) and all information is released. All information disclosed is Grossman's result as explained in Section 1.

As \( p \) approaches one, that is, testing becomes more inconclusive, \( t \) approaches \( q_{\text{min}} + \frac{1}{2} \), only information on products of above average quality is disclosed. Also the trigger is a monotonically increasing function of \( p \). Thus, the less reliable the testing, the more information the firm withholds.

2.2. Disclosure with Risk Averse Consumers

On average, risk neutral consumers do not suffer because of the failure of firms to release bad information. Consumers pay average price of \( q_{\text{min}} + \frac{1}{2} - k \) which equals \( c \), and receive an average quality of \( q_{\text{min}} + \frac{1}{2} \). If consumers are risk averse, however, utility is lowered by the amount \( \text{rvar}(q|"QU",t) \). The distribution of products labeled "QU"
Figure 3

Equilibrium and Disequilibrium Values for the Trigger

\[ \frac{1}{p + T(1-p)} \]

\[ \frac{p}{p + T(1-p)} \]

3a—Equilibrium
Figure 3 - Continued

3b--trigger too high

3c--trigger too low
is not the same when consumers are risk averse because the equilibrium condition is no longer \( a(t) = t \) but rather \( a(t) - r\text{var}(q|"QU", t) = t \). Consumers who purchase the undisclosed quality must be compensated for the increased variance. Figure 4 shows the equilibrium trigger when consumers are risk averse. The more risk averse consumers are, the more information is disclosed in equilibrium. This result can be seen by looking at the condition for an equilibrium trigger. For a trigger to be an equilibrium, 

\[ E[q] - r\text{var}[q] = T \]

must hold, where,

\[ E[q] = (\frac{T+p+pT}{2}T + p(1-T)), \]
\[ E[q^2] = (\frac{(1-p)T^3+p}{3}T + p)(1-p)T), \]

and 

\[ \text{var}[q] = E[q^2] - (E[q])^2. \]

(All expectations are for a product labeled "QU" given \( p \) and \( T \).) Totally differentiating the equilibrium trigger condition with respect to \( T \) and \( r \) yields:

\[ [d(E(q)/dT]DT - \text{var}(q)Dr - r[d(\text{var}(q)/dT]DT = DT. \]

Evaluating at \( r=0 \) yields:

\[ \frac{dT}{Dr} = \text{var}(q)/(p^1.5 - 1) < 0, \]

since the denominator is less than zero. Therefore, increases in risk aversion, i.e., increases in \( r \), cause a decrease in \( T \), and hence more information is disclosed.

Since risk averse consumers would prefer complete disclosure, firms might attempt to earn a reputation for
Figure 4
Distribution of Quality for Products Labeled "QU" with Risk Averse Consumers

\[ a = \text{mean of distribution} \]
\[ t = \text{trigger} \]
\[ s = \text{variance of distribution} \]
\[ t = a - r \cdot s \text{ is the equilibrium condition} \]
always disclosing information, even bad information, i.e., q<q. However, reputation is unlikely to lead to all information being disclosed unless contracts are written for many periods. Imagine a firm that decides to always disclose product quality. The firm will suffer an expected loss equal to (1-p)T^2/2, that is, the probability of knowing the quality, (1-p), times the probability that the quality is below the trigger used by other firms, T, times the average loss, (T/2), from disclosing the quality of low quality products, rather than labeling the product "QU" and receiving t-k. The firm would have to be able to make up for this loss the p% of the times that the quality is actually unknown by receiving the price of an average product whose quality is actually unknown, q_{min}+(1/2)-r/12-k rather than t-k, the price of a product labeled "QU" when firms withhold information. If a firm has earned this reputation and always discloses quality when it is known, the firm might be able to receive the premium. However, there is an incentive to cheat on the reputation and label some low quality known products "QU".

Consumers will not want to pay the price premium for products labeled "QU" if they believe that firms are cheating and knowingly selling low quality products as "QU" products. Consumers will observe quality after they use the product. They base their judgment about whether a firm is cheating on the percentage of products labeled "QU" and o-
the quality distribution of products labeled "QU" over time. Whenever this percentage is greater than p, or the average quality of products labeled "QU" is less than $q_{\min} + 1/2$, chances are that firms have knowingly sold low quality products as "QU". Firms with a reputation for not cheating will have a strong incentive to cheat if in the previous periods, the percentage of products undisclosed is much less than p and the products which were actually of unknown quality were of quality greater than $q_{\min} + 1/2$. Then knowingly labeling a product of quality $q_{\min}$ as "QU" will not jeopardize the reputation of the firm but will increase revenue from $q_{\min} - k$ to $q_{\min} + 1/2 - r/12 - k$. Thus, firms will sometimes have the incentive to milk their reputations.

When contracts are written for many periods, the firm's profits are unaffected by its disclosure decisions during the period of the contract. However, when the contract is renewed, it is to the advantage of the firm to have earned a reputation for disclosing all information, because when quality is truly unknown is will receive higher prices. Thus, firms have more of an incentive to earn reputations for disclosing all information when contracts are over several time periods.

2.3. Consumers Misperceptions about the Reliability of Testing

Misperceptions about p will not persist in the market
If firms are aware of the misperceptions and if consumers observe the relative number of products labeled "QU" and use these observations to update their estimate of $p$. Assume consumers have incorrect perceptions about the reliability of the testing mechanism, i.e., consumers conjecture the reliability is $1-p_w$, where $p_w \neq P_{true}$. Furthermore, assume firms are aware of these misperceptions. They will then use the trigger,

$$t_w = q_{min} + p_w \frac{5}{1 + p_w^5},$$

in order to maximize profits. (If there are many firms then one firm will not alter its disclosure decision in order to prevent consumers from learning $p$.) However, then the number of products labeled "QU" is $P_{true} + (1-P_{true})T_w$, which does not equal $p_w + (1-p_w)T_w$. Therefore, consumers will update their estimates in the direction of $P_{true}$. If $P_w < P_{true}$ then consumers will expect fewer products to be labelled "QU" than actually are labelled "QU". Therefore, consumers will revise their estimates of $p$ upwards.

Similarly, if $P_w > P_{true}$ consumers will expect more products labelled "QU" than actually are labelled "QU". Therefore, consumers will revise their estimates of $p$ downwards. Only when $P_w = P_{true}$ will consumers' perceptions be borne out by the market. Therefore, consumers need not have a priori knowledge about $p$, as the market equilibrium will reveal it.
3. **Costly Product Testing**

In the previous sections, I assumed that all firms test products and that the testing is costless. In this section, I assume that testing is costly and allow the number of firms who test to be endogenously determined. I show that, for low costs of testing all firms test in equilibrium. For moderate costs, either everyone tests, no one tests, or a certain fraction tests. However, the latter equilibrium is not stable. For high costs of testing, no one tests. I also show that firms have incentives to test more often than is socially optimal.

To simplify the exposition, once again assume that consumers are risk neutral. Furthermore, assume that the test for product quality always reveals the true quality, i.e., $p=0$. However, acquiring the technology costs the firm $c_t$. For a firm not investing in the testing procedure, the product quality is unknown, i.e., $p=1$, but no expense is incurred.

A firm has two decisions to make, whether to acquire the testing procedure, and what information to reveal, if it does test. The first question to be answered is what information will a firm release if it does test. Assume $\Theta$ is the proportion of firms which have not acquired the testing procedure, and $(1-\Theta)$ the proportion which have, and that $\Theta$, is for the moment fixed. Following
the technique employed in Section 2, the trigger, $t$, can be computed. The trigger will depend on $\theta$. A firm which knows quality, $q$, is indifferent between revealing $q=t$ and labeling the product quality unknown. (Once again, $q>t$ is revealed, $q<t$ is concealed.) The consumers is willing to pay the expected value of the product labeled "QU".

Assuming $\theta$ fixed, $\theta$ plays the identical role to $p$. If a product is labeled "QU" either the firm does not have the testing procedure and the conditional mean for $q$ is $q_{\min}+1/2$, or else, the firm has the testing procedure but has decided not to disclose, whereupon the conditional mean is $q_{\min}+T/2$. Thus, the trigger in "equilibrium" for the firm with the testing procedure is $t=\theta^5/(1+\theta^5)$. All information about $q$ greater than $t$ is released, all information about $q$ less than $t$ is concealed. However, the equilibrium $\theta$ must be determined.

No one testing the product, $\theta=1$, is an equilibrium if the gain in expected price from testing is less than the cost of testing. In this case no one would have the incentive to start testing and hence $\theta=1$ is an equilibrium.

Some firms testing and others not, is an equilibrium if firms are indifferent between testing and not testing. In this case, both types of firms would be making the same profit.

All firms testing, i.e. $\theta=0$, is an equilibrium if the loss in expected price to any one firm from ceasing testing
exceeds the cost savings from not testing.

The expected price if the firm tests is:

\[ \text{Prob}(q>t)E[q|a>t] + \text{Prob}(q<t)E[q|"QU",t] - k \]
\[ = (1-T)[q_{\text{min}}+(1+T)/2] + T[q_{\text{min}}+T] - k \]
\[ = (1+T^2)/2 + q_{\text{min}} - k. \]

This price depends on \( \theta \) as \( T \) depends on \( \theta \). (This price is greater than \( q_{\text{min}}+T-k \) which is the price received by the firm that never tests and therefore always labels the product "QU". (Remember, \( T<1/2 \). Therefore, when testing is costless, all firms will test and all information is disclosed.) The gain from testing is the expected price if the firm tests minus the price if the firm does not test. Thus, the gain is \((1+T^2)/2 - T\).

All firms test their product, i.e., \( \theta=0 \), is an equilibrium if the gain in expected price from testing is greater than the cost of testing:

\[(3.1) \quad (1+T^2)/2 - T > c_t. \]

When \( \theta=0 \), \( t=q_{\text{min}} \), \( T=0 \) and \( (3.1) \) becomes \( 1/2 > c_t \). Therefore, all firms test only when \( c_t \) is less than \( 1/2 \).

Some firms test and others not, i.e., \( 0<\theta<1 \), is an equilibrium if firms are indifferent between expected gain in price from testing and the cost of testing.

\[(3.2) \quad (1+T^2)/2 - T = c_t. \]

Equation \( (3.2) \) can hold only for \( c_t \) between \( 1/8 \) and \( 1/2 \) and when \( \theta=(1-(2c_t)^{1/5})^2/2c_t \). (Note: \( c_t=1/8 \) implies \( \theta=1 \), \( c_t=1/2 \) implies \( \theta=0 \).)

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No firms test, i.e., $\theta=1$, is an equilibrium if the expected gain to testing is less than the cost of testing:

\[(3.3) \quad \frac{(1+T^2)}{2} - T < c_t.\]

When $\theta=1$, $t=\theta_{\text{min}}+1/2$, $T=1/2$, and (3.3) is $1/8 < c_t$. Thus, $c_t > 1/8$ is a necessary condition for no firms testing. Therefore, no firms test only when the cost of testing is greater than $1/8$. Alternatively, when the cost of testing is less than $1/8$ firms always test.\(^6\)

These results may be summarized as:

<table>
<thead>
<tr>
<th>Costs</th>
<th>Number of firms not testing</th>
<th>Stability of equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_t &lt; 1/8$</td>
<td>$\theta=0$: all firms test</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>$\theta=0$: all firms</td>
<td>yes</td>
</tr>
<tr>
<td>$1/8 &lt; c_t$</td>
<td>$\theta=(1-(2c_t)^{1/2})/2c_t$: some</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>$\theta=1$: no firms test</td>
<td>yes</td>
</tr>
<tr>
<td>$1/2 &lt; c_t$</td>
<td>$\theta=1$: no firms test</td>
<td>yes</td>
</tr>
</tbody>
</table>

Next the stability of the equilibria are explored. All firms testing, i.e., $\theta=0$, is a stable equilibrium for $c_t < 1/2$. If everyone is testing and several firms decide not to test, they will save $c_t$, but lose approximately $(1+T^2)/2 - T$, which equals $1/2$. Therefore, for $c_t$ strictly less than $1/2$, those firms who have stopped testing will prefer to resume testing. Thus, $\theta$ will decrease back to $\theta=0$ and therefore $\theta=0$ is a stable equilibrium.

Likewise, no firm testing is a stable equilibrium for $c_t > 1/8$. When no one is testing, firms which decide to test
will release information whenever \( q > l/2 \). This trigger leads to an expected gain from testing of \( 1/8 \). However, if the cost of testing is strictly greater than \( 1/8 \), these firms will find it more profitable to stop testing. Thus, \( \theta \) will then increase back to \( \theta = 1 \) and therefore \( \theta = 1 \) is a stable equilibrium.

The equilibrium \( \theta = (1 - (2c_t)^{5})^{2}/2c_t \), for \( 1/8 < c_t < 1/2 \), is not stable. If \( \theta \) increases slightly, i.e., a few firms decide not to test, then all the remaining firms prefer not to test. This instability arises because as more firms decide not to test, the remaining firms are better off not testing. As \( \theta \) increases, the gain from testing decreases, and thus firms that previously tested will find the cost of testing greater than the gains. Then \( \theta \) will continue to increase until \( \theta = 1 \). Thus, for small increases in \( \theta \), the market moves to the stable equilibrium \( \theta = 1 \).

The same argument can be made for small decreases in \( \theta \), i.e., a few firms which previously did not test decide to test. Then the gain to testing increases, and the remaining firms benefit from testing. The market then moves to the stable equilibrium \( \theta = 0 \). (See Figure 5.)

When consumers are risk neutral, there is no social gain from the testing of quality. However, the individual firm often has the incentive to test its product and disclose when quality is greater than the trigger.
Figure 5

Benefits and Costs of Testing When θ Percent Are Not Testing

θ - Percent Not Testing
Therefore, the market equilibria with $\theta < 1$ are inefficient.

When consumers are risk averse, the social gain from testing is the reduction in the risk borne by consumers. However, if the social gain from testing is greater than the cost of testing, then it follows that the private gain from testing is necessarily greater than the cost of testing. The private gain from testing, when no one is testing, is the reduction in the variance, the social gain, plus $1/8$, the expected gain from revealing that quality is above average. Thus, firms always test if the social gain is greater than the cost and may also test when the social gain is less than the cost of testing.$^8$

For moderate costs of testing $(1/8 < \sigma_t < 1/2)$, there are two stable equilibria, all firms testing or no firms testing. The no-test equilibrium is preferable. Thus, it would lower social welfare for the government to mandate complete disclosure of product quality.

4. The Incentive to Improve Quality: A Lemons Market

In previous sections, I assumed that the production technology was given, that is, quality ranged between $q_{\text{min}}$ and $q_{\text{min}} + 1$. In this section, I assume that firms choose the production process. I show that average quality is too low in this environment.

Rather than $q_{\text{max}}$ always equaling $q_{\text{min}} + 1$, firms can set $q_{\text{max}}$ at any level above $q_{\text{min}}$. Thus, firms choose $m$, the
amount by which \( q_{\text{max}} \) exceeds \( q_{\text{min}} \), i.e., \( m = q_{\text{max}} - q_{\text{min}} \). The cost of production is \( c(m) \), where \( c' > 0 \) and \( c'' > 0 \). It is increasingly costly to improve the production process. The production process is still assumed uniform. Therefore, the distribution of quality is:

\[ f(q) = \frac{1}{m} \quad q_{\text{min}} < q < q_{\text{min}} + m. \]

For the socially efficient production process, the marginal cost of improving quality equals the marginal benefit from improving quality. Therefore, the efficient \( m, m^* \), must satisfy \( c'(m^*) = \frac{1}{2} \). (The marginal benefit to society from changing the production process from \( m_0 \) to \( m_0 + \Delta m \) is \( E(q | m_0 + \Delta m) - E(q | m) = \Delta m / 2 \).)

The private incentive to improve quality is not the same as the social incentive when there is no testing or when testing is imperfect. With no testing, firms never have the incentive to improve quality. A pure Akerlof (1970) lemons market would result, i.e., \( m = 0 \) and all products would be of quality \( q_{\text{min}} \). However, when firms test the product, average quality is still too low if the testing is imperfect. Improving the production process from \( m_0 \) to \( m_0 + \Delta m \) generates an expected increase in price of

\[ (1-p)(1-T)\Delta m / 2 \]

which equals \( (1-p^5)\Delta m / 2 \). The first term, \( (1-p) \), is the probability of knowing the quality. The second term, \( (1-T) \), is the probability of wanting to release the information, i.e., \( q > t \). The third term, \( \Delta m / 2 \), is the average increase in quality from improving the production process.
process. Therefore, firms that are maximizing profits over the choice of production processes will set \( m \) so as to satisfy:

\[
(1-p^5)/2 = c'(m).
\]

Since \((1-p^5)\) is strictly less than one and \(c^* > 0\), \( m \) is less than \( m^* \). (Remember \( c'(m^*) = 1/2 \).) Therefore, firms produce products with lower quality on average than is socially optimal when firms do not test or when testing is imperfect.

5. **Mandated Testing**

In Section 2, I showed that firms have a propensity to test too often. However, in that section the production process was exogenously determined. In this section, I show that a market may have an equilibrium where no firms test the product and average quality is below the socially optimal level. However, mandating testing can induce an equilibrium with higher average quality and higher social welfare.

The result that mandating testing can improve welfare is shown though an example. The structure of a more general model for analyzing whether mandated testing increases or decreases welfare is presented in the appendix. Assume that there are two production processes. One has quality ranging between \( q_{\text{min}} \) and \( q_{\text{min}} + 1 \) \((f(q) = 1)\) and the other has quality ranging between \( q_{\text{min}} + 1/2 \) and \( q_{\text{min}} + 1 \) \((f(q) = 2)\). The average quality of the first process is \( q_{\text{min}} + 1/2 \).
Figure 6

Distributions of Quality
with the Low and the High Production Processes

\[ q_{\text{min}} \quad q_{\text{min}+1/2} \quad q_{\text{min}+1} \]

quality
quality of the latter process is $q_{\text{min}} + 3/4$. Thus, average quality is higher by 1/4 with the latter process. (See Figure 6.) To manufacture products with the high quality process costs $Y$ more than to manufacture with the lower quality process. As before, the reliability of the testing is $(1-p)$, and the cost of the testing is $c_t$. Below are the private and social gains from testing and from switching from the low quality to the high quality production process when the low quality process is being used and no one is testing.

<table>
<thead>
<tr>
<th>Test</th>
<th>Net Private Gain</th>
<th>Net Social Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1/8 - c_t$</td>
<td>$-c_t$</td>
</tr>
<tr>
<td>Improve quality and not test</td>
<td>$-Y$</td>
<td>$1/4 - Y$</td>
</tr>
<tr>
<td>Improve quality and test</td>
<td>$1/4(1-p)-Y-c_t$</td>
<td>$1/4 - Y-c_t$</td>
</tr>
</tbody>
</table>

If $p=1/4$, $c_t=13/64$, and $Y=1/64$ the chart becomes:
<table>
<thead>
<tr>
<th></th>
<th>Net Private Gain</th>
<th>Net Social Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>-5/64</td>
<td>-13/64</td>
</tr>
<tr>
<td>Improve quality</td>
<td>-1/64</td>
<td>+15/64</td>
</tr>
<tr>
<td>and not test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Improve quality</td>
<td>-1/32</td>
<td>+1/32</td>
</tr>
<tr>
<td>and test</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ideally society would benefit the most (15/64) from firms improving quality and not testing. However, short of strict output regulation of every firm by an outside party, firms will not improve quality because the private gain is negative (-1/64). In this example, firms will not improve quality and test even though society would gain (1/32) because the private gain is -1/32. Therefore, production with the low quality process and with no firms testing is a stable equilibrium.

If the government mandated testing of the product, the incentive for firms to improve quality would change. Given that all firms are testing, the private gain from improving quality is $(1-p)(1/4) - Y$ where 1/4 is the increase in average quality from changing the production process. For the values given above, the gain is 11/64. Therefore, firms have the incentive to improve quality. To check that once all firms have improved quality, no firm has the incentive to revert back to the low quality production.
process, the gain from reverting back is $E(\text{Price using low quality process}) + Y - E(\text{Price using high quality process})$. When all firms are using the high quality process the trigger is $q_{\text{min}} + 2/3$. Thus, the expected price using the low quality process is:

$$
\text{Prob}(q < 2/3) \cdot \text{Price}("Q0") + \text{Prob}(q > 2/3) \cdot E(q | q > 2/3) - k
$$

$$
= 2/3 (q_{\text{min}} + 2/3) + 1/3 (q_{\text{min}} + 5/6) - k
$$

$$
= q_{\text{min}} + 26/36 - k.
$$

The expected price using the high quality process is $q_{\text{min}} + 3/4 - k$, the average quality minus $k$. Therefore, the gain from using the low quality process is:

$$
q_{\text{min}} + 26/36 + 1/64 - (q_{\text{min}} + 3/4)
$$

$$
= 1/64 - 1/32 < 0.
$$

Since the gain is negative, all firms will choose to continue with the high quality process. Therefore, in this example, mandating testing induces firms to improve quality, and as a result social welfare increases by $1/32$. However, if the cost of testing were $1/4$, social welfare would be lowered by $1/64$. Therefore, mandated testing can either raise or lower social welfare, depending on how the gain from improving average quality compares with the cost of testing.

6. **Conclusion**

I have shown that information on products of low quality is withheld when there is imperfect information
about product quality. I modeled the imperfect information as an imperfect testing technology. However, there are other causes of imperfect information. For example, if certain claims about a car are too technical to be understood by the consumer, then the manufacturer will not be able to disclose certain attributes about the car. Therefore, the manufacturer is able to conceal bad information by pretending that the information was too technical. The result that all information about product quality is not disclosed and that the undisclosed information is more likely to be information on low quality is consistent with much of the real world. Consumers would be better off with the undisclosed information. A liability rule could discourage firms from acquiring the information, i.e., discourage testing of the product. It would be useful to integrate a liability rule into the model to determine when welfare is raised because more information is released and when welfare is lowered because firms are discouraged from acquiring information on product quality.

I have shown an example where welfare can be raised or lowered by mandated testing. A propensity to overtest arises from the desire by a firm to prove that its product have above average quality. There is no social gain from this testing. However, the incentives to improve quality are intertwined with the question of testing. Mandating testing can alleviate a lemons market when testing is
imperfect. Thus, for a market where average quality is too low or innovation is too slow because improved quality cannot be signaled, welfare can be raised by mandating testing. However, given that firms have a propensity to overtest and that mandated testing can lower welfare, the costs and benefits should be carefully weighed before testing is required.

There are also important aspects of testing that my model does not capture. For example, when consumers are heterogeneous in their taste for risk or quality, then testing is an important mechanism for matching consumers and products. However, as long as firms can capture these social benefits of testing in the price then introducing heterogeneous consumers would not change most of the basic results of my models.

This model also captures important characteristics of other problems. Advertising a product may be an important mechanism for convincing consumers of product quality. If advertising is costly and certain claims are difficult to make, then I would expect the unadvertised goods to vary in quality, but on average to be of lower quality than the advertised goods. However, the price of unadvertised goods would be lower. A fuller analysis of private label/brand name markets could be built upon the model in this paper.
APPENDIX

Extended Model for Determining Whether Mandated Testing Increases or Decreases Social Welfare

Assume that a firm can improve average quality by eliminating the lower tail of the quality distribution, i.e.,

\[ f(q|x) = \frac{1}{1-x} \quad x < q < 1. \]

The cost of producing a product using \( f(q|x) \) is \( c(x) \) where \( c(0) \) is assumed to equal zero. For a firm to test a product costs \( c_t \). The reliability of the test is \( (1-p) \).

For a no-test equilibrium to exist, no firm must have the incentive to test regardless of its production process, i.e.,

(a.1) \[ \Pr[\text{test is conclusive}]\Pr[\text{desire to disclose}] \]

\[ [\text{gain if disclose}] - c_t - c(x) < 0 \]

for all \( x \).

Alternatively,

\[ (1-p)\min[1,1/(2(1-x))]\max[x/2,1/4] - c_t - c(x), \]

for all \( x \).

Given mandated testing, the equilibrium \( x \) is defined where:

(a.2) \[ t[\arg\max_{x} \text{Profits}(x,t')] = t'. \]

Firms maximize profits by choice of the production process given the trigger \( t' \). If the actual trigger arising from a production process, \( x^* \), is \( t' \) then the production process \( x^* \)
is in equilibrium. With no test, welfare is $1/2$. Welfare with mandated testing is:

$$(a.3) \quad 1/2 + x*/2 - c(x*) - c_t.$$ 

Welfare is lowered by mandated testing if:

$$(a.4) \quad x*/2 - c(x*) - c_t < 0,$$

and increased if the inequality is reversed.

The direction of the inequality depends on $p$, $c(x)$, and $c_t$. Rather than mandating testing, the government might want to subsidize testing by an amount $s$. The subsidy will lead to a testing equilibrium if:

$$(a.5) \quad (1-p) \min[1,1/(2(1-x))] \max[x/2,1/4] - c_t + s - c(x) > 0 \text{ for some } x.$$ 

The subsidy will increase welfare if:

$$(a.6) \quad x*/2 - c(x*) - c_t > 0.$$ 

For every $x^*$, $c(x^*)$, there exists a $\bar{c}_t$ below which a testing equilibrium is preferable to a no-test equilibrium. Thus, the government can set a subsidy $\bar{s}$ so as to cause a switch only if $c_t < \bar{c}_t$. (Note, the subsidy may actually be a tax, i.e., $\bar{s} < 0$.) However, it is important to note that $\bar{s}$ depends on $x^*$, $c(x^*)$, and $p$. ($x*/2 - c(x^*)$ is the gain from curing the lemons market.) Thus, if the government knows the loss from the lemons market and the reliability of the testing, it need not observe the cost of testing in order to arrive at a subsidy that will either improve welfare if testing is beneficial or keep welfare constant if testing is not beneficial.
Figure 7

Stable Interior Equilibrium

θ - Percent Not Testing
In order for the disclosure to be of value to consumers, it must be comprehensible and believable. A disclosure is comprehensible if consumers understand how the disclosure, if true, will translate into product performance. Thus, the expected utility to be derived from the product can be calculated from the disclosure. A disclosure is believable if there is some mechanism working to ensure that the disclosure is true. Possible mechanisms are verification by publication of reliable test results, direct inspection of the product by consumers, antifraud laws, and reputation of the producer. This paper assumes that all disclosures on the part of firms are comprehensible and believable.

I also assume throughout that one firm cannot disclose the quality of a competitor's product. An interesting extension would allow comparative claims.

With risk aversion the end result is the same, in each iteration, however, more firms disclose, since they gain not only from showing above average-quality, but also from reducing the risk to the consumer.

When firms do not disclose product quality they could be held to a "should have known" standard, a standard that imposes liability when the product turns out to be lower than average quality. A should of known standard is equivalent to mandating a warranty. However, if the market relied on disclosure, rather than warranties, prior to regulation, the costs associated with warranties are likely to be higher than the cost associated with disclosure. Otherwise, firms would have voluntarily offered the warranty and saved the cost of the disclosure. Therefore imposing a warranty may lower welfare.
When \( t = q_{\text{min}} \), i.e., all information that a firm has is released, the expected quality of a "QU" product is the \textit{ex ante} average quality because all product labeled "QU" are actually unknown. Thus, \( a(q_{\text{min}}) = q_{\text{min}} + 1/2 \).

The values "1/8" and "1/2" are artifacts of the range, \( (q_{\text{max}} - q_{\text{min}}) \), being equal to one. 1/2 is the difference between average quality and \( q_{\text{min}} \), and hence is the expected gain from testing if everyone tests. In general, this gain is:

\[
\frac{1}{2} (q_{\text{min}} + q_{\text{max}}) - q_{\text{min}} = \frac{1}{2} (q_{\text{max}} - q_{\text{min}}).
\]

Similarly, 1/8 is the gain if no one tests. In general, this gain would equal:

\[
(1/2) \left( \frac{1}{2} (q_{\text{max}} + q) - q \right) = (1/4) (q_{\text{max}} - q) = \frac{1}{8} (q_{\text{ma}} - q_{\text{min}}),
\]

where \( q \) is the average quality. Thus, whether a testing or no-testing equilibrium exists depends on the range of qualities as well as the cost of testing.

Interior solutions with some firms testing and others not testing need not be unstable equilibria. If certain firms have cost advantages in testing then the firms which can test most cheaply will test and the others will not. Then the firms testing will sometimes not disclose the quality of their product but rather pretend that they did not test. See Figure 7.

The formal calculations follow. If no one is testing the social gain is \( r/12 - c_t \). As firms will disclose if \( q > q_{\text{min}} + 1/2 - r/12 \), the private gain from testing is

\[
(1/2 - r/12)^2 + [1 - (1/2 - r/12)] [1 + 1/2 - r/12] /2 - c_t = r^2 /288 + r/24 + 1/8 - c_t. \]

If \( r/12 - c_t > 0 \), then \( r^2 /288 + r/24 + 1/8 - c_t > 0 \).
BIBLIOGRAPHY


