CYCLES IN NONRENEWABLE NATURAL-RESOURCE COMMODITY PRICES: AN ANALYSIS OF THE FREQUENCY DOMAIN

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CYCLES IN NONRENEWABLE NATURAL-RESOURCE COMMODITY PRICES: 
AN ANALYSIS OF THE FREQUENCY DOMAIN

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I. Introduction

Metal markets are well known for being unstable. The instability of these markets has been studied by industry members, consumers, and governments throughout the world. For example, the U.S. Nonfuel Minerals Policy Review (1979) found both prices and production of nonfuel minerals in the United States to be much more volatile than prices and production in the economy as a whole. Each of these groups is concerned with the nature and causes of severe price fluctuations and the gluts and shortages that characterize these markets. An understanding of mineral-commodity price oscillations is important to the developed nations, because they depend on imports of many minerals as basic inputs to industrial production. It is even more important to developing nations, because they may depend on sales of mineral commodities as their principal source of foreign exchange.

There are many reasons why mineral-commodity prices may be volatile. In the short run, price elasticities of supply and demand are generally low. Short-run supply elasticities are low because mineral-industry capacities cannot be varied quickly or at low cost. Demand elasticities are low because mineral commodities are typically used as intermediate inputs, and their cost often constitutes only a small fraction of the price of final goods. Therefore, moderate price changes may have little effect on mineral-commodity production or consumption in the short run. Prices are also unstable because income elasticities of demand
for these commodities are generally high. Income elasticities are high because metals are consumed almost entirely for industrial purposes. Therefore, changes in economic activity may lead to sizeable fluctuations in mineral-commodity prices. Finally, prices are volatile because mineral-industry investment projects are typically large, and because there are often long delays between the initiation and completion of these projects (perhaps as much as ten years). Such lags may cause very long-run price cycles which are the principal focus of this paper.

In this paper, a model is derived that relates price cycles to investment behavior in the mineral industries. From the model, we can predict that price cycles will be roughly twice as long as the investment period (the time between the investment decision and the project completion). Spectral techniques are used to estimate the principal cycles in metal and fuel prices. For the metals, the estimated cycle lengths reinforce the model predictions. However, no significant cycles are found in the prices of the fuels. The lead-lag relationships between prices of different commodities are also investigated and are found to be significant for those metals and fuels that are jointly produced (as coproducts or byproducts).

The organization of the paper is as follows. In the next section, the theoretical model is derived, and in section III, the data are described. In the fourth section, the spectral techniques used in the empirical analysis of the cyclical behavior of
mineral-commodity prices are discussed and the results of using these techniques are reported. In section V, the relationship between prices of commodities that are jointly produced is developed and tested. And finally, in the last section, the findings are summarized and conclusions are drawn.

II. The Price-Cycle Model

One possible explanation for the observed fluctuations in mineral-commodity prices relates to the nature of investment in these industries. Mineral-industry investment projects typically involve large sums of money. For example, Coe (1978) noted that more than $1 billion might be required to bring an integrated mine-through-refinery copper operation online. In addition, a single investment project often accounts for a large fraction of productive capacity in the industry. Therefore, when a new mine is opened, the additional output may depress price. It is often noted that investment projects are initiated when prices are rising but are completed when prices are falling. A simple difference-equation model may help to explain and summarize this behavior.

Suppose we have a reduced-form price equation,

\[ P_t = f(S_t, D_t), \quad P_s < 0, \quad P_d > 0, \]  \hspace{1cm} (1)

where

- \( P_t \) is price at time \( t \)
- \( S_t \) is supply at time \( t \)
- \( D_t \) is demand at time \( t \).
In the simple case where we have a fixed-coefficient production function and units are chosen so that one unit of capacity ($K_t$) produces one unit of output ($S_t$), $S_t$ can be substituted for $K_t$. In addition, if there is no rationing in the market, demand ($D_t$) can be replaced by consumption ($C_t$). After the appropriate substitutions, equation 1 becomes

$$P_t = f(K_t, C_t), \quad P_K < 0, \quad P_C > 0,$$

where $K_t$ is capacity at time $t$ and $C_t$ is consumption at time $t$.

For empirical purposes, we must specify a functional form for $f(\cdot, \cdot)$. If price is a function of capacity utilization, that is, if price is high when consumption is close to capacity and low when the reverse is true, equation 2 can be written as

$$P_t = a_0 + a_1 \frac{K_t}{C_t}, \quad a_0 > 0, \quad a_1 < 0.$$  \hspace{1cm} (3)

In order to focus on investment behavior, we make one final simplifying assumption; we assume that, except for short-term fluctuations, consumption is stable over time (a no-growth model). Then equation 3 becomes

$$P_t = \gamma_0 + \gamma_1 K_t, \quad \gamma_0 > 0, \quad \gamma_1 < 0.$$  \hspace{1cm} (4)

Investments are made in an industry when future profits are expected to be high. If long-run average cost is constant, then, in the long run, profits are determined by prices and expected profits are related to expected prices. Therefore investment will be a function of expected price,$^2$
It = β_0 + β_1 Pt^*, \quad \beta_1 > 0, \quad (5)

where \( I_t \) is investment initiated at time \( t \)
and \( Pt^* \) is the price that is expected to prevail in the future.

If price expectations are determined by the rate of change of current price, equation 5 becomes

\[
I_t = \beta_0 + \beta_1 (dP/dt). \quad (6)
\]

Differentiating equation 4 with respect to time we obtain

\[
(dP/dt)_t = \gamma_1 (dK/dt)_t = \gamma_1 I_{t-i}, \quad (7)
\]

where \( i \) is the time required for the investment project to reach completion. Combining equations 6 and 7 results in

\[
(dP/dt)_t = \gamma_1 \beta_0 + \gamma_1 \beta_1 (dP/dt)_{t-i}, \quad \gamma_1 \beta_1 < 0. \quad (8)
\]

Equation 8 is a first-order difference equation for price changes. The solution to this equation oscillates with period 2i. The oscillations are damped, steady, or explosive, depending on whether \( |\gamma_1 \beta_0| \) is less than, equal to, or greater than one. If price changes oscillate with period 2i, prices oscillate with the same period about an upward-sloping, constant, or downward-sloping trend line, depending on whether \( \gamma_1 \beta_0 \) is positive, zero, or negative.

So far we have assumed that long-run average costs are constant over time. However, as discussed in Slade (1981), average costs may follow a U-shaped time path. Equation 8 can be generalized to include a U-shaped long-run trend. If price varies with both time and capacity, then equation 4 becomes
\[ p_t = \delta_0 + \delta_1 t + \delta_2 t^2 + \delta_3 K t \delta_1, \delta_3 < 0, \delta_2 > 0, \] \quad (9) \]

and

\[
(dP/dt)_t = \delta_1 + 2\delta_2 t + \delta_3 (dK/dt)_t \\
= \delta_1 + 2\delta_2 t + \delta_3 I t - i \\
= \delta_1 + 2\delta_2 t + \delta_3 B_0 + \delta_3 B_1 (dP/dt)_{t-i}, \quad \delta_3 B_1 < 0. \quad (10)
\]

If we differentiate equation 10 with respect to time, we obtain a simple first-order difference equation in the second derivative of price. Price changes then oscillate about a linear trend and prices oscillate about a quadratic trend. Cycles still have period $2i$.

The length of the investment period, $i$, is thus an important determinant of price behavior. This period consists of several stages—exploration and deposit definition, execution of an engineering feasibility study, procurement of project financing, and, finally, construction of the facility and any needed infrastructure. The total time required can be fifteen years or more. However, several of these steps can be shortened. Many companies own well-explored deposits that can be developed when economic conditions are favorable, and financing is facilitated if internal funds are available. Therefore, particularly in the United States, mines can be brought online in as little as five years. Thus, we might expect to see price cycles of ten years or more in length. In section IV, an empirical analysis of the cyclical behavior of mineral-commodity prices is described.
III. DATA SOURCES

The data consist of annual time series for the period 1870 (or year of earliest available figures) to 1978 for all the major metals and fuels with the exception of nickel and gold. Prices were deflated by the U.S. wholesale-price index (1967 = 1) and are thus in 1967-constant dollars. For some commodities, prices of both ore and metal are available (bauxite and aluminum, for example). However, for consistency, metal prices were always used.

Table I lists the 10 commodities, the units of measurement of their prices, and their data sources. Seven of the 10 price series were taken from Manthy (1978). These series were updated to 1978 when possible by using the sources listed in Manthy (tables MP-3 and MP-6, pp. 211-212).

IV. Empirical Results--Cyclical Price Behavior

Spectral techniques can be used to detect periodicities in time-series data and to obtain systematic estimates of cycle lengths. Spectral techniques decompose a time series into a set of orthogonal components, each of which is associated with a frequency. The power spectrum shows the contribution of each frequency band to the total variance of the variable. These techniques are described in detail elsewhere (see Bloomfield, 1976, for example) and will therefore not be discussed here at great length. Estimated power spectra of economic time series generally
### TABLE I

**DATA SOURCES**

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Units</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>Cents/lb</td>
<td>Schurr, Metal Statistics</td>
</tr>
<tr>
<td>Copper</td>
<td>Index (1951-53=100)</td>
<td>Manthy</td>
</tr>
<tr>
<td>Iron</td>
<td>Cents/lb</td>
<td>Manthy</td>
</tr>
<tr>
<td>Lead</td>
<td>Cents/lb</td>
<td>Manthy</td>
</tr>
<tr>
<td>Silver</td>
<td>Cents/oz</td>
<td>Schurr, Metal Statistics</td>
</tr>
<tr>
<td>Tin</td>
<td>Cents/lb</td>
<td>Metal Statistics</td>
</tr>
<tr>
<td>Zinc</td>
<td>Cents/lb</td>
<td>Manthy</td>
</tr>
<tr>
<td>Coal</td>
<td>$/short ton</td>
<td>Manthy</td>
</tr>
<tr>
<td>Natural gas</td>
<td>Cents/1000 ft³</td>
<td>Manthy</td>
</tr>
<tr>
<td>Petroleum</td>
<td>$/bbl</td>
<td>Manthy</td>
</tr>
</tbody>
</table>
show that most of the power is concentrated at low frequencies (Granger, 1966). However, the spectra examined here do not all taper off uniformly but may contain noticeable peaks that correspond to the most important cycles in the data. Even when there are pronounced peaks, the series cannot be described as truly cyclical in the sense of possessing a well-defined spectral peak, statistically different from spectral values at higher or lower frequencies. However, the noticeable peaks indicate powerful (but somewhat irregular) cyclical tendencies in the price series.

The estimation technique used in this study was as follows. A quadratic trend was removed from each deflated price series. Next, the residual series were tapered by use of a split cosine bell window with 20 percent tapering. The data were then padded with zeroes until there were 128 data points. The periodogram (the modulus squared of the discrete Fourier transform of the series) was calculated at the Fourier frequencies, $2\pi j/128$, $j=0,\ldots,64$. (Cycle lengths, in years, are then $2\pi$ divided by the frequency). Finally, the periodogram was averaged using a Daniel window of length seven to obtain a consistent estimate of the spectral density function.

Figures 1-4 show the estimated spectral densities for aluminum, copper, iron, and zinc. The arrows on the figures mark the frequencies of the most powerful cycles and any harmonics (integer multiples) of these frequencies that are pronounced (denoted by $H_i$). Table II shows the estimated lengths of the
principal cycles for the major metals. It should be emphasized that the data do not permit exact resolution; the estimates of cycle lengths merely correspond to the means of the most important frequency bands. In addition, the deviation of the data from exact sinusoidal behavior prevents accurate prediction of the future. The estimates, however, are consistent with the hypothesis that cycles will be ten years or more in length.

In contrast to the metal-price spectra, in which peaks could be detected, the spectra of fuel prices showed no marked periodicities (the spectral estimates tapered off uniformly at higher frequencies as shown in figures 5 and 6). For many metals, a very few mines account for a large share of annual production; however, a single oil well, gas well, or coal mine supplies an insignificant fraction of its respective market. Therefore, in contrast to metal-production capacity, fuel-production capacity can be increased by a small amount in a fairly short time. Thus, the lack of cycles in the fuel-price series is not surprising.

V. The Relationship Between Coproduct Price Series

Metals that tend to occur together in nature (lead and zinc, and copper and silver, for example) are often produced as coproducts or byproducts. When this happens, investment is joint. The joint nature of investment gives rise to an interesting question. If a high price of lead, for example, triggers investment in lead production, it will also trigger investment in zinc production.
FIGURES 1-2
ESTIMATED SPECTRAL DENSITIES FOR ALUMINUM AND COPPER

Frequencies: $2\pi j/128$, $j=0,...,64$

Aluminum

Copper
FIGURES 3-4
ESTIMATED SPECTRAL DENSITIES FOR PIG IRON AND ZINC

Iron

Frequencies: $\frac{2\pi j}{128}, j=0,\ldots,64$

Zinc

Frequencies: $\frac{2\pi j}{128}, j=0,\ldots,64$
<table>
<thead>
<tr>
<th>Commodity</th>
<th>Cycle length (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>10</td>
</tr>
<tr>
<td>Copper</td>
<td>10</td>
</tr>
<tr>
<td>Iron</td>
<td>10</td>
</tr>
<tr>
<td>Lead</td>
<td>10</td>
</tr>
<tr>
<td>Silver</td>
<td>12</td>
</tr>
<tr>
<td>Tin</td>
<td>13</td>
</tr>
<tr>
<td>Zinc</td>
<td>11</td>
</tr>
</tbody>
</table>
FIGURES 5-6
ESTIMATED SPECTRAL DENSITIES FOR
COAL AND PETROLEUM

Coal

Frequencies: $2\pi j/128$, $j=0,...,64$

Petroleum
when the two metals occur together. Therefore, we might ask whether present zinc prices are related to past lead prices as well as to past zinc prices, a hypothesis that can be tested as follows. Suppose $X_1$ and $X_2$ are price series of two commodities that are jointly produced and that the series $X_1$ and $X_2$ have been processed to achieve covariance stationarity. $X_1$ and $X_2$ will vary with changes in aggregate economic activity and will, therefore, be contemporaneously correlated. In addition, $X_1$ and $X_2$ will most likely be autocorrelated. We can estimate an equation of the form

$$X_{1t} = c + \sum_{i=1}^{n_1} a_i X_{1t-i} + \sum_{i=0}^{n_2} b_i X_{2t-i} + \epsilon_t$$

by ordinary least squares if $\epsilon_t$ is uncorrelated with the variables on the right-hand side of the equation. The hypothesis that the history of the series $X_2$ contains information about $X_1$ that is not contained in the realization of the series $X_1$ can be tested by computing the standard F-test of the significance of $\{b_i\}$. The null hypothesis is therefore

$$H_0: \{b_i = 0, i=1,\ldots,n_2\}.$$  

The principal metals that are produced as coproducts in the United States are lead and zinc. In addition, silver is often trapped by isomorphous substitution in the ores of base metals (particularly copper, but also in lead, zinc, and gold ores) and is produced as a byproduct of these metals. The F-statistic to test the hypothesis that past lead prices affect present zinc
prices was computed to be 3.6, and the F-statistic to test the
effect of past copper prices on present silver prices was 2.4.
With (3,97) degrees of freedom, an F value of 2.7 is significant
at the 95 percent confidence level. Therefore, the first rela-
tionship can be considered significant whereas the second one is
marginal (a reasonable result when we know that silver is also a
byproduct of metals other than copper). By contrast, when the
same statistic was computed for metals where investment is separ­
ate, it was not found to be close to the critical vlaue (for
copper and iron, for example, the F-statistic was .99).

Results for the fuels were similar. The principal fuels that
are produced as coproducts are petroleum and natural gas. The
F-statistic to test the effect of past petroleum prices on present
natural gas prices was 2.9, whereas the same statistic relating
past petroleum prices to present coal prices (commodities that are
related in demand but not in supply) was only 1.7. We can, there­
fore, conclude that significant feedback relationships exist
between the prices of coproducts, a result that is consistent with
the model relating price cycles to investment behavior.

VI. Summary and Conclusion

The many reasons why mineral-commodity prices tend to be
unstable include low short-run price elasticities of supply and
demand, high income elasticities of demand, long delays between
initiating and completing investment projects, and lumpy or
large-scale investment. Each of these phenomena can contribute to the cyclical behavior of prices. However, the length of the price cycle will vary, depending on its cause. For example, high income elasticities of demand might lead to price cycles that average four to five years in length (corresponding to the "business cycle"). In contrast, if the nature of investment in these industries is responsible for the dominant cycle, the period may be much longer. In this paper, a simple model is derived that relates the periodic behavior of mineral-commodity prices to investment in the industry. The model consists of a difference equation in the second derivative of price which when integrated varies cyclically about a quadratic trend. The predicted period of price cycles is twice the length of time required for the completion of investment projects.

The empirical analysis of periodicities in mineral-commodity prices revealed that cycles are superimposed on the long-run U-shaped price paths of the major metals. The cycles, the length of which were estimated by spectral techniques, are much too long (10 to 13 years) to be related to inventory accumulation and depletion or to aggregate economic activity. The estimated cycle lengths are precisely what we would expect if investment is responsible for the periodic behavior of metal prices. In contrast to the metals, no systematic cyclical behavior was found
in the prices of the fuels, a result to be expected because fuel-production capacity can be increased in smaller increments (relative to the total market) and with shorter delays.

Although fairly pronounced cycles were detected in the prices of the major metals, the estimated cycle lengths are merely the means of the most important frequency bands. The behavior of the price paths is sufficiently nonsinusoidal that the empirical results cannot be used for accurate forecasting of upturns or downturns in commodity prices. However, the empirical results do contribute to an overall understanding of the dynamic behavior of metal markets.

In addition to examining the periodic behavior of single price series, the time dependence between prices of commodities that are jointly produced was investigated. Significant feedback relationships were found between prices of metals or fuels that are produced as coproducts where investment is joint. In contrast, no such time dependence was found for commodities where investment is unrelated, even when the commodities are related in demand (i.e., when they are substitutes for one another). The empirical results thus reinforce the conclusion that price cycles are related to investment activity in the industry.
FOOTNOTES

1 Fuel prices are also volatile but are more stable than metal prices.

2 In this simple model, investment can be negative (i.e., when price is falling, mines are closed or allowed to depreciate).

3 Note that $\Delta t$ for this difference equation is $i$, not 1.

4 To use spectral techniques, the price series should be detrended whether or not one accepts the argument for a U-shaped time path for costs.

5 For some commodities, data for the last 2 or 3 years of the period were not available.

6 The classification of metals as major follows Peterson and Maxwell (1979). Gold was eliminated because its price was linked to the dollar for most of the period under consideration. Its deflated price is thus proportional to the reciprocal of the deflating index. Spectral densities for nickel were not computed because the price series is too short.

7 Removing the trend has the side effect of removing the power at very low frequencies.
8 Padding was necessary so that all series would be evaluated at the same Fourier frequencies. In addition, the fast Fourier-transform calculation is more efficient if the number of data points is a power of two.

9 The model does not specify how to choose \( n_1 \) and \( n_2 \); \( n_1 \) and \( n_2 \) were chosen to equal five because, after five lags, \( a_i \) and \( b_i \) were small and not statistically significant.

10 This test is similar to one used by Clemhout and Neftci (1979) in their study of housing-market cyclicality.
REFERENCES CITED


