receiver type $v^S > \pi r$. Consider the receiver's ex-ante utility,

$$E\left[U^R\right] = \alpha E\left[v^R - d\left(\theta, q\right)\right] + (1 - \alpha) Pr\left(q \in c_\theta\right) E\left[v^R - d\left(\theta, q\right) \mid q \in c_\theta\right] \tag{8}$$

As before, the first term captures the fact that any informed sender is able to ensure a match in equilibrium, and the second one captures the fact that with probability $1 - \alpha$ the sender induces a match if and only if $q \in c_\theta$. Based on these two identities and the communication policy from the previous section, we establish the following result:

LEMMA 1 (*Willful Ignorance*) *The level of information acquisition associated with the* sender's *first-best payoff is given by*

$$\overline{\alpha}^* = \left[\frac{v^R}{\pi r - v^R}\right]^2 \tag{9}$$

The intuition for Lemma 1 is that the expected receiver utility conditional on any message is always bounded below by zero, which means that the ex-ante utility must also be bounded below by zero. Hence, if the first-best level of information acquisition α^* were to be strictly above $\overline{\alpha}^*$ then there must exist some message m_θ that yields negative expected utility to the receiver. However, this is in contradiction with the strategic assumption of our receiver's behavior. Interpreted differently, $\overline{\alpha}^*$ is the highest level of information acquisition that enables an informed sender to pool with other desirable types to induce a match. If in contrast α were strictly higher than $\overline{\alpha}^*$ then c_θ would be empty, i.e. no message could ensure a match in equilibrium. Finally, threshold $\overline{\alpha}^*$ falls between zero and one and is increasing in the receiver's valuation v^R and decreasing in the market differentiation

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parameter r. Clearly, the sender is able to engage in higher levels of information acquisition as the receiver values a match more. Relatedly, the information does not depend on v^S because the sender is always better off obtaining a match, and so the receiver's expected utility is the only relevant constraint. While other equilibria exist, we will later show that the sender's first-best outcome is focal.

We now show that there exists a messaging policy $m^*(\cdot)$ that can implement the information acquisition level $\overline{\alpha}^*$ while keeping region c_θ nonempty:

THEOREM 1 *The sender can attain his first-best outcome by selecting the message policy that induces the probability density function*

$$f^*_{m|\theta,q} = (1 - \alpha^*) \ \frac{v^R - d(\theta,m)}{\pi(\pi r - 2v^R)} \mathbf{1} \ \ d(\theta,m) \leq v^R \ + \delta(m - q) \qquad (10)$$

where $\alpha^* = \overline{\alpha}^*$ is the utility-maximizing level of information acquisition of the sender.

The optimal communication policy for the sender involves sampling from different messages at different rates. The informed sender prefers to send attractive messages to the receiver, and samples from more attractive messages more frequently than from less attractive ones. Mixing across messages enables informed senders to pool with all attractive uninformed types, and allows attaining the sender's first-best level of information acquisition.

The result above follows from the following considerations. First, an informed sender has an incentive to pool with attractive uninformed types

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to the largest possible extent. Because attractive truth-telling types send messages in region $s_\phi = \{m : d(\theta, m) \leq v^R\}$, informed types prefer to pool over the same region. Second, in the case of informed senders we need only consider communication policies that are invariant to the sender's location. In order to determine policy $f_{m^*|\theta,q,\alpha}$ in equation (6) we need only look for function $\phi(\cdot)$ from set

$$\Phi = \{\phi'(m, \theta, \alpha) : E_q[U^R(m \in s_\phi, \theta, \alpha)] = 0\} \tag{11}$$

which is independent of the sender's location q. The solution is attainable by setting the receiver's expected utility equal to zero point by point, and the result follows.

Depending on the receiver's beliefs, there may exist other equilibria. For example, if the receiver believes that the sender mixes among messages uniformly, then the sender may prefer to engage in such a policy. However, equilibria that do not attain the sender's first-best level of information acquisition are not robust to forward induction. To see this, consider an equilibrium outcome with fixed beliefs $f_{m|\theta,q,\alpha}$, which induces a level of $\alpha' < \overline{\alpha}^*$. Now suppose the sender deviates from this outcome and chooses level $\alpha'' \in (\alpha', \overline{\alpha}^*]$ instead. Under forward induction the receiver ascribes strategic behavior to the sender, and so is willing to 'revisit' her beliefs in order to rationalize the sender's choice of information acquisition. Consequently, any equilibrium with level $\alpha' < \overline{\alpha}^*$ does not survive forward induction because by choosing level $\overline{\alpha}^*$ instead, the sender can induce more advantageous beliefs. This is summarized in the following corollary.

COROLLARY 1 *Only the equilibrium outcome associated with the sender's first-best level of information acquisition survives forward induction.*

In sum, forward induction allows us to rule out other equilibria that yield lower levels of information acquisition as well as different payoff levels. Theorem 1 also has implications to the payoff of the receiver:

COROLLARY 2 *The sender's first-best information acquisition policy makes the receiver's ex-ante utility, and expected utility conditional on any given message, equal to zero.*

This result follows from the discussion above: by selecting his messaging policy appropriately the sender is able to ensure that the receiver always expects to earn zero utility. If the receiver were to expect a higher utility upon receiving a given message, the sender could alter his mixing distribution to increase the information acquisition level as well as his payoffs.

As a result, payoffs are given by

$$E\left[U^R\right]_{\alpha=\overline{\alpha}^*} = 0 \tag{12}$$

and

$$E\left[U^S\right]_{\alpha=\overline{\alpha}^*} = \frac{v^S - v^R}{\pi r - v^R} v^R \tag{13}$$

where, as expected, the sender is better off as v^S and v^R increase and as r decreases.

We now consider the case in which the sender does not hold ex-ante

transparent motives, i.e. $v^S < \pi r$. For example, in this case the sender may face high communication costs.

4 Costly Persuasion / Low Sender Valuation

In many contexts senders are required to incur a communication cost, say $c > 0$, in order to send a message to receivers. In this case engaging in communication yields gross utility $v^{S'} \equiv v^S - c$ for the sender. As communication costs increase, the match utility of the sender becomes lower and as a result he may no longer want to engage in communication with all receivers. For this reason we now allow the sender not to communicate in case he prefers to avoid a match with an unattractive receiver.

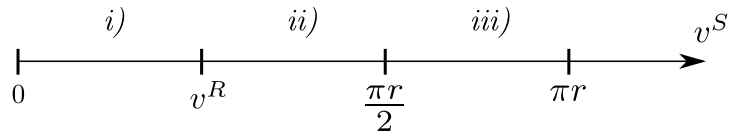Figure 3: Partitions of the Parameter Space

Figure 3 divides the parameter space into different partitions according to the value of v^S. In this section we characterize the cases in which the sender derives utility $v^S < \pi r$ from a match, i.e. the sender no longer holds ex-ante transparent motives. As before, we consider the case of decisive communication when $v^R < E\left(d\left(\theta, q\right)\right)$.

4.1 Region *i)* $v^S < v^R$

When communication costs are high, or equivalently when v^S is low, the sender does not engage in misrepresentation because all attractive receiver

types profit from a match. Consequently the sender engages in full information acquisition and reveals his type whenever his match utility is above zero. If an informed sender's match value is too low, then he prefers not to communicate.

LEMMA 2 *When $v^S < v^R$ the sender engages in full information acquisition and the sender and receiver ex-ante payoffs are given by, respectively,*

$$
\begin{aligned}
E\left[U^S\right] &= Pr\left[v^S \geq d\left(\theta, q\right)\right] E\left[v^S - d\left(\theta, q\right) \mid v^S \geq d\left(\theta, q\right)\right] \quad (14)\\
&= \frac{\left(v^S\right)^2}{2\pi r}
\end{aligned}
$$

and

$$
\begin{aligned}
E\left[U^R\right] &= Pr\left[v^S \geq d\left(\theta, q\right)\right] E\left[v^R - d\left(\theta, q\right) \mid v^S \geq d\left(\theta, q\right)\right] \quad (15)\\
&= \frac{v^S\left(2v^R - v^S\right)}{2\pi r}
\end{aligned}
$$

In this case the comparative statics behave as expected and in particular the receiver is better off as v^S increases in this region because this leads to a higher likelihood of a match.

4.2 Region *ii)* $v^R < v^S < \frac{\pi r}{2}$

When v^S is slightly higher there exist unattractive types of senders that prefer to misrepresent themselves in order to induce matches. However, in this region uninformed senders prefer not to communicate because they expect negative utility from matches. As a result, the receiver understands

that if she receives a message it must originate from an informed sender. Upon reception of a message the receiver expects match utility

$$E\left[U^{R} \mid received\,message\right] = E\left[v^{R} - d\left(\theta, q\right) \mid v^{S} \geq d\left(\theta, q\right)\right] = v^{R} - \frac{v^{S}}{2}$$

(16)

which yields the following result:

LEMMA 3 *When $v^{R} < v^{S} < \frac{\pi r}{2}$ communication and matches take place if and only if $2v^{R} \geq v^{S}$, in which case the sender engages in full information acquisition and payoffs are equal to those of case i).*

When $2v^{R} \geq v^{S}$ the sender engages in full information acquisition because that decision has no bearing on the receiver's expected utility of a match, conditional on receiving a message. However, when $2v^{R} < v^{S}$ no matches occur because the sender's incentive for misrepresentation is too great. The reason is that under cheap-talk communication the sender is unable to commit not to send attractive messages when the value of the match is low.

4.3 Region *iii)* $\frac{\pi r}{2} < v^{S} < \pi r$

In this case uninformed senders are willing to communicate, but informed senders located far from the receiver's location may not be. This case is similar to the main model, with the added complexity that now informed types may prefer not to engage in communication. The receiver's utility

conditional on receiving a message is

$$E\left[U^R \mid message\right] = \gamma E\left[v^R - d\left(\theta, q\right) \mid v^S \geq d\left(\theta, q\right)\right] \tag{17}$$

$$+ \ (1 - \gamma) Pr\left[v^R \geq d\left(\theta, q\right)\right] E\left[v^R - d\left(\theta, q\right) \mid v^R \geq d\left(\theta, q\right)\right]$$

where γ is the probability that the sender is informed conditional on prefer-ring to communicate; $\gamma = \frac{\alpha Pr\left(v^S \geq d(\theta, q)\right)}{\alpha Pr(v^S \geq d(\theta, q)) + 1 - \alpha}$. Equating $E\left[U^R \mid message\right]$ to zero yields an upper bound on the sender's level of information acquisition

$$\overline{\alpha}'^* = \min\left\{1, \ \left[\frac{v^R}{v^S - v^R}\right]^2\right\} \tag{18}$$

By the same methodology used to prove Lemma 1 and Theorem 1, we find the following results:

LEMMA 4 *There exists a message policy that enables the sender to attain level of information acquisition $\overline{\alpha}'^*$ when $\frac{\pi r}{2} < v^S < \pi r$. Forward induction equilibrium payoffs are given by*

$$E\left[U^R\right] = \overline{\alpha}'^* Pr\left[v^S \geq d\left(\theta, q\right)\right] E\left[v^R - d\left(\theta, q\right) \mid v^S \geq d\left(\theta, q\right)\right] \tag{19}$$

$$+ \ (1 - \overline{\alpha}'^*) Pr\left[v^R \geq d\left(\theta, q\right)\right] E\left[v^R - d\left(\theta, q\right) \mid v^R \geq d\left(\theta, q\right)\right]$$

$$= \ 0$$

and

$$E\left[U^S\right] = \overline{\alpha}'^* Pr\left[v^S \geq d\left(\theta, q\right)\right] E\left[v^S - d\left(\theta, q\right) \mid v^S \geq d\left(\theta, q\right)\right] \tag{20}$$

$$+ \ (1 - \overline{\alpha}'^*) Pr\left[v^R \geq d\left(\theta, q\right)\right] E\left[v^S - d\left(\theta, q\right) \mid v^R \geq d\left(\theta, q\right)\right]$$

$$= \ \frac{v^R v^S}{\pi r}$$

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As in the main model, the sender is able to appropriate all of the receiver's expected utility by setting $\alpha = \overline{\alpha}'^{*}$.

In this parameter region both the cost as well as the content of the message act as signals to the receiver. The mere presence of communication is relevant for the receiver because she understands that not all sender types are willing to communicate. The content of the message provides further information: if the message is close to the receiver's location then she weighs the relative probabilities of types of senders, but when the message is far away the receiver understands the sender is uninformed and takes appropriate action. Lemma 4 unites prior work on informative and dissipative communication, allowing these mechanisms to have complementary rather than substitute roles.

# 5   Discussion and Welfare Implications

Having characterized the equilibrium outcomes across different parameter regions, we now present general results on the first-best information levels:

THEOREM 2 *The receiver's first-best level of information acquisition is always less or equal to the sender's. Moreover, both parties' preferred levels weakly decrease in v^S and r and increase in v^R.*

This is because the sender always prefers a higher level of information acquisition in order to engage in persuasive communication. However, unless information acquisition is necessary for a match to occur, the receiver prefers to be approached only by uninformed senders, who in turn prefer

to communicate truthfully. Figure 4 summarizes the receiver and sender first-best information levels.

Figure 4: First-Best Information Levels for Sender and Receiver
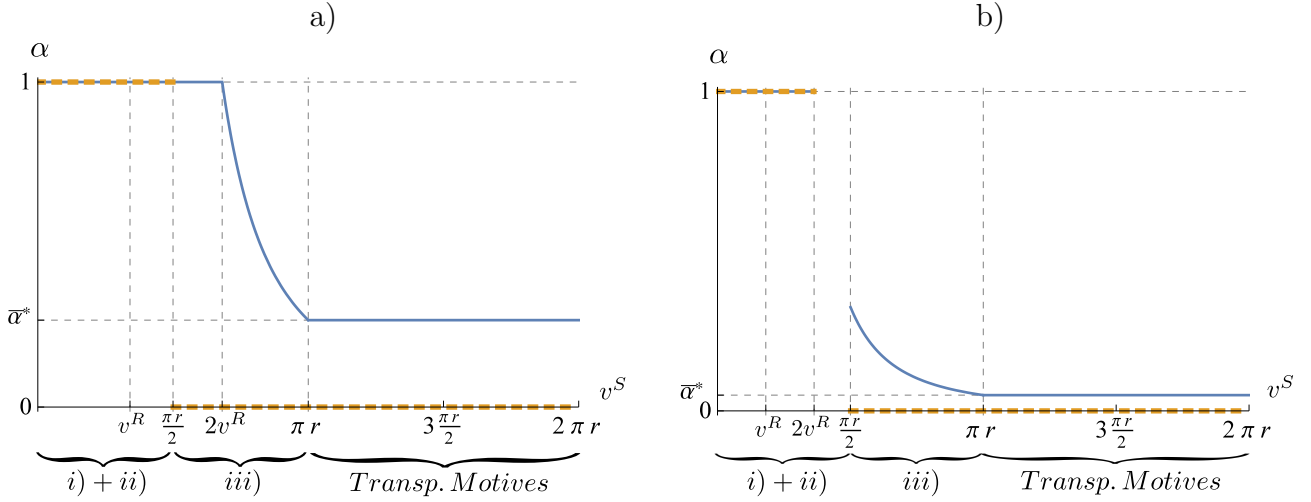
Note: In both panels, $r = 1$. Panel a) $v^R = \frac{\pi r}{2} - \frac{1}{2}$; Panel b) $v^R = \frac{\pi r}{2} - 1$. Solid and dashed lines represent first-best levels of information acquisition for sender and receiver, respectively.

The left panel of Figure 4 shows that when $v^S < \frac{\pi r}{2}$, information acquisition is necessary to incentivize the sender to engage in communication, and so both parties prefer $\alpha = 1$. However, as v^S increases past this range the sender has an incentive to tailor the message, and as a result the receiver prefers to avoid being identified. In general, the receiver's preferences over information collection are as follows: if information acquisition is a necessary condition to incentivize communication then the receiver prefers full disclosure. Otherwise, the receiver prefers complete privacy. In contrast, the sender always prefers high levels of information but is forced to reduce the information level as v^S increases in order to satisfy the receiver's participation constraint. After $v^S > \pi r$ the sender is willing to attract any receiver, and so the actual level of v^S no longer affects the level of

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information acquisition.

The right panel of Figure 4 depicts the case of a lower v^R, in which case region $v^S \in \left[2v^R, \frac{\pi r}{2} \right]$ with no matches emerge. In this region uninformed senders are not willing to communicate whereas informed senders may be, but cannot commit not to take advantage of the information they collect. Therefore, receivers expect negative utility from informed senders and no matches emerge. Theorem 2 has additional implications in case the receiver is also uncertain about the sender's gross valuation v^S:

COROLLARY 3 *Suppose the sender's gross valuation $v^S \in [\underline{v}, \overline{v}]$ is uncertain to the receiver. In that case there exists a belief $\widehat{v^S} \in \left[\underline{v}, v^S \right]$ that is (weakly) more favorable to the sender.*

This result describes a belief ordering over sender types. The sender prefers to be believed to derive low gross value from matches in order to engage in information acquisition to a higher extent. Hence, if senders were able to communicate the value of v^S credibly we should expect them to disclose them in ascending order, much as in the spirit of Milgrom (1981). However, if there is no way of credibly communicating v^S then cheap-talk mechanisms are ineffective because all sender types prefer to claim a low value of v^S. We now inspect the case of joint welfare maximization:

COROLLARY 4 *The level of information acquisition that maximizes total welfare is equal to one when $v^S < v^R$ and equal to $\overline{\alpha}^*$ when $v^S > \pi r$. In the intermediate range $v^S \in \left[v^R, \pi r \right]$ all information levels yield the same level of total welfare.*

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Figure 5: Welfare-Maximizing Levels of Information
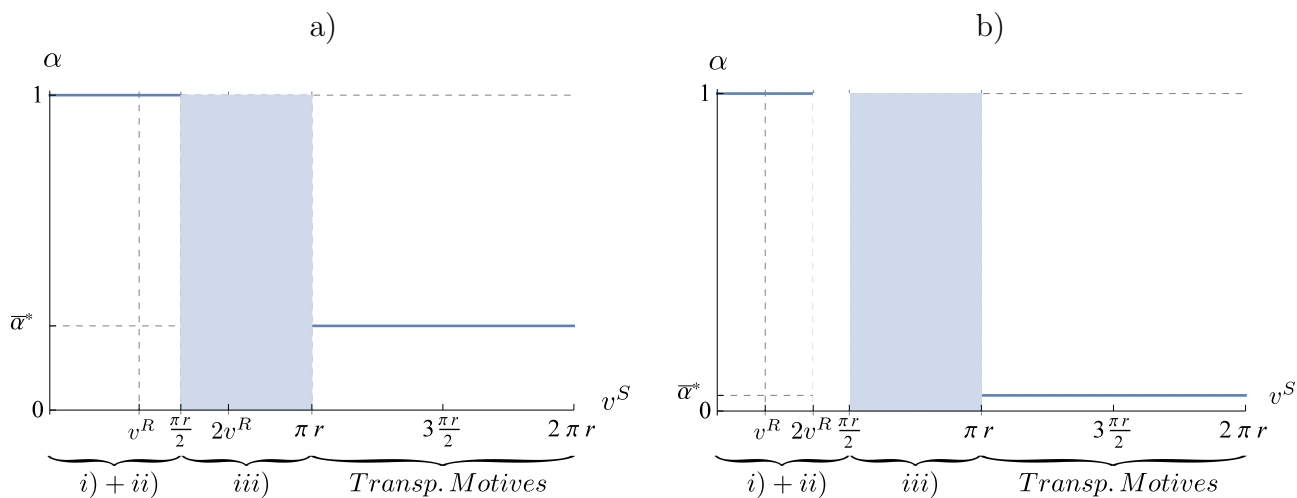
Note: In both panels, $r = 1$. Panel a) $v^R = \frac{\pi r}{2} - \frac{1}{2}$; Panel b) $v^R = \frac{\pi r}{2} - 1$. The solid line represents welfare-maximizing level of information, and in the solid region all levels yield the same joint welfare.

Figure 5 depicts the first-best level of information. In the left panel information acquisition is equal to 1 when $v^S < \frac{\pi r}{2}$, consistent with the results described in Figure 4. However, in region $v^S \in \left[\frac{\pi r}{2}, \pi r\right]$ changing the level of information acquisition transfers utility efficiently between the receiver and the sender while keeping the match probabilities constant. When $v^S > \pi r$, total utility is maximized when $\alpha = \overline{\alpha}^*$ because the sender has much to gain from a match.

The right panel of Figure 5 considers a case with a lower value of v^R. It illustrates that when $v^S \in \left[2v^R, \frac{\pi r}{2}\right]$ no level of information acquisition enables a match because the sender cannot commit not to tailor the message to his advantage.

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6   Concluding Remarks

We propose a model of communication in which the sender is able to engage in information acquisition about the receiver's preferences. The main result is that the sender may prefer to remain in a state of partial willful ignorance in order to ensure credibility. When the sender features ex-ante transparent motives he prefers to remain partially ignorant about the receiver's preferences and is able to attain his first-best payoff in the forward-induction equilibrium. In contrast, the receiver would be better off shrouding her preferences altogether in this case.

When the sender's valuation is low, information acquisition may be essential for matches to take place. In this case both parties benefit from information acquisition and prefer the highest possible level. Finally, in an intermediate range different levels of information efficiently transfer payoffs between the agents. We uncover two additional results. First, the sender's first-best outcome always maximizes joint welfare. Second, dissipative and cheap-talk communication mechanisms may complement each other rather than act purely as substitutes.

Our results are relevant to matching markets and shed light on current market trends and policy debates related to consumer privacy, personalized communication and online advertising in particular. We have found that information acquisition increases consumers' welfare only when it is pivotal for communication. For example, consumers may be better off sharing their preferences with niche firms but should shroud them from those willing to attract the average consumer. Our results also point to the flip-side of obtaining better information, which is essentially the deterioration of communication credibility.

Another implication of our model is that receivers also have preferences over the amount of information available to senders. In settings such as the job and dating markets, the receiver (e.g. a firm comparing applicants' vitae or an individual being romantically pursued) may have an incentive not to share too much information about what she is looking for, because the sender may use such information to persuade her that he possesses the skills or shares the right set of interests that ensure a successful match. In short, agents should provide only the information necessary to peak interest, but no further information that may be used for misrepresentation by their suitors.

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The right-hand side of (29) is concave in Δ, and its unique maximizer is

$$\Delta^* = \frac{v^R}{r} \tag{30}$$

such that the support of $\phi(m, \theta, \alpha)$ is equal to c_θ. Finally, substituting Δ^* into (29) yields $\alpha^* = \overline{\alpha}^*$, which completes the proof.

A.1.3 COROLLARY 1

Corollary 1 is easily shown by contradiction. Fix some equilibrium beliefs associated with a forward-induction equilibrium level $\alpha' < \overline{\alpha}^*$. Under forward induction the receiver is willing to 'revisit' her beliefs over the messaging policy upon observation of an 'unexpected' α. Hence, a sender who deviates to level $\alpha'' = \overline{\alpha}^*$ can increase payoffs by inducing beliefs $f_{m^*|\theta,q,\alpha} = f_{m^*|\theta,q,\alpha}$, as defined in equation (10), and therefore no level of level $\alpha' < \overline{\alpha}^*$ survives forward induction.

A.1.4 COROLLARY 2

The receiver's payoffs follow directly from the derivation of Theorem 1. The sender's payoffs are given by

$$
\begin{aligned}
E\left[U^S\right] &= \overline{\alpha}^* E\left(v^S - d(\theta,q)\right) + (1-\overline{\alpha}^*) Pr\left[q \in \left[\theta - \tfrac{v^R}{r}, \theta + \tfrac{v^R}{r}\right]\right] E\left[v^S - d(\theta,q)\big| q \in \left[\theta - \tfrac{v^R}{r}, \theta + \tfrac{v^R}{r}\right]\right] \\
&= \overline{\alpha}^*\left(v^S - \tfrac{\pi r}{2}\right) + (1-\overline{\alpha}^*) \int_{\theta - \frac{v^R}{r}}^{\theta + \frac{v^R}{r}} \left(v^S - d(\theta,q)\right) \tfrac{1}{2\pi} d\theta \\
&= \overline{\alpha}^*\left(v^S - \tfrac{\pi r}{2}\right) + \tfrac{1-\overline{\alpha}^*}{2\pi} \int_{\theta - \frac{v^R}{r}}^{\theta} v^S - r(\theta-q) d\theta + (1-\overline{\alpha}^*) \int_{\theta}^{\theta + \frac{v^R}{r}} v^S - r(q-\theta) d\theta \\
&= \tfrac{v^S - v^R}{\pi r - v^R} v^R
\end{aligned}
$$