On the one hand, a merger may increase the profit gap between the leader and followers—increasing the incentives to innovate—and, on the other hand, it reduces the number of firms performing R&D. Figures 2.b and 2.c show that the dominance of one effect over the other may change as a function of the number of firms, creating an inverted-U- or even an N shaped relationship between the pace of innovation and the number of firms.

Result 2. *The relationship between the pace of innovation and the number of firms can be monotonic or non-monotonic (e.g., inverted-U or N shaped).*

Table 3.b shows an example where a profit gap that decreases in the number of firms is insufficient for a merger to increase the pace of innovation. In Proposition 5, however, we argue that for a sufficiently large number of research labs, m, a decreasing profit gap becomes sufficient for a merger to increase the pace of innovation. Using the same parameters as in Table 3.b, we find that $m = 2$ research labs are sufficient for a merger to increase the pace of innovation whenever the number of large firms ranges between 2 and 8. We report these results in Table 3.d (see Figure 2.d). This result suggests that even a small number of labs may transform the relationship between the number of firms and the pace of innovation. Therefore, even in concentrated industries, a decreasing profit gap may be sufficient to invalidate the argument that a merger will reduce the pace of innovation.

Result 3. *A small number of labs may be sufficient for a profit gap that decreases in the number of firms to increase the pace of innovation with a merger.*

Finally, Figure 3 shows the relationship between the elasticity of a firm's R&D curve and some of the key parameters of the model, β and γ_1. Even though the profit gap increases with the cost advantage of the leader β, the elasticity of a follower's R&D with respect to the number of firms, $e_{x_n^*,n}$, does not have a monotone comparative static in β. Similarly, more inelastic R&D cost technologies (i.e., a higher γ_1) also affect $e_{x_n^*,n}$ non-monotonically. This non-monotonicity captures the complex interaction that exists between the different components of the model and further highlights the value of our conditions for the alignment of the static and dynamic merger-review criteria.

5 Concluding Remarks

We studied the impact of mergers in innovative industries. We found that a merger between two large firms affects R&D outcomes both directly by reducing the number of firms performing R&D and indirectly by changing the product market profits. The relationship among these effects is complex and may lead to scenarios where a merger increases an industry's pace of innovation and consumer surplus in the long run.

Based on properties of the product market competition game, we provide conditions for when a merger increases or decreases the pace of innovation. These conditions are based on product market payoffs and provide valuable information on whether the (common) argument that a merger reduces incentives to innovate really applies.[13] Moreover, these conditions are simple to check—in the sense that they only require information that is commonly used for merger simulations or demand estimation. Based on these results, we provide conditions for when rejecting or approving a merger using a static merger-review criterion (i.e., based on static price effects) is aligned with a dynamic merger-review criterion, which considers effects on both the price and innovation processes. Finally, we provide a necessary and sufficient condition for when a merger benefits consumers in the long run despite any short run price effects. This latter condition is helpful when the above conditions suggest that the static and dynamic merger-review criteria may be unaligned.

Our theoretical results together with empirical evidence suggesting that reduced product market competition may increase innovation rates—e.g., see Aghion et al. (2005)—stress the relevance and importance of analyzing the dynamic effects of mergers in innovative industries. As mentioned above, checking our sufficient conditions for whether a merger increases innovation rates does not require estimating or solving a dynamic model. We believe these conditions are simple enough to be easily brought into merger evaluation.

Finally, our results also highlight the importance of product market payoffs for the analysis of the impact of mergers on R&D outcomes. For this reason, empirical studies should carefully specify demand models and the rules of the product market competition game. A lack of flexibility in the model may prevent the data from showing the true relationship between the profit gap and the number of firms, which may lead the researcher to erroneously conclude that a merger will decrease

[13] See footnote 1.

(or increase) the pace of innovation.

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Appendix

A Preliminary Results

Lemma 1. *The function $f(z)$ implicitly defined by $c'(f(z)) = z$ satisfies:*

1. *$f(z) > 0$ for all $z > 0$ and $f(0) = 0$.*
2. *$f'(z) > 0$ for all $z \geq 0$.*
3. *Let $h(z) = (n+1)zf(z) - c(f(z))$ for $z \geq 0$. Then $h'(z) = (n+1)f(z) + nzf'(z) > 0$ for all $z \geq 0$.*

Proof. 1. $c(x)$ being strictly increasing and differentiable implies $c'(x) > 0$ for all $x > 0$. $c(x)$ being strictly convex implies $c''(x) > 0$ for all $x \geq 0$. Thus, $c'(x)$ is unbounded above and for each z there exists a unique value of $x = f(z) > 0$ such that $c'(x) = z$. Moreover, because $c'(0) = 0$, then $f(0) = 0$.
2. The result follows from the derivative of the inverse function being equal to $f'(z) = 1/c''(f(z))$ in conjunction with the strict convexity of $c(x)$.
3. Differentiating h and using $c'(f(z)) = z$ delivers $h'(z) = (n+1)f(z) + nzf'(z)$, which is positive by claims 1 and 2. □

Lemma 2. *The discounted expected consumer surplus is given by equation (6).*

Proof. Consider an asset that pays the consumer surplus flow at every instant of time. Starting from a consumer surplus cs_n, the value of this asset is given by

$$rA(cs_n) = cs_n + \lambda_{n,m}(A(cs'_n) - A(cs_n)) \tag{8}$$

where cs'_n is the consumer surplus after an innovation arrives. Using the condition that $cs'_n = cs_n + \delta_n$, we guess and verify that equation (6) solves equation (8), i.e., $A(cs_n) = CS_n$, proving the result. □

B Proofs

Proof of Proposition 1 Using the first order condition (see equation (4)), we find that the equilibrium values for the leader and followers are given by

$$\begin{aligned} rV_{n,m} &= \pi_n^l - (n+m)(V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) \\ rW_{n,m} &= \pi_n^f + (V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) - c(f(V_{n,m} - W_{n,m})). \end{aligned}$$

Subtracting these equations and defining $Z_{n,m} \equiv V_{n,m} - W_{n,m}$ we obtain

$$rZ_{n,m} = \Delta\pi_n - (n+m+1)Z_{n,m}f(Z_{n,m}) + c(f(Z_{n,m})). \tag{9}$$

To prove existence and uniqueness of an equilibrium with $Z_{n,m} > 0$, note that the left-hand side of equation (9) is strictly increasing in $Z_{n,m}$ and ranges from 0 to ∞. Lemma 1.1 implies that the right-hand side of equation (9) is strictly decreasing in

$Z_{n,m}$, taking the value of $\Delta\pi_n + c(0) > 0$ when $Z_{n,m} = 0$. Thus, the two functions intersect once at a positive value of $Z_{n,m}$, proving the result.

Proof of Proposition 2 Using implicit differentiation in equation (9), we reach the following results:

i) The derivative of $Z_{n,m}$ with respect to $\Delta\pi_n$ is given by

$$\frac{dZ_{n,m}}{d\Delta\pi_n} = \frac{1}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.$$

Since $x^*_{n,m} = f(Z_{n,m})$ and $\lambda_{n,m} = (n+m)f(Z_{n,m})$, Lemma 1.2 implies that both are increasing in $\Delta\pi_n$.

ii) The derivative of $Z_{n,m}$ with respect to m is given by

$$\frac{dZ_{n,m}}{dm} = \frac{-Z_{n,m}f(Z_{n,m})}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} < 0.$$

Thus, an increase in m decreases a firm's R&D investment. The derivative of the pace of innovation with respect to m is

$$\begin{aligned}\frac{d\lambda_{n,m}}{dm} &= f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dm} \\ &= \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.\end{aligned}$$

proving that the pace of innovation increases with m.

Proof of Proposition 3 Using implicit differentiation in equation (9), we find that the derivative of the pace of innovation with respect to n is

$$\frac{d\lambda_{n,m}}{dn} = f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dn}. \tag{10}$$

This derivative is positive when

$$\frac{n}{n+m} < -\frac{n}{f(Z_{n,m})}\frac{df(Z_{n,m})}{dZ_n}\frac{dZ_{n,m}}{dn} = -\frac{dx^*_{n,m}/x^*_n}{dn/n} \equiv e_{x^*_{n,m},n}, \tag{11}$$

which proves the result.

Proof of Proposition 4 Using implicit differentiation in equation (9) we obtain $dZ_{n,m}/dn$. Replacing it in (10), we find

$$\frac{d\lambda_{n,m}}{dn} = \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2 + (n+m)f'(Z_{n,m})\frac{d\Delta_n}{dn}}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})}. \tag{12}$$

If Δ_n satisfies $d\Delta_n/dn > 0$ (i.e., if Δ_n has an increasing profit gap), then the derivative is positive. Hence, a reduction in the number of large firms leads to a reduction in the pace of innovation.

Proof of Proposition 5 A necessary condition for equation (12) to be negative is $d\Delta_n/dn < 0$. For sufficiency, we need to show that there exists an $\bar{m}$ such that $m > \bar{m}$ implies $d\lambda_{n,m}/dn < 0$. Since the denominator of (12) is positive, $d\lambda_{n,m}/dn < 0$ is equivalent to

$$\frac{r}{n+m}\frac{f(Z_{n,m})}{f'(Z_{n,m})} + \frac{n+m+1}{n+m}\frac{f(Z_{n,m})^2}{f'(Z_{n,m})} < -\frac{d\Delta\pi_n}{dn}.$$

$d\Delta\pi_n/dn < 0$ guarantees that right-hand side of the inequality is always positive. Given that $f(0) = 0$ and $f'(0) > 0$ (see Lemma 1), and $dZ_{n,m}/dm < 0$, it is sufficient to show that $\lim_{m\to\infty} Z_{n,m} = 0$ for the inequality to hold.

For any small $\epsilon > 0$, pick $Z_\epsilon \in (0, \epsilon)$. By Proposition 1, equation (9) has a unique solution. Using (9), define m_ϵ to be

$$m_\epsilon = \frac{\Delta\pi_n + c(f(Z_\epsilon)) - (r + (n+1)f(Z_\epsilon))Z_\epsilon}{f(Z_\epsilon)Z_\epsilon},$$

which is always well defined (but possibly negative). Thus, take any decreasing sequence of Z_ϵ converging to zero. For each element of the sequence, there exists an increasing sequence m_ϵ that delivers Z_ϵ as an equilibrium. Thus, $\lim_{m\to\infty} Z_{n,m} = 0$ and the result follows.

Proof of Proposition 6 The derivative of CS_n with respect to n is given by

$$\frac{dCS_n}{dn} = \frac{dcs_n}{dn} + \frac{1}{r}\left(\frac{d\lambda_{n,m}}{dn}\delta_n + \lambda_{n,m}\frac{d\delta_n}{dn}\right).$$

Using equation (12), we note that

$$(n+m)\frac{d\lambda_{n,m}}{dn} = \lambda_{n,m}\left(1 - \frac{m+n}{n}e_{x_n^*,n}\right).$$

By replacing this expression into dCS_n/dn, we find that a merger increases consumer surplus if and only if condition (7) holds.