Contracting, Exclusivity and the Formation of Supply Networks with Downstream Competition

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Abstract

This paper studies the endogenous formation of supply networks in bilateral oligopoly by analyzing a model of contracting with transfers in which each firm optimizes its entire set of contracts across multiple bilateral negotiations. Because of downstream competition, industry profits are not necessarily maximized when all supply links are active and the supply networks that constitute coalition-proof Nash equilibria of the contracting game may differ from those that maximize industry profits. I first demonstrate that, in the absence of public commitment, all marginal input prices in any self-enforcing supply network are equal to the marginal cost of production. I then explore how a number of factors – such as supplier and retailer differentiation and the availability of exclusive contracts – affect the structure of equilibrium supply networks, profits and welfare. I also explore how the analysis changes if firms cannot use transfers or long-term contracts at the network-formation stage and must instead engage in ex-post bilateral bargaining with the associated hold-up problems.

Keywords: Bilateral oligopoly, contracting with transfers, exclusive contracts, supply network formation.

JEL Classification: D43, D85, L13, L14.

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1 Introduction

In many industries a number of differentiated but competing upstream firms ("suppliers") distribute their products through a number of differentiated but competing downstream firms ("retailers"). In some of these industries most retailers distribute the products of most suppliers, i.e., most potential supply links are active. In other industries, instead, each retailer distributes the products of a different supplier and some important potential supply links remain inactive. Examples of the latter include the exclusive distribution of many sports events and films in the pay TV industry – see, e.g., OECD (2013) and Weeds (2016) – and, until a few years ago, of the iPhone and other models of smartphones in the wireless telecommunication industry – see, e.g., Sinkinson (2014). Moreover, in some industries the structure of supply networks appears to be changing over time. For example, the wireless telecommunication industry has recently moved away from smartphone exclusivity, whereas the healthcare industry appears to be moving in the opposite direction, with an increasing number of health insurance companies offering networks that restrict the ability of patients to choose among different healthcare providers (see, e.g., Ho and Lee, 2017).

This paper presents a model of contracting and competition that can shed some light on the endogenous emergence of different structures of supply networks in markets like the ones discussed above. In this model, first all suppliers and retailers simultaneously have an opportunity to negotiate nonlinear vertical contracts, then all retailers with at least one contract compete in the downstream market. Importantly, when negotiating its contracts in the first stage, each firm makes use of all the information at its disposal and optimizes its strategy across multiple bilateral negotiations. The model is used to study the effects of nonexclusive and exclusive contracts on equilibrium supply networks, profits and welfare in environments with varying degrees of supplier and retailer differentiation.

Bilateral trade and contracting in settings with multiple firms on both sides of a market have been studied by at least two streams of literature. A substantial theoretical literature has studied trade in buyer-seller networks (e.g., Kranton and Minehart, 2001; Ostrovsky, 2008; and Elliott, 2015) and contracting with multiple principals and agents (e.g., Prat and Rustichini, 2003). However, this literature has focused exclusively on the case in which buyers do not compete in the downstream market. Downstream competition, and specifically the ability of supply contracts to affect such competition, plays instead a crucial role in this paper.

A more applied literature has studied the division of surplus between suppliers and retailers in a number of markets, including the television market (e.g., Crawford and Yurukoglu, 2012) and the healthcare market (e.g., Gowrisankaran, Nevo and Town, 2015). This literature relies on a "Nash-in-Nash" approach, in which each pair of firms engages in Nash bargaining, taking as given the agreements reached by all other pairs (see Collard-Wexler, Gowrisankaran and Lee (2017) for a thorough discussion of, and theoretical foundations for, this approach). Although this approach can accommodate downstream competition, it relies on a number of fairly strong assumptions about contracting. In particular, it assumes that, when negotiating their contracts, firms take all other contracts (including other contracts to which they themselves are a party) as given, thus not making use of all the information at their disposal, and that there are gains from trade associated with every potential supply link. These assumptions make it possible to derive, and take to the data, precise and tractable implications regarding the division of surplus over given supply networks. They are, however, less well-suited to studying how firms may affect downstream competition by implementing networks in which some supply links may remain inactive.[1] For example, this approach does not account adequately for the fact that a pair of firms may not find it profitable to rescind a supply link if all other links remain active, but may find it profitable to do so if other supply links are also rescinded. Another limitation of this literature is that it typically constrains payments from retailers to suppliers to be either lump-sum (e.g., Gowrisankaran et al., 2015; Collard-Wexler et al., 2017) or linear without any lump-sum component (e.g., Horn and Wolinsky, 1988; Crawford and Yurukoglu, 2012).

In this paper I advance this literature by combining insights from the literature on network formation with transfers (e.g., Bloch and Jackson, 2007; Jackson, 2008; Bloch and Dutta, 2011) with insights from the literature on vertical contracting with a single supplier (e.g., O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Vergé, 2004) or a single retailer (e.g., O'Brien and Shaffer, 1997; Bernheim and Whinston, 1998). Relative to the "Nash-in-Nash" literature discussed above, this approach provides less precise predictions regarding the division of surplus between upstream and downstream firms, but can shed some light on other important aspects, such as the structure of vertical contracts, the importance of multilateral deviations, the endogenous emergence of narrow supply networks, and the effects of the latter on the intensity of downstream competition.

Overview – The starting point of the analysis is an exploration of how the structure of sup-

[1] Liebman (2016) and Ho and Lee (2017) study situations in which health insurance companies commit ex-ante to exclude one or more health care providers from their networks to increase their bargaining leverage. However, they either do not allow for downstream competition (Ho and Lee, 2017) or do not clearly discuss the implications of exclusivity for such competition (Liebman, 2016). Other important differences, discussed in further detail in Section 6.2, are that my framework allows for both upstream and downstream exclusivity and does not allow for ex-ante commitment to exclude.

ply networks affects industry profits.[2] Networks in which a relatively large number of potential distribution channels remain inactive, because some retailers are excluded or because active retailers distribute different products, have two opposite effects on industry profits. On the one hand, when suppliers and retailers are differentiated, the absence of some product-retailer combinations reduces the demand or willingness to pay expressed by some consumers, thus lowering total industry revenues. On the other hand, it softens downstream competition by reducing the number or increasing the effective differentiation of active retailers. The relative importance of these two effects determines which type of network maximizes industry profits. For example, when retailers are close substitutes, softening downstream competition is more important than preserving retailers' variety. As a result, industry profits are maximized by networks with relatively few retailers or with retailers carrying different products. The opposite is true when retailers are intrinsically highly differentiated.

Although a number of articles has developed similar insights regarding the effects of different supply networks on downstream competition, these articles have either taken such networks as exogenously given (e.g., O'Brien and Shaffer, 1993; Besanko and Perry, 1994) or analyzed very stylized contracting games that, effectively, impose exogenous limits on the types of networks that can arise (e.g., Dobson and Waterson, 1996; Hermalin and Katz, 2013; Nocke and Rey, 2016).[3] I advance this literature by developing and solving a full-fledged contracting model that does not preclude any contracting channel, and thus does not exogenously rule out any type of supply network. In particular, I study how externalities arising from upstream and downstream competition may prevent decentralized, bilateral contracting from yielding equilibrium supply networks that maximize industry profits, and how firms can use exclusive contracts to address these externalities, thus affecting equilibrium supply networks, profits and welfare.

The formal model on which I rely is an adaptation of Bloch and Jackson's (2007) model of network formation with transfers to a bilateral oligopoly setting with vertical contracting. In this model, all firms on either side of the market (i.e., all suppliers and all retailers) simultaneously and secretly submit contract proposals to all firms on the other side of the market. A contract proposal includes proposals regarding i) the wholesale price at which the retailer will purchase

[2] Another factor that may affect industry profits, besides the structure of supply networks, is the extent to which suppliers can soften downstream competition by publicly committing to marginal input prices. As I discuss further below, in this paper I assume that suppliers do not have any ability to commit publicly to such prices and thus to affect equilibrium industry profits through this channel.

[3] Hermalin and Katz (2013) and Nocke and Rey (2016) consider simple contracting games in which each firm on one side of the market is exogenously matched with one and only one firm on the other side of the market and, therefore, cannot organize multilateral deviations in which it negotiates with, and possibly requires exclusivity from, two or more firms at the same time. Analogously, Dobson and Waterson (1996) assume that each firm is exogenously limited to requiring exclusivity from at most one other firm.

the product from the supplier, ii) exclusivity clauses (if any), and iii) an upfront transfer to be paid by one party to the other at the time the contract is signed. A supplier and a retailer enter a contract only if their proposals regarding all three of these elements are consistent. Once all contracting is concluded, retailers with at least one supply contract compete in prices or quantities in the downstream market.

There are three main differences between my model and that of Bloch and Jackson (2007). First, in Bloch and Jackson's model, and in much of the literature on network formation, the only relevant choice for a pair of players is whether to form or sever a link. In the vertical contracting setting of this paper, instead, each supplier-retailer pair must also specify a wholesale price (see i) above), which affects the price or quantity chosen by the retailer and, through this channel, the payoffs of the network formation game. I show that, in any equilibrium with secret contracts all wholesale prices are equal to marginal cost. Second, the application of the model to a bilateral oligopoly setting implies some inescapable restrictions on payoffs, arising mainly from supplier and retailer substitutability, that make it impossible to rely on some of the assumptions (e.g., nonnegative externalities and link-separability of payoffs) used by Bloch and Jackson to derive some of their results. Finally, as discussed below, in order to deal with the pervasive horizontal externalities in my model, I use Bernheim, Peleg and Whinston's (1987) coalition-proof Nash equilibrium (CPNE) as a solution concept. CPNE allows for multilateral deviations and is a stronger equilibrium concept than the pairwise Nash equilibrium used by Bloch and Jackson (2007), in which firms can only add one new link at a time, or the contract equilibrium proposed by Crémer and Riordan (1987). I solve the model described above by first deriving some general results and then applying them to a bilateral duopoly with linear demand. This allows me to answer the following questions.

First, in an environment without exclusive contracts, can some supply links remain inactive in equilibrium? And under what conditions do equilibrium supply networks maximize industry profits? I show that – with the exception of Bertrand competition between closely substitutable retailers, for which each retailer sells a different product in equilibrium – all potential supply links are generally active in equilibrium, even when supply networks with fewer links would maximize industry profits. This is the case because the inability of firms to use exclusive contracts makes it impossible to prevent certain supplier-retailer pairs from behaving opportunistically and forming new links at the expense of other supplier-retailer pairs.

Second, how does the availability of exclusive contracts affect equilibrium supply networks, welfare and profits? By requiring the consent of broader coalitions of firms for deviations that expand a supply network, exclusive contracts solve the opportunism problem discussed above and make it possible to support equilibria with narrow networks when these maximize industry profits, although pure-strategy equilibria do not always exist. When exclusive contracts affect the equilibrium structure of supply networks, they always reduce variety and the intensity of downstream competition, resulting in lower consumer and overall welfare. Moreover, the availability of exclusive contracts, by affecting the disagreement payoffs of suppliers and retailers differently, redistributes profits from retailers to suppliers, even when such contracts are not adopted in equilibrium.

Finally, how do constraints on the firms' ability to use upfront transfers or long-term contracts and to create instantaneously new supply links at the network formation stage affect equilibrium outcomes? In the baseline model discussed above there are no constraints on such ability. As a result, the division of the profits generated by new supply links takes place at the same time that the network is formed and is not affected by hold-up problems. Instead, if firms must form a complete supply network, for example by making specific investments, *before* starting to negotiate supply contracts and agreeing on any transfers, the division of profits resulting from ex-post bargaining is affected by hold up, as in Lee and Fong (2013) and Rey and Vergé (2016) (discussed at the end of this introduction). I show that, relative to an environment with upfront transfers, ex-post bargaining with hold up makes it easier to support equilibria with inactive supply links because firms find it more difficult to share the joint profits from deviations that form new supply links. This has ambiguous effects on industry profits, but unambiguously reduces consumer and overall welfare.

From a methodological point of view, solving the model of simultaneous contracting with transfers presented in this paper poses a number of challenges. One such challenge lies in applying the CPNE solution concept to an environment with transfers. CPNE requires coalitional deviations to be immune to further deviations by subcoalitions. When firms can use transfers, this implies that the transfers that support successive deviations are related, and one must keep track of their relationships. Specifically, the transfers that make a given deviation profitable for all firms involved in that deviation may depend on the transfers that made the previous deviation in the sequence profitable, and so on, potentially all the way back to the transfers that support the candidate equilibrium. I tackle this issue by proposing a general algorithm that I subsequently use to characterize the CPNE of a tractable bilateral duopoly model.

Another challenge is the determination of equilibrium wholesale prices. As is well-known, in the presence of downstream competition equilibrium wholesale prices depend on the extent to which a supplier can publicly commit to his offers and, if public commitment is not available, on the beliefs held by retailers about that supplier's dealings with rival retailers. In this paper

I assume that firms cannot publicly commit to their contract proposals and show that all CPNE wholesale prices are equal to marginal cost. Similar results have already been established by the existing literature in different settings (e.g., a single supplier or a single retailer) and/or for different equilibrium concepts (e.g., PBE or contract equilibrium). Here I extend those findings to the CPNE of a bilateral oligopoly. I also show that, by requiring multilateral deviations to be self-enforcing, CPNE avoids the issues identified by Rey and Vergé (2004) for the existence of perfect Bayesian equilibrium wholesale prices with Bertrand downstream competition.

Other related literature – This paper is also related to Rey and Vergé (2016) and Lee and Fong (2013). As in this paper, Rey and Vergé (2016) allow firms to engage in multiple bilateral contract negotiations. However, contrary to this paper, they do not allow for the use of transfers or exclusive contracts at the stage in which supply relations are initially formed, limiting their attention to the case in which the surplus from any relation can only be divided ex-post under conditions of hold up. Lee and Fong (2013) present a model of dynamic network formation in which, at any point in time, firms can only bargain over the existing network and, like in Rey and Vergé (2016), cannot create new supply links. However, they allow firms to create new supply links in subsequent periods. When the cost from doing so is low and the time between periods is short, Lee and Fong's (2013) environment approaches the environment without hold up that I study in this paper. Notwithstanding this similarity, their model and mine are quite different. Lee and Fong (2013) emphasize intrinsically dynamic aspects, such as the response of networks to shocks in the presence of adjustment costs, and simplify other aspects by, e.g., assuming that firms can only use lump-sum transfers and that exclusive contracts are not available. Instead, I adopt a static model of simultaneous contracting and emphasize the role played by the structure of vertical contracts and the degree of supplier and retailer differentiation. By relying on coalition-proof Nash equilibrium, I also propose a more systematic refinement of the set of equilibria than Lee and Fong (2013). Finally, I provide an in-depth analysis of the role played by exclusive contracts, which are not addressed in Rey and Vergé (2016) and Lee and Fong (2013), and are more naturally studied in an environment like mine, in which firms can use upfront payments to purchase exclusivity.

Organization of the paper – Section 2 introduces a formal model with upfront transfers. Section 3 presents the solution method and some general results. Sections 4 through 6 study supply networks with and without exclusive contracts in a bilateral duopoly model with linear demand. Section 7 explores the effects of ex-post bargaining and hold up. Section 8 concludes. All proofs are in Appendix A. Supplemental material is contained in Appendix B (available online).[4]

[4]This online appendix will be available shortly at `https://sites.google.com/site/paoloramezzana/`.

2 Model and equilibrium concept

This section first introduces a model of contracting and competition in bilateral oligopoly and then discusses why Bernheim, Peleg and Whinston's (1987) coalition-proof Nash equilibrium (CPNE) is an appropriate solution concept for this model.

2.1 Model

There are $S \geq 2$ suppliers, each producing a different product at constant marginal cost c. The products are imperfect substitutes and are distributed to consumers by $R \geq 2$ differentiated and competing retailers at no additional costs besides their payments to suppliers (introduced further below). With a slight abuse of notation, S and R denote both the number and the set of suppliers and retailers, respectively. Let q_{sr} denote the quantity and p_{sr} the retail price of product $s \in S$ sold by retailer $r \in R$, with $q \in \mathbb{R}^{S \times R}$ and $p \in \mathbb{R}^{S \times R}$ denoting the vectors of all quantities and prices. The direct demand for product s at retailer r is $q_{sr} = D_{sr}(p)$, with inverse demand $p_{sr} = P_{sr}(q)$. In Section 4, I will introduce a linear demand system that allows me to parametrize the degree of supplier and retailer differentiation.

Given this environment, I study a two-stage game, in which first suppliers and retailers negotiate bilateral supply contracts and then retailers compete in the downstream market. Specifically, in the first stage suppliers and retailers engage in the following simultaneous contracting game with transfers, which is an adaptation of the game of network formation with transfers introduced by Bloch and Jackson (2007). Denote by $K = \{S, R\}$ one side of the market (i.e. the set of suppliers or the set of retailers) and by $K' = \{S, R\}$, $K' \neq K$, the other side of the market. Each firm $i \in K$ announces a contract proposal $x_i^j = \left\langle t_i^j, w_i^j, \theta_i^j \right\rangle$ for each firm $j \in K'$, where $t_i^j \gtreqless 0$ and $w_i^j \geq 0$ are, respectively, the proposed transfer and wholesale price that the retailer must pay to the supplier, and θ_i^j the type of exclusive arrangement, if any, that governs the relationship between i and j. Exclusivity arrangements may involve one-way exclusivity, with which i commits to be exclusive to j or vice versa, or mutual exclusivity and will be discussed in further detail in Section 6. I assume that firms cannot publicly commit to their contract proposals. I discuss the details of this assumption below, after having discussed the equilibrium concept adopted.

Denoting by s the supplier and by r the retailer in the pair (i, j), s and r reach an agreement if $w_s^r = w_r^s$, $\theta_s^r = \theta_r^s$, and $t_r^s \geq t_s^r$, where the latter condition means that the retailer is willing to offer a transfer t_r^s that meets the transfer request t_s^r of the supplier. When s and r reach an agreement, a contract with $w_{sr} = w_s^r = w_r^s$, $\theta_{sr} = \theta_s^r = \theta_r^s$ and $t_{sr} = t_r^s$ enters into effect and a supply link, denoted by $\ell_{sr} = 1$, is formed.[5] Otherwise no link is formed and $\ell_{sr} = 0$. The collection

[5]Note that in any equilibrium it will always be $t_r^s = t_s^r$, otherwise either s or r would have a profitable

$\ell = \langle \ell_{sr} \rangle_{s \in S, r \in R}$ of all supply links, or lack thereof, gives rise to a supply network $g = g(\ell)$.

When bilateral contracting is concluded, retailers observe the resulting supply network g, but not the wholesale prices in the contracts signed by other retailers, and compete in the downstream market in stage 2. Since the general principles of the analysis that follows apply equally well to Cournot or Bertrand downstream competition, I allow for either mode of competition. I assume that for any supply network g and profile of wholesale prices $w \in \mathbb{R}^{S \times R}$, downstream competition results in a unique equilibrium profile of retail prices $p(g, w)$ and quantities $q(g, w)$. Given a profile of contracts $x = \langle t, w, \theta \rangle$, and the resulting supply network g, the payoffs of supplier s and retailer r are therefore, respectively,

$$\pi_s(g, t, w) = \sum_{r \in R} \ell_{sr} \left[t_{sr} + (w_{sr} - c) q_{sr}(g, w) \right], \tag{1}$$

$$\pi_r(g, t, w) = \sum_{s \in S} \ell_{sr} \left[(p_{sr} - w_{sr}) q_{sr}(g, w) - t_{sr} \right]. \tag{2}$$

Throughout the paper it will also be helpful to keep track of the total vertical profits (gross of any payment to suppliers) generated by retailer r, which are given by

$$\Pi_r(g, w) = \sum_{s \in S} \ell_{sr}(g) \left[p_{sr}(g, w) - c \right] q_{sr}(g, w). \tag{3}$$

For future reference it is important to note that, because of downstream competition, total industry profits, $\sum_{r \in R} \Pi_r(g, w)$, are not necessarily maximized when all suppliers trade with all retailers. Networks in which some supply links are not active, such as downstream monopoly or pairwise exclusivity, sacrifice some variety but reduce the intensity of downstream competition between retailers and may thus yield higher overall industry profits.

In order to derive some of the results in subsequent sections, it is also helpful to ensure that secret changes in w_{sr}, i.e., changes in w_{sr} that are observed only by retailer r, cause well-behaved responses in the derived demand for product s. This is guaranteed by the following assumption.

Assumption 1 (Derived demand) *For all g, and all $s \in S$ and $r \in R$ for which $\ell^g_{sr} = 1$, secret changes in w_{sr} have the following effects on the derived demand for product s:*

$$\frac{dq_{sr}}{dw_{sr}} < 0, \quad \frac{dq_{sj}}{dw_{sr}} \geq 0 \text{ for all } j \in R \text{ with } \ell^g_{sj} = 1 \text{ and } j \neq r, \text{ and } \frac{dq_s}{dw_{sr}} = \sum_{j \in R} \ell^g_{sr} \frac{dq_{sj}}{dw_{sr}} < 0$$

A change in w_{sr} that is unobserved by other retailers $j \neq r$ can affect directly only the vector of quantities q_r (when competition is Cournot) or retail prices p_r (when competition is Bertrand) chosen by retailer r for the products that she carries. When competition is Cournot, no other

unilateral deviation. It is, therefore, immaterial whether the transfer paid in equilibrium is that offered by the retailer (i.e., $t_{sr} = t^s_r$, as assumed here) or that requested by the supplier (i.e., $t_{sr} = t^r_s$).

changes in quantities occur and thus $dq_{sj}/dw_{sr} = 0$ for all $j \neq r$. The only "bite" of Assumption 1 in this case is therefore to ensure that $dq_{sr}/dw_{sr} < 0$ and thus $dq_s/dw_{sr} < 0$. When competition is Bertrand, the changes in p_r induced by changes in w_{sr} cause instead the quantities sold by other retailers to change, even if the retail prices charged by those other retailers do not change, so that $dq_{sj}/dw_{sr} > 0$ for all $j \neq r$. In this case Assumption 1 ensures that the direct effect, $dq_{sr}/dw_{sr} < 0$, dominates the indirect effects, $\sum_{j \neq r} dq_{sj}/dw_{sr} > 0$, of a change in w_{sr} on the demand for product s and thus that the overall market demand for product s is downward sloping in w_{sr}. Assumption 1 holds for the linear demand system that will be introduced in Section 4.

2.2 Equilibrium concept

When suppliers and retailers act unilaterally and cannot coordinate their strategies (i.e., their contract proposals), the contracting game described above has a large number of Nash equilibria, many of which are supported by coordination failures. For example, as in all games of network formation, vertical coordination failures may give rise to Nash equilibria in which a supplier s and a retailer r do not form a supply link, even though it would be jointly profitable for them to do so. Moreover, in the simultaneous contracting model studied in this paper, even when s and r manage to form a link, they may still fail to coordinate on the terms of the contract, w_{sr} or θ_{sr}, that would maximize their joint profits. These vertical coordination failures may be avoided by adopting "bilateral" solution concepts, such as contract equilibrium (e.g., Crémer and Riordan, 1987; O'Brien and Shaffer, 1992), in which each sr pair maximizes its joint profits, given all other contracts (including other contracts to which s and r are a party). Although such equilibrium concepts are appropriate in some settings, they cannot address two aspects that play an important role in this paper.

First, contract equilibrium is not sufficient to rule out the horizontal coordination failures that may arise in a setting with bilateral oligopoly and exclusive contracts. For example, consider the bilateral duopoly model illustrated in Figure 1, in which solid lines represent active supply links and dashed lines represent potential supply links that are inactive.[6] Focus on a candidate equilibrium in which $S1$ and $R1$ and $S2$ and $R2$ adopt mutually exclusive contracts, resulting in the pairwise exclusive supply network shown in Figure 1(c). Assume that the double common agency supply network in Figure 1(a) and/or the mixed supply network in Figure 1(b) yield higher overall industry profits than a pairwise exclusive network. A deviation from pairwise exclusivity to one of these two networks could benefit all firms but cannot be achieved unilaterally

[6]This bilateral duopoly model will also form the basis for other examples introduced below and for the analysis of Sections 4 through 7.

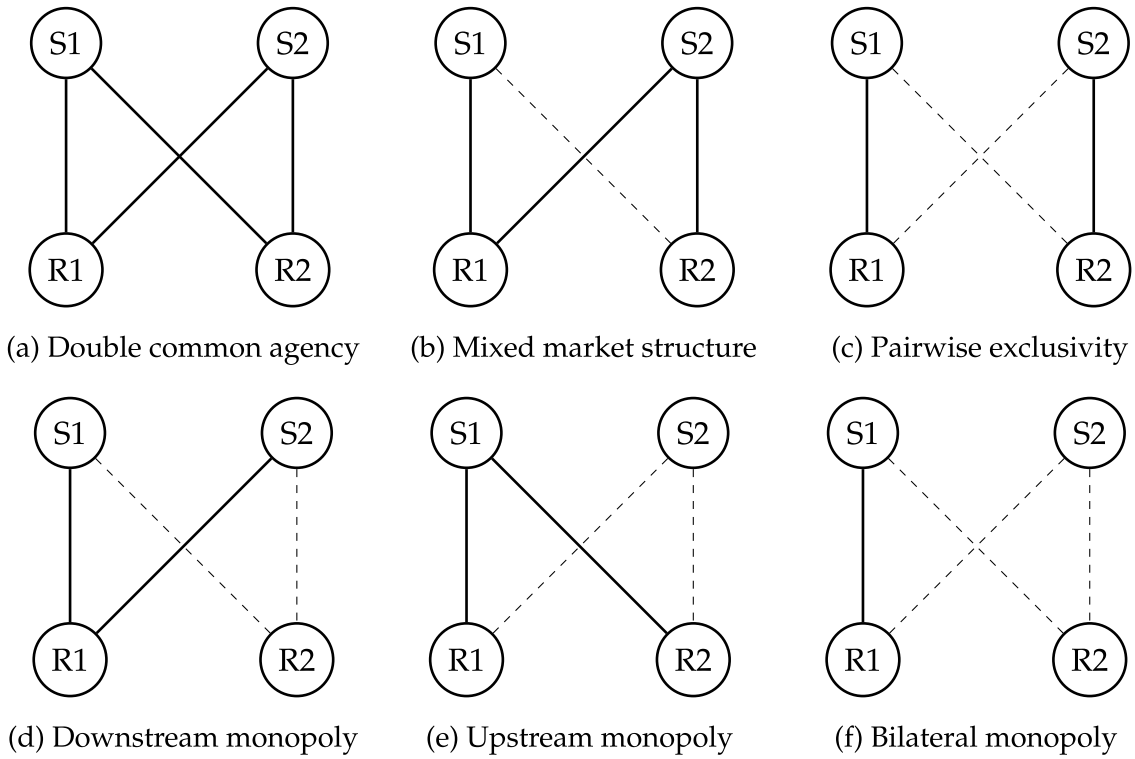


Figure 1: Supply networks in a bilateral duopoly.

or bilaterally, as it requires coordination among all four firms. This is the case because, given the mutually exclusive contracts that support the candidate pairwise exclusive equilibrium, $S2$ and $R1$ cannot, individually or bilaterally, form a supply link while maintaining their other supply links. For this to be possible, $S1$ must forgo the exclusivity he demands of $R1$, and $R2$ must forgo the exclusivity she demands of $S2$. Moreover, a deviation to double common agency also requires the pairs $S1 - R2$ and $S2 - R1$ to coordinate the formation of the two new diagonal links. One can also construct examples, with our without exclusive contracts, where firms may profit from coordinating deviations in which they rescind more than one existing link at a time, instead of creating more than one new link at a time.

Second, as already discussed in the introduction, contract equilibrium does not allow for multilateral deviations in which firms modify more than one contract at a time. This is particularly problematic in a setting with exclusive contracts as the one of this paper, because it makes sense for a supplier s and a retailer r to enter a (one-way or two-way) exclusive contract only if at least one of them can modify (i.e., rescind) its other contracts. Besides this almost mechanical point, many deviations in which one or more firms attempt to modify a supply network are profitable only if such deviations entail modifying more than one contract at a time. For example, starting from a candidate equilibrium with double common agency as in Figure 1(a), $R1$ may find it profitable to sign up $S1$ to exclusivity if he can also sign up $S2$ to exclusivity, thus implementing a downstream monopoly as in Figure 1(d), but may not find it profitable to do so if he must continue in its nonexclusive contract with $S2$ (as mandated by contract equilibrium), thus implementing a mixed network as in Figure 1(b).

To avoid the issues discussed above, I adopt as a solution concept Bernheim, Peleg and Whinston's (1987) coalition-proof Nash equilibrium (CPNE), which allows for multilateral deviations and (nonbinding) coordination. A profile of strategies (i.e., of contract proposals in this paper) constitutes a CPNE if it is immune to mutually profitable and self-enforcing *nonbinding* agreements to deviate by any coalition of firms, taking as given the strategies of firms that do not participate in the coalition. An agreement is self-enforcing if it is itself immune to further self-enforcing deviations by subcoalitions, where these further deviations are also required to be self-enforcing, and so on (see Bernheim, Peleg and Whinston (1987) for a formal definition).[7] CPNE imposes some degree of consistency on deviations, ruling out deviations that should not be considered "credible" (i.e., self-enforcing).

Finally, even though firms cannot publicly commit to their contract proposals, I do not need to specify the beliefs held by firms involved in deviations. This is because in a CPNE of the game of simultaneous contracting introduced in Section 2.1 no firm participating in a deviation can unilaterally modify its contracts with firms that do not participate in that deviation. As discussed further in Section 3.1, this feature of CPNE has implications that are analogous to those of contract equilibrium for the determination of equilibrium wholesale prices.

3 Some general properties of equilibria

One can characterize the CPNE of this model in two stages, by first characterizing the self-enforcing profile of wholesale prices $w(g)$ for any network g, and then studying which networks g can be supported as equilibria.

3.1 Equilibrium wholesale prices

When Assumption 1 on derived demand holds, one can establish the following result, which takes as given the supply links in a given supply network g and characterizes wholesale prices in the contracts governing those links. This (intermediate) result is, therefore, obtained not by asking whether there exist deviations to contracts that would implement different supply networks,

[7] Although the definition is recursive, it avoids cycles because it limits further deviations (but not the original deviation from the candidate equilibrium) to subcoalitions and has thus a natural end point. This aspect will be discussed in further detail in Section 3.2.

but rather whether there exist deviations to contracts that implement the same supply network with different wholesale prices.

Proposition 1 (Equilibrium wholesale prices) *When firms cannot publicly commit to their contract proposals, for any supply network g there always exists a unique coalition-proof Nash equilibrium profile of wholesale prices $w(g)$, with $w(g) = c$, regardless of the mode of downstream competition.*

Proposition 1 extends results obtained in different settings and for different equilibrium concepts by the vertical contracting literature (briefly discussed below) to the CPNE of the bilateral oligopoly model studied in this paper. Its logic can be understood in two steps. First, since any jointly profitable bilateral deviation by s and r to a different w_{sr} is self-enforcing, a CPNE must be immune to any such deviation and is therefore also a contract equilibrium. By extending O'Brien and Shaffer's (1992) analysis of contract equilibria with a single supplier to a setting with multiple suppliers, one can therefore prove that in any CPNE it must be $w_{sr} = c$ for all s and r. If this were not the case, any supplier-retailer pair for which $w_{sr} > c$ could profitably engage in a bilateral deviation to $w_{sr} = c$, which would allow this pair to appropriate some of the retail margins of other retailers. Second, the above implies that any multilateral deviation that changed the wholesale prices of two or more retailers away from $w = c$ at the same time would never be self enforcing, since it would always be blocked by a further self-enforcing bilateral deviation to marginal cost pricing, regardless of the mode of downstream competition.

This last aspect is what distinguishes Proposition 1 from existing literature, especially for the case of Bertrand downstream competition. As shown by Rey and Vergé (2004), when retailers compete à la Bertrand, are sufficiently close substitutes and hold passive beliefs, multilateral deviations in which suppliers raise the wholesale prices of two or more retailers above marginal cost at the same time may become profitable. If one adopts an equilibrium concept such as perfect Bayesian equilibrium, which allows for such multilateral deviations without requiring them to be self enforcing, this implies that pure-strategy equilibria may fail to exist, because there exist profitable deviations both when all wholesale prices are equal to marginal cost and when some of them are not. The vertical contracting literature typically addresses these equilibrium existence issues by limiting attention to contract equilibrium concepts that allow only for bilateral deviations (see, e.g., O'Brien and Shaffer (1992) and McAfee and Schwartz (1994) for models without upstream competition and Rey and Vergé (2016) for a bilateral oligopoly model with upstream competition) or to Cournot competition (see, e.g., Nocke and Rey (2016) for a bilateral oligopoly model with upstream competition).[8] The CPNE solution concept adopted in this paper

[8]The existence issue does not arise with Cournot downstream competition because in that case multilateral deviations do not present any advantage relative to bilateral deviations.

does not face this stark choice, as it overcomes the existence problems associated with Bertrand competition while allowing for multilateral deviations.

3.2 Equilibrium supply networks

Having shown that, when firms cannot commit to their wholesale price proposals, equilibrium wholesale prices are equal to marginal cost in all possible networks, the next step is to determine which of these networks can be supported as equilibria by some profile of transfers. As will become clear below, when a network g can in fact be supported as an equilibrium there typically exists a (possibly broad) range of transfers t^g for which this is the case. The model is, therefore, better suited to shedding light on what types of supply networks arise as equilibria under different conditions, rather than to predicting exactly how profits will be split between firms in those equilibria. As the division of profits resulting from t^g is irrelevant for consumer and overall welfare, because it does not directly affect product or retailer variety or the intensity of downstream competition, I do not view this as a significant shortcoming of the model.[9] Moreover, as shown in Section 6.2, certain changes in the environment, such as the availability of exclusive contracts, cause the range of suppliers or retailers' equilibrium profits to shift entirely to the right or the left of their initial range. When this is the case, the approach taken in this model is sufficient to determine unambiguously the distributional effects of those changes.

In order to predict the exact level of t^g one would have to study a different model in which suppliers and retailers engage in some form of coalitional bargaining at the contracting stage.[10] However, most existing models of coalitional bargaining – see, e.g., Shapley (1953) for a cooperative model and Chatterjee et al. (1993) for a noncooperative model – assume that the grand coalition always forms and that there are no externalities between coalitions; or, when they allow for externalities between coalitions (e.g., Ray and Vohra, 1999), they rely to a large extent on symmetry among all players to obtain tractable results.[11] These assumptions do not describe well the environment studied in this paper, in which externalities resulting from downstream competition and exclusive contracts, as well as asymmetries between suppliers and retailers,

[9] It should, however, be noted that the division of profits among firms can have welfare effects if it affects the investment incentives of firms, an aspect that is not modeled in this paper.

[10] Such a model would be different, and have different implications, from the model studied in Section 7, in which firms first form a supply network in the absence of upfront transfers and then engage in ex-post bilateral bargaining over the existing supply network. Whereas hold-up plays no role in the coalitional bargaining game with upfront transfers described in the text above, it plays a crucial role in the model of ex-post bilateral bargaining of Section 7.

[11] For a discussion of these issues see Bloch and Dutta (2011) and Maskin (2003). In particular, Maskin argues that the assumption that the grand coalition always forms is one of the main shortcomings of noncooperative game theory.

play a crucial role. In light of this, developing a full-fledged coalitional bargaining game that is well-suited to this complex environment, only for the purpose of obtaining precise predictions regarding transfers, would be well beyond the scope of this paper and is not attempted here.[12]

Having clarified the scope of the analysis, I can proceed with a characterization of equilibrium supply networks. I first introduce formal definitions of self-enforcing agreements and of CPNE supply networks in the specific model with transfers of this paper, and then derive some helpful results that allow me to make these definitions operational.

Denote by $\mathcal{Z}_{g\to h}$ the set of all coalitions that, starting from a network g, can implement a network $h \neq g$ without requiring the consent of firms outside the coalition (i.e., without needing firms outside the coalition to modify their strategies). One can then define a self-enforcing nonbinding agreement as follows.

Definition 1 (Self-enforcing agreement) *A nonbinding agreement among the members of coalition $Z \subseteq S \cup R$ to implement a supply network g with transfers t^g is self-enforcing if there does not exist any other self-enforcing agreement among the members of any subcoalition $Z' \subseteq Z$, $Z' \in \mathcal{Z}_{g\to h}$, that implements a supply network $h \neq g$ with transfers t^hsuch that, for all $i \in Z'$,*

$$\pi_i(h, t^h) > \pi_i\left(g, t^g\right) \tag{4}$$

A few aspects of Definition 1 are worth noting. First, the definition is recursive: an agreement is self-enforcing if and only if it cannot be improved upon by another self-enforcing agreement. The problem remains, however, well defined because deviations from agreements are restricted to subcoalitions Z' of the coalition Z that reaches the original agreement. This limits the number of successive deviations that one needs to consider and a solution can be reached in a finite number of steps. In the bilateral duopoly applications discussed in Sections 5 through 7 this number of steps is generally small and the problem remains tractable.

Second, the ability of firms to implement a new supply network h starting from a supply network g, and the size and composition of the coalitions that can do so, captured by $\mathcal{Z}_{g\to h}$, depend on a number of factors, such as the extent to which firms are allowed to communicate with one another and whether exclusive contracts are allowed or not. For example, it is generally

[12]One can, however, conjecture that an efficient coalitional bargaining game in which coalitions always reach agreements that are mutually profitable relative to alternative *self-enforcing* outcomes would yield the same equilibrium networks as the simultaneous contracting game that I study in this paper. Specifically, this should be the case for bargaining games in which firms i) can continue to make proposals to each other until all possibilities have been explored and ii) cannot credibly commit ex-ante to deal with a limited number of counterparts in order to elicit better offers from those counterparts. If this conjecture is correct, the analysis in this paper characterizes the bargaining set to which the outcomes of such efficient bargaining games must belong.

more difficult for firms to implement a new supply network when nonbinding communication is partially restricted (e.g., when communication between firms on the same side of the market is prohibited). As for exclusive contracts, their adoption makes it mode difficult to deviate from a given network g to broader networks h that add new supply links to g without rescinding existing links. Such deviations would require consent from broader coalitions than in the absence of exclusive contracts, because some firms that have exclusive rights in network g must consent to the expansions of the network from g to h. These aspects will be discussed in further detail in Sections 5 and 6, where they will play important roles.

Building on Definition 1, one can define a CPNE supply network as follows.

Definition 2 (CPNE with transfers) *A supply network g^* with transfers t^{g^*} constitutes a CPNE if and only if there does not exist any self-enforcing nonbinding agreement among the members of any coalition $Z \in \mathcal{Z}_{g^* \to g}$ that implements a supply network $g \neq g^*$ with transfers t^g such that $\pi_i(g, t^g) > \pi_i\left(g^*, t^{g^*}\right)$ for all $i \in Z$.*

To avoid confusion, note that the initial deviations from a candidate equilibrium supply network g^* to a different network g in Definition 2 can be carried out by any coalition $Z \in \mathcal{Z}_{g^* \to g}$, whereas successive deviations from g can only be carried out by subcoalitions $Z' \subseteq Z$ (see Definition 1).

Definitions 1 and 2 can be made operational for use in the applications presented in Sections 5 through 7 as follows. Since, by Proposition 1, $w(g) = c$ for all networks g, (1) and (2) take the following simple forms.

$$\pi_s(g, t^g) = \sum_{r \in R} \ell_{sr}^g t_{sr}^g \tag{5}$$

and

$$\pi_r(g, t^g) = \Pi_r^g - \sum_{s \in S} \ell_{sr}^g t_{sr}^g, \tag{6}$$

where $\Pi_r^g \equiv \Pi_r(g, c)$ is given by (3) and denotes the gross vertical profits generated by retailer r in supply network g when all wholesale prices are equal to marginal cost. Using these expressions, one can establish the following result regarding the mutual profitability of coalitional deviations from any network g to any network h.

Lemma 1 *Given a network g with transfers t^g, there exists a network $h \neq g$ and profile of transfers t^h such that $\pi_i\left(h, t^h\right) > \pi_i(g, t^g)$ for all $i \in Z$ if and only if*

$$\sum_{r \in Z} \left[\Pi_r^h - \Pi_r^g\right] > \sum_{s \in Z} \sum_{r \notin Z} \left(\ell_{sr}^g - \ell_{sr}^h\right) t_{sr}^g - \sum_{r \in Z} \sum_{s \notin Z} \left(\ell_{sr}^g - \ell_{sr}^h\right) t_{sr}^g \tag{7}$$

Lemma 1 is used below to construct an algorithm to solve for the CPNE of the model. Before doing so, however, it may be helpful to discuss a few intuitive aspects that play a role in that solution. The left-hand side of (7) represents the incremental gross vertical profits generated by the retailers that participate in a deviation from network g to network h. These incremental profits can be positive or negative. The right-hand side represents, instead, the net loss of transfers from firms with which the deviating coalition rescinds existing links or forms new links. For example, with reference to Figure 1, if $S1$ and $R1$ form a coalition and deviate from Figure 1(a) to Figure 1(c) by rescinding their links with $R2$ and $S2$, respectively, the right-hand side of (7) corresponds to the difference between the transfer that $S1$ receives from $R2$ in Figure 1(a) and the transfer that $R1$ pays to $S2$ in Figure 1(a).[13] In general, this net loss can also be positive or negative. Lemma 1 states that there exists a mutually profitable deviation from a network g with transfers t^g to a different network h if and only if the incremental vertical profits exceed the net loss in transfers.

Of particular interest is the question of whether a coalition Z can engage in mutually profitable deviations from a network g that maximizes industry profits. In this respect, two types of deviations play an important role in the applications that follow. In the first type of deviations the members of the deviating coalition Z do not modify their links (if any) with firms outside Z, i.e., $\ell_{sr}^g = \ell_{sr}^h$ if $s \notin Z$ or $r \notin Z$ and the right-hand side of (7) is thus equal to zero for any t^g. If the number of retailers involved in the deviation is strictly less than the total number of retailers R and $\sum_{r\in Z} \Pi_r^h > \sum_{r\in Z} \Pi_r^g$ this deviation is mutually profitable for the members of Z, even though $\sum_{r\in R} \Pi_r^h < \sum_{r\in R} \Pi_r^g$, because of the negative externalities imposed on the firms that do not participate in the deviation. As an example, which foreshadows the analysis in Section 5, refer to the pairwise exclusive network in Figure 1(c) and assume that exclusive contracts are not allowed. Consider a deviation in which the two firms in coalition $Z = \{R1, S2\}$ form a link with each other while preserving their existing links with other firms, thus causing the market to move to the mixed network in Figure 1(b). This deviation does not alter any links with firms outside coalition Z and, by condition (7), is profitable if and only if it increases the vertical profits generated by the only retailer in the coalition, $R1$, even if total industry profits, i.e., the sum of the vertical profits generated by $R1$ and $R2$, are lower in Figure 1(b) than in Figure 1(c). As will be discussed in Section 6, exclusive contracts can render such a deviation mutually unprofitable, by making the consent of all four firms necessary (i.e., by restricting the set $\mathcal{Z}_{g\to h}$ of coalitions

[13]In this context a coalition is simply a set of firms discussing whether and how to coordinate their strategies (i.e., their transfer and exclusivity proposals t and θ) to achieve a certain outcome, taking as given the strategies of firms outside the set. Firms that belong to a coalition can also continue, stop, or start dealing with firms outside the coalition by deciding to agree or not to the *given* transfer and exclusivity proposals of those firms. In other words, a coalition formed for the purpose of coordinating the strategies of its members does not necessarily coincide with the set of firms with which its members have a supply link.

that can implement this deviation).

The second type of deviations of interest is one in which the members of Z rescind their links with some firms that do not participate in the deviation, i.e., in which $\ell_{sr}^g = 1$ and $\ell_{sr}^h = 0$ for some $s \in Z$ and $r \notin Z$ or some $s \notin Z$ and $r \in Z$. This type of deviations is mutually profitable for the members of Z if, given an initial profile of transfers t^g, the firms dropped from network g appropriate more than their contribution to g relative to some other network h that can be implemented without their participation. Importantly, the profile of transfers t^g may not provide enough degrees of freedom to ensure that *all* firms extract less than their contribution to a given network g relative to *every* alternative network, and thus g may always be vulnerable to at least one mutually profitable (though not necessarily self-enforcing) deviation.[14] This is, for example, the case for some parameter values in the bilateral duopoly model with exclusive contracts studied in Section 6, for which pure-strategy equilibria may fail to exist.

As already mentioned above, Lemma 1 can be used to construct the following algorithm to solve for the CPNE of the contracting model with transfers introduced in Section 2. For any possible candidate equilibrium supply network g:

Step 1 – Determine the set of all alternative networks h that can be implemented by any coalition $Z \in \mathcal{Z}_{g \to h}$. If there exists a profile of transfers t^g such that (7) fails for all h and $Z \in \mathcal{Z}_{g \to h}$ (i.e., such that no deviation is mutually profitable), one can stop and conclude that g constitutes a CPNE (and, in fact, a strong equilibrium). If instead no such t^g exists, for each t^g and each mutually profitable deviation by any coalition $Z \in \mathcal{Z}_{g \to h}$ implementing a network $h \neq g$, determine the range of transfers $I^h(t^g) = \left\{ t^h : \pi_i \left(h, t^h \right) > \pi_i (g, t^g), \forall i \in Z \right\}$ that can make each member of the deviating coalition better off and move to Step 2.

Step 2 – For each network h identified above, determine all the alternative networks k that can be implemented by any *subcoalition* Z' of Z and verify whether, for each t^g, there exists any $\tilde{t}^h \in I^h(t^g)$ such that (7) fails for all k and $Z' \in \mathcal{Z}_{h \to k}$, $Z' \subseteq Z$, when one substitutes Z' for Z, h for g, k for h, and $\tilde{t}^h$ for t^g (i.e., such that there exists no mutually profitable deviation from h to k). If there exists such a $\tilde{t}^h$ one can stop here and conclude that the original deviation to h is self-enforcing for each t^g and thus that g is not a CPNE. If instead there does not exist such a $\tilde{t}^h$ for at least some t^g, then some deviations from h to k are mutually profitable and one should apply Step 2 to k to determine whether any of these deviations are self-enforcing. If the answer is yes, the original deviation to h is not self-enforcing and

[14] See also Bloch and Jackson (2007) for a discussion of this issue.

g is a CPNE. If the answer is no, the original deviation to h is self-enforcing and g is not a CPNE.

The solution algorithm introduced above is well defined for any arbitrary numbers of suppliers, S, and retailers, R. In particular, since successive deviations are limited to subcoalitions, it always converges to an end point. However, using it to solve a model with more than a few firms on each side of the market would be unwieldy because the numbers of possible supply networks and coalitions grow exponentially with S and R. Related to this, large values of S and R would also give rise to an unmanageably large number of deviations from deviations that would need to be checked, i.e. of iterations of Step 2 above. For this reason, in the rest or the paper I restrict attention to a more tractable bilateral duopoly model with two symmetrically differentiated suppliers and two symmetrically differentiated retailers that satisfies the assumptions laid out in Section 2.

4 A bilateral duopoly model

The possible types of supply networks that can arise in bilateral duopoly are listed below, together with the maximum vertical profits Π_r^g that each retailer r can generate in supply network g when all retailers obtain products at wholesale prices $w(g) = c$ and compete in the downstream market.

- *Double common agency* (denoted by $g = dca$ and illustrated in Figure 1(a)). Both retailers deal with both suppliers. Given the symmetry of the model, each retailer generates the same vertical profits $\Pi_r^{dca} = \Pi^{dca}$.

- *Mixed network* ($g = mix$, Figure 1(b)). One of the retailers deals with both suppliers, while the other retailer only deals with one supplier. The vertical profits generated by the retailer that deals with both suppliers are Π^{mix2}, whereas those of the retailer that only deals with one supplier are Π^{mix1}.

- *Pairwise exclusivity* ($g = pe$, Figure 1(c)). Each retailer deals with a different supplier and generates the same vertical profits $\Pi_r^{pe} = \Pi^{pe}$.

- *Downstream monopoly* ($g = dm$, Figure 1(d)). Both suppliers only deal with the same retailer, thus excluding the other retailer. The active retailer generates vertical profits Π^{dm}, whereas the excluded retailer generates zero vertical profits.

- *Upstream monopoly* ($g = um$, Figure 1(e)). Both retailers only deal with the same supplier, excluding the other supplier. Each retailer generates the same vertical profits $\Pi_r^{um} = \Pi^{um}$.

- *Bilateral monopoly* ($g = bm$, Figure 1(f)): A retailer and a supplier only deal with each other, while the other retailer and supplier are excluded. The active retailer generates vertical profits Π^{bm}, whereas the excluded retailer generates zero vertical profits.

Although one could in principle use a fairly general demand system to rank unambiguously the vertical profits generated by retailers under some of the supply networks listed above, this is not the case for all of these networks.[15] Moreover, even if one could obtain an ordinal ranking of vertical profits for all supply networks, this would be insufficient because the analysis in subsequent sections involves linear functions of these vertical profits and requires these profits to be cardinally comparable. In the interest of concreteness, therefore, I use the following (inverse) linear demand system, which allows me to parametrize the degrees of supplier and retailer differentiation

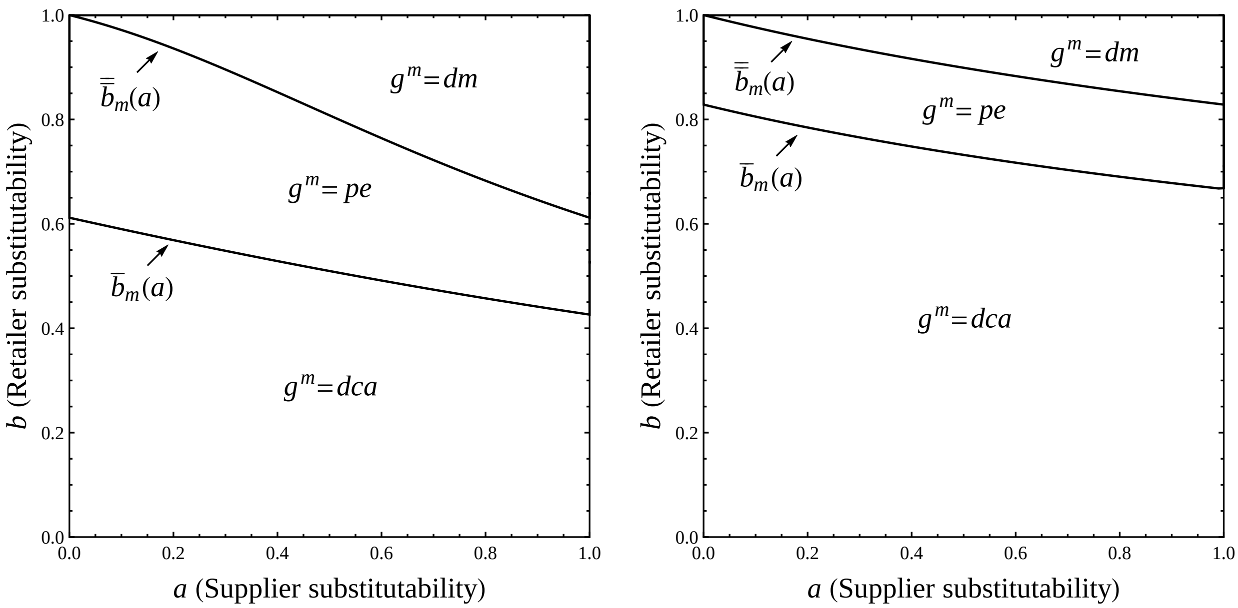
$$p_{sr} = v - (q_{sr} + aq_{s'r}) - b\,(q_{sr'} + aq_{s'r'})\,, \tag{8}$$

where $v > c$ and $a, b \in [0, 1]$.[16] Lower values of a and b indicate, respectively, higher supplier and retailer differentiation, with $a = 0$ and $b = 0$ corresponding to the extreme case in which, respectively, suppliers or retailers are completely independent in demand and $a = 1$ and $b = 1$ to the extreme case in which they are perfect substitutes. Note that supplier differentiation and retailer differentiation are "cumulative", in that the coefficient ab on $q_{s'r'}$ in (8) is smaller than the coefficients a and b on $q_{s'r}$ and $q_{sr'}$, respectively. A complete solution of the downstream market game played by retailers in stage 3 with this demand system and $w\,(g) = c$ under both Cournot and Bertrand competition is presented in an online appendix (enclosed with this submission). In the body of the paper I will rely directly on the values of the vertical profits Π^g resulting from that solution.

[15] For example, under mild assumptions on demand, one can generally conclude that the profits generated by a retailer are decreasing in the number of products carried by the other retailer, which implies, e.g., $\Pi^{mix2} \geq \Pi^{dca}$ and $\Pi^{dm} \geq \Pi^g$ for all $g \neq dm$; or that, given the number of products carried by the two retailers, profits are higher when the retailers carry different products, which implies $\Pi^{pe} \geq \Pi^{um}$. However, other comparisons are more ambiguous. For example, one cannot necessarily conclude that the equilibrium vertical profits generated by a retailer increase in the number of products carried by that retailer, because the equilibrium prices charged by the other retailer are likely to fall, or the quantities likely to increase, when the first retailer introduces new products. This implies that, in general, there is no unambiguous comparison between, e.g., Π^{mix2} and Π^{pe} or Π^{dca} and Π^{mix1}.

[16] See the online appendix for a derivation of this demand system from underlying consumer preferences and for a verification that it satisfies Assumption 1.

This framework is used in Figure 2 to characterize the supply networks that maximize total industry profits, conditional on retailers competing in the downstream market. Comparisons between these networks and the networks that arise in equilibrium will prove particularly helpful in the sections that follow, as they will provide insights into the nature and magnitude of the externalities that prevent equilibrium networks from maximizing industry profits. Figure 2 divides



(a) Bertrand competition (b) Cournot competition

Figure 2: Supply networks that maximize industry profits.

the unit square representing all possible combinations of supplier substitutability, a, and retailer substitutability, b, into three regions and shows that industry profits are maximized by a different supply network in each of these regions. Specifically, denoting with $\overline{b}_m(a)$ the value of b for which $2\Pi^{pe} = 2\Pi^{dca}$ (i.e., for which industry profits are the same under pairwise exclusivity and double common agency) and with $\overline{\overline{b}}_m(a)$ the value of b for which $\Pi^{dm} = 2\Pi^{pe}$ (i.e., for which industry profits are the same under downstream monopoly and pairwise exclusivity), the figure shows that industry profits are maximized by double common agency for $b \leq \overline{b}_m(a)$, pairwise exclusivity for $\overline{b}_m(a) \leq b \leq \overline{\overline{b}}_m(a)$, and downstream monopoly for $b \geq \overline{\overline{b}}_m(a)$. Industry profits under other supply networks, such as a mixed network or upstream monopoly, are always dominated by industry profits under one or more of the three networks shown in the figure.

The intuition for Figure 2 can be explained by noting that the elimination of supply links has two opposite effects on industry profits. On the one hand, it reduces the intensity of downstream competition. Specifically, a move from double common agency to pairwise exclusivity increases the degree of effective retailer differentiation, since the two retailers go from both carrying the same products to each carrying a different product, whereas a move from pairwise exclusivity to downstream monopoly completely eliminates any residual downstream competition. On the other hand, when both suppliers and retailers are differentiated, eliminating supply links causes loss of variety. As b increases, more intense downstream competition causes greater dissipation of industry profits and thus the former effect (softening of downstream competition) becomes progressively more important than the latter effect (loss of variety). Therefore, as b increases, industry profits are maximized by networks with less downstream competition and less variety.

5 Equilibria without exclusive contracts

Having characterized the supply networks that maximize industry profits, one can study the extent to which bilateral contracting between suppliers and retailers can implement these networks under different assumptions about upfront transfers and exclusive contracts. In this section, I start by considering the case in which firms can use upfront transfers but exclusive contracts are banned or not enforceable.

Proposition 2 (No exclusive contracts) *Consider a bilateral duopoly model where firms cannot publicly commit to contract proposals and face the symmetric linear demand system in (8). When firms can use transfers but not exclusive contracts there exists a unique coalition-proof Nash equilibrium supply network $g^*_{t,ne}$ with*

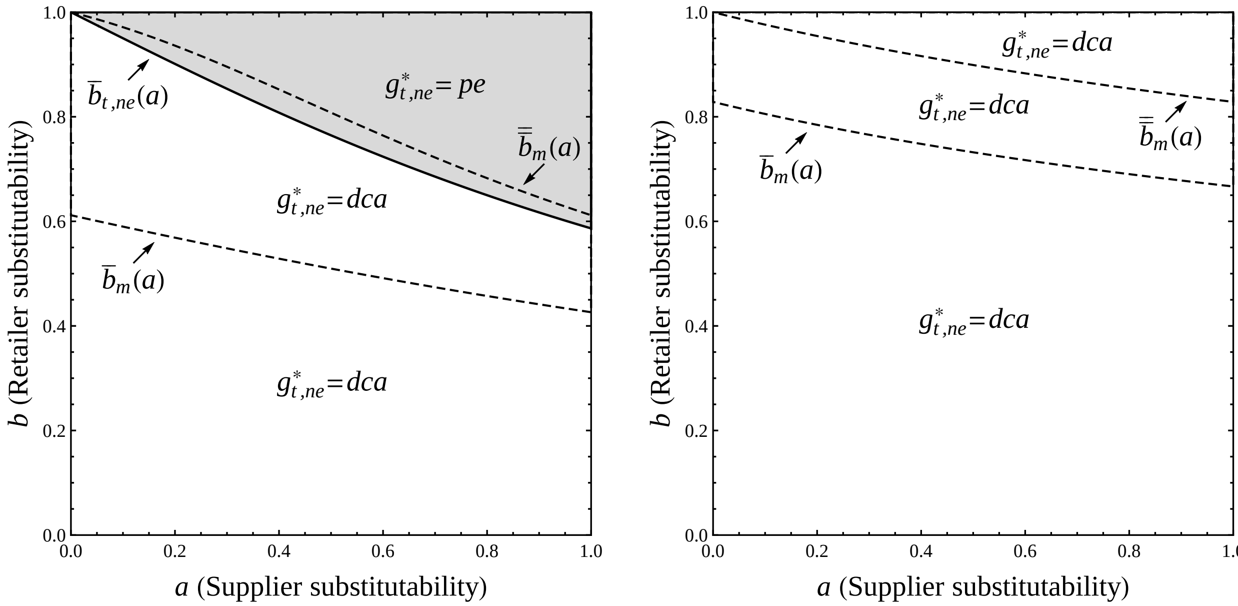
1. $g^*_{t,ne} = pe$ *and transfers* $t^{pe}_{ne} \in (0, \Pi^{pe} - \Pi^{um})$ *if and only if*

$$\Pi^{pe} \geq \Pi^{mix2}. \tag{9}$$

2. $g^*_{t,ne} = dca$ *and* $t^{dca}_{ne} \in \left(0, \Pi^{dca} - \Pi^{mix1}\right)$ *otherwise.*

With differentiated Bertrand competition (9) holds if and only if $b \geq \overline{b}_{t,ne}(a)$ *(see shaded area in Figure 3(a) below). With differentiated Cournot competition (9) never holds.*

Although the formal proof of Proposition 2 is quite involved, the intuition for the result is fairly straightforward. Without exclusive contracts, it is difficult (though not impossible) for supply networks with some inactive links to be self enforcing, even when these networks maximize industry profits. This is because, starting from a candidate equilibrium network with inactive links (e.g., $g = dm$ or $g = pe$), under most configurations of parameters at least one supplier-retailer pair has bilateral incentives to deviate by activating one of the inactive links and, in the absence of exclusive contracts, can do so without the consent of other trading partners. As a



(a) Bertrand competition

(b) Cournot competition

Figure 3: Equilibria with transfers and no exclusive contracts.

result, in equilibrium firms tend to form "too many" supply links from the point of view of industry profit maximization.

Specifically, as illustrated in Figure 3, downstream monopoly can never be supported as an equilibrium, even though it maximizes industry profits for $b > \overline{\overline{b}}_m(a)$, because, without exclusive contracts, a supplier can accept an offer from the excluded retailer, r', without having to forego the profits he earns from the other retailer, r, and can thus always be induced to do so by r'. Analogously, pairwise exclusivity cannot be supported as an equilibrium in large parts (with Bertrand competition) or the totality (with Cournot competition) of the region with $b \in \left[\overline{b}_m(a), \overline{\overline{b}}_m(a)\right]$ in which it maximizes industry profits. The relevant deviation from a candidate equilibrium with pairwise exclusivity is one in which a supplier, say s, opens a new link with a second retailer, say r', thus giving rise to a mixed network in which r', who now carries two products, generates vertical profits Π^{mix2}. This deviations is mutually profitable and self enforcing whenever $\Pi^{mix2} > \Pi^{pe}$, which is the case for $b < \overline{b}_{t,ne}(a)$ with Bertrand competition and always the case with Cournot competition. The difference between Bertrand and Cournot competition is explained by the fact that, for any given degree of intrinsic retailer substitutability b, the former is generally more competitive than the latter. The softening of downstream competition caused by pairwise exclusivity has, therefore, a greater positive effect on profits in the former than in the latter.

The results in Proposition 2 have been obtained under the assumption that firms can engage in *any* type of nonbinding pre-play communication, including nonbinding (but possibly self enforcing) reciprocal agreements between firms on the same side of the market not to deal with their competitor's suppliers or retailers. Since these agreements effectively amount to market allocation agreements, which are illegal in most jurisdictions even when nonbinding, one may wonder how the results would change if one prohibited deviations relying on such agreements. The following remark, which maintains the same assumptions as Proposition 2 except for the types of nonbinding agreements that are allowed, addresses this question.

Remark 1 (No exclusive contracts, no market allocation agreements) *When firms cannot discuss (nonbinding) market allocation agreements, in addition to the equilibria in Proposition 2 there also exist equilibria with* $g^*_{t,ne} = dca$ *when condition (9) holds. This gives rise to multiple equilibria for Bertrand competition and* $b \geq \overline{b}_{t,ne}(a)$ *in the shaded area in Figure 3(a).*

As one would expect, limiting the type of nonbinding communication that can take place expands the set of equilibria by reducing the ability of firms to coordinate deviations. In the specific case of the bilateral duopoly model of this section, when firms can engage in any type of communication, as was the case in Proposition 2, and condition (9) holds, the two retailers can reach a mutually profitable and self-enforcing nonbinding agreement to deviate from a candidate equilibrium with $g = dca$ to a network $g = pe$ in which they allocate the market by each refusing to sell a different product. This is, however, no longer possible under the restriction in Remark 1.

6 Equilibria with exclusive contracts

In this section I study the implications of allowing firms to use exclusive contracts. In principle one could adopt a general framework in which a contract between firm i and firm j can be made contingent on all other supply links in the market, i.e., on all the links that firm i has with other firms $h \neq j$, all the links that firm j has with other firms $k \neq i$, and all the links that other firms $h, k \neq i, j$ have with one another. Such contracts are, however, rarely enforceable for practical and legal reasons. Therefore, I focus on the case in which the only contingencies allowed in a contract between i and j are those requiring i and/or j to be exclusive to the other.[17]

[17] I also assume that the tariff in a nonexclusive contract between a firm i and a firm j cannot be made contingent on the volume traded by either firm with third parties $k \neq i, j$, as would instead be the case with market-share discounts or retail price parity agreements. These restraints would not make a difference in the environment with secret contracting studied in this paper (see Rey and Vergé (2016), because the fact that upstream margins are equal to zero (i.e., $w = c$) rules out any competition for *marginal* sales between suppliers. They would, however, have competitive effects in environments in which firms can publicly commit to contracts, as in such environments equilibrium upstream margins are generically different from zero (see Ramezzana, 2016).

Specifically, I assume that a contract between i and j can either require no exclusivity at all, or i to be exclusive to j, or j to be exclusive to i, or mutual exclusivity between i and j.

Moreover, to prevent firms from making mutually inconsistent exclusive offers, I assume that a firm i that offers an exclusive contract to a firm j is not allowed to offer contracts to other firms $k \neq j$. However, if i has promised exclusivity to j, and j engages in a further deviation in which it rejects i's proposal, i is again free to submit proposals to other firms $k \neq j$ that do not belong to the deviating coalition that includes j.[18] This last assumption, which I call the "no stranding" assumption, rules out the possibility that j can eliminate i from the market by first convincing i not to discuss a contract with any other firm $k \neq j$ and subsequently leaving i stranded without a partner (i.e., by essentially playing a "bait-and-switch" strategy).

Such bait-and-switch strategies would allow j to implement in a roundabout way supply networks that it could not implement directly and that are unrealistic in the unfettered contracting environment of this paper. To see this, consider the following example based on Figure 1. Assume that, starting from an equilibrium with $g = dca$, as in panel 1(a), a coalition $\{S1, S2, R1\}$ reaches a nonbinding agreement to deviate to $g = dm$, with both suppliers agreeing to submit exclusive contract offers to $R1$, and consequently no contract offers to $R2$, as in panel 1(d). Once this nonbinding agreement has been reached, $S1$ and $R1$ may deviate further by committing to mutual exclusivity, thus excluding $S2$, as in panel 1(f). Absent the "no stranding" assumption introduced above, $S2$ would remain without a trading partner, as would $R2$, who was excluded in the initial deviation to $g = dm$. As a result, $S1$ and $R1$ would be able to implement a bilateral monopoly, an outcome they could not have achieved by deviating to mutual exclusivity directly from the candidate equilibrium with $g = dca$. This is an unrealistic outcome, as it is difficult to see how $S1$ and $R1$ could expect the two stranded firms, $S2$ and $R2$, not to find a way to reach an agreement to form a supply link. The "no stranding" assumption allows $S2$ and $R2$ to form such a link and, therefore, implies that a deviation to mutual exclusivity by $S1$ and $R1$ results in a pairwise exclusive supply network, as in panel 1(c), instead of resulting in a bilateral monopoly network.[19]

Another issue that one needs to address is the extent to which a given supply network can

[18] Alternatively, one can assume that when i promises exclusivity to j it can also simultaneously submit to other firms $k \neq j$ contract proposals that become valid only if j rejects i's initial exclusive offer. This alternative assumption would yield the same results.

[19] The issue of how players that are excluded from a coalitional deviation will react to such a deviation has received considerable attention in the literature on coalitional equilibria and coalition formation, where it is known as the "prediction problem" (see, e.g., Bloch and Dutta (2011) for a discussion). The "no stranding" assumption adopted in the specific context of this section is reminiscent of the solution to this problem proposed by Ray and Vohra (1997), who allow players that are excluded from a deviating coalition to reorganize their strategies optimally (i.e., to play a best response).

be supported by different combinations of exclusive clauses (including, possibly, no exclusive clauses at all). Since a link between two firms can remain inactive also in the absence of contractual exclusive clauses, provided that at least one of the two firms refuses to trade with the other, exclusive clauses are not necessary to obtain any give supply network g. For example, the supply network with pairwise exclusivity shown in Figure 1(c) can be implemented through any type of one-way or mutual exclusive clauses between $S1$ and $R1$ and between $S1$ and $R2$ or by all firms refusing to engage in "cross trade," without any contractual exclusive clauses. As a result, the set of possible supply networks in a bilateral duopoly remains the same as in the absence of exclusive contracts (see Figure 1 and Section 4). This does not, however, mean that the adoption of exclusive clauses is irrelevant for the equilibria of the model. On the contrary, different combinations of exclusive clauses have different implications for the feasibility or profitability of deviations from a given network, and can therefore support different networks as equilibria.

In particular, networks that are implemented with a more extensive use of exclusive clauses are generally less vulnerable to deviations in which firms attempt to expand the network by adding new supply links, because such deviations require the consent of a broader set of firms. For example, consider the supply network with pairwise exclusivity shown in Figure 1(c). If this network is supported only by a refusal by all firms to "cross trade", without any contractual exclusive clauses, a deviation that adds a link between $S2$ and $R1$ (thus implementing the mixed network in Figure 1(b)) only requires the consent of $S2$ and $R1$. If instead the same network is supported by one-way contractual exclusivity (e.g., $S1$ and $S2$ committing to be exclusive to $R1$ and $R2$, respectively, but not the other way around) the same deviation requires the consent of the coalition $\{S2, R1, R2\}$, since $R2$ must now consent to $S2$ dealing also with $R1$. Finally, if the same network is supported by mutual contractual exclusivity between $S1$ and $R1$ and between $S1$ and $R2$, the deviation requires the consent of the grand coalition $\{S1, S2, R1, R2\}$, since every firm must consent to its trading partner in the candidate equilibrium starting to deal with a new firm.

In order to study the full effects that firms can achieve by resorting to contractual exclusive clauses, I resolve the ambiguity discussed above by assuming that whenever firms i and j do not trade with each other in a candidate equilibrium network g, they are prevented from doing so by the most extensive combination of exclusive clauses consistent with network g. For example, I assume that a candidate equilibrium with the pairwise exclusive supply network in Figure 1(c) is supported by mutual exclusivity between $S1$ and $R1$ and between $S1$ and $R2$.[20]

[20] It is important to note that this assumption applies only to candidate equilibrium networks, not to the networks that result from deviations. Deviations are not restricted in any way and can rely on any possible

6.1 Effects of exclusive contracts on equilibrium supply networks

Under the assumptions introduced above one can establish the following result.

Proposition 3 (Exclusive contracts) *In a bilateral duopoly model with the same assumptions as in Proposition 2, but in which firms can use exclusive contracts, there exist pure-strategy coalition-proof Nash equilibria with the following supply networks and transfers:*

1. $g^*_{t,e} = dm$ *and* $t_e^{dm} = \Pi^{dm}/2$ *if and only if*

$$\Pi^{dm} \geq 2\Pi^{pe}, \tag{10}$$

which is the case if and only if $b \geq \overline{\overline{b}}_m(a)$ *(see Figure 4).*

2. $g^*_{t,e} = pe$ *and* $t_e^{pe} \in \left[\Pi^{dm} - \Pi^{pe}, \min\left\{2\left(\Pi^{pe} - \Pi^{um}\right), \Pi^{pe}\right\}\right]$ *if and only if (10) fails and*

$$2\left(\Pi^{pe} - \Pi^{um}\right) \geq \Pi^{dm} - \Pi^{pe}, \tag{11}$$

which is the case if and only if $\overline{\overline{b}}_{t,e}(a) \leq b < \overline{\overline{b}}_m(a)$ *(see Figure 4).*

3. $g^*_{t,e} = dca$ *and* $t_e^{dca} \in \left[\left(\Pi^{dm} - \Pi^{dca}\right)/2, \Pi^{dca} - \Pi^{mix1}\right]$ *if and only if*

$$2\left(\Pi^{dca} - \Pi^{mix1}\right) \geq \Pi^{dm} - \Pi^{dca}, \tag{12}$$

which is the case if and only if $b \leq \overline{b}_{t,e}(a)$ *(see Figure 4).*

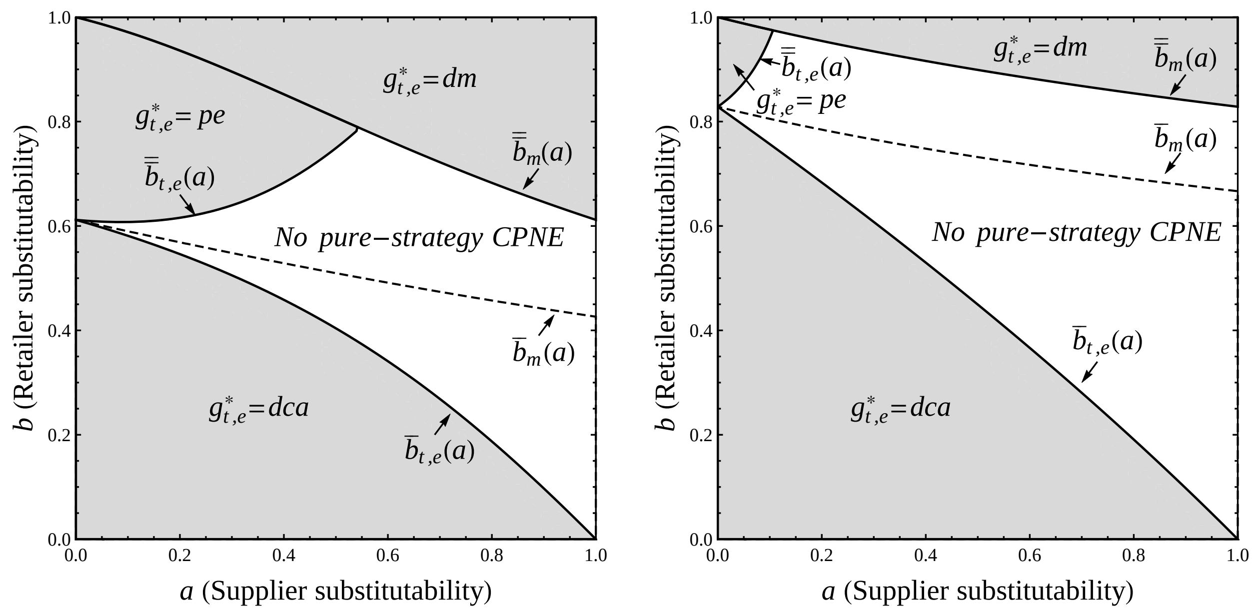
There exist no other pure-strategy equilibria.

For those combinations of the supplier and retailer substitutability parameters, a and b, for which a pure-strategy equilibrium supply network exists, such a network is unique and maximizes industry profits.[21] The latter can be seen by noting that the lines $\overline{b}_m(a)$ and $\overline{\overline{b}}_m(a)$ in Figure 4 are the same as those used in Figure 2 to illustrate the supply networks that maximize industry profits for different values of a and b. The intuition for this result is as follows.

Starting from a candidate equilibrium with some inactive supply links (e.g., $g^*_{t,e} = pe$), consider a deviation in which two firms i and j want to form a new link. In the absence of exclusive contracts, they can do so without having to ask for permission from, or forego their relationship

combination of exclusive clauses. However, deviations that use the most restrictive combination of exclusive clauses are the most likely to be self enforcing, and therefore play a prominent role in the proofs of the results that follow.

[21] Indeed, as can be seen from the proof of Proposition 3, the CPNE of the model with exclusive contracts correspond to its strong equilibria. In a model with transferable utility like the present one, when strong equilibria exist, they must always maximize the sum of the players' payoffs (i.e., of the firms' profits).



(a) Bertrand competition

(b) Cournot competition

Figure 4: Equilibria with exclusive contracts and upfront transfers.

with, any other firm $k \neq i, j$. This is indeed the reason that, as shown in Section 5, equilibria with downstream monopoly or pairwise exclusivity might not be supportable without exclusive contracts even when they maximize industry profits. Things are, however, quite different if in the candidate equilibrium one or both of i and j have committed to be exclusive to other firms. For example, assume that firm i is part of the candidate equilibrium network and has committed to be exclusive to firm $k \neq j$. If i wishes to deviate by forming a supply link with j it must now either i) obtain consent from k, possibly in exchange for compensation, or ii) forego its relationship with k, effectively swapping j for k. Starting from equilibria with $g = dm$ and $g = pe$, i) and ii) make forming a link with j unprofitable for i whenever it is unprofitable for the supply network as a whole. Specifically, starting from $g = dm$ a supplier may wish to form a link with the excluded retailer, and starting from $g = pe$ any supplier or retailer may wish to form a second supply link. However, exclusive contracts, through the effects in i) and ii) discussed above, make this unprofitable when $g = dm$ and $g = pe$ maximize industry profits.

Besides supporting equilibria with some sort of exclusivity when these maximize industry profits, exclusive contracts also eliminate (pure-strategy) equilibria with nonexclusive networks, such as $g = dca$, when these networks do not maximize industry profits, as is the case for $\overline{b}_m(a) \leq b < \overline{b}_{t,ne}(a)$ in Figure 3, where exclusive contracts were not available. The reason for this is that exclusive contracts tend to make deviations that rescind some links, such as deviations to $g = dm$ or to $g = pe$, self-enforcing.

The latter is also the reason that pure-strategy equilibria may fail to exist for high values of a and intermediate values of b.[22] For these parameter values, the availability of exclusive contracts makes both deviations in which firms exclude one of the retailers (such as a deviation to $g = dm$) and deviations in which they exclude one of the suppliers (such as a deviation to $g = mix$) become self enforcing. As preventing the first type of deviation requires large transfers t^g from retailers to suppliers (to ensure that retailers do not extract to much), whereas preventing the second type of deviations requires small transfers t^g (to ensure that suppliers do not extract too much), there may exist no t^g that can prevent all deviations.

This is, instead, not a problem for low values of a or extreme (low or high) values of b. For example, when a is low (i.e., suppliers are highly differentiated), a supply network with $g = pe$ allows retailers to inherit the high degree of supplier differentiation, thus softening downstream competition without sacrificing much variety. Starting from such a network, deviations to $g = dm$ or $g = mix$ would not be profitable, as they would only provide a modest reduction in downstream competition and would entail a significant sacrifice in variety. Analogously, deviations from $g = dca$ to $g = dm$ would be very costly for low values of b, and deviations from $g = dm$ to $g = pe$ would be very costly for high values of b especially when a is also high.

As shown in the online appendix, the nonexistence of pure-strategy equilibria would not constitute a problem if one dropped the "no stranding" assumption. Specifically, dropping this assumption would make deviations from $g = dca$ or $g = pe$ to $g = dm$ less likely to be self-enforcing, and would thus result in a weaker equilibrium concept, with pure-strategy equilibria existing for a broader range of parameters. This is because, absent the "no stranding" assumption, a further deviation from $g = dm$ to mutual exclusivity by a supplier and a retailer would implement a bilateral monopoly, $g = bm$, which is more profitable than the pairwise exclusive network, $g = pe$, that the same deviation would instead implement with the "no stranding" assumption. As shown in the online appendix, without the "no stranding" assumption the model would have multiple pure-strategy equilibria for high values of a and intermediate values of b, instead of having no pure-strategy equilibria. Regardless of its consequences for the existence and uniqueness of equilibria, as explained above, I believe that the "no stranding" assumption

[22] As noted by Bernheim, Peleg and Whinston (1987), the existence coalition-proof Nash equilibria, even in mixed strategies, cannot be guaranteed for general classes of games. Specifically, finding sufficient conditions for the existence of such equilibria is far from obvious, unless the game played by any subset of players, given any set of actions of the remaining players, has a unique Nash equilibrium. Since this uniqueness condition does not necessarily apply to the bilateral oligopoly game presented here, one cannot assert with certainty that there exist mixed-strategy equilibria in the parameter region where pure-strategy equilibria do not exist, although this may be the case. Given the complexity of the analysis, I have not attempted a characterization of mixed-strategy equilibria.

(or some other assumption with a similar function) is necessary for a realistic analysis of the long-term structure of supply networks.

6.2 Effects of exclusive contracts on welfare and firms' profits

The results obtained above have the following welfare implications.

Proposition 4 (Welfare) *Whenever exclusive contracts are adopted and affect the equilibrium structure of supply networks they reduce consumer and overall welfare.*

The result in Proposition 4 is straightforward (a proof relying on the utility function underlying the inverse linear demand in (8) is provided in the online appendix enclosed with this submission). Specifically, with Bertrand downstream competition, exclusive contracts cause the equilibrium network to switch from double common agency to pairwise exclusivity for $\bar{\bar{b}}_{t,e}(a) \leq b < \bar{b}_{t,ne}(a)$ and from pairwise exclusivity to downstream monopoly for $b \geq \bar{\bar{b}}_{m}(a)$. With Cournot downstream competition, instead, they cause the equilibrium network to switch from double common agency to pairwise exclusivity for $\bar{b}_{m}(a) \leq b < \bar{\bar{b}}_{m}(a)$ and from double common agency to downstream monopoly for $b \geq \bar{\bar{b}}_{m}(a)$. In all these cases they soften downstream competition, thus leading to higher prices, and reduce the variety of supplier retailer combinations available in the market. Both effects unambiguously reduce consumer and overall welfare.[23]

Less straightforward are, instead, the effects of the availability of exclusive contracts on the equilibrium profits earned by individual suppliers and individual retailers. Although, as explained in Section 3.2, the simultaneous contracting model used in this paper does not yield exact predictions regarding the equilibrium level of transfers, it nevertheless provides ranges within which such transfers, and thus the equilibrium profits of individual suppliers and retailers, must lie. If a change in the environment, such as the availability of exclusive contracts, causes these ranges to shift entirely to the right or the left of their initial position, the model adopted in this paper is sufficient to determine the distributional effects of that change. This is the approach used in deriving the following result.

Proposition 5 (Distribution of profits) *When exclusive contracts become available and are adopted in equilibrium, so that the resulting equilibrium supply network is $g^*_{t,e} = pe$ or $g^*_{t,e} = dm$, they make*

[23] As is well known, under certain conditions (e.g. when the seller and/or the buyer can make relationship-specific investments) exclusive contracts can enhance welfare by aligning investment incentives. In order to focus on the competitive effects of exclusive contracts, these potential efficiencies are not addressed in this paper.

suppliers strictly better off and retailers strictly worse off. When exclusive contracts become available but are not adopted in equilibrium, so that the equilibrium supply network remains $g^*_{t,e} = dca$, they make suppliers no worse off (and possibly strictly better off) and retailers no better off (and possibly strictly worse off).

The intuition for this results can be understood by noting that, besides possibly affecting the level of equilibrium industry profits, the availability of exclusive contracts may also affect the shares of any given amount of profits that suppliers and retailers are able to extract. Specifically, the upper bound on the profits that the firms on one side of the market can extract in a given equilibrium are determined by the extent to which the firms on the opposite side of the market can profitably drop them from their supply network *in deviations that are self enforcing.* In other words, the profits of the firms on one side of the market are determined by the credible disagreement payoffs available to the firms on the opposite side of the market. The availability of exclusive contracts makes a broader range of deviations self enforcing relative to an environment without exclusive contracts. For example, deviations to $g = um$, in which a supplier is excluded, or to $g = dm$, in which a retailer is excluded, can be self enforcing only if exclusive contracts are available. As shown in the proof of Proposition 3, whereas deviations to $g = um$, which limit the bargaining power of suppliers, are not sufficiently profitable to be self-enforcing (i.e., credible), deviations to $g = dm$, which limit the bargaining power of retailers, are generally sufficiently profitable to be self enforcing. Loosely speaking, this is the case because deviations to $g = um$ sacrifice product variety without softening downstream competition, whereas deviations to $g = dm$, though also costly in terms of variety, can increase profits by eliminating downstream competition. As a result, by making deviations to $g = dm$ self enforcing, the availability of exclusive contracts improves the credible disagreement payoffs of suppliers and shifts the balance of power in their favor.

Note that, when exclusive contracts are allowed, retailers have no way of preventing this outcome. Each individual retailer would have incentives to accept contracts that made her a downstream monopolist if she were offered such contracts, and (at least in this model) retailers cannot credibly commit not to accept such contracts. One would, however, expect that if retailers were given a say in public policy towards exclusive contracts before the contracting game is played, they would generally oppose the availability of such contracts.

The mechanism by which the availability of exclusive contracts can affect the distribution of profits in the present model is different from the mechanism discussed in O'Brien and Shaffer (1997) and at work in Liebman (2016) and Ho and Lee (2017). In those papers, the firms on one side of the market (typically downstream firms) can extract higher profits by (credibly) committing ex-ante to accept only a limited number of offers and honoring that commitment by actually excluding some firms in equilibrium. The mechanism in this paper does not rely on ex-ante commitment to exclude some firms in equilibrium, but rather on the effects of exclusive contracts on out-of-equilibrium alternatives. It is, therefore, closer in spirit to that studied by Bernheim and Whinston (1998) in a different setting with entry deterrence and without downstream competition.

7 Ex-post bargaining and hold up

To understand better the role that upfront transfers play in shaping supply networks, it is helpful to compare the equilibria characterized in Sections 5 and 6 with those of a modified game in which firms cannot use transfers at the network formation stage and must instead bargain bilaterally under some degree of hold-up only after a supply network has been formed.[24] This modified game develops in three stages. In stage 1, firms form supply links by playing Myerson's (1991) network formation game. Specifically, all firms simultaneously announce a list of firms on the other side of the market with which they are willing to deal, and a supply link between two firms is formed if and only if each firm has indicated the other as a partner. In Subsection 7.1 I assume that firms cannot commit to exclusivity as part of their announcements, whereas in Subsection 7.2 I allow for such commitment. No transfers are possible at this stage. In stage 2, all pairs of firms with a supply link bargain bilaterally, simultaneously and secretly over nonlinear supply contracts with fixed fee F_{sr} and wholesale price w_{sr}, splitting the surplus according to the generalized Nash bargaining solution, with a share β going to the supplier and a share $(1-\beta)$ to the retailer. Importantly, bilateral bargaining can take place only between firms with an existing link and no new links can be formed at this stage, which exposes firms to hold-up in negotiations. Finally, in stage 3, after having observed which supply links resulted from stage 1 (i.e., the prevailing supply network) but not the specific terms (e.g., the wholesale prices) that resulted from contracting in stage 2, retailers compete in the downstream market.

This game is similar to that studied by Rey and Vergé (2016) and is analyzed here mostly to provide context for the results obtained in previous sections, rather than as an original contribution in itself. This notwithstanding, the version of the model presented in this section features

[24]As already discussed, complete long-term contracts would play a role similar to upfront transfers, because they would allow firms to commit to leave a certain level of profits to their future trading partners at the moment in which supply networks are formed. The analysis of contracting under hold-up presented in this section should therefore be interpreted as referring to environments in which both transfers and complete long-term contracts are unavailable or severely limited.

some differences and additional contributions relative to Rey and Vergé (2016). Specifically, I assume that retailers observe the full set of active supply links (i.e., the prevailing supply network) before competing in the downstream market,[25] and also study the cases, not covered by Rey and Vergé, in which firms can adopt exclusive contracts and in which downstream competition is Cournot.

Given the gross vertical profits generated by downstream competition in stage 3, which remain the same as in Sections 4 through 6 (and are derived in the online appendix for the case of linear demand), one can solve for the contracts that result from bilateral negotiation in stage 2. As in previous sections, all wholesale prices are equal to marginal cost. The profits earned by a supplier s in supply network g correspond, therefore, to the sum of fixed fees, $\sum_{r\in R} \ell_{sr} F^g_{sr}$ charged by s in that network. For each network g in which $\ell_{sr} = 1$ the fixed fee F^g_{sr} is determined according to the generalized Nash bargaining solution by solving the following equation for F^g_{sr}, with $g\backslash sr$ denoting the supply network obtained from g when s and r do not form a supply link,

$$F^g_{sr} + \sum_{j\neq r} \ell_{sj} F^g_{sj} = \sum_{j\neq r} \ell_{sj} F^g_{sj} + \beta \left[\left(\sum_{j\neq r} \ell_{sj} F^g_{sj} + \Pi^g_r - \sum_{i\neq s} \ell_{ir} F^g_{ir} \right) - \left(\sum_{j\neq r} \ell_{sj} F^g_{sj} + \Pi^{g\backslash sr}_r - \sum_{i\neq s} \ell_{ir} F^g_{ir} \right) \right],$$

which reduces to

$$F^g_{sr} = \beta \left(\Pi^g_r - \Pi^{g\backslash sr}_r \right). \tag{13}$$

Using (13) one can calculate the following supplier (left column) and retailer (right column) profits for each possible supply network in the bilateral duopoly model introduced in Section 4.

$$\begin{aligned}
\pi^{dca}_s &= 2\beta \left(\Pi^{dca} - \Pi^{mix1} \right), & \pi^{dca}_r &= \Pi^{dca} - 2\beta \left(\Pi^{dca} - \Pi^{mix1} \right), \\
\pi^{pe}_s &= \beta \Pi^{pe}, & \pi^{pe}_r &= (1-\beta)\, \Pi^{pe}, \\
\pi^{dm}_s &= \beta \left(\Pi^{dm} - \Pi^{bm} \right), & \pi^{dm}_r &= \Pi^{dm} - 2\beta \left(\Pi^{dm} - \Pi^{bm} \right), \\
\pi^{um}_s &= \beta \Pi^{um}, & \pi^{um}_r &= (1-\beta)\, \Pi^{um}, \\
\pi^{bm}_s &= \beta \Pi^{bm}, & \pi^{bm}_r &= (1-\beta)\, \Pi^{bm}, \\
\pi^{mix1}_s &= \beta \left(\Pi^{mix2} - \Pi^{um} \right), & \pi^{mix1}_r &= (1-\beta)\, \Pi^{mix1}, \\
\pi^{mix2}_s &= \beta \left(\Pi^{mix1} + \Pi^{mix2} - \Pi^{pe} \right), & \pi^{mix2}_r &= \Pi^{mix2} - \beta \left(2\Pi^{mix2} - \Pi^{pe} - \Pi^{um} \right).
\end{aligned} \tag{14}$$

[25] Rey and Vergé (2016) assume that retailers do not observe the prevailing downstream market structure before setting retail prices and, following an unobserved deviation involving one of their rivals, behave as if the market structure was the one prescribed by the candidate equilibrium of the supply network formation game.

The profits in (14) constitute the payoffs of the link formation game played by firms in stage 1, which I turn to study next, first for the case in which commitment to exclusivity at the network formation stage is not possible and then the case in which it is.[26]

7.1 No ex-ante commitment to exclusivity

The main difference between the contracting game with upfront transfers studied in Section 5 and the game with ex-post bargaining studied in this subsection lies in the ability of firms to induce other firms to participate in deviations *in which new supply links are formed*. Specifically, when firms can use upfront transfers as in Section 5 and a coalition of firms would find it jointly profitable to enter into new supply contracts, there always exists a profile of upfront transfers that makes it individually profitable for each firm in the coalition to enter into those contracts. In the game with ex-post bargaining studied in this section, instead, the division of the joint profits resulting from the formation of a new supply link is determined by the firms' relative ex-post bargaining power and may not make each firm better off even if forming the new link is jointly profitable.

In the absence of exclusive contracts, upfront transfers do not play any role in facilitating deviations *in which firms rescind one or more supply links*. Without exclusive contracts, any agreement between a supplier s and a retailer r that, say, supplier s will not trade with another retailer $r' \neq r$ is nonbinding and must therefore be self enforcing. This implies that any transfer from r to s in exchange for a nonbinding promise by s to be exclusive with r is either unnecessary (when the promise is self-enforcing and thus s would have complied even without the transfer) or ineffective (when the promise is not-self enforcing).

The fact that upfront transfers facilitate agreements to create new links and, in the absence of exclusive contracts, do not have any effect on agreements to rescind existing links suggests that, at least for some parameter regions, equilibria with upfront transfers should tend to have more supply links than equilibria without upfront transfers. This intuition is confirmed by the following result.

Proposition 6 (No upfront transfers, no commitment to exclusivity) *Consider the same bilateral duopoly model as in Proposition 2 but assume that firms cannot use upfront transfers at the network*

[26] In this game, firms may have incentives to commit to exclusivity at the time of forming the supply network in stage 1, not at the time of engaging in ex-post bilateral bargaining over supply contracts in stage 2. Once firms reach stage 2, commitment to exclusivity is of no value because, by assumption, firms would not be able to form new links anyhow, and no firm would find it profitable to reduce its outside options by contractually renouncing some of the links that it has already formed. Therefore, the exclusivity studied in this subsection is better thought of as being achieved through some sort of ex-ante technological commitment in stage 1, rather than through contractual clauses in stage 2.

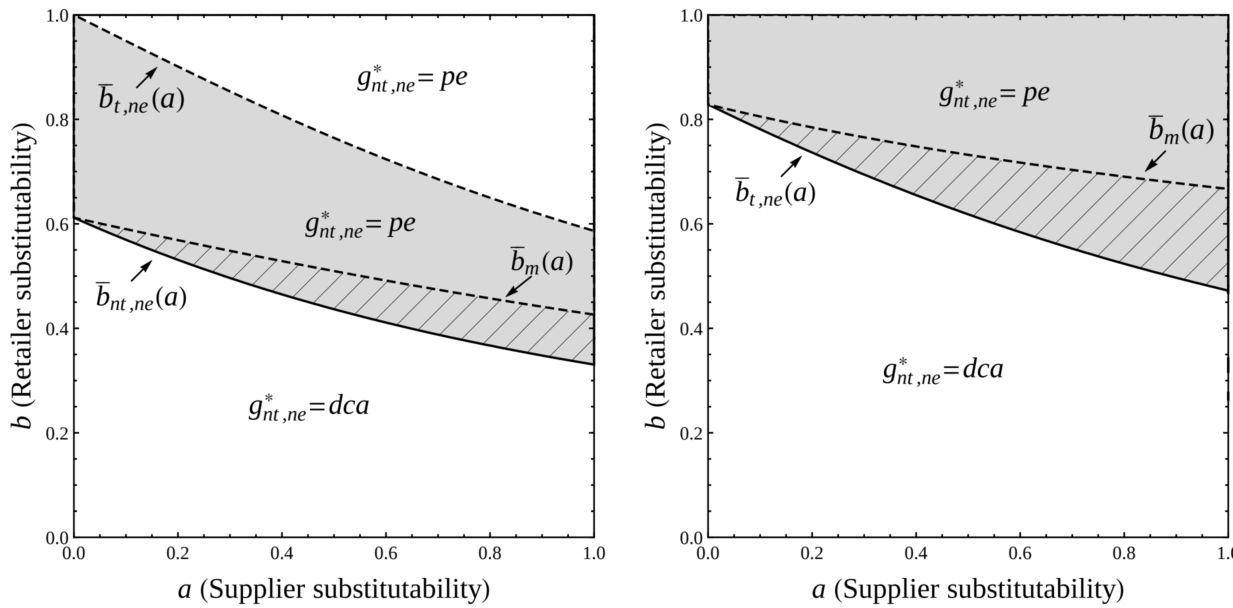
formation stage. There always exists a unique coalition-proof Nash equilibrium supply network $g^*_{nt,ne}$ *with*

1. $g^*_{t,ne} = pe$ *if and only if*

$$2\Pi^{pe} \geq \Pi^{mix1} + \Pi^{mix2}. \tag{15}$$

2. $g^*_{nt,ne} = dca$ *otherwise.*

Condition (15) holds for $b \geq \overline{b}_{nt,ne}(a)$*, where* $\overline{b}_{nt,ne}(a) < \overline{b}_{t,ne}(a)$ *is shown in Figure 5.*



(a) Bertrand competition

(b) Cournot competition

Figure 5: Equilibria without transfers (no exclusive contracts).

Figure 5 compares the equilibrium supply networks in the game with ex-post bargaining studied in this section to the equilibrium supply networks in the game with upfront transfers characterized in Section 5 and to the supply networks that maximize industry profits characterized in Section 4. As in a model with upfront transfers, in the absence of exclusive contracts downstream monopoly is not an equilibrium in a model with ex-post bargaining either, even though it maximizes industry profits for $b \geq \overline{\overline{b}}_m(a)$.[27] In particular, although the absence of upfront transfers hinders to some extent a deviation from downstream monopoly in which the excluded retailer forms a new link with one (or more) of the suppliers for the reasons discussed above, such deviation continues to be mutually profitable also with ex-post bargaining.

[27]To minimize clutter, I did not include the region in which downstream monopoly maximizes industry profits in Figure 5.

Ex-post bargaining does, instead, make a difference for the regions in which pairwise exclusivity and double common agency can be supported as equilibria. Specifically, with ex-post bargaining the region of parameters for which pairwise exclusivity arises in equilibrium expands, with the lower bound on b in Figure 5 shifting downward from $\overline{b}_{t,ne}(a)$ to $\overline{b}_{nt,ne}(a)$ (where $\overline{b}_{t,ne}(a) = 1$ for Cournot competition) and pairwise exclusivity replacing double common agency as the unique equilibrium outcome in the shaded regions.

The intuition for this is closely related to the discussion that precedes Proposition 6. Specifically, just as in the case with upfront transfers, the relevant self-enforcing deviation from a candidate equilibrium with pairwise exclusivity involves a supplier s starting to trade with a second retailer r' (or vice versa), thus implementing a mixed network. Such a deviation yields incremental joint profits of $\left(\Pi^{mix2} - \Pi^{pe}\right)$ for s and r'. With upfront transfers, s and r' can find a way to profit individually from this deviation provided that $\Pi^{mix2} > \Pi^{pe}$ (see Proposition 2). With ex-post bargaining, instead, the formation of the new link between s and r' also affects the bargaining power of these two firms in their negotiations over other links and thus the (inframarginal) profits that they can extract from those other links. The result is that both parties profit from a deviation from pairwise exclusivity to a mixed network only if $\Pi^{mix1} + \Pi^{mix2} > 2\Pi^{pe}$, which is more stringent than $\Pi^{mix2} > \Pi^{pe}$. The region of parameters where pairwise exclusivity is an equilibrium is therefore larger relative to that for a model with upfront transfers. This is accompanied by a corresponding reduction of the region where double common agency is an equilibrium, due mainly to that fact that deviations that move the market away from double common agency by rescinding links are more self-enforcing, though not necessarily more jointly profitable, when upfront transfers are not available.

Finally, ex-post bargaining affects overall industry profits differently in different parameter regions. As discussed above, ex-post bargaining shifts the equilibrium from double common agency to pairwise exclusivity in the entire shaded region in Figure 5, i.e., for $b \in \left[\overline{b}_{nt,ne}(a), \overline{b}_{t,ne}(a)\right]$. This shift increases total industry profits in the region without hashing in which $b \in \left[\overline{b}_{m}(a), \overline{b}_{t,ne}(a)\right]$ and reduces them in the hashed region in which $b \in \left[\overline{b}_{nt,ne}(a), \overline{b}_{m}(a)\right]$. Intuitively, the fact that ex-post bargaining makes it harder to form new links has two opposite effects on overall industry profits. On the one hand, it makes it harder for supplier-retailer pairs to impose negative externalities on other firms by forming new links that reduce the profits of those firms (this positive effect dominates in the shaded region without hashing). On the other hand, it also makes it harder for firms to realize gains from trade when these exist (this negative effect dominates in the hashed region). The fact that upfront transfers that cannot be made contingent on exclusivity do not necessarily lead to the maximization of total payoffs in games of network formation with externalities has also been stressed in a different context by Bloch and Jackson (2007).[28]

7.2 Ex-ante commitment to exclusivity

Assume now that, when firms announce which other firms they are willing to form a link with at the network formation stage, they can make that announcement contingent on the different type of exclusivity studied in Section 6. For example, a supplier s can announce that he is willing to form a link with a retailer r only if he obtains exclusivity from r or, alternatively, that he is willing to offer exclusivity to r. As above, once the supply network has been formed, firms that are connected by a supply link proceed to bargain bilaterally under some degree of hold-up. The following proposition characterizes the equilibria that arise in this environment.

Proposition 7 (No upfront transfers, with commitment to exclusivity) *In a bilateral duopoly model where firms can commit to exclusivity but not upfront transfers at the network formation stage, there exist coalition-proof Nash equilibria with*

1. $g^*_{nt,e} = pe$ *if and only if*

$$\Pi^{pe} \geq 2\left(\Pi^{dca} - \Pi^{mix1}\right). \tag{16}$$

which is the case if and only if $b > \overline{b}_{nt,e}(a)$ *in Figure 6 below.*

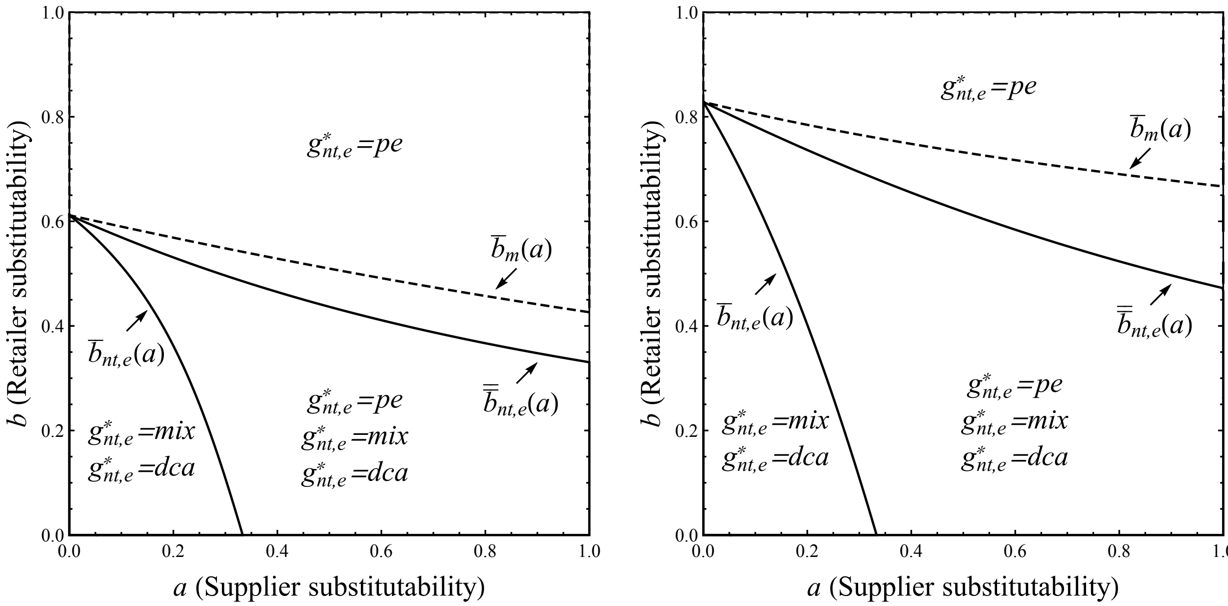
2. $g^*_{nt,e} = dca$ *and* $g^*_{nt,e} = mix$ *if and only if*

$$\Pi^{mix1} + \Pi^{mix2} \geq 2\Pi^{pe}, \tag{17}$$

which is the case if and only if $b \leq \overline{\overline{b}}_{nt,e}(a) = \overline{b}_{nt,ne}(a)$ *in Figure 6 below, with* $\overline{\overline{b}}_{nt,e}(a) > \overline{b}_{nt,e}(a)$.

The availability of exclusive contracts and the absence of upfront transfers make it more difficult for firms to deviate profitably by forming new supply links. Exclusive contracts tend, therefore, to expand the region of parameters where networks with some type of exclusivity, such as a mixed network or pairwise exclusivity, can be supported as equilibria. Specifically, in addition to the equilibrium with double common agency that exists also in the absence of

[28]Elliott (2015) also explores the effects of upfront transfers (relative to ex-post bargaining under hold up) on the efficiency of networks. The environment and issues that he studies are, however, quite different from those in this paper. Specifically, he considers markets without externalities (i.e., in which downstream firms do not compete with one another) and focuses on the firms' incentives to undertake relation-specific investments. In such an environment transfers also have two opposite effects on overall profits: they alleviate underinvestment by solving the hold-up problem but can encourage firms to overinvest in new links in order to boost their outside options.



(a) Bertrand competition

(b) Cournot competition

Figure 6: Equilibria with exclusive contracts and no transfers.

exclusive contracts for $b < \overline{\overline{b}}_{nt,e}(a)$, exclusive contracts support an equilibrium with a mixed network for $b < \overline{b}_{nt,e}(a)$ and equilibria with a mixed network and pairwise exclusivity for $b \in \left[\overline{b}_{nt,e}(a), \overline{\overline{b}}_{nt,e}(a)\right]$ in Figure 6.

One may wonder why there exist equilibria with exclusivity even when $b = 0$ in Figure 6, i.e., even when there is no downstream competition between retailers and thus exclusivity always reduces industry profits by reducing variety. The reason for this is that some firms have more ex-post bargaining power under certain supply networks with exclusivity than under alternative networks without exclusivity. For example, suppliers have more ex-post bargaining power under pairwise exclusivity, in which retailers have no ex-post alternative to their only supplier, than under double common agency, in which retailers can shift ex-post some of their business to the other supplier in case of disagreement during bilateral bargaining. In the absence of upfront transfers, the firms that would see their bargaining power fall as a consequence of deviations to networks with less exclusivity cannot be compensated for their loss and would thus block such deviations.

8 Conclusion

This paper has studied the formation of supply networks in environments with upstream and downstream competition in which firms can use upfront transfers and exclusive contracts. I have shown that, when contracts are secret, all (coalition-proof) equilibria are characterized by marginal input prices equal to marginal cost. Moreover, when exclusive contracts are not available and retailers are sufficiently close substitutes, equilibrium supply networks tend to have more supply links and more intense downstream competition than the networks that maximize industry profits. Exclusive contracts make it easier to support equilibrium networks with fewer supply links, thus eliminating the divergence between equilibrium and industry-profit maximizing networks and harming welfare through lower variety and higher prices. Finally, if the division of profits must take place through ex-post bargaining, for example because firms must make specific investments before starting to negotiate, it is more difficult for firms to organize mutually profitable deviations to broader networks and is thus easier to support equilibria with fewer supply links relative to an environment with upfront transfers or long-term contracts.

The model presented in this paper has been developed for the main purpose of studying the determinants and welfare effects of different supply networks, not of precisely predicting the division of profits between suppliers and retailers. As a result, it only characterizes lower and upper bounds on the transfers that support different equilibrium supply networks. This approach is sufficient to determine unambiguously the qualitative effects of certain changes in the environment on the distribution of profits. For example, it predicts that the availability of exclusive contracts unambiguously favors suppliers over retailers, by allowing the former to threaten credibly to exclude the latter. To obtain more precise predictions on the exact division of profits one would, however, need to adopt a specific coalitional bargaining game at the network-formation stage. Such an extension would add further complexity to an already complex environment and would be unlikely to affect the equilibrium structure of networks, unless the specific coalitional bargaining protocol adopted i) limits the firms' ability to continue making one another proposals until all possibilities have been explored, or ii) allows firms to credibly commit *ex-ante* (i.e., before starting negotiations) to restrict the number of counterparts with which they are willing to contract in order to elicit better offers from the other side of the market as in, e.g., Liebman (2016). Further work on this aspect may, however, prove useful.

Finally, this paper has focused on the case in which suppliers cannot publicly commit to marginal input prices. However, in some industries suppliers can attain some form of public commitment through various means.[29] This commitment can affect the structure of equilibrium

[29]Examples of regulatory obligations include the healthcare industry, in which some U.S. states (e.g. New Hampshire) have recently started to collect and publish the bilateral reimbursement rates agreed for every procedure by healthcare providers and insurers; the alcohol distribution industry, in which some U.S. license states require wholesalers to post and commit to wholesale prices for 30 days; and the automobile industry, in which the franchise laws of many states require manufacturers to offer the same price to all dealers and

supply networks because its potential to soften downstream competition and increase industry profits is greater in certain networks than in others. Moreover, since it typically yields wholesale prices that are different from the marginal cost of production, it introduces externalities between suppliers, in addition to the externalities between retailers that are also present in the environment without public commitment of this paper. I am studying these and other implications of public commitment in ongoing work.

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APPENDIX A
Proofs of Lemmas and Propositions

Proof of Proposition 1 – I first show that, for any network g, any profile of self-enforcing wholesale prices must be equal to marginal cost, i.e., $w(g) = c$. I then argue that this implies that there does not exist any self-enforcing (bilateral or multilateral) deviation from $w(g) = c$ and therefore that there always exist a unique CPNE profile of wholesale prices with $w(g) = c$.

A CPNE profile of wholesale prices must be immune to jointly profitable bilateral deviations by any supplier-retailer pair sr. Such bilateral deviations are always self-enforcing when jointly profitable because, in a CPNE, further deviations can only be undertaken by s alone or r alone and cannot modify the wholesale prices in the contracts with other firms. Therefore, if in their initial bilateral deviation s and r agreed on a contract $\langle t_{sr}, w_{sr}\rangle$ that left both of them better off given the strategies of all other players, neither s nor r can improve on that contract by deviating unilaterally.

The above implies that a CPNE profile of wholesale prices is always a contract equilibrium (see O'Brien and Shaffer, 1992), i.e., it maximizes the joint profits of each supplier-retailer pair, taking as given the contracts entered into by other pairs. In any contract equilibrium, and thus in any CPNE, it must be $w_{sr}(g) = c$ for all $s \in S$ and $r \in R$ with $\ell_{sr} = 1$. This can be demonstrated as follows. Consider a deviation by a coalition that involves only s and r. Given the assumptions of the model and the equilibrium concept adopted, the only wholesale price that can be changed by this bilateral deviation is w_{sr}. This is the case because, given the wholesale price proposals $w^s_{r'}$ by any other retailer $r' \neq r$, s cannot unilaterally change the wholesale price $w_{sr'}$ in his contract with r'. Moreover, since r' is not part of the original deviating coalition $\{s, r\}$, and CPNE restricts further deviations to subcoalitions, s cannot orchestrate further deviations to persuade r' to change her proposal $w^s_{r'}$. The fact that in this deviation s cannot change wholesale prices other than w_{sr}, and the fact that this is common knowledge, makes it unnecessary to specify beliefs for r in this deviation. Finally, since the change in w_{sr} is unobserved to other retailers, it does not affect the quantities (with Cournot competition) or prices (with Bertrand competition) chosen by those retailers. A small change in w_{sr} has, therefore, the following effect on the joint profits $\pi_{sr} = \pi_s + \pi_r$ of s and r,

$$\frac{d\pi_{sr}}{dw_{sr}} = \sum_{i \in S_r} \frac{\partial \pi_r}{\partial x_{ir}} \frac{dx_{ir}}{dw_{sr}} + \cancel{q_{sr}} + \sum_{j \in R_s} \left(w_{sj} - c\right) \frac{dq_{jr}}{dw_{sr}} - \cancel{q_{sr}}, \tag{A-1}$$

where x_{ir} represents the choice variable of retailer r and is equal to q_{ir} for Cournot competition and p_{ir} for Bertrand competition, and S_r is the set of suppliers with which retailer r has a link (i.e., $s \in S_r$ if $\ell_{sr} = 1$) and R_s is the set of retailers with which supplier s has a link (i.e., $r \in R_s$ if $\ell_{sr} = 1$). In a contract equilibrium, and thus in a CPNE, the wholesale price w_{sr} must maximize the joint profits of any sr pair with $\ell_{sr} = 1$, otherwise there would exist a profitable and self-enforcing bilateral deviation for at least one of these pairs. It must therefore be $d\pi_{sr}/dw_{sr} = 0$ for all s and r. Moreover, profit maximization by retailer r implies that $\partial\pi_r/\partial x_{ir} = 0$ for all $i \in S_r$, therefore the first term in (A-1) is always equal to zero by the envelope theorem. This implies that the following $R_s \times R_s$ system of equations must hold in a CPNE for all $s \in S_r$

$$\frac{d\pi_{sr}}{dw_{sr}} = \sum_{j \in R_s} \left(w_{sj} - c\right) \frac{dq_{sj}}{dw_{sr}} = 0, \quad \text{for all } r \in R_s. \tag{A-2}$$

If downstream competition is Cournot, $dq_{sj}/dw_{sr} = 0$ for all $j \neq r$ and, for all $s \in S_r$, (A-2) reduces to

$$\left(w_{sr} - c\right) \frac{dq_{sr}}{dw_{sr}} = 0, \quad \text{for all } r \in R_s. \tag{A-3}$$

Since $dq_{sr}/dw_{sr} < 0$ this implies $w_{sr} = c$ for all $s \in S_r$ and $r \in R_s$.

If downstream competition is instead Bertrand, $dq_{sj}/dw_{sr} \geq 0$ for $j \neq r$. In particular, although $dp_{ij}/dw_{sr} = 0$ for $j \neq r$, one still has $dp_{ir}/dw_{sr} \neq 0$ for all $i \in S_r$, and changes in the prices p_{ir} affect all quantities, including q_{sj} for $j \neq r$. The relevant conditions remain therefore those given in (A-2). These conditions can be written in matrix form as $\left(\mathbf{w}_s - c\right)\mathbf{M} = \mathbf{0}$, where $\mathbf{w}_s$ is a $1 \times R_s$ vector, and $\mathbf{M}$ is an $R_s \times R_s$ matrix with the expression in (A-2) constituting the typical element for column j and row r. Given Assumption 1, $\mathbf{M}$ has a dominant diagonal, i.e., $\left|dq_{sr}/dw_{sr}\right| > \sum_{j \neq r} \left|dq_{sj}/dw_{sr}\right|$ for all $r \in R_s$. By the Levy-Desplaques theorem $\mathbf{M}$ is therefore invertible and the unique solution to (A-2) is $w_{sr} = c$ for all $r \in R_s$.

The results above can be used to establish that $w\left(g\right) = c$ is not only a necessary, but also a sufficient, condition for $w\left(g\right)$ to constitute a CPNE profile of wholesale prices in a network g. Even if there existed a mutually profitable multilateral deviation that set the wholesale prices of two or more retailers at a level different from marginal cost (as might, for example, be profitable in the case of Bertrand competition and highly substitutable retailers), such a deviation would not be self-enforcing, because there would always exist further profitable and self-enforcing bilateral deviations to marginal cost pricing for at least one supplier-retailer pair. ■

Proof of Lemma 1 – I first prove necessity and then sufficiency. Consider a network g and a mutually profitable deviation by a coalition $Z \subseteq N$ that implements a network $h \neq g$. Using the expressions for the profits of suppliers and retailers provided in (5) and (6), the fact that the deviation is mutually profitable implies that, for all $s \in Z$,

$$\sum_{r\in R} \ell^h_{sr} t^h_{sr} > \sum_{r\in R} \ell^g_{sr} t^g_{sr}, \tag{A-4}$$

and, for all $r \in Z$,

$$\Pi^h_r - \sum_{s\in S} \ell^h_{sr} t^h_{sr} > \Pi^g_r - \sum_{s\in S} \ell^g_{sr} t^g_{sr}. \tag{A-5}$$

Note that, for $k \in \{h, g\}$, one can write $\sum_{r\in R} \ell^k_{sr} t^k_{sr} = \sum_{r\in Z} \ell^k_{sr} t^k_{sr} + \sum_{r\notin Z} \ell^k_{sr} t^k_{sr}$ and $\sum_{s\in S} \ell^k_{sr} t^k_{sr} = \sum_{s\in Z} \ell^k_{sr} t^k_{sr} + \sum_{s\notin Z} \ell^k_{sr} t^k_{sr}$. Using these identities and adding (A-4) over $s \in Z$ and (A-5) over $r \in Z$, one obtains

$$\sum_{s\in Z} \left[\sum_{r\in Z} \ell^h_{sr} t^h_{sr} + \sum_{r\notin Z} \ell^h_{sr} t^h_{sr} \right] > \sum_{s\in Z} \left[\sum_{r\in Z} \ell^g_{sr} t^g_{sr} + \sum_{r\notin Z} \ell^g_{sr} t^g_{sr} \right], \tag{A-6}$$

$$\sum_{r\in Z} \Pi^h_r - \sum_{r\in Z} \left[\sum_{s\in Z} \ell^h_{sr} t^h_{sr} + \sum_{s\notin Z} \ell^h_{sr} t^h_{sr} \right] > \sum_{r\in Z} \Pi^g_r - \sum_{r\in Z} \left[\sum_{s\in Z} \ell^g_{sr} t^g_{sr} + \sum_{s\notin Z} \ell^g_{sr} t^g_{sr} \right]. \tag{A-7}$$

Adding up (A-6) and (A-7) and rearranging terms, one obtains

$$\sum_{r\in Z} \left[\Pi^h_r - \Pi^g_r \right] > \sum_{s\in Z} \sum_{r\notin Z} \left(\ell^g_{sr} t^g_{sr} - \ell^h_{sr} t^h_{sr} \right) - \sum_{r\in Z} \sum_{s\notin Z} \left(\ell^g_{sr} t^g_{sr} - \ell^h_{sr} t^h_{sr} \right) \tag{A-8}$$

For all links sr in which one of the firms belongs to Z and the other does not, as is always the case in the right-hand side of (A-8), one has $\ell^h_{sr} t^h_{sr} = \ell^h_{sr} t^g_{sr}$. This is trivially true for $\ell^h_{sr} = 0$. For $\ell^h_{sr} = 1$, instead, the deviating firm must meet the transfer proposal t^g_{sr} of the firm that does not participate in the deviation, since the latter is not given a chance to modify such proposal and if that proposal were not met one would have $\ell^h_{sr} = 0$. This implies that $\ell^h_{sr} = 1$ if and only if $t^h_{sr} = t^g_{sr}$. The fact that $\ell^h_{sr} t^h_{sr} = \ell^h_{sr} t^g_{sr}$ in the right-hand side of (A-8) yields the result in (7).

Next, I show that (7) is a sufficient condition for there to exist a mutually profitable deviation by a coalition $Z \in \mathcal{Z}_{g\to h}$ to a network h with transfers t^h. For each $r \in Z$, choose a profile of links ℓ^h_{sr} and transfers t^h_{sr} with $s \notin Z$ such that (A-5) just holds, so that all retailers in the deviating coalition are just better off in network h than in network g. That is, for each $r \in Z$, choose $\sum_{s\notin Z} \ell^h_{sr} t^h_{sr}$ and an arbitrarily small $\varepsilon_r > 0$ such that

$$\sum_{s\notin Z} \left(\ell^g_{sr} t^g_{sr} - \ell^h_{sr} t^h_{sr} \right) = \left(\Pi^g_r - \Pi^h_r \right) - \sum_{s\in Z} \left(\ell^g_{sr} t^g_{sr} - \ell^h_{sr} t^h_{sr} \right) - \varepsilon_r. \tag{A-9}$$

If (7) holds then, for the reasoning above, so does (A-8). Substituting (A-9) into (A-8) and simplifying one obtains

$$\sum_{s\in Z} \left[\sum_{r\in R} \left(\ell^h_{sr} t^h_{sr} - \ell^g_{sr} t^g_{sr} \right) \right] > \sum_{r\in Z} \varepsilon_r. \tag{A-10}$$

The terms in square brackets in the left-hand side of (A-10) are independent from one another, since each refers to a different supplier $s \in Z$, and can be chosen freely, since the restrictions imposed by (A-9) constrain ℓ_{sr}^{h} and t_{sr}^{h} for $s \notin Z$ but not for $s \in Z$. Therefore, for $\sum_{r \in Z} \varepsilon_r \to 0^+$, (A-10) implies that one can always find ℓ_{sr}^{h} and t_{sr}^{h} for each $s \in Z$ such that (A-4) holds for all $s \in Z$. This proves that, if (7) holds, one can always find a network h with profile of transfers t^h such that all suppliers and retailers in some coalition $Z \in \mathcal{Z}_{g \to h}$ are better off. ■

Proof of Proposition 2 (No exclusive contracts, with transfers) – The proof proceeds by first characterizing the conditions under which there exist coalition-proof Nash equilibria ("equilibria", for short) with $g_{t,ne}^{*} = pe$ and $g_{t,ne}^{*} = dca$, and then proving that there never exist equilibria with $g_{t,ne}^{*} \in \{bm, dm, um, mix\}$.

Existence of equilibria with $g_{t,ne}^{*} = pe$

Consider a candidate equilibrium with $g = pe$ (see Figure 1(c)), in which no firm is willing to engage in "cross trade," i.e., in which s and r' refuse to trade with each other and the same applies to s' and r. Equilibria with $g = pe$ can also be supported by having only one of s and r' (or one of s' and r) refuse to trade with the other. However, such candidate equilibria would be easier to break than candidate equilibria supported by strategies in which both firms refuse to trade with the other. The restriction to the latter is, therefore, without loss of generality. Consider a symmetric profile of transfers $t_{sr} = t_{s'r'} = t^{pe}$ for the pairs of firms that agree to trade. This restriction is also without loss of generality because candidate equilibria with asymmetric transfers are easier to break than candidate equilibria with symmetric transfers, since, when transfers are asymmetric, the supplier and/or retailer with the lowest payoff would be more amenable to a deviation.

For there to be no profitable unilateral deviations in which a supplier or a retailer rescinds his or her link it must be

$$0 \leq t^{pe} \leq \Pi^{pe}. \tag{A-11}$$

No firm can unilaterally create a new link, since this requires consent from a supplier-retailer pair. Consider then joint deviations by supplier s and retailer r'. This pair can implement the following deviations:

A deviation to $g = dm$, with s agreeing to trade with r' and stop trading with r, while r' continues to trade with s'. For $t^{pe} > 0$, this deviation is not self-enforcing in the absence of exclusive contracts, since s would always have an incentive to re-form his link with r. For $t^{pe} = 0$, this deviation is instead self enforcing, since s would have no incentive to reform his link with

r, and mutually profitable, since $\Pi^{dm} \geq \Pi^{pe}$. For an equilibrium with $g = pe$ to exist it must therefore be $t^{pe} > 0$.

A deviation to $g = mix$, with r' agreeing to trade with both s and s', while s continues to trade with r. This deviation is self-enforcing (i.e., supportable as a Nash equilibrium of the two-player game between s and r') if and only if $t^{pe} \leq \Pi^{mix2} - \Pi^{um}$ and strictly Pareto dominates $g = pe$ if and only if $\Pi^{mix2} > \Pi^{pe}$. Therefore, there does not exist a mutually profitable and self-enforcing deviation to $g = mix$ if and only if at least one of the following holds

$$\Pi^{pe} \geq \Pi^{mix2} \tag{A-12}$$

$$t^{pe} > \Pi^{mix2} - \Pi^{um} \tag{A-13}$$

A deviation to $g = um$, with r' agreeing to start trading with s and stop trading with s', while s continues to trade with r. This deviation is self-enforcing if and only if $t^{pe} \geq \Pi^{mix2} - \Pi^{um}$ and Pareto dominates $g = pe$ if and only if $t^{pe} > \Pi^{pe} - \Pi^{um}$. This implies that there exists no mutually profitable and self-enforcing deviation to $g = um$ if and only if

$$t^{pe} \leq \max\left\{\Pi^{mix2}, \Pi^{pe}\right\} - \Pi^{um}, \tag{A-14}$$

with strict inequality if $\Pi^{mix2} > \Pi^{pe}$.

One can summarize the analysis above as follows. If (A-12) does not hold, there never exists an equilibrium with $g = pe$, since both (A-13) and (A-14) would need to hold, and there does not exist any t^{pe} for which this can be the case. If instead (A-12) holds, the only other condition required for there to exist no self-enforcing deviation by $\{s, r'\}$ or $\{s', r\}$ is that (A-14) be satisfied by some $t^{pe} > 0$, which is always the case since $\Pi^{pe} > \Pi^{um}$. Therefore, for $\Pi^{pe} \geq \Pi^{mix2}$, there always exist profiles of transfers $t^{pe} \in (0, \Pi^{pe} - \Pi^{um}]$ that make $g = pe$ immune to deviations by $\{s, r'\}$ or $\{s', r\}$.

Given that, in the candidate equilibrium, s and r', as well as s' and r, refuse to trade with each other, coalitions of three firms can do no better than $\{s, r'\}$ or $\{s', r\}$. The only coalition that can achieve deviations not available to $\{s, r'\}$ or $\{s', r\}$ is the grand coalition $\{s, s', r, r'\}$, which can implement $g = dca$. Such a deviation is, however, never profitable for all firms in $\{s, s', r, r'\}$ when $\Pi^{pe} \geq \Pi^{mix2}$, since, with linear demand, this condition implies $\Pi^{pe} > \Pi^{dca}$.

Taken together, all of the above implies that there exist equilibria with $g = pe$ and $t^{pe} \in (0, \Pi^{pe} - \Pi^{um}]$ if and only if $\Pi^{pe} \geq \Pi^{mix2}$.

Existence of equilibria with $g^*_{t,ne} = dca$

Consider a candidate equilibrium with $g = dca$ as in Figure 1(a) and $t^{dca}_{sr} = t^{dca}$ for all s and

r. As explained above, the restriction to symmetric transfers is without loss of generality. For a supplier or retailer to have no incentives to rescind unilaterally one of their supply links, thus implementing a market structure with $g = mix$ as in Figure 1(b), it must be

$$0 \leq t^{dca} \leq \Pi^{dca} - \Pi^{mix1}. \tag{A-15}$$

Moreover, when (A-15) holds, in the candidate equilibrium a retailer earns $\Pi^{dca} - 2t^{dca} \geq 2\Pi^{mix1} - \Pi^{dca} > 0$ (where the last inequality follows from the products being substitutes) and has thus no incentive to withdraw from the market completely by rescinding both of its links (a supplier has obviously no incentives to do so for $t^{dca} \geq 0$.)

Consider a deviation to $g = pe$ by the grand coalition $\{s, s', r, r'\}$ in which all firms refuse to engage in cross trade (e.g., s and r' refuse to deal with each other and s' and r refuse to deal with each other). Two aspects of this deviation are worth mentioning. First, as discussed in relation to Remark 1 in the main text, this deviation could be viewed as a (nonbinding) market allocation agreement. The consequences of prohibiting such agreements (and thus the deviation considered here) are discussed in Remark 1, which is proved further below in this appendix. For the time being, however, I allow for this type of nonbinding agreements. Second, a deviation to $g = pe$ can also be implemented by smaller coalitions (e.g., $\{r, r'\}$ agreeing not to "cross trade" with suppliers) but such deviations would be less likely to be self enforcing than a deviation by $\{s, s', r, r'\}$, as in such deviations not all firms would withdraw their cross offers (e.g., in a deviation by $\{r, r'\}$ only the retailers would withdraw their cross offers, whereas the suppliers would continue to offer contracts to all retailers). Such deviations by smaller coalitions would thus not add anything to the analysis presented here. Having clarified these aspects, denote by $\widetilde{t}^{pe}$ the transfers in this deviation. The deviation can be mutually profitable if there exists a $\widetilde{t}^{pe}$ such that

$$2t^{dca} < \widetilde{t}^{pe} < \Pi^{pe} - \Pi^{dca} + 2t^{dca} \tag{A-16}$$

The deviation is never self enforcing for $\Pi^{mix2} > \Pi^{pe}$ because, as shown above, when this condition holds a network with $g = pe$ is never self-enforcing even with unrestricted transfers t^{pe}. Therefore, it is a fortiori never self-enforcing with transfers $\widetilde{t}^{pe}$ that are restricted as in (A-16). The deviation is, instead, always self enforcing for $\Pi^{pe} \geq \Pi^{mix2}$, as there always exists a $\widetilde{t}^{pe} > 0$ that satisfies both (A-14) and (A-16). The latter requires $\Pi^{pe} > \Pi^{dca}$ (which is always the case when $\Pi^{pe} \geq \Pi^{mix2}$, as $\Pi^{mix2} > \Pi^{dca}$) and $2t^{dca} < \Pi^{pe} - \Pi^{um}$. For there to be no $\widetilde{t}^{pe} > 0$ that satisfies (A-14) and (A-16) one would therefore need $2t^{dca} \geq \Pi^{pe} - \Pi^{um}$. However, with linear demand and when $\Pi^{pe} \geq \Pi^{mix2}$, there never exists a t^{dca} satisfying both this condition and condition (A-15) for no unilateral deviations.

Consider next a deviation to $g = dm$ by a coalition $\{s, s', r\}$. For $t^{dca} > 0$, this deviation is not self-enforcing in the absence of exclusive contracts, since either supplier would always have incentives to re-form his link with r'. For $t^{dca} = 0$, this deviation is instead self enforcing, since no supplier would have incentives to reform his link with r', and mutually profitable, since $\Pi^{dm} \geq \Pi^{dca}$. For an equilibrium with $g = dca$ to exist it must therefore be $t^{dca} > 0$.

Finally, consider a deviation to $g = um$ by a coalition $\{s, r, r'\}$. This deviation is self enforcing if and only if $t^{dca} > \Pi^{mix2} - \Pi^{um}$ and always mutually profitable whenever it is self enforcing. A necessary condition for there to exist an equilibrium with $g = dca$ is therefore $t^{dca} \leq \Pi^{mix2} - \Pi^{um}$. Since with linear demand $\Pi^{mix2} \geq \Pi^{dca}$ and $\Pi^{um} = \Pi^{mix1}$, this condition is never binding when (A-15) holds.

Taken together, all of the above implies that there exist equilibria with $g = dca$ and $t^{dca} \in \left(0, \Pi^{dca} - \Pi^{mix1}\right]$ if and only if $\Pi^{mix2} > \Pi^{pe}$.

There do not exist equilibria with $g^*_{t,ne} \in \{bm, dm, um, mix\}$

There cannot exist an equilibrium with $g = bm$ because the excluded supplier and retailer would always have self-enforcing incentives to form a link and earn positive profits.

There cannot exist an equilibrium with $g = dm$ because the excluded retailer, r', can always profitably deviate by entering into a contract with at least one supplier. Without exclusive contracts, that supplier would be able to keep his contract with the other retailer, r, and would thus be willing to accept any additional positive transfer from r'. Analogously, there cannot exist an equilibrium with $g = um$ either. The excluded supplier, s', can always profitably deviate by offering a contract with a positive and arbitrarily small transfer request to at least one of the retailers. Since that retailer can keep his link with s and since $\Pi^{mix2} > \Pi^{um}$, he accepts the proposal by s'.

Finally, consider a candidate mixed equilibrium with $g = mix$ where all links are active except the link between s' and r'. There always exists a self-enforcing deviation in which s' and r' form that link. If in the candidate equilibrium $t^{mix}_{sr'} \leq \Pi^{dca} - \Pi^{mix1}$, the self-enforcing deviation involves r' maintaining her link with s and implementing $g = dca$ (this deviation is jointly profitable for s' and r' since $\Pi^{dca} > \Pi^{mix1}$). If instead in the candidate equilibrium $t^{mix}_{sr'} > \Pi^{dca} - \Pi^{mix1}$, the self-enforcing deviation involves r' rescinding her link with s and implementing $g = mix$ (this deviation, which effectively involves switching s' for s, is jointly profitable for s' and r' since $t^{mix}_{sr'} > \Pi^{dca} - \Pi^{mix1} > 0$). As a result, there never exists an equilibrium with a mixed network. ■

Proof of Remark 1 (No exclusive contracts, no market allocation agreements) – Regarding the existence of equilibria with $g = pe$, as the conditions obtained in the proof of Proposition 2 do not rely on market allocation agreements, prohibiting such agreements has no effects.

Regarding the existence of equilibria with $g = dca$, prohibiting market allocation agreements rules out the deviation to $g = pe$ by the grand coalition in the proof of Proposition 2. The only feasible deviation to $g = pe$ remains one by a supplier s and a retailer r, but such a deviation is not self enforcing as, without exclusive contracts, the supplier has always incentives to reform his link with the other retailer, r'. Since, as established in the proof of Proposition 2, there always exists a t^{dca} that prevents self-enforcing deviations to networks other than $g = pe$, there always exist equilibria with $g = dca$. ■

Proof of Proposition 3 (Exclusive contracts and transfers) – The proof proceeds by first characterizing the conditions under which there exist equilibria with $g^*_{t,e} \in \{dca, pe, dm\}$ and then proving that there never exist equilibria with $g^*_{t,e} \in \{bm, um, mix\}$.

Existence of equilibria with $g^*_{t,e} = dca$

As in the proof of Proposition 2, one can focus without loss of generality on symmetric equilibria with $t^{dca}_{sr} = t^{dca}$ for all s and r. Suppliers and retailers do not find it profitable to rescind unilaterally one of their links, thus implementing $g = mix$, if and only if (A-15) holds. Deviations to $g = mix$ can be also carried out by coalitions including more than one firm, but such deviations would be less likely to be jointly profitable or self-enforcing than a unilateral deviation and thus do not play any role when (A-15) holds.

Consider a deviation by a coalition $\{s, r\}$ in which these two firms commit to mutual exclusivity, thus implementing $g = pe$. This deviation is mutually profitable if and only if there exists a $\widetilde{t}^{pe}$ such that

$$2t^{dca} < \widetilde{t}^{pe} < \Pi^{pe} - \Pi^{dca} + 2t^{dca} \tag{A-17}$$

When mutually profitable, this deviation is always self-enforcing. To see why, note that the adoption of mutual exclusivity by $\{s, r\}$ implies that if either s or r deviates further it must stop trading with the other. The only possible further deviations from $g = pe$ are, therefore, s trading only with r' (thus implementing $g = dm$, since s' continues to trade with r') or r trading only with s' (thus implementing $g = um$, since r' continues to trade with s'.) These deviations are not mutually profitable, and thus the original deviation is self-enforcing, if and only if

$$t^{dca} \leq \widetilde{t}^{pe} \leq \Pi^{pe} - \Pi^{um} + t^{dca} \tag{A-18}$$

Since in any equilibrium with $g = dca$ condition (A-15) must hold and since with linear demand $\Pi^{um} = \Pi^{mix1}$, it is straightforward to verify that (A-18) always holds when (A-17) holds, and thus the original deviation to $g = pe$ is always self-enforcing when mutually profitable. Using (A-17), this implies that there does not exist a self-enforcing deviation to $g = pe$ by $\{s, r\}$ if and only if

$$\Pi^{dca} \geq \Pi^{pe} \tag{A-19}$$

Deviations to $g = pe$ can also be implemented by broader coalitions, such as the grand coalition $\{s, s', r, r'\}$ in the proof of Proposition 2, but these deviations do not add anything to the conditions derived from the deviation by $\{s, r\}$ studied above.

Consider now a deviation to $g = um$ in which r and r' commit to be exclusive to s. This deviation is mutually profitable if and only if the joint profits that $\{s, r, r'\}$ can earn in the deviation, $2\Pi^{um}$, are greater than the joint profits that $\{s, r, r'\}$ earns in the candidate equilibrium, $2\Pi^{dca} - 2t^{dca}$, which is the case if and only if $t^{dca} > \Pi^{dca} - \Pi^{um}$. This condition never holds, and thus a deviation to $g = um$ is never profitable, when condition (A-15) for no unilateral deviations holds, since, with linear demand, $\Pi^{um} = \Pi^{mix1}$.

Finally, consider a deviation to $g = dm$ in which s and s' commit to be exclusive to r. This deviation is mutually profitable if and only if there exists a $\tilde{t}^{dm}$ such that

$$4t^{dca} < 2\tilde{t}^{dm} < \Pi^{dm} - \Pi^{dca} + 2t^{dca}. \tag{A-20}$$

Moreover, the deviation is self enforcing if and only if none of the following subsequent deviations is profitable and self enforcing: i) one of the suppliers rejects r's proposal and accepts the candidate equilibrium proposal t^{dca} of r', which implements $g = pe$; ii) a coalition of one supplier and one retailer, say $\{s, r\}$, agrees to mutual exclusivity, with r rejecting the deviation proposal of s', which, after s' enters a new contract with r', implements $g = pe$; iii) a coalition of one supplier and one retailer, say $\{s, r\}$, agrees that r rejects the deviation proposal of s', while s accepts the candidate equilibrium proposal t^{dca} of r', which, after s' enters a new contract with r' as a consequence of the "no stranding" assumption, implements $g = mix$; iv) a coalition of one supplier and one retailer, say $\{s, r\}$, agrees that r accepts the deviation proposal of s', while s accepts the candidate equilibrium proposal t^{dca} of r', which implements $g = mix$. Subsequent deviations i) and ii) are not profitable, and thus the original deviation to $g = dm$ is self enforcing, if and only if $2t^{dca} < 2\tilde{t}^{dm} < 2\left(\Pi^{dm} - \Pi^{pe}\right)$. In the relevant case where $t^{dca} \geq 0$ (see condition (A-15) above) and $\Pi^{dca} \geq \Pi^{pe}$ (see condition (A-19) above) this is always true when $\tilde{t}^{dm}$ satisfies (A-20), i.e., when the initial deviation to $g = dm$ is mutually profitable. Deviations iii) and iv) are instead never mutually profitable and self enforcing when the original deviation to $g = dm$ is mutually profitable and condition (A-15) for the existence of an equilibrium with $g^*_{t,e} = dca$ holds. These deviations do not, therefore, add anything to the analysis. The detailed proof of this fact is tedious and is contained in the online appendix. All of this implies that, for there to exist no mutually profitable and self-enforcing deviation to $g = dm$, (A-20) must fail for any $\widetilde{t}^{dm}$, which is the case if and only if

$$2t^{dca} \geq \Pi^{dm} - \Pi^{dca} \tag{A-21}$$

The analysis above implies that there exist equilibria with $g = dca$ if and only if (A-19) holds and there exists a t^{dca} such that (A-15) and (A-21) also hold. The latter is the case if and only if

$$2\left(\Pi^{dca} - \Pi^{mix1}\right) \geq \Pi^{dm} - \Pi^{dca}. \tag{A-22}$$

Intuitively, for there to exist an equilibrium with $g = dca$ there must exist intermediate transfers t^{dca} that ensure that i) either supplier does not extract too much, otherwise one or both retailers would drop him and ii) either retailer does not extract too much, otherwise the other retailer could convince both suppliers to drop her. As can be seen in Figure 4, this is possible only if suppliers and retailers are sufficiently differentiated.

Existence of equilibria with $g^*_{t,e} = pe$

Consider an equilibrium with $g = pe$ supported by mutual exclusivity between s and r and between s' and r'. Although equilibria with $g = pe$ can also be supported by one-way commitments to exclusivity, equilibria supported by mutual exclusivity are more difficult to break and thus more likely to exist. As in the proof of Proposition 2, focus without loss of generality on symmetric transfers $t_{sr} = t_{s'r'} = t^{pe}$ for the pairs of firms that agree to trade. Suppliers and retailers do not have incentives to deviate unilaterally by becoming inactive if and only if (A-11) holds.

Consider first a deviation to $g = um$ in which r and r' commit to be exclusive to s. This deviation is mutually profitable if and only if there exists a $\widetilde{t}^{um}$ such that $t^{pe} < 2\widetilde{t}^{um} < 2\left(\Pi^{um} - \Pi^{pe} + t^{pe}\right)$ and is (trivially) self enforcing whenever it is profitable (in the candidate equilibrium, supplier s' does not have a proposal out to retailer r and cannot be involved in further negotiations because it does not belong to the original deviating coalition that includes r). Therefore, for there to exist no mutually profitable and self-enforcing deviation to $g = um$ it must be

$$t^{pe} \leq 2\left(\Pi^{pe} - \Pi^{um}\right) \tag{A-23}$$

Consider next a deviation to $g = dm$ in which s and s' commit to be exclusive to r and r pays a transfer $\widetilde{t}^{dm}$ to each supplier. This deviation is mutually profitable if and only if there exists a $\widetilde{t}^{dm}$ such that $2t^{pe} < 2\widetilde{t}^{dm} < \Pi^{dm} - \Pi^{pe} + t^{pe}$ and, as above, is (trivially) self enforcing whenever it is profitable (in the candidate equilibrium, retailer r' does not have a proposal out to supplier s and cannot be involved in further negotiations because it does not belong to the original deviating coalition that includes s). Therefore, for there to exist no mutually profitable and self-enforcing deviation to $g = dm$ it must be

$$t^{pe} \geq \Pi^{dm} - \Pi^{pe}. \tag{A-24}$$

Finally, consider deviations to $g = mix$ or $g = dca$. Since the candidate equilibrium with $g = pe$ is supported by mutual exclusivity, these deviations can only be implemented by the grand coalition $\{s, s', r, r'\}$, because all the contract proposals in the candidate equilibrium would need to be modified. This deviation can therefore be mutually profitable only if, respectively, $\Pi^{mix1} + \Pi^{mix2} > 2\Pi^{pe}$ and $2\Pi^{dca} > 2\Pi^{pe}$. With linear demand, this is never the case in the region of parameters where (A-23) and (A-24) hold.

In conclusion, there exists an equilibrium with $g = pe$ if and only if there exists a t^{pe} such that (A-11), (A-24) and (A-23) hold, which is the case if and only if

$$2\Pi^{pe} \geq \Pi^{dm}, \tag{A-25}$$

$$2\left(\Pi^{pe} - \Pi^{um}\right) \geq \Pi^{dm} - \Pi^{pe}. \tag{A-26}$$

Condition (A-25) follows from the need for t^{pe} to satisfy (A-11) and (A-24), which can be the case only if pairwise exclusivity yields higher profits than downstream monopoly, whereas condition (A-26) follows from the need for t^{pe} to satisfy (A-23) and (A-24) (the requirement that $t^{pe} \geq 0$ is satisfied when (A-24) holds because $\Pi^{dm} - \Pi^{pe} > 0$).

Existence of equilibria with $g^*_{t,e} = dm$

Consider a candidate equilibrium with $g = dm$ in which s and s' commit to be exclusive to r and assume, without loss of generality, symmetric transfers $t^{dm}_{sr} = t^{dm}_{s'r} = t^{dm}$. For there to be no jointly profitable and self-enforcing deviation in which the excluded retailer, r', and one of the suppliers enter a mutually exclusive contract that implements $g = pe$ it must be $t^{dm} \geq \Pi^{pe}$. Moreover, for there to be no jointly profitable and self-enforcing deviation in which the monopolistic retailer, r, and one of the two suppliers, say s, enter a mutually exclusive contract it must be $t^{dm} \leq \Pi^{dm} - \Pi^{pe}$ (note that, by the "no stranding" assumption, the rejection of s''s exclusive proposal by r would allow s' to enter a new contract with r' and the market structure resulting from this deviation would thus be $g = pe$). There exists a t^{dm} that satisfies these two conditions if and only if (A-25) fails (or holds with equality).

In this equilibrium both retailers must earn zero profits, which implies that the active retailer must pay transfers $t^{dm} = \Pi^{dm}/2$. If $t^{dm} < \Pi^{dm}/2$, there would exist a profitable deviation in which the excluded retailer, r', steals the entire business of the active retailer, r, by offering both suppliers transfers $\tilde{t}^{dm} \in \left(t^{dm}, \Pi^{dm}/2\right]$ in exchange for exclusivity and earns profits $\Pi^{dm} - 2\tilde{t}^{dm} > 0$. Such a deviation would be self enforcing because the only feasible further deviation, i.e., r' and one of the two suppliers, say s', committing to mutual exclusivity, is not profitable. This is the case because, by the "no stranding" assumption, such a deviation would cause the two excluded firms, s and r, to enter a contract and would thus implement $g = pe$, and $\Pi^{pe} \leq \Pi^{dm} - \tilde{t}^{dm}$ for $\tilde{t}^{dm} \leq \Pi^{dm}/2$ when $\Pi^{dm} \geq 2\Pi^{pe}$.

Finally, when (A-25) fails (or holds with equality), one also has that $\Pi^{dm} > 2\Pi^{dca}$ and $\Pi^{dm} > \Pi^{mix1} + \Pi^{mix2}$ so that deviations in which the grand coalition deviates to $g = dca$ or a coalition of $\{s', r, r'\}$ deviates to $g = mix$ (with r' carrying product s' and s carrying both products) are not mutually profitable.

There do not exist equilibria with $g^*_{t,e} \in \{bm, um, mix\}$

There cannot exist an equilibrium with $g = bm$ because the excluded supplier and retailer would always have self-enforcing incentives to form a link and earn positive profits.

Consider a candidate equilibrium with $g = um$ in which r and r' commit to be exclusive to s and assume, without loss of generality, symmetric transfers $t^{um}_{sr} = t^{um}_{sr'} = t^{um}$. Unless $t^{um} \leq \Pi^{um} - \Pi^{pe}$ there always exists a self-enforcing deviation in which the excluded supplier, s', and one of the retailers, say r, enter a mutually exclusive contract that implements $g = pe$. Since $\Pi^{um} - \Pi^{pe} < 0$ and suppliers have incentives to rescind their candidate equilibrium supply link if $t^{um} < 0$, there does not exist any equilibrium with $g = um$.

Finally, consider a candidate equilibrium with $g = mix$ in which all links are active except the link between s' and r'. Specifically, s' commits to exclusivity with r, r' commits to exclusivity with s, and s and r trade with each other on a nonexclusive basis. In such a candidate equilibrium it must be

$$t^{mix}_{sr'} = 0. \tag{A-27}$$

To prove this, assume by contradiction that in equilibrium $t^{mix}_{sr'} > 0$. A coalition $\{s', r, r'\}$ can then engage in a mutually profitable deviation to a different mixed network with s' switched for s and transfers $\widetilde{t}^{mix}_{sr} = t^{mix}_{sr}$, $\widetilde{t}^{mix}_{s'r} = t^{mix}_{s'r}$ and $\widetilde{t}^{mix}_{s'r'} = t^{mix}_{sr'} - \varepsilon$, $\varepsilon > 0$. Given symmetry of demand, the network resulting from this deviation yields the same structure of payoffs (with s' switched for s) as the equilibrium network, up to an arbitrarily small difference ε. Since the equilibrium network is self-enforcing by definition, this deviation is also self-enforcing, which contradicts the assumption that $t^{mix}_{sr'} > 0$ constitutes an equilibrium . It must therefore be $t^{mix}_{sr'} \leq 0$. Moreover, for supplier s not to rescind his link with r' unilaterally it must be $t^{mix}_{sr'} \geq 0$. This establishes

(A-27). Intuitively, as the suppliers are symmetric, either supplier can provide the same profits by supplying his product to the retailer that carries only one product, and no supplier can thus extract any profits by doing so.

Consider now a deviation by a coalition $\{s, r\}$ in which s commits to be exclusive to r, thus implementing $g = dm$. This deviation is jointly profitable if and only if there exists a $\tilde{t}_{sr}^{dm}$ such that

$$t_{sr'}^{mix} + t_{sr}^{mix} < \tilde{t}_{sr}^{dm} < \Pi^{dm} - \Pi^{mix2} + t_{sr}^{mix}, \tag{A-28}$$

and self-enforcing if and only if neither s nor r find it profitable to deviate further by stopping dealing with the other, which is the case if and only if

$$t_{sr'}^{mix} \leq \tilde{t}_{sr}^{dm} \leq \Pi^{dm} - \Pi^{pe}. \tag{A-29}$$

Since in any candidate equilibrium with $g = mix$ it must be $t_{sr}^{mix} \geq 0$, (A-28) and (A-29) can be consolidated into the following condition for the existence of a profitable and self-enforcing deviation to $g = dm$

$$t_{sr'}^{mix} + t_{sr}^{mix} < \min\left\{\Pi^{dm} - \Pi^{mix2} + t_{sr}^{mix}, \Pi^{dm} - \Pi^{pe}\right\}, \tag{A-30}$$

If t_{sr}^{mix} is such that the right-hand side of (A-30) is $\Pi^{dm} - \Pi^{mix2} + t_{sr}^{mix}$, for there to exist no profitable and self-enforcing deviation to $g = dm$ it must be $t_{sr'}^{mix} \geq \Pi^{dm} - \Pi^{mix2}$, which can never be the case, since $\Pi^{dm} - \Pi^{mix2} > 0$ and (A-27) must hold. If instead t_{sr}^{mix} is such that the right-hand side of (A-30) is $\Pi^{dm} - \Pi^{pe}$, for there to exist no profitable and self-enforcing deviation to $g = dm$ it must be $t_{sr'}^{mix} \geq \Pi^{dm} - \Pi^{pe} - t_{sr}^{mix}$. Since in any equilibrium with $g = mix$ it must be $t_{sr}^{mix} \leq \Pi^{mix2} - \Pi^{pe}$ (otherwise r would drop s), the previous condition implies, again, $t_{sr'}^{mix} \geq \Pi^{dm} - \Pi^{mix2}$, which, as above, can never be the case. Therefore, when (A-27) holds there always exist profitable and self-enforcing deviations to $g = dm$ by $\{s, r\}$. Intuitively, since s is not compensated for providing his product to the single-product retailer, r', he can be easily induced to help the two-product retailer, r, monopolize the downstream market by not selling to r'. ■

Proof of Proposition 5 (Distribution of profits) – It is straightforward to see that the availability of exclusive contracts can never make suppliers unambiguously worse off, as the lower bound on suppliers' profits without exclusive contracts is always zero (see Proposition 2) and suppliers cannot be forced to earn negative profits. The fact that the availability of exclusive contracts can (and, for b sufficiently high, does) make suppliers unambiguously better off and retailers unambiguously worse off is, however, less immediate.

Consider first the case in which exclusive contracts result in an equilibrium supply network with $g^*_{t,e} = pe$. There are two cases: i) the equilibrium supply network is $g^*_{t,e} = g^*_{t,ne} = pe$ regardless of whether exclusive contracts are available or not, which is the case for $\overline{b}_{t,ne}(a) \leq b \leq \overline{\overline{b}}_m(a)$ (note that this case makes sense only with Bertrand downstream competition, as with Cournot downstream competition there is no b for which $g^*_{t,ne} = pe$ is an equilibrium without exclusive contracts); ii) exclusive contracts cause the equilibrium supply network to switch from $g^*_{t,ne} = dca$ to $g^*_{t,e} = pe$, which is the case for $\overline{\overline{b}}_{t,e}(a) \leq b < \overline{b}_{t,ne}(a)$.

Case i): As total industry profits remain the same, an unambiguous increase in suppliers' profits implies an unambiguous decrease in retailers' profits (and vice versa). It is therefore sufficient to analyze the profits of only one type of firm. In the absence of exclusive contracts, suppliers' profits have an upper bound $\overline{\pi}^{pe}_{s,ne} = \Pi^{pe} - \Pi^{um}$, whereas when exclusive contracts are available they have a lower bound $\underline{\pi}^{pe}_{s,e} = \Pi^{dm} - \Pi^{pe}$. With linear demand $\underline{\pi}^{pe}_{s,e} > \overline{\pi}^{pe}_{s,ne}$, and suppliers are therefore unambiguously better off (with retailers being unambiguously worse off) when exclusive contracts are available.

Case ii): As the equilibrium changes, so do total industry profits. One needs therefore to carry out separate profit comparisons for suppliers and retailers. Suppliers' profits have an upper bound $\overline{\pi}^{dca}_{s,ne} = 2\left(\Pi^{dca} - \Pi^{mix1}\right)$ without exclusive contracts and a lower bound $\underline{\pi}^{pe}_{s,e} = \Pi^{dm} - \Pi^{pe}$ with exclusive contracts. Retailers' profits have a lower bound $\underline{\pi}^{dca}_{r,ne} = 2\Pi^{mix1} - \Pi^{dca}$ without exclusive contracts and an upper bound $\overline{\pi}^{pe}_{r,e} = 2\Pi^{pe} - \Pi^{dm}$ with exclusive contracts. With linear demand, $\underline{\pi}^{pe}_{s,e} > \overline{\pi}^{dca}_{s,ne}$ in the region of parameters where $g^*_{t,e} = pe$ (see point 2 in Proposition 3) and $\overline{\pi}^{pe}_{r,e} < \underline{\pi}^{dca}_{r,ne}$ for all parameter values, which implies that the availability of exclusive contracts makes suppliers unambiguously better off and retailers unambiguously worse off.

Consider next the case in which exclusive contracts cause the equilibrium supply network to switch to $g^*_{t,e} = dm$, which is the case for $b \geq \overline{\overline{b}}_m(a)$. The analysis differs between i) Bertrand competition, for which the equilibrium network switches from $g^*_{t,ne} = pe$ to $g^*_{t,e} = dm$; ii) Cournot competition, for which the equilibrium network switches from $g^*_{t,ne} = dca$ to $g^*_{t,e} = dm$.

Case i) (Bertrand competition): Suppliers' profits have an upper bound $\overline{\pi}^{pe}_{s,ne} = \Pi^{pe} - \Pi^{um}$ without exclusive contracts and a lower bound $\underline{\pi}^{dm}_{s,e} = \Pi^{dm}/2$ with exclusive contracts. In the region where $g^*_{t,e} = dm$ can be supported as an equilibrium, $\Pi^{dm} \geq 2\Pi^{pe}$ and thus $\underline{\pi}^{dm}_{s,e} > \overline{\pi}^{pe}_{s,ne}$, which implies that exclusive contracts make suppliers unambiguously better off. Retailers' profits have a lower bound $\underline{\pi}^{pe}_{r,ne} = \Pi^{um}$ without exclusive contracts and are always equal to zero in an equilibrium with exclusive contracts and $g^*_{t,e} = dm$, which implies that exclusive contracts make retailers unambiguously worse off.

Case ii) (Cournot competition): Suppliers' profits have an upper bound $\overline{\pi}^{dca}_{s,ne} =$

$2\left(\Pi^{dca}-\Pi^{mix1}\right)$ without exclusive contracts and a lower bound $\underline{\pi}^{dm}_{s,e}=\Pi^{dm}/2$ with exclusive contracts. With linear demand, in the region where $g^*_{t,e}=dm$ can be supported as an equilibrium, $\underline{\pi}^{dm}_{s,e}>\overline{\pi}^{dca}_{s,ne}$, which implies that exclusive contracts make suppliers unambiguously better off. Retailers' profits have a lower bound $\underline{\pi}^{dca}_{r,ne}=2\Pi^{mix1}-\Pi^{dca}>0$ without exclusive contracts and are always equal to zero in an equilibrium with exclusive contracts and $g^*_{t,e}=dm$, which implies that exclusive contracts make retailers unambiguously worse off.

Finally, consider the case in which the equilibrium supply network is $g^*_{t,e}=g^*_{t,ne}=dca$ regardless of whether exclusive contracts are available or not, which is the case for $b<\overline{b}_{t,e}(a)$. This is the case when condition (12) in Proposition 3 holds. The lower bound on the suppliers' profits is strictly greater when exclusive contracts are available than when they are not, i.e., $\underline{\pi}^{dca}_{s,e}=\Pi^{dm}-\Pi^{dca}>0=\underline{\pi}^{dca}_{s,ne}$, whereas the upper bound is the same regardless of whether exclusive contracts are available or not, i.e., $\overline{\pi}^{dca}_{s,ne}=\overline{\pi}^{dca}_{s,e}=2\left(\Pi^{dca}-\Pi^{mix1}\right)$. Therefore, when the equilibrium remains $g^*_{t,e}=g^*_{t,ne}=dca$, the availability of exclusive contracts never makes suppliers worse off and can make them strictly better off. As industry profits are unaffected by the availability of exclusive contracts, the opposite conclusion applies to retailers' profits, which are no higher, and possibly strictly lower, when exclusive contracts are available. ■

Proof of Proposition 6 (No exclusive contracts, no transfers) – The proof proceeds by first characterizing the conditions under which there exist equilibria with $g^*_{nt,ne}=dca$ and $g^*_{nt,ne}=pe$; and then proving that there never exist equilibria with $g^*_{nt,ne}\in\{bm,dm,um,mix\}$. The expressions for the supplier and retailer profits in network g, π^g_s and π^g_r, are given by (14) in the main text.

Existence of equilibria with $g^*_{nt,ne}=dca$

Since in the candidate equilibrium all firms are willing to trade with all other firms, all deviations involve rescinding links and can be implemented unilaterally by a single supplier or retailer, or by coalitions of two firms, namely $\{s,s'\}$, $\{r,r'\}$ or $\{s,r\}$.

Unilateral deviations are not profitable if and only if $\pi^{dca}_r>\pi^{mix1}_r$ and $\pi^{dca}_s>\pi^{mix1}_s$. The first condition corresponds to

$$\left(2-\frac{1}{\beta}\right)\left(\Pi^{dca}-\Pi^{mix1}\right)<\Pi^{mix1} \tag{A-31}$$

and always holds for all $\beta\in[0,1]$, since $\Pi^{mix1}-\Pi^{dca}>\Pi^{mix1}>0$. The second condition corresponds to $2\left(\Pi^{dca}-\Pi^{mix1}\right)>\Pi^{mix2}-\Pi^{um}$ and, since with linear demand $\Pi^{mix1}=\Pi^{um}$, can be written as

$$2\Pi^{dca}>\Pi^{mix1}+\Pi^{mix2}. \tag{A-32}$$

Consider then a deviation by the coalition $\{s, s'\}$. This coalition can carry out deviations that implement $g = dm$, $g = mix$ or $g = pe$. A deviation that implements $g = dm$ is never self enforcing, since $\pi_s^{pe} > \pi_s^{dm}$ and thus, when supplier s' trades only with r, supplier s prefers trading only with r' to also trading only with r. A deviation that implements $g = mix$ is equivalent to a unilateral deviation in which one of the suppliers rescinds one of his links and is thus not self enforcing when (A-32) holds. A deviation that implements $g = pe$ is self enforcing if it is a best response for s to deal only with r when s' deals only with r', i.e., if $\pi_s^{pe} > \max\left\{\pi_s^{mix2}, \pi_s^{dm}\right\}$. When (A-32) holds, $\pi_s^{mix2} > \pi_s^{dm}$ and this is therefore the case if and only if $2\Pi^{pe} > \Pi^{mix1} + \Pi^{mix2}$. When this deviation is self-enforcing it is also mutually profitable for $\{s, s'\}$, since $\Pi^{pe} > 2\left(\Pi^{dca} - \Pi^{mix1}\right)$. Therefore, there exists no mutually profitable and self-enforcing deviation by $\{s, s'\}$ if and only if

$$\Pi^{mix1} + \Pi^{mix2} \geq 2\Pi^{pe}. \tag{A-33}$$

In what follows I restrict attention to the region of parameters where (A-33) holds and show that, in this region, there exist no self-enforcing deviations by the coalitions $\{r, r'\}$ and $\{s, r\}$. The coalition $\{r, r'\}$ can carry out deviations that implement $g = um$, $g = mix$ and $g = pe$. A deviation that implements $g = um$ is never self-enforcing because $\Pi^{pe} > \Pi^{um}$ implies $\pi_r^{pe} > \pi_r^{um}$; and a deviation to $g = mix$ is never self-enforcing when unilateral deviations are not profitable, i.e. when (A-32) holds. A deviation that implements $g = pe$ is never self-enforcing when (A-33) holds, since (A-33) implies $\pi_s^{mix2} > \pi_s^{pe}$.

A coalition $\{s, r\}$ can carry out a deviation that implements $g = pe$. As above, such a deviation is never self-enforcing when (A-33) holds, since (A-33) implies $\pi_s^{mix2} > \pi_s^{pe}$. The same coalition can also carry out deviations that implement $g = mix$ but, as argued above, these deviations are never self enforcing when unilateral deviations are unprofitable.

In summary, since (A-32) is not binding when (A-33) holds, (A-33) is the only necessary and sufficient condition for the existence of an equilibrium with $g^*_{nt,ne} = dca$.

Existence of equilibria with $g^*_{nt,ne} = pe$

Consider a candidate equilibrium with $g = pe$ in which s is willing to trade only with r (and vice versa) and s' is willing to trade only with r' (and vice versa). As in the proof of Proposition 2, the only relevant deviation is one by the two-firm coalition formed by s and r' (or s' and r). This coalition can implement deviations to $g = dm$, $g = um$ and $g = mix$. A deviation to $g = dm$ in which r' is the only active retailer is not self-enforcing because, when r' is willing to deal with both suppliers, s prefers exclusivity with r to exclusivity with r', since $\Pi^{pe} > \Pi^{dm} - \Pi^{bm}$ implies $\pi_s^{pe} > \pi_s^{dm}$. A deviation to $g = um$ in which s is the only active supplier is not self-enforcing because, when s is willing to deal with both suppliers, r' prefers exclusivity with s' to exclusivity with s, since $\Pi^{pe} > \Pi^{um}$ implies $\pi_{r'}^{pe} > \pi_{r'}^{um}$. Finally, a deviation to $g = mix$ is self enforcing if and only if r' responds to the willingness of s to deal with both retailers by being willing to deal with both suppliers, and vice versa, i.e., if and only if $\pi_s^{mix2} \geq \max\left\{\pi_s^{pe}, \pi_s^{dm}\right\}$ and $\pi_{r'}^{mix2} \geq \max\left\{\pi_{r'}^{pe}, \pi_{r'}^{um}\right\}$. The first conditions corresponds to $\pi_s^{mix2} \geq \pi_s^{pe}$, since $\pi_s^{pe} \geq \pi_s^{dm}$, and is satisfied if and only if (A-33) holds. The second condition corresponds to $\pi_{r'}^{mix2} \geq \pi_{r'}^{pe}$, since $\pi_{r'}^{pe} \geq \pi_{r'}^{um}$, and, rearranging terms, yields

$$\left(2 - \frac{1}{\beta}\right)\left(\Pi^{mix2} - \Pi^{pe}\right) \leq \Pi^{um} \tag{A-34}$$

Condition (A-34) always holds when (A-33) does. To see this note that, when (A-33) holds, $\Pi^{mix2} - \Pi^{pe} > 0$, given that $\Pi^{mix1} < \Pi^{pe}$. Since $(2 - 1/\beta)$ is increasing in β, this implies that (A-34) always holds if it holds for $\beta = 1$. The claim then follows from the fact that with linear demand $\left(\Pi^{mix2} - \Pi^{pe}\right) \leq \Pi^{um}$ with both Bertrand and Cournot competition. Condition (A-33) is, therefore, the only relevant necessary and sufficient condition for there to exist a self-enforcing deviation to $g = mix$ by the coalition $\{s, r'\}$. Since this deviation is mutually profitable when self-enforcing, there exists an equilibrium with $g^*_{nt,ne} = pe$ if and only if (A-33) fails.

There do not exist equilibria with $g^*_{nt,ne} \in \{bm, dm, um, mix\}$

There cannot exist an equilibrium with $g = bm$ because the excluded supplier and retailer would always have self-enforcing incentives to form a link and earn positive profits. There cannot exist an equilibrium with $g = dm$ in which, say, r is the only active retailer because $\min\left\{\pi_s^{pe}, \pi_s^{mix2}\right\} > \pi_s^{dm}$ and thus one of the suppliers can profit by opening a link with r', who would also profit from the deviation. There cannot exist an equilibrium with $g = um$ in which, say, s is the only active supplier because $\min\left\{\pi_r^{pe}, \pi_r^{mix2}\right\} > \pi_r^{um}$ and thus one of the retailers can profit by opening a link with s', who would also profit from the deviation.

Finally, consider a candidate equilibrium with $g = mix$ in which the link between, say, s and r is inactive. If (A-32) holds there exists a mutually profitable and self-enforcing deviation in which s and r form a link with each other, thus implementing $g = dca$. If (A-33) holds there exists a mutually profitable and self-enforcing deviation in which s' and r' rescind their link, thus implementing $g = pe$. Since one of (A-32) and (A-33) always holds, there does not exist any equilibrium with $g = mix$. ■

Proof of Proposition 7 (Exclusive contracts, no transfers) – The proof proceeds by first characterizing the conditions under which there exist equilibria with $g^*_{nt,e} = dca$, $g^*_{nt,e} = pe$ and $g^*_{nt,e} = mix$; and then proving that there never exist equilibria with $g^*_{nt,e} \in \{bm, dm, um\}$.

Existence of equilibria with $g^*_{nt,e} = dca$

First, note that all the deviations that were profitable and self-enforcing without exclusive contracts remain so with exclusive contracts. Therefore, in light of Proposition 6, an equilibrium with $g = dca$ can exist only for $b \leq \overline{b}_{nt,ne}(a)$. Exclusive contracts may also render additional deviations self-enforcing.

A deviation by a coalition $\{s, r\}$ in which these two firms commit to mutual exclusivity, thus implementing $g = pe$, is profitable for s if and only if

$$\Pi^{pe} > 2\left(\Pi^{dca} - \Pi^{mix1}\right) \tag{A-35}$$

and for r if and only if

$$(1-\beta)\,\Pi^{pe} > \Pi^{dca} - 2\beta\left(\Pi^{dca} - \Pi^{mix1}\right) \tag{A-36}$$

This deviation is never profitable when $b \leq \overline{b}_{nt,ne}(a)$. To see this, rewrite (A-36) as

$$(1-\beta)\left[\Pi^{pe} - 2\left(\Pi^{dca} - \Pi^{mix1}\right)\right] > \Pi^{dca} - 2\left(\Pi^{dca} - \Pi^{mix1}\right) \tag{A-37}$$

When (A-35) holds the term in square brackets in the left-hand side of (A-37) is positive. The condition is therefore most likely to hold for $\beta = 0$, where it becomes $\Pi^{pe} > \Pi^{dca}$, which is therefore a necessary condition for the deviation to be profitable for r. Since $\Pi^{pe} > \Pi^{dca}$ never holds for $b \leq \overline{b}_{nt,ne}(a)$, this deviation to mutual exclusivity by $\{s, r\}$ does not further restrict the region of parameters for the existence of an equilibrium with $g = dca$ relative to the case without exclusive contracts.

Finally, deviations to $g = dm$ and $g = um$ are not self enforcing since, as will be proven below, there always exist profitable and self-enforcing deviations from these networks also with exclusive contracts.

Existence of equilibria with $g^*_{nt,e} = pe$

As in Proposition 3, consider an equilibrium with $g = pe$ supported by mutual exclusivity between s and r and between s' and r'. Consider first a deviation to $g = dca$ by the grand coalition. This deviation is profitable for suppliers and retailers if and only if, respectively, (A-35) and (A-36) fail (or, more precisely, if the sign in those conditions is changed from $>$ to $<$). With a reasoning analogous to that conducted above in relation to (A-37) one can demonstrate that the deviation is always profitable for retailers when it is profitable for suppliers, which implies that there exists a jointly profitable deviation to $g = dca$ if and only if (A-35) fails, which is the case if and only if $b \leq \overline{\overline{b}}_{nt,e}(a)$. Since $\overline{\overline{b}}_{nt,e}(a) < \overline{b}_{nt,e}(a)$, the deviation to $g = dca$ is self-enforcing whenever it is profitable. Therefore, the existence of an equilibrium with $g = pe$ requires (A-35) to hold, i.e., $b > \overline{\overline{b}}_{nt,e}(a)$.

Consider next a deviation to $g = mix$ by the grand coalition. This deviation is never profitable for the supplier that would be left dealing with only one retailer, since it can be verified that with linear demand $\beta\left(\Pi^{mix2} - \Pi^{um}\right) > \beta\Pi^{pe}$. Moreover, it can be verified that a deviation to $g = dm$ is never profitable for suppliers, because $\Pi^{dm} - \Pi^{bm} \leq \Pi^{pe}$, and a deviation to $g = um$ is never profitable for retailers, because $\Pi^{um} \leq \Pi^{pe}$. Intuitively, although a deviation to $g = dm$ may increase total industry profits when $\Pi^{dm} > 2\Pi^{pe}$, it reduces the bargaining power of suppliers, who, after committing to a single retailer, do not have an outside option when bargaining in stage 2. A deviation to $g = um$ always reduces industry profits (as well as the bargaining power of retailers) and is thus never profitable.

Existence of equilibria with $g^*_{nt,e} = mix$

As in Proposition 3, consider a candidate equilibrium with $g = mix$ in which s' commits to exclusivity with r, r' commits to exclusivity with s, and s and r trade with each other on a nonexclusive basis. A deviation to $g = dca$ requires consent by all firms and can never be profitable for both suppliers. Specifically, the deviation is profitable for s (i.e., the supplier dealing with two retailers in the candidate equilibrium) if and only if $2\left(\Pi^{dca} - \Pi^{mix1}\right) > \Pi^{mix1} + \Pi^{mix2} - \Pi^{pe}$and for s' (i.e., the supplier dealing with only one retailer in the candidate equilibrium) if and only if $2\left(\Pi^{dca} - \Pi^{mix1}\right) > \Pi^{mix2} - \Pi^{um}$. It can be verified that with linear demand these two conditions cannot hold in the same parameter region.

Consider now a deviation by a coalition $\{s, r\}$ in which r commits to exclusivity with s, thus implementing $g = um$ and excluding s' is never profitable for r. To see this note that such a deviation is profitable for r if and only if $(1-\beta)\Pi^{um} > \Pi^{mix2} - \beta\left(2\Pi^{mix2} - \Pi^{pe} - \Pi^{um}\right)$, which can be re-written as $(2 - 1/\beta)\left(\Pi^{mix2} - \Pi^{um}\right) > \Pi^{pe}$.Since the left-hand side is increasing in β, this condition is most likely to hold when $\beta = 1$. When this is the case, the condition becomes $\Pi^{mix2} > \Pi^{pe} + \Pi^{um}$, which is never the case with linear demand. This implies that this condition never holds and thus that there never exists a mutually profitable deviation to $g = um$.

Consider next a deviation by a coalition $\{s, r\}$ in which s commits to exclusivity with r, thus implementing $g = dm$ and excluding r'. This deviation is profitable for s if and only if $\Pi^{dm} - \Pi^{bm} > \Pi^{mix1} + \Pi^{mix2} - \Pi^{pe}$, which is never the case when (A-33) holds.

Finally, consider deviations in which s unilaterally rescinds his link with r or vice versa. Both deviations implement $g = pe$. The unilateral deviation by s is not profitable if and only if (A-33) holds and the unilateral deviation by r is not profitable if and only if (A-34) holds. As demonstrated above, (A-34) holds for all $\beta \in [0, 1]$ when (A-33) holds. Therefore there exist no profitable unilateral deviations to $g = pe$ if and only if (A-33) holds.

The analysis above implies that there exists an equilibrium with $g = mix$ if and only if (A-33) holds.

There do not exist equilibria with $g^*_{nt,e} \in \{bm, dm, um\}$

This part of the proof is analogous to that of the proof of Proposition 6. Specifically, there does not exist equilibria with $g = dm$ because $\pi_s^{pe} > \pi_s^{dm}$ and equilibria with $g = um$ because $\pi_r^{pe} > \pi_r^{um}$. ■