WORKING
PAPERS

WHY PRICE CORRELATIONS DO NOT DEFINE ANTITRUST MARKETS:

ON ECONOMETRIC ALGORITHMS FOR MARKET DEFINITION

Jonathan B. Baker

WORKING PAPER NO. 149

January 1987

BUREAU OF ECONOMICS
FEDERAL TRADE COMMISSION
WASHINGTON, DC 20580

Why Price Correlations

Do Not Define Antitrust Markets:

On Econometric Algorithms for Market Definition

Jonathan B. Baker

Abstract

This paper compares two econometric methods that have been proposed for market definition: price correlations and residual demand curve estimation. Econometric theory is used to demonstrate that price correlations among firms will likely contain little or no information relevant to defining antitrust markets, under the assumption that a hypothetical cartel facing a downward sloping residual demand curve constitutes an antitrust market (defined according to the DOJ Guidelines). Hence price correlation analyses are likely to have little value for antitrust market definition. In terms of the literature on empirical techniques for market definition, this paper shows that if the econometric market definition algorithm based on residual demand curve estimation of Scheffman and Spiller (1985) is correct, then the econometric market definition algorithms based on price correlations of Stigler and Sherwin (1985) and Horowitz (1982) will not be valuable for antitrust enforcement. In the process of establishing these results, the paper clarifies the significance for antitrust market definition of reduced form price equations for single firms.

Why Price Correlations

Do Not Define Antitrust Markets:

On Econometric Algorithms for Market Definition

Jonathan B. Baker[1]

In a competitive market where sellers and buyers have full information and goods are homogeneous, the ability of market participants to practice arbitrage ensures that all sales at any one time will tend to be made at the same price.[2] Relying on this price theory proposition, some antitrust commentators have argued that, for the purpose of applying antitrust law, a market should be defined as a group of products in a geographic area

---

[1]The Amos Tuck School of Business Administration, Dartmouth College, Hanover N.H. Research underlying this paper was performed while the author was employed by the Federal Trade Commission. The views expressed in this paper do not necessarily reflect those of the Federal Trade Commission or any individual Commissioner. The author is indebted to Oliver Grawe, Robert Hansen, John Howell, Gale Mosteller, Monica Noether and Steven Salop.

[2]For an econometric method of assessing transportation and other transaction cost differentials limiting arbitrage of a homogeneous product, see Spiller and Huang (1986).

When products are differentiated by having specific locations in geographic or characteristics space, prices in a monopolistically competitive market will tend to differ only by a constant differential representing transportation costs (when differentiation is geographic) or quality differences for marginal consumers (when differentiation is in terms of product characteristics), assuming perfect buyer information.

among which prices tend to be uniform.[3] Several statistical tests implementing this proposed definition have been developed, all based generally on correlations of price levels or price changes among candidate market members over time. The most widely known price correlation tests are those proposed by Stigler and Sherwin (1985) and by Horowitz (1981). This paper explains why the price correlation methodology is a suspect tool for antitrust market definition, notwithstanding the possible utility of the price uniformity approach for defining markets for research purposes unrelated to antitrust.

An antitrust market is defined for one reason: because courts and government enforcers analyze the ability of firms to exercise market power within such markets. Hence the proper definition of an antitrust market must be related to the antitrust goal of interdicting the exercise of market power. This insight is incorporated in the U.S. Department of Justice Merger Guidelines (DOJ Guidelines), which define a market for the purpose of evaluating acquisitions under the antitrust laws as a group of products in a geographic region that could raise price profitably if that group were a cartel, with its output controlled by a hypothetical monopolist.[4]

---

[3]See, e.g., Stigler and Sherwin (1985). A comparison of this antitrust market definition approach with the leading alternatives can be found in ABA Antitrust Section (1986), pp. 89-110.

[4]"Formally, a market is defined as a product or group of products and a geographic area in which it is sold such that a hypothetical, profit-maximizing firm, not subject to price regulation, that was the only present and future seller of those

A market defined by a tendency toward price uniformity need have no relation to an antitrust market, properly defined according to the DOJ Guidelines definition. In an important recent paper, Scheffman and Spiller (1985) consider the relation between these market definition concepts.[5] They define "antitrust markets" in accordance with the DOJ Guidelines and point out that price correlation tests are predicated on a different market notion, "economic markets." Economic markets are defined by the presence of arbitrage, a market mechanism tending to produce price uniformity.

Scheffman and Spiller explain that economic markets need not be the same as antitrust markets, for two simple reasons. An economic market will be smaller than an antitrust market if

---

products in that area would impose a `small but significant and nontransitory' increase in price above prevailing or likely future levels. The group of products and geographic area that comprise a market will be referred to respectively as the `product market' and the `geographic market.'" U.S. Dept. of Justice Merger Guidelines §2.0 (June 14, 1984). Although an infinite number of markets, each larger than the last by the addition of other products in the economy or other geographic regions, will satisfy this test, the Department generally considers the relevant market to be the smallest group of products or regions which satisfies this test. DOJ Guidelines §§2.11, 2.31. In most contexts, a "small but significant and nontransitory" price increase means a 5% increase lasting for one year. DOJ Guidelines §2.11. Since the Department of Justice adopted it in 1982, this approach to antitrust market definition has been endorsed by a number of commentators. See generally, ABA Antitrust Section (1986), pp. 106, 106n.531, & 106n.532.

[5]Scheffman and Spiller (1985), pp.10 -11; see also Spiller and Huang (1986). Scheffman and Spiller also show how to operationalize the DOJ Guidelines approach to antitrust market definition by applying an econometric technique for estimating residual demand elasticities developed by Baker and Bresnahan (1984).

potential competition from firms not presently engaged in sales subject to the possibility of arbitrage constrains incumbent producers from supracompetitive pricing; the antitrust market then includes the potential competitors while the economic market does not.[6] On the other hand, an economic market will be larger than an antitrust market if some participants in an economic market are unable to expand supply following a price increase above competitive levels by the other market participants; the subgroup of firms excluding the capacity constrained producers would be free of competitive discipline so would constitute an antitrust market.[7]

The present paper makes related points through an econometric argument comparing price correlations tests, a tool for identifying economic markets, with residual demand estimation, the tool employed by Scheffman and Spiller to identify antitrust markets. This comparison shows that the two market definition algorithms will likely lead to very different

---

[6]Scheffman and Spiller (1985), p. 4n.6.

[7]Scheffman and Spiller (1985), pp. 4-6. Spiller and Huang (1986) estimate the typical maximum price differential between two physically homogeneous goods subject to arbitrage, and use this price differential as an estimator of the transactions costs of arbitrage. This information is a lower bound to the arbitrage costs relevant to antitrust market definition, however. The observed maximum price differential is less than the maximum that a hypothetical cartel including all present competitors could profitably raise price, unless the arbitraging competitors are known to have a perfectly elastic supply curve, or unless potential competitors are capable of arbitrage at the same margin sufficient to induce arbitrage by incumbent competitors. Hence Spiller and Huang's method of inferring the extent of economic markets cannot be used to infer the extent of antitrust markets. See Spiller and Huang (1986), p. 143.

market definitions. On the assumption that antitrust markets are of primary interest, the employment of a tool for identifying economic markets, price correlations tests, will likely create substantial errors in market definition. The particular price correlation tests studied involve both the price level correlations tests advocated by Stigler and Sherwin, and the price change correlations tests employed by Horowitz.

Section A of this paper summarizes the primary results by cataloguing the ways price correlations tests can mislead as to antitrust market definition. Section B analyzes the relationship between single firm reduced form price equations and antitrust market definition. This analysis underlies the remainder of the paper becuase price correlations tests can be thought of as inferring the extent of markets from reduced form price equations. Through this discussion, the conceptual experiment for antitrust market definition is clarified. A new concept, the "market defining' cost shift variable, is defined as am exogenous variable that shifts the supply curves of all members of the smallest antitrust market including a given firm, without affecting the supply curves of any other firms in the economy. This disucussion also incorporates an exposition of the residual demand curve estimation techniques employed by Baker and Bresnahan (1984) and Scheffman and Spiller (1985), and indicates what can be learned about antitrust markets from reduced form price equations for single firms.

With this background, Sections C, D, and E study the implications of price level correlations tests and price change correlations tests for market definition. These sections derive what econometric theory predicts would be revealed by the correlation of single firm reduced form price equations. The correlation results are then compared to what must be revealed in order for the statistical test to provide antitrust market definition information, according to the prior analysis of Section B. The main conclusion from this comparison is that price correlation tests contain little or no information relevant to the issue of antitrust market definition.

A. Summary of Argument

The econometric argument of this paper shows that the price correlation approach can mislead as to market definition. The price correlation methodology creates errors both by excluding firms which should properly be included in any antitrust market involving a given producer, and by including firms which are not in the smallest antitrust market involving that producer.

Incorrect exclusion will occur if the prices of two products are imperfectly correlated, yet the two exert competitive discipline upon each other. Suppose, for example, Chevy and Ford each (individually) face demand curves that are not perfectly elastic, but have some downward slope reflecting product differentiation. If so, cost increases or decreases limited to Chevy could lead to changes in the relative price at which the two firms sell their products. If firm-individuated cost shifts are the only reason for price changes, as would be true if industry-wide cost or demand shift variables did not fluctuate over the sample period, then price levels need not be correlated highly and price changes will be uncorrelated. Using price correlation tests, one might conclude that the two products are not found in the same antitrust market. Yet nothing in this hypothetical example precludes the possibility that the two firms could raise price substantially if they collude, either just with each other or in a cartel incorporating other producers such as Chrysler; the aggregate (industry) demand curve may well be substantially less elastic than either firm's demand curve. If so, the two automobile brands, perhaps in league with a handful of other producers, could form an antitrust market defined by the DOJ algorithm. Yet these firms might be improperly placed in separate markets if a price correlation methodology is applied.[8]

Incorrect inclusion will instead occur when prices are highly correlated for reasons unrelated to the economic forces tending to create price uniformity within a market.[9] High

---

[8]This econometric argument reasons from assumptions which in effect presume that Chevy and Ford are potential, not actual, competitors over the sample period. Hence the two firms would not be placed in a market defined by arbitrage, but should be incorporated within an antitrust market. In this way, the present econometric argument for underinclusion by price correlation tests has an analogue in the economic argument for underinclusion of economic markets relative to antitrust markets made by Scheffman and Spiller.

[9]Another, less significant mechanism will cause price correlations test to incorrectly include firms in antitrust markets. Suppose Ford's automobile output cannot increase beyond a low level, but Chevy faces no capacity constraint. The price charged by Ford and the price charged by Chevy will be perfectly

correlations may reflect instead similar economic forces affecting disparate markets. If so, a market definition based on price correlations will overstate the extent of the market.[10]

For example, assume that automobile and truck prices, both levels and differences, are highly correlated over some sample period as a result of the effect of changing real rates of interest on the demand for durable goods. Further, assume that the two products are not demand substitutes for most purchasers, and that they are built on dedicated production lines so they are not supply substitutes over a period of several years. Under these assumptions, the price correlation analyst will place automobiles and trucks in the same market, yet the products provide no competitive discipline for each other and do not satisfy the market definition algorithm of the DOJ Guidelines. This statistical difficulty with the price correlation test arises because the prices are correlated for reasons unrelated to

---

correlated if arbitrage can occur, so the price correlation analyst would place the two firms in the same market. Yet if Ford and Chrysler were able to raise price through coordinated behavior, Chevy's price would also increase but additional Chevy production would not be forthcoming. Chevy would therefore exert no competitive discipline on anticompetitive action by Ford and Chrysler. Hence Chevy would not be a part of the smallest antitrust market which includes Ford. Under these assumptions, the price correlation methodology overstates the scope of the relevant antitrust market. This econometric argument is based on the assumption that Ford's production capacity is constrained; it has an analogue in the economic argument for overinclusion by price correlation tests made by Scheffman and Spiller.

[10]This difficulty has been remarked upon by several commentators. See Giffin and Kushner (1982); R. Rogowsky & W. Shugart (1982); Uri, Howell, & Rifkin (1985); see generally ABA Antitrust Section (1986), pp. 104, 104n.519.

arbitrage. Under the above assumptions, automobiles and trucks are neither in the same economic market nor in the same antitrust market, yet the price correlation methodology would improperly place them in the same market.

The remainder of this paper applies econometric theory to demonstrate how the difficulties difficulties described above infect market definition analyses based on price correlations tests.

B. Residual Demand Curves and Antitrust Market Definition

To evaluate the utility of price correlation tests, their statistical properties will be compared with the properties of the econometric tool employed by Scheffman and Spiller to identify antitrust markets: the estimation of a residual demand curve for a proposed market aggregate. This section describes the relation between reduced form price equations and residual demand curves. The relation is first discussed in the context of the residual demand curve defined for a single firm, where it is most easily understood, and then considered in the context of the residual demand curve defined for a market aggregate, where it will be applied to the antitrust market definition problem. The primary significance of Section B is to indicate how each cost shift variable with a non-zero coefficient in the reduced form price equation for a single firm can be used to define an antitrust market including that firm, and how the smallest such antitrust market can be identified.

1. The Residual Demand Curve for the Single Firm[11]

The residual demand curve for any single firm is identified by the conceptual experiment of shifting costs for the single firm alone. This experiment gives the firm an incentive to raise price and allows the market forces imposing competitive discipline on that price rise to work. We can then observe whether on balance the firm is able to raise its price, showing a downward sloping residual demand curve, or whether instead the firm must absorb the cost increase without raising price, exhibiting a flat residual demand curve. This conceptual experiment is significant because it has an important consequence for the interpretation of the reduced form price equation.

Consider the residual demand curve for a single firm denoted firm a, defined by equation (1).

(1) $Q^a = R(P^a, Z, Y)$

In equation (1), Q^a and P^a represent the quantity and price chosen by firm a. Z and Y are cost and demand shift variables respectively.[12] The demand shift variable Y affects firm a either directly or indirectly through its effect on the behavior

---

[11]The Appendix to this paper proves the assertions of Section B.1 for the case of a firm in a linear duopoly, by deriving a residual demand curve and reduced form equations for price and quantity for a single firm and demonstrating the relation among these functions. A general analysis is found in Baker & Bresnahan (1984).

[12]For simplicity, the cost and demand shift variables W, Z, and Y are taken to be scalars; the points made in this paper would not change were they instead vectors of exogenous variables.

of firm a's rivals. The cost shift variable Z appears in the residual demand curve only by altering the behavior of the firm's rivals; it is either an industry-wide cost shift variables or a variable increasing the costs of rivals without affecting firm a's costs. The oligopoly solution concept is assumed stable and supressed. Equation (1) is assumed differentiable, so that its slope is always defined.

The reduced from price and quantity equations take the following form:

(2) $P^a = P^a(W,Z,Y)$

(3) $Q^a = Q^a(W,Z,Y)$

These equations include a variable W not present in the residual demand function; W is a cost shift variable affecting firm a without affecting any other firm.

The slope of the residual demand curve (1) facing firm a is related to derivatives of the reduced form equations (2) and (3). As the conceptual experiment desrcribed above suggests, the following relation holds[13]:

---

[13] In general, given two simultaneous equations in X and Y, $X = f(Y,A,B)$ and $X = g(Y,A,C)$, then $f_Y = \delta f/\delta Y = [\delta X/\delta C]/[\delta Y/\delta C]$. This can be seen by solving the two simultaneous equations created by totally differentiating the functions f( ) and g( ), then setting the differentials $dA = dB = 0$. The resulting system of differential equations can be written as follows: $dX - f_Y dY = 0$ and $dX - g_Y dY = g_C dC$. These equations solve for f_Y.

The econometric analogue of equation (4) -- relating the coefficient of an endogenous variable in the ordinary least squares estimation of one of a system of simultaneous equations to the ratio of coefficients of an exogenous instrumental variable in reduced form equations -- is well known. See Haavelmo (1943).

(4) $\delta R/\delta P^a = (\delta Q^a/\delta W)/(\delta P^a/\delta W)$.

Equation (4) explains that the slope of the residual demand curve equals the ratio of the partial derivative of the reduced form quantity equation with respect to a firm individuated cost shift variable to the partial derivative of the reduced form price equation with respect to the same cost shift variable.

Equation (4) has an important implication for the analysis of this paper: the partial derivative of the reduced form price equation with respect to a firm individuated cost shift variable ($\delta P^a/\delta W$) is a sufficient statistic for identifying single firm market power. Firm a has no market power if and only if $\delta P^a/\delta W = 0$. Only then can the ratio $(\delta Q^a/\delta W)/(\delta P^a/\delta W)$ grow without limit. Appendix A demonstrates this point for a linear duopoly example.

2. Reduced Form Price Equation for a Collusive Group

This section describes how a residual demand curve for a multi-firm aggregate identifies an antitrust market. The present exposition differs from that of Scheffman and Spiller (1985) in order to highlight the relation between a residual demand curve and a reduced form price equation for a hypothetical collusive group of firms. Further, the present discussion treats in the margin several econometric issues raised by the residual demand curve methodology for market definition not addressed by Scheffman and Spiller.

An antitrust market is identified, the DOJ Guidelines instruct, if a hypothetical collusive group of firms would be able to exercise market power. Whether a hypothetical cartel possesses market power depends upon the elasticity of the residual demand curve facing the group.[14] If the demand curve facing the group is perfectly elastic, the provisional antitrust market must be expanded to include additional producers. If the demand curve facing the group is inelastic, the collusive group has market power.[15] In that case, the collusive group forms an antitrust market, although a smaller collusive group might also form an antitrust market.

The conceptual experiment that identifies a residual demand curve for a hypothetical collusive group is analogous to the experiment which identifies a residual demand curve for a single firm. The collusive group's demand curve is identified in the following way: raise costs for all members of the hypothetical group but for no other firms, and see if the equilibrium price charged by the group increases. If the firms, acting in

---

[14]See Landes & Posner (1981); Scheffman & Spiller (1985).

[15]It is not the purpose of this paper to determine how inelastic the demand curve must be before an antitrust market is inferred. However, it may be useful to note that if the hypothetical cartel at issue acts as a Stackelberg leader, and if the collusive group faces a demand curve of constant elasticity, then the Lerner Index markup [(price - marginal cost)/price] likely to be achieved by the cartel equals the inverse of the negative of the elasticity. See Baker & Bresnahan (1984), pp. 12-15; Scheffman & Spiller (1985), p. 30. Under these assumptions, a demand elasticity of -20 for the collusive group translates into a 5% markup, and a demand elasticity closer to zero translates into a greater markup. An example of antitrust geographic market definition undertaken by estimating the residual demand curve for the hypothetical collusive group appears in Scheffman & Spiller (1985).

coordination, are able to pass the cost increase through to customers, they face a downward sloping demand curve and, as a group, would possess market power. If they are unable to pass through the price increase -- whether that competitive discipline arises from demand substitution, the threat of entry, or the nature of interfirm rivalry -- they do not as a group possess market power so do not form an antitrust market.

It is noteworthy that the conceptual experiment identifying a residual demand curve for a hypothetical collusive group is the very market definition algorithm employed by the DOJ Guidelines. This equivalence provides the intuitive justification for an empirical approach to antitrust market definition relying on the estimation of the residual demand curve for an aggregate composed of the members of a proposed antitrust market.[16]

To formalize this discussion, equation (1) can be reinterpreted as describing the residual demand curve facing entity a, an aggregation of candidate members of an antitrust market. Consistent with this reinterpretation, P^a represents the average price received by the aggregate entity[17], and Q^a represents the total output of the aggregate. This reinterpretation supressess the nature of the rivalry (oligopoly

---

[16]Scheffman & Spiller (1985) were the first to employ this approach to an actual market definition problem by identifying geographic markets for refined gasoline in the Eastern U.S. When used as an empirical test for identifying antitrust markets, the methodology is subject to several qualifications described in notes 18 and 19, infra. These qualifications are assumed away in the present discussion.

[17]If the goods are homogeneous, P^a will be the market price.

behavior) within entity a[18] and is subject to several other qualifications ignored in the present discussion.[19] Y is reinterpreted as a variable which shifts demand for any of the products aggregated in entity a, either directly or indirectly by affecting the behavior of rival firms outside the aggregate. The variable Z is reinterpreted as shifting costs common both to entity a and to firms outside that aggregate, or as shifting

---

[18]A cartel will typically have an incentive to reduce the quantity of some goods by more than others. For a brief discussion of the effect of this incentive on econometric estimates of the market power gains from merger, see Baker & Bresnahan, The Gains from Merger or Collusion in Product-Differentiated Industries (1985), p. 441n.20. This possibility is ignored in the aggregation presumed to form the hypothetical collusive group in the text.

[19]Aggregation in residual demand curve estimation raises several econometric issues assumed away in the present discussion as not related to the significance of price correlation tests for market definition. The analysis here and in Scheffman and Spiller (1985) aggregates the market demand curves of many individual firms, the optimizing actors in the model, into a multi-firm hypothetical cartel as suggested by the DOJ market definition algorithm. Estimated residual demand elasticities for multi-firm aggregates will overestimate market power if collusion leads to entry by new competitors of a type not previously threatened, hence not apparent in the data. Baker & Bresnahan (1985), p. 427n.1. In addition, in differentiated product industries, an aggregate price and quantity may not be well defined, particularly when the goods are not close substitutes in demand. If so, it preferable in theory to infer gains from hypothetical collusion by estimating partial residual demand curves, as is undertaken in Baker & Bresnahan (1985). Even when aggregation across firms is sensible, estimates of residual demand elasticities may be biased. Baker & Bresnahan (1984), p. 49 (considering bias in residual demand curve estimation arising "if firm-individuated demand curves are aggregated into hypothetical mergred firms or an industry cartel"). Further, the econometric analysis in both the single firm and aggregated entity contexts presumes that product attributes and the oligopoly solution concept are stable over the sample period. For an example of a correction for this problem, see Baker & Bresnahan (1984), pp. 19-20 (role of Lite beer).

costs borne by firms outside entity a but not borne by firms in that group. W raises costs for one, several, or all firms within group a, but not for rival firms.

The slope of the residual demand curve for group a will be perfectly elastic if and only if the hypothetical collusive group does not form an antitrust market. If instead the residual demand curve has slope, then group a forms an antitrust market (although not necessarily the smallest such market).

As in the single firm case, the slope of the reduced form price equation for group a is a test statistic for the presence of group market power. As the previous discussion of equation (4) makes clear, if the derivative of this reduced form price equation with respect to a group individuated cost shift variable is non-zero, the multi-firm aggregate would face a downward sloping residual demand curve, implying that group a would presence of market power were it to act as a collusive group.

An example of the interpretation of a reduced form price equation for a market aggregate is presented below and will reappear in modified form to illustrate points made in later sections. Although this example involves antitrust product market definition, the example could equally well have involved geographic market definiiton.

Suppose the following reduced form equation is estimated for an average beer price:

(5) P(beer) = a + bY(income) + cP(hops)

+ dP(bottling machinery) + eP(labor)

Equation (5) relates the price of beer to one demand shift variable (income) and three cost shift variables (the price of hops, the rental value for bottling machinery, and the wage rate in the United States). Hops are a factor of production for brewers but not for any other firms. Bottling machinery and labor are costs to brewers and to producers of substitutes to beer. Assume that equation (5) does not omit any relevant cost or demand shift variables and that its functional form is correctly specified[20]. If the coefficient on the price of hops, a brewing industry individuated cost shift variable, is significantly greater than zero in equation (5), then the group of brewers in the aggregate have a downard sloping residual demand curve. Hence brewing would constitute an antitrust product market.[21] If the price of hops instead has a coefficient

---

[20]The assumption that the analyst can specify the full list of demand and cost shift variables affecting the firms in an industry does not require that the analyst perform a market definition exercise prior to the reduced form market definition analysis. The assumption requires no more than familiarity with the range of plausible production technologies for the products at issue, and a plausible set of candidate demand substitutes. This general prior information does not mandate any particular boundary for an antitrust market.

[21]Before making this inference in practice, it would be important to confirm that hops are indeed a significant cost shift variable for brewers. Even if hops are known to be part of the recipe for beer, they might not appear in the reduced form price equation if they have an insignificant cost share in beer production or if their price does not vary over the sample period. (Indeed, while hops provide a convenient vehicle for explaining the significance of cost shift variables because they are an input used only by brewers, they account for a tiny fraction of brewing cost so are unlikely to have empiricial significance in a reduced form price equation for any firm in the brewing industry.) The significance of hops as a cost shift variable for brewers can be confirmed by examining whether the

of zero, a hypothetical collusive group formed by all brewers would not be able to raise price, so would not form an antitrust market.

3. Reduced Form Price Equation for a Single Firm

This section argues that the reduced form price equation for a single firm contains the same market-identifying information as is found in the reduced form price equation for a proposed market aggregate including that firm. If so, the correlation of two product prices could contain some information relevant for antitrust market definition. However, later sections of this paper will argue that the relevant information is not in practice revealed by price correlations.

Consider equation (6), a reduced form price equation for Budweiser beer.

(6) P(Budweiser) = a + bY(income) + cP(hops)

+ dP(bottling machinery) + eP(labor)

The reduced form price equation for Budweiser must include as independent variables all variables that appear in the reduced form price equation for a brewing industry aggregate, equation (5). Any variable affecting the demand or supply of beer, and hence the price of beer, necessarily also affects the price of Budweiser. The converse must also hold: all variables affecting

---

price of hops enters into the parallel reduced form equation for quantity. If the analysis establishes that changes in the price of hops alter the quantity of beer sold without affecting the price of beer, then brewing would not form an antitrust product market.

the demand or supply of Budweiser, hence present in the reduced form price equation for that brand, necessarily affect the price of beer.

For the purpose of antitrust market definition, equation (6) can be interpreted as a misspecification of equation (5) with one crucial difference: the price of one brand, here Budweiser, is employed as a proxy for the true dependent variable, the price of beer.[22] Hence if the coefficient of a brewing individuated cost shift variable, here hops, is non-zero, then a collusive group of brewers would be able to exercise market power. Therefore, if the coefficient on the price of hops in equation (6) is significantly different from zero, Budweiser can be said to be a member of an antitrust product market that includes all other hops users, namely all brewers. If the price of hops has a coefficient of zero, the smallest antitrust product market including Budweiser is broader than brewers.[23]

When viewed as an econometric problem, the misspecification of equation (5) as equation (6) reduces the power of the resulting coefficient estimates but is not likely to bias them[24].

---

[22]It is possible that the two reduced form price equations will differ in their functional form. If this were to occur, it would be a consequence of a problem in aggregation, assumed away in the present discussion.

[23]This analysis continues to assume that shifts in the price of hops affect Busweiser quantity even if the price of hops has no effect on Budweiser price. See note 21, infra.

[24]It is unlikely that changes in any hypothetical cartel member's price will be systematically related to the firm's price level if quality differentials among differentiated products are related to tastes independent of prices, so will be preserved by

If the hypothetical cartel has market power, every member firm will likely share in a price increase generated by an increase in a common cost, so the coefficient on the common cost shift variable will be non-zero in the reduced form price equation for each member. If the hypothetical cartel lacks market power, an increase in a cost variable common to the group but not affecting outside firms will not generate a price increase for any group member; all will absorb the cost increase by reducing quantity.

Under this analysis, each cost shift variable in a single firm's reduced form price equation can be interpreted as redefining the scope of the corresponding hypothetical collusive group. If the reduced form price equation for a single firm is correctly specified and estimated, and the coeffcient of a cost shift variable W in that equation is positive, then the single firm must be part of an antitrust market that includes <u>all</u> other firms for which W is also a cost shift variable, although this antitrust market need not be the smallest possible antitrust market. In this way, each common cost shift variable identifies a hypothetical collusive group.[25]

---

the hypothetical cartel. Hence the price of any brand of beer can be viewed as having a fixed differential from the industry price.

[25]This reduced form price equation approach to antitrust product market definition is a useful conceptual device for understanding the significance of correlations of prices. Yet the approach may well be more difficult to implement than the equivalent residual demand curve methodology. The experience of Baker and Bresnahan (1984) in estimating residual demand curves suggests that the residual demand elasticity (quantity coefficient) may often be precisely estimated while estimated coefficients on factor prices may not be robust to specification

The antitrust market identified by a positive coefficient on a cost shift variable is not necessarily the smallest antitrust market. If the price of bottling machinery, another factor of production, enters equation (6) with a positive coefficient, Budweiser is properly shown to be included in an antitrust product market along with bottled water, bottled soft drinks, and other bottled products, regardless of whether some smaller antitrust market containing Budweiser can also be defined. If the average wage for U.S. production workers is included as an exogenous cost shift variable and has a positive coefficient, as is likely, this would imply that a collusive group formed by all firms in the economy which employ labor as a factor of production would successfully be able to raise price. Then it would be proper to conclude that the entire economy forms an antitrust market, but this is almost certainly not the smallest antitrust

---

changes. This may occur because estimates of the coefficients on factor prices are more sensitive to biases from omitted cost and demand shift variables than are estimates of the coefficients on quantity. If coefficients on factor prices in reduced form equations are similarly not robust to specification changes, or if the likely collinearity of factor prices makes it difficult to discover whether the coefficient of any individual factor price in a reduced form equation is significantly different from zero, then the residual demand elasticity approach to antitrust market definition will be substantially preferable to the reduced form price equation approach. Further, the reduced form approach never allows inference of the extent of market power achievable by a hypothetical cartel, unlike the residual demand approach. (The extent of potential anticompetitive gains may be an appropriate consideration in the exercise of antitrust enforcement discretion.)

The reduced form approach does have one important advantage over residual demand curve estimation. The reduced form approach avoids the need to correct for simultaneity; ordinary least squares estimates of reduced form price and quantity equations are unbiased.

market that includes Budweiser. In the discussion below, the cost shift variable with non-zero coefficient in the reduced form price equation that identifies the smallest antitrust market including the particular firm at issue will be termed a "market defining" variable for that firm.[26] As the practical market definition exercise in antitrust cases involves the smallest antitrust market, it is proper to limit attention to market defining cost shift variables.

In contrast to the significance of cost shift variables, no inference about market definition can be made from the coefficient of the demand shift variable income in the estimated reduced form price equation. This is because demand shift variables are not excluded exogenous variables from the perspective of the residual demand curve facing a hypothetical collusive group.[27]

---

[26]If some producers in an industry are capacity constrained, their output should not be aggregated into the smallest hypothetical cartel. In this situation there may well be no cost shift variable which is unique to the members of the smallest antitrust marke, as any candidate will also increase costs and prices of the capacity constrained producers. If so, the apparent "market defining" cost shift variable will identify the smallest antitrust market observable using this method, but not the smallest such market that actually exists. This difficulty has little practical significance if the slope of the supply curve of firms placed in an antitrust market can be ascertained, and will be ignored in the remaining analysis.

[27]The "supply relation" that defines equilibrium in conjuction with the residual demand curve is defined in note 47, supra. It includes includes parameters from the demand curve so long as the hypothetical collusive group acts other than as a price taker. For example, an exogenous group-individuated demand shift variable appears in both the residual demand curve and the supply relation, so will not identify either. Hence if beer demand rises because of a decline in the drinking age for beer

The analysis of the significance of single firm reduced form price equations for antitrust market definition is summarized by the following propositions:

(P.1) If the price of good a increases following an increase in a cost shift variable W affecting products a and b, then the two products, along with all other products whose costs also increase when W increases, collectively form an antitrust market (not necessarily the smallest antitrust market).

(Definition) A cost shift variable W* that defines the smallest antitrust market including product a will be termed a "market defining" cost shift variable for product a.

(P.2) If the price of good a does not increase following an increase in a cost shift variable W affecting only products a and b, while the quantity of good a sold falls, then the two products, along with all other products with costs which also increase when W increases, do not collectively form an antitrust market.

(P.3) Variation in the price of good a arising from any other source, including price variation arising from variation in

---

(while the drinking age for all other alcoholic beverages remains unchanged), then the average price of beer will increase regardless of whether brewers as a hypothetical collusive group could exercise market power, so long as the marginal cost curve of the brewing industry is upward sloping.

demand shift variables, provides no information for antitrust market definition.

4. Reduced Form Price Equations for Two Firms

The previous discussion of the significance of reduced form price equations for antitrust market definition is applied in this section to identify the circumstances under which simultaneous variation in the prices received by two firms suggests that the two fall within the same antitrust market. For obvious reasons, this issue underlies the analysis of price correlation tests of antitrust market definition.

Suppose the prices (or the price changes) of Budweiser and Miller, the flagship brands of the two leading U.S. brewers, move together. If similar movements in the two firm's prices arise from variation in a common cost shift variable, such as the price of hops, and the common cost shift variable affects only the producers in a narrowly defined group, as the price of hops might affect the prices received by brewers alone, then the price movement comparison correctly suggests that Budweiser and Miller are part of the same antitrust market. Under these assumptions, a price correlation analysis is not misleading; brewing will indeed constitute an antitrust product market. The price of all other beer brands can be expected to vary similarly with that of these two firms, as all will be affected by the price of hops. However, the price of wine or soft drinks, which do not respond to variation in the price of hops, may well vary independently from the price of beer.

If instead the similar price movement arises from variation in a common cost shift variable, like the wage rate, that also affects a large number of other firms in the economy, no antitrust market definition inference can be made from the price correlation. This interpretive difference from the case of variation in the price of hops arises because, if brewing is an antitrust market, the price of hops is a market defining cost shift variable for both Budweiser and Miller, while the wage rate is not.

If the prices of Budweiser and Miller are highly correlated because price movements in both are driven by a common demand shift variable, the degree of price correlation gives no information about market definition. Further, if price movements are uncorrelated, yet common cost shift varibles are unchanged over the sample period, the firms may or may not lie in the same antitrust market. In that case price correlations are uninformative concerning market definition.

In short, the significance of price correlations for antitrust market definition depends first upon the source of observed price correlations or the reason for the absence of price correlations, and second, if the price correlations arise from a common cost shift variable, upon whether that variable is a market-defining cost shift variable. Unfortunately, as later sections of this paper will demonstrate, it is impossible to identify the information in price correlations relevant to antitrust market definition without bringing to the analysis sufficient additional information tantamount to performing a residual demand analysis. It will then be the residual demand analysis and not the price correlation which performs the antitrust market definition.

An example of outside information will clarify this important point. One common method of importing outside information into the price correlation analysis is the simultaneous examination of the price correlations among many related brands. In defining a market that includes Budweiser, for example, the analyst would likely correlate its price with the prices of a number of other beverages such as Miller, Pabst, Coke, and Maxwell House. These are not randomly selected goods from throughout the economy. They are chosen because the analyst recognizes, explicitly or implicitly, that they likely share common cost or demand shift variables with Budweiser. If high price correlations are found among products employing hops as an input, but not between those products and products not employing hops, it may be appropriate to conclude that hops users (brewers) form an antitrust market. As the previous discussion has demonstrated, this conclusion can properly be reached if the analyst also establishes that hops form a significant cost share for those firms using them as an input, that the factor cost of hops varies over the sample period, and no significant demand shift variables affect the hops users without affecting products among the other brands studied. These additional pieces of information, however, implicitly convert the price correlation study into a rough and ready residual demand analysis of antitrust market definition. Further, it is unlikely that the outside information necessary to convert the price corrlation study into a market definition test will be unambiguous without statistical analysis. In particular, only when the rare "natural experiment" occurs is it likely to be possible to assign a single primary cause to price variation. In general, statistical techniques such as estimation of the reduced form price equation are likely to be necessary in order to isolate the contributions of a market-defining cost shift variable in affecting price variation.[28]

---

[28]The analysis of the previous section also suggests that if the (correctly specified) reduced form price equations for two goods, such as Budweiser and Miller, contain different exogenous demand or cost shift variables, then the two goods are in separate antitrust markets. This can be seen by supposing that equation (6) is a noisy but unbiased misspecification of equation (5), because Budweiser is in the beer market and beer is the smallest antitrust market containing Budweiser. Then equation (6a) below, a reduced form price equation for a second brewer, would also proxy equation (5).

(6a) P(Miller) = a + bY(income) + cP(hops) + dP(bottling machinery) + eP(labor)

Under these assumptions, equation (6a) should have identical coefficients as equation (6), the reduced form price equation for Budweiser, except the two intercepts will vary to reflect the quality differential to the marginal customers.

If instead the reduced form price equations for Budweiser and Miller have different independent demand or cost shift variables, then each brand would be capable of sustaining a price increase that would not be competed away by its rival. Hence the smallest antitrust markets containing each brand must be separate.

Unfortunately, this analysis does not lead to a practical empirical technique for market definition because it would place firms in different antitrust markets whenever a single firm has

C. Time Series Representation of Reduced Form Price Equations

The main conclusions of this paper are derived from linear reduced form equilibrium equations for the price of two products. The two products studied, possibly differentiated, are denoted a and b. These products should be thought of as candidates for being included in the same antitrust market, varying in either geographic or product characteristic space. The equations analyzed may be understood as linear approximations to reduced form price equations of unspecified functional form. To further motivate these equations, the Appendix shows that linear reduced forms arise directly from a linear duopoly model. This section sets forth their time series representation under simple assumptions about the evolution of the exogenous variables.

1. Structural Representation of Reduced Form Price Equations

The particular reduced form price equations assumed are stated as equations (7) and (8). The expression P^{i}_{t} represents the price of good i at time t.

(7) $P^{a}_{t} = a_0 + a_1R_t + a_2S_t + e_t$

(8) $P^{b}_{t} = b_0 + b_1R_t + b_2T_t + v_t$

The variables R, S and T represent all the exogenous demand and supply shift variables affecting price, including such

---

market power, implicitly identifying single firms with market power as antitrust markets. Single firms cannot constitute markets in antitrust analysis, however.

variables as factor prices and income. Without loss of generality, the analysis is limited to the case of one common exogenous variable R, and two product individuated variables S and T. R, S, and T could each represent either a cost or demand shift variable.[29] This notation emphasizes that without additional information, the price correlation analyst does not know the source of price movements and correlations.

In equations (7) and (8), the a's and b's with numbered subscripts are parameters. The subscript t identifies the time period of the observation. These equations are stochastic, where e and v are independently and identically distributed errors with mean zero and variance σ^2_e and σ^2_v respectively. The covariance of e and v is assumed to be zero.

2. Significance of Parameters of Reduce Form Price Equations for Antitrust Market Definition

Propositions P.1 to P.3 of the previous section can be

---

[29]This formulation is more general than arises from the linear duopoly model of the Appendix because each reduced form price equation (7) and (8) includes a shift variable not present in the other reduced from price equation. Under the oligopoly model of the Appendix, the two reduced form price equations (A.7) and (A.8) include the same variables, but one can imagine other plausible models in which some variables would affect the price charged by some but not all firms. For example, a factor price affecting the costs of a dominant firm but not the costs of its competitive fringe would appear in the dominant firm's reduced form price equation but not in the reduced form equation for fringe pricing. Further, in the linear model of the Appendix, the reduced form quantity equations exclude variables directly affecting only the demand curve faced by the rival producer; it is likely that a simple alteration of the solution concept would produce reduced form price equations with similarly excluded variables.

applied to the interpretation of the reduced form price equations (7) and (8) in order to indicate the significance of the parameters of those equations for antitrust market definition. Those propositions imply that entity b is in some antitrust market (not necessarily the smallest antitrust market) that includes entity a if and only if (i) the common variable R is a cost shfit variable for both entities, and (ii) the parameters a_1 and b_1 are non-zero. Only if R is a market defining cost shift variable for firm a is entity b a member of the smallest antitrust market including entity a. The remainder of this paper assesses what can be learned about the parameters a_1 and b_1, and consequently what can be learned about antitrust market definition, from the correlation of P^a with P^b.

3. Time Series Representation of Reduced Form Price Equations

By assumption, each of the exogenous variables (R, S, and T) evolves according to a first order autoregressive process. Although more general time series processes might be postulated, this simple assumption, an approximation to the behavior of many economic time series, is sufficient to show the potentially misleading nature of price correlation tests.

(9) $R_t = c_0 + c_1 R_{t-1} + \Theta_t$

(10) $S_t = d_0 + d_1 S_{t-1} + \mu_t$

(11) $T_t = e_0 + e_1 T_{t-1} + \varphi_t$

Here Θ, μ, and φ are independent and identically distributed random variables with no covariance across time and no covariance either with each other or with e or v. They each have mean zero and variance σ^2_i, where i indexes the random variable at issue.

Equations (9), (10), and (11) can be written in the following equivalent moving average forms, assuming that c_1, d_1, and e_1 are less than unity in absolute value (that is, that each autoregressive process is stationary):

$$(9')\quad R_t = c_0(1 + c_1 + c_1^2 + c_1^3 + \ldots) + (\Theta_t + c_1\Theta_{t-1} + c_1^2\Theta_{t-2} + c_1^3\Theta_{t-3} + \ldots)$$

$$(10')\quad S_t = d_0(1 + d_1 + d_1^2 + d_1^3 + \ldots) + (\mu_t + d_1\mu_{t-1} + d_1^2\mu_{t-2} + d_1^3\mu_{t-3} + \ldots)$$

$$(11')\quad T_t = e_0(1 + e_1 + e_1^2 + e_1^3 + \ldots) + (\varphi_t + e_1\varphi_{t-1} + e_1^2\varphi_{t-2} + e_1^3\varphi_{t-3} + \ldots)$$

Substituting (9'), (10'), and (11') into equations (7) and (8) allows the reduced form equilibrium equations for price to be represented in a form entirely dependent upon the parameters and random innovations. The resulting equations, (7') and (8'), are indicated below.

$$(7')\quad P^a_t = a_0 + a_1[c_0(1 + c_1 + c_1^2 + c_1^3 + \ldots) + (\Theta_t + c_1\Theta_{t-1} + c_1^2\Theta_{t-2} + c_1^3\Theta_{t-3} + \ldots)] + a_2[d_0(1 + d_1 + d_1^2 + d_1^3 + \ldots) + (\mu_t + d_1\mu_{t-1} + d_1^2\mu_{t-2} + d_1^3\mu_{t-3} + \ldots)] + e_t$$

$$(8')\quad P^b_t = b_0 + b_1[c_0(1 + c_1 + c_1^2 + c_1^3 + \ldots) + (\Theta_t + c_1\Theta_{t-1} + c_1^2\Theta_{t-2} + c_1^3\Theta_{t-3} + \ldots)] +$$

$$b_2[e_0(1 + e_1 + e_1{}^2 + e_1{}^3 + \ldots)$$
$$+ (\varphi_t + e_1\varphi_{t-1} + e_1{}^2\varphi_{t-2} + e_1{}^3\varphi_{t-3} + \ldots)] + v_t$$

Equations (7') and (8') will allow the identification of the source of price correlations in later sections of this paper.

D. Price Level Correlations

This section analyzes the significance of price correlations for market definition. The standard price correlations approach involves the estimation of the simple correlation coefficient between the prices charged by two firms. This approach implicitly assumes that the two prices are connected by a linear relation such as equation (12).[30]

(12) $P^b{}_t = \beta_0 + \beta_1 P^a{}_t + \varepsilon_t$

Those who employ price correlations to define markets believe that two products are in the same market if the estimated simple correlation coefficient for their two price series is near unity[31], while an estimated correlation coefficient near zero indicates that the products are in separate markets.

The price level correlation approach described above incorporates the essence of the various price correlation studies conducted by Stigler and Sherwin. At various times, Stigler and

---

[30]Under this approach, the constant term β_0 would represent the equilibrium transportation cost or quality differential for the marginal purchaser.

[31]This interpretation presumes that the products at issue are substitutes. Market definition issues for compelements in demand or supply are analyzed in ABA Antitrust Section (1986), pp. 138-141.

Sherwin correlate price levels and first differences in price levels; further, they generally transform prices into logarithms before undertaking the correlation analysis.[32] The interpretation of the variables P, R, S, and T in the above equations can be varied so that equation (12) represents each such method employed. (The implict functional form of the reduced form price equation (7) and the interpretation of the autoregressive process followed by each exogenous variable would also vary in a corresponding manner.)

Equations (7') and (8') allow the identification of the large sample properties of the crucial correlation coefficient. This statistic is consistently estimated by the ratio of the covariance of P_a and P_b to the product of the standard deviations of P_a and P_b.[33] This ratio of moments of the price variable distributions can be computed from the time series representation of the reduced form price equations (7') and (8'), again assuming that the autoregressive processes describing the evolution of the

---

[32]See, e.g. Stigler & Sherwin (1985), pp. 559 (first differences), 566 (levels and first differences of logarithms), 570 (first differences of logarithms), 574 (levels and first differences of logarithms), and 576 (first differences of logarithms). Stigler and Sherwin's Appendix identifies the correlation between first differences in logarithms of price series as the "critical statistic" for determining whether two locations are in the same geographic market.

[33]In the two variable linear model of equation (12), the simple correlation coefficient between the two price series (r^2) is related to the slope coefficient because, in probability limit, the correlation equals the expression $\beta_1\sigma_a/\sigma_b$, where σ_i is the standard error of P^i. In large samples, β_1 will equal $\sigma_{ab}/\sigma^2{}_a$, where σ_{ab} is the covariance between prices, and r^2 will equal $\sigma_{ab}/(\sigma_a\sigma_b)$. J. Johnston (1972), pp. 34-35.

exogenous variables are stationary:

(13) $\text{plim } r^2 = \sigma_{ab}/\sigma_a\sigma_b =$

$$[a_1 b_1 f(c_1)\sigma^2_\Theta] \; / $$
$$\{[a_1{}^2 f(c_1)\sigma^2_\Theta + a_2{}^2 f(d_1)\sigma^2_\mu + \sigma^2_e]^{1/2}[b_1{}^2 f(c_1)\sigma^2_\Theta + b_2{}^2 f(e_1)\sigma^2_\varphi + \sigma^2_v]^{1/2}\}$$

where $f(x) = (1 + x^2 + x^4 + \ldots)$

The remainder of this section demonstrates that the test statistic r^2 can be near zero even if the products a and b are in the same antitrust market, and near unity even if they are in different antitrust markets. It is evident from equation (13) that only in one situation can r^2 provide information about the crucial market definition parameters a_1 and b_1, namely when σ^2_Θ is large relative to $\sigma^2\mu$, σ^2_φ, σ^2_e, and σ^2_v. Even then, antitrust market definition will also require that the analyst know that the common variable affecting both prices is a cost shift variable, and that this variable is a market defining variable.

The likely value of r^2, the price level relationship test statistic, will be assessed under three alternative assumptions about the source of innovations, encompassing the complete set of significant influences on price variation in the model. As will be seen, each such assumption determines the value of r^2, yet only one bears any necessary or consistent relationship to an antitrust market identifying experiment, and even that assumption is not sufficient for the price correlation test to identify an antitrust market.

First, assume that σ^2_e is large (or, equivalently, that σ^2_v is large). This assumption presumes that there are many random price changes unrelated to shifts in exogenous variables. This will most plausibly happen if the oligopoly solution concept is unstable or if there are fluctuations in product quality or transaction lot size. Then, as equation (13) shows, r^2 will approach zero regardless of the result of the antitrust market definition identifying conceptual experiment, that is regardless of whether both prices rise in response to an increase in R if R is a market defining cost shift variable for products a and b. The price correlations analyst will infer that the two products are not in the same antitrust market, whether they are in the same market or not. In this situation, the two products may exert competitive discipline on each other, yet their prices will be imperfectly correlated because price changes are largely random.

Next, suppose that σ^2_μ or σ^2_φ is large. This will occur if most of the price variation is caused by variation in product individuated cost shift variables.[34] Again, equation (13) shows that the estimate of r^2 will likely be low, approaching zero, regardless of whether products a and b are in the same antitrust market. These products may exert competitive discipline on each other, yet their prices will be imperfectly correlated because no

[34]In a differentiated product industry, it is not implausible that individual firms have a slight amount of market power, sufficient for variations in firm-individuated costs to produce price changes, without having sufficient market power to constitute a collusive group on their own.

market defining experiment occured in the data.

Finally, suppose that σ^2_φ, σ^2_μ, σ^2_v, and σ^2_e are dominated by σ^2_Θ. This assumes that most price variation arises from variation in the common exogenous variable R. Equation (13) then implies that r^2 approaches unity in probability limit. The price correlation analyst will then conclude that the two products are in the same antitrust market. If R is a common demand shift variable, however, that conclusion could easily be erroneous, as the proper market definition experiment relies on common cost shift variables. This suggests the most likely error to arise from using price correlation tests for market definition: the price of shoes and the price of automobiles might be highly correlated in a period of rising income if the demand for both rises substantially as a result, yet the two products are not in the same antitrust market.

If instead R is a common cost shift variable, and if it affects the costs of both products comparably, the price correlation test properly implies that a and b are in the same antitrust market. However, the implied antitrust market is not limited to products a and b. It also includes all other products for which R is a cost shift variable. If the price of Budweiser and the price of Pabst are highly correlated because increases in bottling costs raise the marginal cost of both, fruit juice and other bottled products are also in the implied product market; we cannot infer from an r^2 of unity that the product market is limited to beer. If instead the prices are correlated because price variation results primarily from the changing price of hops, a likely market defining cost shift variable, the brewing industry forms an antitrust market.

Without imposing additional information, the price correlation analyst has no way of identifying the cost shift variable inducing the correlation, even if, somehow, he were aware that the correlation resulted from a common supply side variable rather than a demand side variable. He will therefore be unable to determine the breadth of the implied antitrust market if one is in fact suggested. Nor can the analyst know that the market so identified is the smallest antitrust market; the smallest such market may not involve the two products a and b even if they are in some larger market and have correlated prices as a result. The information needed to make these judgments is found in the reduced form price equation. If the analyst is willing to import such information into the price correlation analysis, he can use that information for market definition.[35] However, he could employ that same information more efficiently

---

[35]For example, if one were willing to assume that (a) all price variation for product a and substitutes for it arises from changes in input prices and other cost shift variables, and no price variation arises from movements in demand shift variables, and (b) random price variation for each product has the same variance, then prices of substitute products will be less correlated with product a as the other goods become less close substitutes. This would occur because the two goods would share fewer cost shift variables in their reduced form price equations as they become more distant substitutes. Yet even with these strong assumptions, equivalent to applying a substantial amount of outside information, the price correlation approach would not substitute for a residual demand analysis because it would not compel the line in the observed chain of substitutes that would define a market boundary.

and systematically by estimating the reduced form price equation directly and using it for market definition.

E. Price Difference Correlations

The potential for misleading market definition inferences is similar if price differences rather than price levels are correlated. This approach to market definition was first advocated by Horowitz.[36]

Define differences across products or regions D_t by the expression $(P^a{}_t - P^b{}_t)$. Then, assuming the structural representations of the reduced form price equations specified in equations (7) and (8):

(14) $D_t = (a_0 - b_0) + (a_1 - b_1)R_t + a_2S_t - b_2T_t + (e_t - v_t)$

The equivalent form (14') can be derived from (7') and (8'), assuming each exogenous variable evolves according to the stationary first order autoregressive processes defined previously.

(14') $$D_t = (a_0 - b_0) + (a_1 - b_1)c_0g(c_1) + a_2d_0g(d_1) - b_2e_0g(e_1) + (a_1 - b_1)h(c_1,\Theta_t) + a_2h(d_1,\mu_t) - b_2h(e_1,\varphi_t) + e_t - v_t,$$

where $g(x) = (1 + x + x^2 + x^3 + \ldots)$,

$$h(x,\pi_t) = (\pi_t + x\pi_{t-1} + x^2\pi_{t-2} + \ldots)$$

The typical market definition study employing price differences estimates a linear relationship between current and past price differences:

(15) $D_t = \alpha_0 + \alpha_1D_{t-1} + \tau$

[36] Horowitz (1981); see Uri, Howell & Rifkin (1985).

Here the α's are parameters and τ is a stochastic term.

Those employing price difference correlation analysis for market defiition interpret the regression (15) in the following way. If the products a and b are in the same market, the price difference correlation analyst argues that changes in D_t reflect the adjustment of prices to a new equilibrium following shocks. Under this view, if equation (15) correctly specifies the dynamics of equilibration, then the long run equilibrium price difference between P^a and P^b is $\alpha_0/(1-\alpha_1)$.[37] Further, under this view the parameter α_1 will lie in the open interval (-1,1) if the adjustment process to long run equilibrium is stable. The closer $|\alpha_1|$ is to unity, the speedier the adjustment.[38]

The regression suggested by Horowitz, equation (15), will be reinterpreted on the assumption that equation (14') holds. This interpretation of the evolution of D_t is sensible so long as the reduced form equations (7') and (8') characterize the evolution of equilibrium prices. This reinterpretation will demonstrate, in contrast to what is generally presumed in the literature based on Horowitz, that it is not necessary to postulate that any, some, or most market transactions reflect temporary disequilibria in order for equation (15) to constitute a stable empirical regularity.

The analysis begins with the presumption that equation (14') is correct, and uses that equation to interpret equation (15).

---

[37]If $ED_t = ED_{t-1}$, then $ED_t = \alpha/(1-\alpha_1)$.

[38]See generally, Uri, Howell & Rifkin (1985).

As all the stochastic terms in (14') have means of zero, $ED_t = ED_{t-1}$. Therefore, the parameter α_0 in equation (15) will approximate $(1-\alpha_1)(ED)$, where ED is the mean (expected value) of D_t.[39] Hence, there is only one piece of independent information in the two estimated coefficients of equation (15). Therefore, the remaining analysis will be limited to the slope parameter, α_1.[40]

Equation (14) can be used to demonstrate that the proposed market definition test, based on whether α_1 is in the open invterval (-1,1), is subject to the possibility of substantial error because α_1 will likely lie in that interval regardless of whether the two products whose price differences are correlated are in the same or different antitrust markets. The slope coefficient α_1 will be estimated by the ratio of the covariance of D_t and D_{t-1} with the variance of D_{t-1}. The large sample properties of this expression, derived from equation (14'), are

---

[39]This conclusion is robust to a variety of plusible model specifications in addition to the first order autoregressive processes underlying equation (14'). It holds so long as each exogenous variable in the economy evolves according to <u>any</u> moving average process, not merely the specific processes assumed in equations (9'), (10'), and (11'). It also holds whenever the price series P^a_t and P^b_t are filtered to remove their time trends before equation (15) is estimated, regardless of the process by which the exogenous variables evolve.

[40]Alternatively, if the price series P_a and P_b are first differenced before the regression is run, and if they have been filtered to remove their time trend as suggested by Uri, Howell, and Rifkin (1985), then ED will equal zero by construction so α_0 will be estimated as zero regardless of the estimate of α_1, and regardless of whether product a and b are in the same antitrust market. This point provides further justification for concentrating the analysis on the parameter α_1, ignoring α_0.

indicated by equation (16).

(16) $\text{plim } \alpha_1 =$

$$[(a_1^2 + b_1^2)c_1 f(c_1)\sigma^2_\Theta + a_2^2 d_1 f(d_1)\sigma^2_\mu + b_2^2 e_1 f(e_1)\sigma^2_\varphi] \;/$$
$$[(a_1^2 + b_1^2)f(c_1)\sigma^2_\Theta + a_2^2 f(d_1)\sigma^2_\mu + b_2^2 f(e_1)\sigma^2_\varphi + \sigma^2_e + \sigma^2_v]$$

where $f(x) = (1 + x^2 + x^4 + \ldots)$

As with the analogous discussion of price level correlations, any sign or size coefficient for α_1 is consistent with both hypotheses between which the analyst wishes to discriminate: that products a and b are in the same antitrust market or that they are in different markets. An analysis of equation (16) will demonstrate that α_1 can never provide information about the crucial market definition parameters a_1 and b_1. Even if it could, antitrust market definition would also require that the analyst know that the common variable affecting both prices is a cost shift variable, and that this variable is a market defining variable.

If random fluctuations (as might be created by instability of the oligopoly solution concept or fluctuations in transaction lot size or product quality) are the source of most price movement, so the expression $(\sigma^2_e + \sigma^2_v)$ is large, then plim α_1 will tend toward zero regardless of whether the two products a and b are in the same antitrust market.

If price changes largely result from variation in the innovations to the product individuated demand or cost shift variables S and T, so either σ^2_μ or σ^2_φ is large, then α_1 tends toward estimating the parameters d_1 or e_1, respectively. These parameters reflect the first order autocorrelation in S and T, so will lie in the open interval (-1,1) whenever S and T are stationary time series. In this case, the price difference analyst will conclude that α_1 satisfies the necessary condition for stability of equation (15) whenever S and T are stationary, a condition that is unrelated to the market definition question. For example, if d_1 or e_1 are less than but near to one, as is likely if the sample period is short and the time series of the exogenous variables exhibits stationarity, then α_1 will be estimated near one. Products a and b will meet the conditions postulated by Horowitz for competing in the same market regardless of whether they in fact compete in the same market.

Finally, if price changes result primarily from fluctuations in the common variable R, so that σ^2_Θ is large, then estimates of α_1 will tend toward c_1. Again, the price difference correlation measures stationarity of the time series for the common variable, not whether the common variable is a cost or demand shift variable. Even if the common variable is a cost shift variable, stationarity of its time series has no necessary or consistent connection with whether a and b are in the same antitrust market. Rather, it is necessary to know what the relevant variable R is and to determine whether R is a market defining variable before antitrust market definition can be undertaken with this information. Again, if c_1 is less than but near one, as is likely if the sample period is short, products a and b will pass the Horowitz test regardless of whether they truly compete in the same antitrust market.

Although the empirical literature on the Horowitz market definition algorithm is extremely limited, what does exist suggests that the problems with the price correlations approach described here are serious: the Horowitz test can readily produce inconsistent, misleading, or incorrect antitrust market definitions.[41]

E. Conclusion

This paper created a simple model in which price correlations, whether of levels or differences, provide little information on the extent of antitrust markets, as defined by the DOJ Guidelines. Errors can readily lie in either direction, improperly excluding products from the market or improperly concluding that an overbroad market is the smallest collusive group. Only when the source of the correlation is a market defining cost shift variable, affecting the costs of a small group of firms comprising a plausible market only, does the correlation provide information relevant to antitrust market definition. Unfortunately, the price correlation technique provides no way of discovering the source of the correlation; additional information, as from a residual demand analysis of

---

[41]Rogowsky & Shugart (1982), pp. 12-14. I understand that the antitrust enforcement agencies have obtained inconsistent results in applying the Horowitz approach to defining markets during law enforcement investigations: close substitutes may appear to be in different markets while distant substitutes appear to be in the same market.

market definition, must be employed. This demonstration suggests that antitrust market definition analyses based upon price correlation information, including the analyses advocated by Stigler and Sherwin (1985) and by Horowitz (1982), should not be relied upon. The best systematic (econometric) approach to antitrust market definition is the residual demand elasticity approach, which directly operationalizes the DOJ Guidelines definition.

Appendix

Reduced Form Equations and Residual Demand Curves for a Linear Duopoly Model

This appendix defines a linear duopoly model and uses it to derive linear reduced form equations for price and quantity and a residual demand curve.[42] The derivation of the reduced forms makes plausible the linear functional form for the reduced form equations used in the analysis of price correlations tests in the main body of this paper. Further, this Appendix derives the relation between the residual demand curve and the corresponding reduced form price equation in the linear model.

1. Derivation of Reduced Form Equations

[42]The model presented here is a special case of the general analysis of Baker & Bresnahan (1984). Unlike the present example, the model in that paper allows any number of firms, does not impose symmetry or linearity on demand or cost functions, and is not tied to any specific oligopoly solution concept.

Assume that a two firm industry se..ls differentiated products. The firms have symmetric demand curves, marginal cost curves that are linear in factor prices, and no fixed costs. The oligopoly solution concept is Nash in prices.

Let Q^a and Q^b represent firm outputs, and P^a and P^b represent prices. Y is a demand shift variable such as income, affecting both products. H and J are exogenous firm-individuated demand shift variables, perhaps related to consumer tastes.[43] The two structural demand curves have linear functional forms:

(A.1) $$Q^a = \alpha_0 - \alpha_1 P^a + \alpha_2 P^b + \alpha_3 Y + \alpha_4 H$$

(A.2) $$Q^b = \alpha_0 - \alpha_1 P^b + \alpha_2 P^a + \alpha_3 Y + \alpha_4 J$$

Each firm employs two variable factors of production: a common factor with price X, and a firm-individuated factor with price W (for firm 1) or Z (for firm 2). The two marginal cost curves are linear in factor prices but do not vary with output:

(A.3) $$C^a = \beta_0 + \beta_1 X + \beta_2 W$$

(A.4) $$C^b = \beta_0 + \beta_1 X + \beta_2 Z$$

Each parameter α_i and β_i is assumed to be a positive number. Further, the own price effect on demand is assumed to dominate the cross price effect: $|\alpha_1| > |\alpha_2|$.

Firm 1 earns profits equal to $(P^a - C^a)Q^a$. The behavior of a profit maximizing firm playing a Bertrand game can be determined by setting equal to zero the derivative of firm a's profits with

---

[43] H and J could represent the effect of firm advertising if that variable is statistically exogenous, as appears to be true for demand curves in the U.S. brewing industry. Baker & Bresnahan (1984), pp. 32-34.

respect to price, assuming no price reaction by firm b. The resulting first order condition, equating marginal revenue with marginal cost, and its analogue for firm b, appear below.[44]

(A.5) $\alpha_0 - 2\alpha_1 P^a + \alpha_2 P^b + \alpha_3 Y + \alpha_4 H = \alpha_1(\beta_0 + \beta_1 X + \beta_2 W)$

(A.6) $\alpha_0 - 2\alpha_1 P^b + \alpha_2 P^a + \alpha_3 Y + \alpha_4 J = \alpha_1(\beta_0 + \beta_1 X + \beta_2 Z)$

Equations (A.5) and (A.6) solve for the two reduced form equations for price, written in terms of the parameters, the exogenous cost shift variables X, W, and Z, and the exogenous demand shift variables Y, H, and J. These equations define the equilibrium for the model.

(A.7) $P^a = [(2\alpha_1 + \alpha_2)(\alpha_1\beta_0 - \alpha_0) + (2\alpha_1 + \alpha_2)\alpha_1\beta_1 X + 2\alpha_1{}^2\beta_2 W$
$+ \alpha_1\alpha_2\beta_2 Z - (2\alpha_1 + \alpha_2)\alpha_3 Y - 2\alpha_1\alpha_4 H - \alpha_2\alpha_4 J]/D$

(A.8) $P^b = [(2\alpha_1 + \alpha_2)(\alpha_1\beta_0 - \alpha_0) + (2\alpha_1 + \alpha_2)\alpha_1\beta_1 X + 2\alpha_1{}^2\beta_2 Z$
$+ \alpha_1\alpha_2\beta_2 W - (2\alpha_1 + \alpha_2)\alpha_3 Y - 2\alpha_1\alpha_4 J - \alpha_2\alpha_4 H]/D$

where $D = \alpha_2{}^2 - 4\alpha_1{}^2$

Comparable reduced form equations for quantity are derived by substituting equations (A.7) and (A.8) into structural demand curves (A.1) and (A.2). The resulting reduced form equation for the output of firm a appears as equation (A.9).[45]

(A.9) $Q^a = [\alpha_0 + ((\alpha_2 - \alpha_1)(2\alpha_1 + \alpha_2)(\alpha_1\beta_0 - \alpha_0)/D]$
$+ [(\alpha_2 - \alpha_1)(2\alpha_1 + \alpha_2)(\alpha_1\beta_1)/D]X$

---

[44]The second order conditions for an interior maximum require $\alpha_1 > 0$. For price and quanitity to be positive, it is necessary that $|\alpha_1| > |\alpha_2|$. These conditions are satisfied by assumption.

[45]In this model the reduced form quantity equation, unlike the reduced form price equation, excludes the firm-individuated cost shift variable for the other firm.

$$+ [(\alpha_1\alpha_2{}^2\beta_2 - 2\alpha_1{}^3\beta_2)/D]W$$

$$+ [(\alpha_1{}^2\alpha_2\beta_2)/D]Z$$

$$+ [\alpha_3 + ((\alpha_1 - \alpha_2)(2\alpha_1 + \alpha_2)\alpha_3)/D]Y$$

$$+ [\alpha_4 + \alpha_4(2\alpha_1{}^2 - \alpha_2{}^2)/D]H$$

The reduced form equations for price and quantity are noteworthy because they are linear in factor prices and income. In this way, a duopoly model with linear demand and with marginal cost linear in factor prices produces a linear reduced form equation for price. The text exploits the linearity of these reduced forms, but does not impose other restrictions resulting from the specific model solved here. In particular, cross equation restrictions on the reduced form equations -- generated as a result of the symmetry assumptions, specific functional forms assumed, and the oligopoly solution concept chosen -- are not imposed.

2. Derivation of Residual Demand Curve

Baker and Bresnahan (1984) have shown that the conceptual experiment of raising firm-individuated costs identifies the residual demand curve for the single firm. This section of the Appendix will rederive that result for the linear duopoly model in order to analyze the relationship between the reduced form price equations and the residual demand curves. That relationship is employed in the text to assess the relationship between the two market definition tools compared: price correlation tests and residual demand analysis.

The residual demand curve for firm a is defined as the demand curve that takes into account the response of the firm's rivals. As a result, it takes into account all the market mechanisms imposing competitive discipline on a firm's ability to exercise market power: the effect of demand substitutability to products outside the provisional market, the nature of rivalry among producers in that market, and the tempering effect of the prospect of new entry on the behavior of incumbent producers. In the linear duopoly model, the residual demand curve for firm a is derived from the structural demand curve for firm a, equation (A.1), by substituting in the reactions of firm a's rival:

(A.10) $Q^a = \alpha_0 - \alpha_1 P^a + \alpha_2 P^b(P^a, C^b) + \alpha_3 Y + \alpha_4 H$

In equation (A.10), the expression $P^b(P^a, C^b)$ is implied by equation (A.6), the first order condition defining the behavior of firm b.[46] Therefore, the residual demand curve facing firm a

[46]This definition of the residual demand curve incorporates the second firm's <u>actual</u> reaction function. That function will not equal the first firm's <u>perception</u> of that reaction unless the first firm is a Stackelberg leader with respect to its environment. Baker & Bresnahan (1984) show that the residual demand elasticity allows the inference of markup if the distinction between perceived and actual reactions can be ignored. When that distinction is important, a residual demand curve with downward slope implies the presence of market power, but does not allow the inference of markup. In the linear duopoly model solved above, the two firms achieve a Bertrand equilibrium, so the difference between perceived and actual reaction functions is important. As a result, the residual demand elasticity, that is, the elasticity of quantity with respect to price in equation (A.11), will not allow the inference of firm a's behavior (markup). If instead firm a were a competitor, were a dominant firm, were in an industry characterized by extreme product differentiation, or were in an industry which has achieved a Consistent Conjectures Equilibrium, firm a's markup could have been been inferred from the elasticity of residual demand.

can be written

$$(A.11) \quad Q^a = [\alpha_0 + ((\alpha_0 - \alpha_1\beta_0)\alpha_2)/(2\alpha_1)] + [-\alpha_1 + (\alpha_2{}^2)/(2\alpha_1)]P^a + \alpha_3 Y - [(\alpha_2\beta_1)/2]X - [(\alpha_2\beta_2)/2]Z + \alpha_4 H$$

The slope of the firm a's residual demand curve, which defines whether that firm possesses market power, is $[-\alpha_1 + (\alpha_2{}^2)/(2\alpha_1)]$, the coefficient of own price in equation (A.11). If this expression grows (negative) without limit, then the combined efforts of consumer demand substitution and rival response force firm a to act as a price taker. If the expression is a finite negative number, firm a faces a downward sloping demand curve even after these competitive forces are taken into account.

This example demonstrates an important relation between the reduced form price equation and the residual demand curve relied upon in the text: the slope of the residual demand curve (A.11) equals the ratio of the partial derivative of the reduced form equation for quantity (A.9) with respect to the firm-individuated exogenous cost shift variable W, or $(\delta Q^a/\delta W)$, to the partial derivative of the reduced form equation for price (A.7) with respect to W, or $(\delta P^a/\delta W)$. As an econometric matter, this procedure for identifying the slope of the residual demand curve works because W is an excluded exogenous variable from the point of view of estimating the residual demand curve (11).[47]

---

[47]This econometric identification issue is considered in detail in Baker & Bresnahan (1984). The residual demand curve model is completed, and equilibrium defined, by a supply relation

of the form $P^1 = C^1(Q^1) - [P^1(Q^1) - MR^1(Q^1)]$, where $P^1(Q^1)$ is the residual demand curve and $MR^1(Q^1)$ is marginal to the residual demand curve. (This supply relation is a transformation of the familiar equilibrium condition equating marginal revenue with marginal cost.) As the supply relation depends upon both marginal revenue and marginal cost, it incorporates all exogenous variables found in either the demand or cost curve.

As Y, H, J, Z, and X are not excluded exogenous variables from the point of view of estimating the residual demand curve for firm a, the ratio of dervivatives of the two reduced form equations with respect to these variables will not identify the slope of the residual demand curve. For example, H, an exogenous demand shift variable, does not identify the slope of the residual demand curve. Rather, equations (A.7) and (A.9) imply that $[\delta Q^1/\delta H]/[\delta P^1/\delta H] = \alpha_1$. (This expression contains no parameters of the marginal cost curve because the example of this Appendix presumes that marginal cost does not depend on output.)

When the residual demand curve (A.11) is estimated directly, consistent estimates of the coefficient of P^1 can be obtained in the usual way, by using the firm-individuated cost shift variable W as an instrumental variable, without need to estimate the two reduced form equations and take the ratio of the appropriate coefficients.

REFERENCES

ABA Antitrust Section (1986), Horizontal Mergers: Law and Policy.

Baker, J. and T. Bresnahan (1984), Estimating the Elasticity of Demand Facing a Single Firm: Evidence on Three Brewing Firms, Research Paper No. 54, Stanford Workshop on Factor Markets.

Baker, J. and T. Bresnahan (1985), The Gains from Merger or Collusion in Product-Differentiated Industries, Journal of Industrial Economics, 33, pp. 427 - 444.

Giffin, P. and J. Kushner (1982), Market Definition in Antitrust Analysis: Comment, Southern Economic Journal, 49, pp. 559 - 562.

Haavelmo, T. (1943), The Statistical Implications of a System of Simultaneous Equations, Econometrica, pp. 1- 12.

Horowitz, I. (1981), Market Definition in Antitrust Analysis: A Regression-Based Approach, Southern Economic Journal, 48, pp. 1-16.

Horowitz, I. (1982), Market Definition in Antitrust Analysis: Reply, Southern Economic Journal, 49, pp. 564 - 566.

Johnston, J. (1972), Econometric Methods (2nd. ed.).

Landes, W. and R. Posner (1981), Market Power in Antitrust Cases, Harvard Law Review, 94, pp. 937 - 996.

Rogowsky, R. and W. Shugart (1982), Market Definition in Antitrust Analysis: Comment, Working Paper No. 77, Bureau of Economics, Federal Trade Commission.

Scheffman, D. and P. Spiller (1985), Geographic Market Definition Under the DOJ Merger Guidelines, Working Paper No. 129, Bureau of Economics, Federal Trade Commission.

Spiller, P. and C. Huang (1986), On the Extent of the Market: Wholesale Gasoline in the Northeastern United States, Journal of Industrial Economics, 35, pp. 131-145.

Stigler, G. and R. Sherwin (1985), The Extent of the Market, Journal of Law and Economics, 28, pp. 555 - 585.

U.S. Department of Justice Merger Guidelines, June 14, 1984.

Uri, N., J. Howell, and E. Rifkin (1985), On Defining Geographic Markets, Applied Economics, 15, pp. 959 - 977.