Volume Discounts, Loss Leaders,  
and Competition for More Profitable Customers

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When some customers are more profitable to serve than others, one might expect sellers to compete more vigorously for the more profitable customers. One way sellers might do this is to sell goods that are purchased primarily by the most profitable customers using a lower mark-up than on other goods. This allows the firms to give discounts to more profitable customers without offering them to less profitable customers.

This paper presents a model in which competing multi-product firms facing customers that purchase different quantities of goods, set prices in this manner. This model suggests a theory of multi-product pricing in which the markup on any particular product is inversely related to the average profitability of the customer that purchases the good.

One interesting implication of this paper is that loss leader pricing might be viewed as a way for firms to compete more vigorously for more profitable customers. Such an explanation offers another characteristic of products that should be used as loss leaders. This explanation provides potentially testable implications about the types of goods that can (or ought to) be used as loss leaders and can explain why grocery stores sell turkeys as loss leaders at Thanksgiving and lower the price of eggs at Easter, but not flowers on Mother's Day, or candy on Valentine's Day.

I thank Dave Reiffen for first introducing me to this topic as well as discussions about possible explanations. I also thank Cindy Alexander, Kyle Bagwell, Robert Frank, Robert Gibbons, Joe Harrington, Robert Mason and Michael Waldman for helpful comments. The views expressed in this paper do not necessarily reflect the views of the Commission.
1. Introduction

Anyone who has purchased a turkey at Thanksgiving probably has noticed that at the time of year when demand is the greatest, the prices of turkeys are at their lowest. Discussions with a number of economists over the years about why this is true have primarily yielded the explanation that grocery store must be pricing the turkey as a "loss leader."

"Loss leader pricing is the practice of setting prices on selected products at low levels that generates less than the usual profit margins... For retailers the objective is to increase store traffic so they can sell other products at traditional profit margins... Products that are used as loss leaders are usually well known brands and frequently purchased." Thus the traditional loss leader story suggest that customer choose the store at which they shop based on the price of one good, and then purchase that good along with others, (on which store owners earn their profit).

This paper offers a somewhat different explanation for loss leader pricing. Rather than being priced to attract all shoppers, loss leaders can (or should) be used as a way of competing for higher profit customers (that shop at a particular store) by offering them discounts that are not available to less profitable customers. Thus, loss leader pricing can be a way of price discriminating among groups of customers in a competitive setting. This explanation, while not inconsistent with the traditional explanations, implies that it is neither necessary nor sufficient for a product to be sold under a well known brand, nor purchased by many customers (as suggested above). Rather a product could be priced as a loss leader if, in a market in which some customers purchase bundles of products that are more profitable than bundles purchased by others, the product is purchased primarily by customers that purchase more profitable bundles. The reason is that, for more profitable customers, sellers have an incentive to compete more vigorously to keep them from purchasing another seller. I will provide conditions under which loss leader pricing is an equilibrium way to offer such a discount in competition.

Applying the explanation to turkey pricing, I will suggest that turkeys are effective loss leaders, not only because almost every customer purchases a turkey at Thanksgiving, but because the customers that do purchase turkeys, on average purchase more units of other

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1 See Busch and Houston (1985) pg. 498
2 It is not the absolute quantity of purchase that creates the incentive for a discount, but rather the relative difference across customers at a store. Thus, both grocery stores and convenience stores can offer quantity discounts even though the absolute quantity of low volume users at grocery stores is likely to exceed the quantity of high volume purchasers at convenience stores.
goods than do customers shopping at the same time that do not buy turkeys, and are thus more profitable. This explanation differs in a very important and potentially testable way from the traditional loss leader story. The logic of the traditional loss leader story suggests that (if turkeys are used as loss leaders at Thanksgiving then) we should expect to see candy or flowers to be priced as loss leaders before Valentine's Day or Mother's Day. However, we typically do not see such discounts. My explanation is consistent with this observation because while a purchaser of a turkey just before Thanksgiving is very likely to purchase more items than the average grocery shopper, a purchaser of candy before Valentine’s Day is not. \(^4\) Therefore, one should expect to see stores offering a discount in the form of discounted turkey prices at Thanksgiving, but no discount in the form of discounted candy prices around Valentine’s Day.

There are two other papers, Simester (1996) and Nagle and Novak (1988) that suggest a price discrimination explanation for loss leader pricing. Each of these papers looks at competition between two sellers in a market with three types of customers, those that are loyal to (will always buy from) the first seller regardless of the price he charges, those that are loyal to (will always buy from) the second seller regardless of the price he charges, and those that will price shop and purchase from the store whose prices offer them the higher customer surplus. They then hypothesize that there are some products that are purchased more often by price shoppers than by loyal customers. In such a case it can be shown that these products will be priced as loss leaders.

As I explain in the next section, this explanation is fundamentally different from the explanation proposed in this paper. For now I simply note that for these explanations to explain why turkeys are priced as loss leaders at Thanksgiving, but candy is not priced as a loss leader before Valentine’s Day, one must believe that the customers that buy turkeys at Thanksgiving are primarily customers that choose to shop at the store offering the most surplus, while those that buy candy before Valentine’s are primarily customers that are loyal to a particular store.

I present several models that look at the pricing of a "target" good (the turkey in our opening example) when the demand of other goods (called side goods) changes. I do this for three reasons. First, it helps explain the use of "seasonal" loss leaders such as turkeys at

\(^3\) If the margin on each product is roughly the same, more profitable customers will be large volume customers.

\(^4\) Hosken and Reiffen (2002) report that eggs typically have a lower price at Easter than at other times. This is consistent with the hypothesis that the average person purchasing large number of eggs at Easter is likely coloring them for their children, and would likely be buying groceries for a family and is therefore a larger than average customer. Chevalier et al (2003) find empirical evidence that lower prices during peak demand period is consistent with loss leader pricing.
Thanksgiving. Second, it is the prediction that loss leader pricing will be used when the demand for other goods is high and that it won’t when such demand is low that distinguishes this theory from the other theories in the literature. That is, my theory predicts that when overall demand for other goods increases for a group of customers, the price of goods bought primarily by these customers will fall. None of the other theories make this prediction. Finally, and most importantly it highlights the more general principle that applies to the more traditional or everyday loss leaders such as diapers and milk, which is that some goods are priced as loss leaders (i.e. close to or even below cost) on a regular basis because customers that purchase them regularly are more profitable than the average customer.

Therefore this theory is not just a theory of loss leader pricing during holidays, rather it is a general addition to the theory of pricing based on a correlation between the total profitability of the bundle of goods purchased by consumers and the propensity of them to purchase a particular good. I use the "holiday setting" as an explanatory device because the change in price of the "holiday good" resulting from a change in the volume of demand highlights the theory, and provides a potential test.

This paper offers another contribution. Recently there has interest in explanations for why seller would grant customers quantity discounts.\(^5\) This paper offers one simple explanation. When the distribution of taste across small volume purchaser is the same as the distribution of taste across large volume users, each firm’s demand from large volume users is more elastic, with respect to quantity\(^6\) than the demand from small volume customers. This results in lower per unit prices for larger volume customers.

2. Literature Review

2.1. Loss Leaders

There is a growing literature studying the phenomena of loss leader pricing. Hess and Gerstner (1987) argue that when customers purchase impulse goods (good purchased by a customer that does not compare the price at other stores), a store may price a shopping good (a good for which a shopper compares price) below marginal cost in order to attract customers who then purchase impulse goods.

Bagwell and Ramey (1994) provide a coordination explanation for loss leader pricing.


\(^6\) The elasticity with respect to customers is identical in this model.
They conjecture that some stores can exploit economies of scale in retailing better than other stores. Then, if such a store obtains many customers, it can offer lower prices because it realizes economies of scale. For this to happen, customers must know which store can take advantage of the economies of scale (thus the need to coordinate). Loss leader pricing (or loss leader pricing along with dissipative advertising) can credibly signal which store has the economies of scale and act as a coordination device.

Lal and Matutes (1991) present a model in which two stores choose how many products to sell, which products to advertise (if any) and what prices to set for each good. Customers can choose the number of stores at which to shop and which good(s) to purchase at each store and can only know the price of an item if it is advertised. They show that there exists an equilibrium in which each store chooses to advertise only one good, which is sold below cost (the loss leader) and that customers make their "shopping decisions" based on this information and rationally formed expectations regarding the unadvertised price(s). In their equilibrium customers purchase the loss leader at a price below cost, but all other goods at a price equal to their reservation value.

Simester considers a market in which there are two sellers. Each seller has a set of customers that is loyal to him. Loyal customers are those that always buy from the same seller regardless of the price, although the quantity they purchase varies inversely with price. There is also a group of customers called price shoppers that will purchase from which ever seller's prices offer the customer the greater surplus. In this market setting Simester shows that in general goods that are priced as loss leaders are those that are purchased primarily by price shoppers and those for whom demand is most elastic.

Nagle and Novak (1988) also hypothesize the existence of price shoppers over whom stores compete as well as a set of "convenience shoppers" associated with each store. They argue that there are certain demographic characteristics such as the number of children in a family, income, number of adults that work, etc., that will make a shopper more likely to be either a price shopper or a convenience shopper. Using grocery store data they then provide empirical evidence suggesting that items most often purchased by customers with demographic characteristics that make them more likely to be price shoppers have lower mark ups than items that are not.

Perhaps a useful way to distinguish the model in this paper from those in Simester (1996) and Nagle and Novak (1988) is that in their works, some customers are more
responsive to a price decrease (in terms of switching sellers) than other customers. That is, a one dollar price reduction offered to price shoppers generates more additional customers than a one dollar price reduction offered to loyal customers (which generates no extra customers). In our model a one dollar price reduction offered to low volume customers will cause just as many low volume customers to switch sellers as will a one dollar price reduction offered to high volume customers. The difference is that high volume customers (all else equal) are potentially more profitable to a seller than a low volume customer so sellers are willing to offer them deeper discounts to steal them from the competing seller.\(^7\)

There are two other works related to loss leader pricing. Denekere, Marvel, and Peck (1993) show that a manufacturer might not want its good to be priced as a loss leader because in a market with uncertain demand, all retailers might not choose to stock the good. There analysis however does not explain why a retailer would choose to sell a particular good as a loss leader.\(^8\) Walters and McKenzie (1988) empirically study the effects of loss leader marketing on firm profit. They show that firms make the same profit when they use loss leader pricing as when they don't and that the use of a loss leader does not significantly increase the amount of customer traffic. As we will discuss later, our theory is consistent with both of these findings.

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\(^7\) As we will see competition will dissipate all of the additional potential profits.

\(^8\) In their model the retailer only sells one type of good. Thus, the model cannot capture the decision regarding which good to sell as a loss leader.
2.2. Quantity Discounts

Recent work explaining why large buyers can command lower prices fall into three main categories. First work by Snyder (1996, 1998) suggests that when sellers tacitly collude, it is more difficult to prevent a firm from shading price with respect to a large customer than with respect to a small customer. Thus colluding firms would set lower prices to large customers to reduce the incentive any one firm would have to lower price to a large firm. My results differ from those of Snyder in that there is no collusion between competing firms. Rather all prices are Nash equilibrium prices of a one shot game.

Second work by Horn and Wolinsky (1988) Stole and Zwiebel (1996) Chipty and Snyder (1999) and Chae and Heidhues (1999a, 1999b) look at bargaining between a monopolist and different size sellers. In these models joint surplus between buyers and the seller is increasing but strictly concave in total output. Each buyer views himself as the marginal buyer and so bargains over the marginal surplus given that all other buyers have completed their bargains. It is assumed that the sell and buyer split the (perceived) surplus, (a la the Nash bargaining solution). The surplus retained by the seller is interpreted as a payment from the buyer to the seller. Because the surplus function is concave, the average surplus per unit of output is smaller for large buyers than for small buyers. Thus when the surplus is split, the seller receives a lower surplus per unit than he receives from small buyers. This lower surplus is interpreted as a quantity discount.

My results differ from these results in that because all customers inelastically demand a fixed number of units, and marginal cost is constant, the joint surplus between a firm and customers is linear. In addition these results rely in the assumption that buyer and seller divide evenly the surplus from a transaction. In my work the division of surplus is determined endogenously.

A third explanation DeGraba (2003) is that sellers who are risk averse and cannot view each customer’s valuation for a good but can observe the number of units they demand find larger customers riskier than smaller customers. To mitigate some of the risk associated with larger customers, sellers offer them lower prices. In the paper that follows I rely neither on risk averse sellers nor incomplete information.

3. The Basic Model

I now present a model of competition between two competing multiproduct firms and
compute equilibrium prices for their products. I will compare prices of products in a “normal” period to those in a test period. I will define loss leader pricing as a price reduction on a given good during the test period\(^9\), and look for characteristic of goods that will be priced in this manner.

Consider two firms indexed by \(i \in \{0, 1\}\) located at opposite ends of a Hotelling line of length 1. Customers are located uniformly along the line and have linear travel cost normalized to 1 per unit traveled. Customers demand two goods, a “target” good (denoted \(t\)) and a side good, (denoted \(s\)). These goods are produced at zero marginal cost by the two firms.

On a "normal week" each customer demands one unit of the target good and \(s_n\) units of the side good. On a test or "holiday week" each customer purchases one unit of \(t\) and \(s_h\) units of \(s\). Assume \(s_h > s_n\) to capture the notion that customers purchase more in preparation for the "holiday" than they do during a normal week.

Given this structure I construct the following game. Firms initially set the prices of the two goods, \(P_{ti}\) and \(P_{si}\) simultaneously. Customers observe \(P_{ti}\) and \(P_{si}\) and purchase from the firm at which the total cost of purchasing, \(P_{ti} + s_w P_{si}\) is lower, where \(w \in \{n, h\}\). The firms’ payoffs are their profits.

In proposition 1 I compare the equilibrium prices charged by each firm during a normal week to the prices charged on a holiday.

**Proposition 1.** Firms charge a lower per unit price during a holiday week than during a normal week.

**Proof:** Let \(z^*\) be the location of the customer purchasing \(s_w\) units of \(s\) (and 1 unit of \(t\)) who is indifferent between purchasing from firm 0 and firm 1. \(z^*\) satisfies

\[
P_{t0} + s_w P_{s0} + z^* = 1 - z^* + P_{t1} + s_w P_{s1}.
\]

Thus firm \(i\)'s objective function is

\[
\Pi_i = (P_{ti} + s_w P_{si})(1 + (P_{i,i} + s_w P_{s,i} - P_{ti} - s_w P_{si}))/2
\]

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\(^9\) More generally, one could calculate the Lerner indices for all equilibrium prices and interpret the products with the lowest index as the loss leader. This would allow for product with small but positive margins to be considered loss leaders.
where -$i$ indicates firm $i$'s competitor. The first order conditions indicate that

$$P_{ti} + s_w P_{si} = 1.$$  \hspace{1cm} (3)

Since $s_h > s_n$, firms charge a lower per unit price during holidays weeks than during normal weeks. \textit{QED}.

This result establishes the idea that when demand per customer increases, competing firms have an incentive to lower their per unit prices in an effort to prevent customers at the margin from purchasing from their competitors. To understand this note that for any given level of customer demand, the profit maximizing price is set such that a marginal decrease in the prices of a seller's goods results in an increase in profits from sales to additional customers that just offsets the decrease in profit due to the reduction in the price/cost margin. When demand per customer increases, the marginal customer becomes more profitable (as long as he buys units whose price is greater than marginal cost) but the cost of the price decrease (the lower price/cost margins) needed to attract him does not change. Thus, after demand per customer increases, firms would strictly prefer to lower their per unit prices, which effectively amounts to a volume discount.\textsuperscript{10}

An alternative explanation for a monopolist's use of volume discounts is to discriminate among different customers who have differing valuation for the marginal unit. We use the Hotelling model to rule out this explanation since all customers demand all units inelastically. Thus, the use of volume discounts in this model is driven entirely by the fact that as customers’ demand increases they become more profitable at the margin so sellers have a greater incentive to lower price.

In proposition 1 the additional price competition caused by higher per customer demand dissipates the entire potential profit from the increase in demand. Thus, customers have the same total payment during normal weeks and holiday weeks, even though they purchase more during the holiday week. The equality of payments across weeks is an artifact of the assumptions that i) quantity is demanded inelastically and ii) the increase in demand per customer occurs uniformly along the line.

\textsuperscript{10} Lal and Matutes 1991 make a similar observation.
The effect of i) can be seen in a model in which the quantity demanded is strictly decreasing in price. In this case it can be shown that a customer's total payment in a holiday week is greater than his payment in normal weeks. I chose the Hotelling framework specifically to avoid this effect. The model highlights the fact that an increase in demand per customer results in lower prices precisely because firms compete more vigorously over the marginal customer, and is independent of the fact that customers buy more at a lower price.

To understand the effects of ii) note that if demand per customer increased only for those customers near the "ends" of the Hotelling line, then equilibrium prices would be higher, and a customer's total payment in a holiday week would be greater than his payment in a normal week. Similarly, if demand per customer increased only for customers in the "middle" of the line then equilibrium prices would be lower, and the total payment in a holiday week would be less than the payment in normal weeks.

This result is particularly noteworthy since in many models an increase in demand will cause an increase in price. This difference is due to the fact that in most models (where equilibrium price is greater than marginal cost) an increase in demand causes the demand curve to be more inelastic leading to an increase in price. In this model an increase in per customer demand causes each firm's demand curve with respect to each item's price to be more elastic (for reasons discussed above) leading to lower equilibrium per unit prices.

While these results indicate that overall prices will fall when per customer demand increases, the previous model with only one type of customer, does not indicate which prices will fall. In the next section I introduce more than one type of customer and show that the distribution of customer types will uniquely determine which prices decrease as a result of a per customer increase in demand.

4. Multiple Customer Types

In the previous section I showed that an increase in demand per customer will lead to discounted prices. I now show why at Thanksgiving stores would offer this discount by reducing the price of turkeys as opposed to lowering the price of all goods. Essentially I will

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11 In models where \( P = MC \) (i.e. perfect competition) the increase in price is not driven by changes in demand elasticity but because the market equilibrium simply move up the supply curve.

12 When a customer buys good \( A \) and \( B \) in the same shopping trip, an increase in the per customer quantity demanded of good \( A \), makes the demand for \( B \) faced by each firm effectively more price elastic because lowering the price of \( B \) now generates more revenue from the sale of \( A \).
argue that on a given holiday week customers that buy a turkey will purchase more side goods than those that do not buy a turkey. Thus, by offering a discount on turkeys, sellers can give discounts to those who buy large quantities while charging small buyers higher prices.

I expand the model in the previous section as follows. There are two types of customers \( T \) and \( S \). Type \( S \) customers demand \( s_S \) units of \( s \) on both normal weeks and holiday weeks and never buy \( t \). They are distributed uniformly along the line and have mass \( m_{Sn} \) during a normal week and \( m_{Sh} \) during a holiday week. Type \( T \) customers always buy one unit of \( t \), \( s_T \) units of \( s \) during a normal week and \( s_{Th} \) units during a holiday week. Type \( T \)'s are uniformly distributed along the line and have mass \( m_{Tn} \) during a normal week and \( m_{Th} \) during a holiday week. Assume \( s_{Th} = s_S - 1 \) so that during a normal week type \( T \)'s and \( S \)'s purchase the same number of units. Let \( c_t \) be the marginal cost of producing the target good and \( c_s \) be the marginal cost of producing the side good.

**Proposition 2.** If \( s_{Th} = s_S \) then sellers offer a volume discount by lowering the price of \( t \). Further \( P^*_{it} \) is a decreasing function of \( s_{Th}/s_S \) and is independent of the mass of type \( S \)'s and type \( T \)'s.

**Proof:**

The objective function of firm \( i \) is

\[
\Pi_i = (P_{ti} - c_t + s_{Tw}(P_{si} - c_s))(1/2)(1 + P_{ti} + s_{Tw}P_{si} - P_{ti} - s_{Tw}P_{si})m_{Tw} + (s_sP_{si} - c_s)(1/2)(1 + s_sP_{si} - s_sP_{si})m_{Sw},
\]

(4)

The first order conditions are

\[
\frac{\partial \Pi_i}{\partial P_{ti}} = \{(1 + P_{ti} + s_{Tw}P_{si} - P_{ti} - s_{Tw}P_{si}) - (P_{ti} - c_t + s_{Tw}(P_{si} - c_s))\}m_{Tw} = 0 \quad (5.a)
\]

\[
\frac{\partial \Pi_i}{\partial P_{si}} = \{(1 + P_{ti} + s_{Tw}P_{si} - P_{ti} - s_{Tw}P_{si})s_{Tw} - (P_{ti} - c_t + s_{Tw}(P_{si} - c_s))s_{Tw}\}m_{Tw} + \{(1 + s_sP_{si} - s_sP_{si})s_s - (s_sP_{si} - c_s)s_s\}m_{Sw} = 0, \quad (5.b)
\]

which reduce to

\[
P^*_{ti} - c_t + s_{Tw}(P^*_{si} - c_s) = 1 \quad (6.a)
\]
\[ s_S(P_{si}^* - c_i) = 1. \] (6.b)

Since \( s_{Th} = s_S - 1 \), the price/cost margin during a normal week is \( 1/s_S \). During a holiday week (6.b) implies the price cost margin on \( s \) is still \( 1/s_S \). \( s_{Th} = s_S \) in (6.a) implies \( P_{ni}^* - c_{ni} = 0 \), yielding the result that the discount is granted solely by reducing the price of \( t \). The fact that \( P_{ni}^* - c_{ni} \) is decreasing in \( s_{Th}/s_S \) obtains immediately from substituting

\[ (P_{si}^* - c_i) = 1/s_S \] into (6.a). The lack of dependence of \( P_{ni}^* \) on \( m_{Tw} \) and \( m_{Sw} \) is immediate from (6.a). \( QED. \)

To understand why the discount is given solely by discounting \( t \), simply note that (as proposition 1 suggests) sellers wish to give discounts relative to the normal week’s prices only to high volume customers (type \( T \)'s during a holiday week). Since all high volume customers purchase \( t \), the discount can be given to them without being given to the low volume customers.

The fact that \( P_{ni}^* \) is decreasing in \( s_{Th}/s_S \) simply reflects the fact that the greater the number of units of \( s \) demanded by a type \( T \) customer (at a mark up of \( 1/s_S \)) the more profitable he is at the margin. Thus competition causes sellers to offer a deeper discount in order to prevent him from switching sellers.

The mass of customers purchasing, \( m_T \), has no affect on the price of the loss leader (or even if the good should be used as a loss leader) in part because in this model an increase in the number of customers uniformly across the line has no effect on the elasticity of demand. This is because such an increase in the number of customers increases that quantity sold by the same proportion that it decreases the slope of the demand curve facing the seller.

The lack of dependence of \( P_{ni}^* \) on \( m_T \) and \( m_S \) is also due to the fact that all high volume customers and none of the low volume customers purchase the target good. Note that in this case the monopolist's profit during the holiday week is the same as during the normal week, as found in Walters and McKenzie (1988). This is because the competition for the higher volume customers in the form of the loss leader price discount completely dissipates the potential additional profits from the higher per customer demand. Of course, in this model the traffic cannot increase since the number of customers is fixed. However, the fact that in such a model loss leader pricing is used in equilibrium does highlight the fact that loss leader pricing can be used by each firm to steal customers from competitors, rather than generate new...
customers in the market.

I now consider pricing when some high volume and some low volume customers purchase the target good. As before the total number of those purchasing the good has no effect on the price, however the proportion of high volume to low volume customers will have an effect on $P_{ni}^*$. 

I expand the model of this section in the following manner. Assume that during a holiday week there are two types of type $T$ customers, high volume customers and low volume customers. High volume customers purchase $s_{Thh}$ units of the side good during a holiday week and low volume customers purchase $s_{Thl}$, where $s_{Thh} > s_{Thl}$. High and low types have mass $m_{Tl}$ and $m_{Th}$ respectively and each are distributed uniformly along the line.

**Proposition 3.** The price of the loss leader is a decreasing function of the proportion of type $T$ customers that purchase high volume.

**Proof:** The objective function for firm $i$ during the holiday week is

$$
\Pi_i = (P_{ni} - c_i + s_{Thh}(P_{si} - c_i))(1/2)(1 + P_{t-i} + s_{Thh}P_{si} - P_{ni} - s_{Thl}P_{si})m_{Th} +
\left(P_{nl} - c_i + s_{Thl}(P_{si} - c_i))(1/2)(1 + P_{t-i} + s_{Thl}P_{si} - P_{nl} - s_{Thh}P_{si})m_{Thh} +
(s_{Sh}P_{si} - c_i)(1/2)(1 + s_{Sh}P_{si} - s_{Sh}P_{si})m_{Sh}
\right)
$$

and arithmetic similar to that of proposition 2 yields

$$
P_{ni}^* - c_i = 1 - \frac{(s_{Thh}^2m_{Th} + s_{Thh}^2m_{Th})/s_{Sh}}{(s_{Thh}m_{Th} + s_{Thh}m_{Th})}.
$$

Since both the numerator and the denominator of (8) are weighted sums of $m_{Tl}$ and $m_{Th}$, $P_{ni}^*$ is a function of the ratio $m_{Tl}/m_{Th}$ and since $s_{Thh} > s_{Thl}$, it is decreasing.

The intuition behind this result is an extension of the intuition developed in proposition 1. In equilibrium competition causes sellers to give discounts to large (and therefore more profitable) buyers, but not to the small buyers. Thus the greater the proportion of type $T$'s that are low volume customers, the more costly it is for the seller to offer volume discounts to the high volume customers, thus the smaller the discount.

One might ask why a store would offer a volume discount implicitly through a reduction
in the price of one good rather than explicitly by announcing a discount schedule that is a function of the quantity purchased. There are three potential reasons. First customers that were willing to make many small purchases over time and pay a high price, might find it in their interest to consolidate purchases to take advantage of the discount, thereby reducing the store's profits. The second is that low volume customers could potentially combine shopping to qualify for the volume discount. Offering the discount on a good prevents this behavior.

Third, in a market in which products have different marginal costs and different markups, discounts based on explicit quantity or revenue levels may be as imperfect a method of discrimination as loss leader pricing. Sellers would like to discount based on the profitability of each customer’s purchases. It would be difficult to establish a pricing policy that is clear to customers that is based on profit margins since customers cannot observe marginal cost. A plan that gives a discount based on total value of the grocery bill will be an imperfect approximation of profit if some customers buy large baskets of goods with small profit margins, while others purchase smaller baskets of goods with higher mark-ups. Thus, it is not clear that an explicit quantity discount based on revenue would necessarily better target profitable customers than a well chosen loss leader.

5. Most frequently purchased product.

As stated earlier a popular belief is that a loss leader should be a product that most people purchase. I now present a model that shows that being the most frequently purchased product by itself is not enough to cause a product to be priced as a loss leader.

Again consider two firms located at opposite ends of a Hotelling line. There are 3 goods that customers can buy, $T$, $s_1$, and $s_2$. The marginal costs of producing these goods are $c_t$, $c_{s1}$ and $c_{s2}$. There are three types of customers each of which is distributed uniformly along the line. The first type buys one unit of $T$ and one unit of $s_1$. The second type buys one unit of $T$ and one unit of $s_2$. The third type buys one unit of $T$, one unit of $s_1$ and one unit of $s_2$.

As before firms set the prices of each good non-cooperatively. Proposition 4 shows that even though all customers purchase a unit of $T$, it is not priced as a loss leader.

**Proposition 4.** In this market $T$ is the only good with a positive mark-up.

**Proof:** Each firm faces a profit function of:

$$
\Pi_i = \left( P_{ti} - c_t + (P_{s1i} - c_{s1}) \right) \left( 1 + P_{t-i} + P_{s1-i} - P_{ti} - P_{s1i} \right) + 
$$
\[(P_t - c_t + (P_{s2i} - c_{s2}))(1/2)(1 + P_{t-i} + P_{s2-i} - P_{s2i}) + \]
\[(P_t - c_t + (P_{s1i} - c_{s1}) + (P_{s2i} - c_{s2}))(1/2)(1 + P_{t-i} + P_{s1-i} + P_{s2-i} - P_{s1i} - P_{s2i}) . \quad (9)\]

The first order conditions are
\[\partial \Pi / \partial P_{t_i} = - (P_t - c_t + (P_{s1i} - c_{s1})) + (1 + P_{t-i} + P_{s1-i} - P_{s1i}) - \]
\[(P_t - c_t + (P_{s2i} - c_{s2})) + (1 + P_{t-i} + P_{s2-i} - P_{s2i}) - \]
\[(P_t - c_t + (P_{s1i} - c_{s1}) + (P_{s2i} - c_{s2})) + (1 + P_{t-i} + P_{s1-i} + P_{s2-i} - P_{s1i} - P_{s2i}) \]
\[= 0\]

\[\partial \Pi / \partial P_{s1i} = - (P_t - c_t + (P_{s1i} - c_{s1})) + (1 + P_{t-i} + P_{s1-i} - P_{s1i}) - \]
\[(P_t - c_t + (P_{s1i} - c_{s1}) + (P_{s2i} - c_{s2})) + (1 + P_{t-i} + P_{s1-i} + P_{s2-i} - P_{s1i} - P_{s2i}) \]
\[= 0\]

\[\partial \Pi / \partial P_{s2i} = - (P_t - c_t + (P_{s2i} - c_{s2})) + (1 + P_{t-i} + P_{s2-i} - P_{s2i}) - \]
\[(P_t - c_t + (P_{s1i} - c_{s1}) + (P_{s2i} - c_{s2})) + (1 + P_{t-i} + P_{s1-i} + P_{s2-i} - P_{s1i} - P_{s2i}) \]
\[= 0\]

which reduce to
\[P_{*t_i} - c_{*t} = 1 \quad (11.a)\]
\[P_{*s1i} - c_{s1} = 0 \quad (11.b)\]
\[P_{*s2i} - c_{s2} = 0. \quad (11.b)\]

**QED**

The easiest way to understand this result is to recall that in equilibrium when a customer type is uniformly distributed along the line, and travel cost equals 1, each customer of that type will pay a total mark up above marginal cost of 1 to a firm regardless of how many units that customer type will purchases. The only way this can be satisfied for all three groups is for \( T \) to be marked up 1 over marginal cost and for \( s_1 \) and \( s_2 \) to be priced at marginal cost.

The important point of this example is that the good purchased by the most customers (and the good that sells the most units) is \( T \), and in competition it has the highest mark up of all goods sold. Thus this result calls into question the popular notion that a good is priced as a loss leader if it is purchased by the most people.

**6. Conclusion**

It is widely believed that a good loss leader is a product that is a well known brand or a
product that is widely purchased. The argument is that lowering the price on such items will attract customers into the store who will then purchase other items at the regular price. I have shown that this reasoning by itself may not be complete. This paper presents a model suggesting that an important consideration for choosing a product as a "loss leader" is that it be purchased primarily by high profit producing customers. To the extent that more profitable customers purchase a larger basket of goods (in dollar terms) I argue that correlation with the size of customer purchases, should be an important characteristic that determines if a good should be priced as a loss leader.

A more general implication of this work is that the intuition developed should not be limited to products used as loss leaders. Rather every product should be priced taking into account not only the fact that this price affects the quantity of the product sold, but also the fact that at the margin it will affect the make up of customers that enter the store and therefore the quantity of other units that will be purchased as well. Thus, in general goods that are bought primarily by high profit (or large volume) customers should have lower mark ups (all else equal) than goods primarily bought by low profit customers. All else equal the larger the basket of goods in which a good is purchased, the lower should be the mark up on it.

This paper suggests (at least) one interesting line of empirical research. The intuition behind the results could be generalized to suggest that if large volume purchasers systematically buy a set of goods that small volume purchasers do not, then we should expect the markup on these items to be less than the average markup throughout the store. So for example if milk, bread and eggs, are purchased mostly by those shopping for families with large grocery orders, and less so by single people with small orders, this theory predicts these items would have smaller than average margins. Thus a testable implication of this theory (to distinguish it from the notion that a product simply need be purchased by most customers or be a popular brand) is that on average, the baskets of goods that contain the loss leader is larger than baskets of goods that do not contain the loss leader.

13 Note that in the model of the previous section, since all side goods had the same price cost margin high revenue producing customers is synonymous with high volume customers.
References


