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Uncertainty and the Value of Information:
An Application of the Le Chatelier Principle

by

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It is well accepted that accurate information has a non-negative value.[1] It is more difficult to determine how that value changes when the uncertainty faced by a decision maker changes. Our intuitive response is that the relation should be positive: an increase in uncertainty increases the expected value of information. This hypothesis is of interest because it may have certain implications for the information-gathering activities of firms and consumers.

For example, suppose a seller's offering is of uniform quality but consumers are uncertain of what this level of quality is. The decision of how much to buy from such a seller must then be made subject to the uncertainty about quality. Suppose, however, that perfect information about the quality can be made available to consumers at some cost. For example, consumers may be able to buy a device that can measure with perfect accuracy the quality of the good before purchase; alternatively, the seller may (at some cost to consumers) be able to "guarantee" claims about the quality of the seller's current offering. If the cost of consumer measurement or of the seller guarantee is independent of the potential variance in quality, the hypothesis above suggests that information produced in either way is more likely to appear the greater is the potential variance of quality.

Unfortunately, the hypothesis that uncertainty and the value of information are positively related is not a straightforward implication of maximization behavior under uncertainty. Instead, such a positive relation can be established only by imposing additional restrictions on preferences.[2] It is not clear, however, how these mathematical restrictions on objective functions correspond to restrictions on behavior.

In this paper, I investigate the relation between uncertainty and the value of information by recasting uncertainty as a constraint on maximization. By doing so, it is possible to apply the Le Chatelier principle, a local property which relates the value of a maximized objective function to the number of constraints placed on the choice variables.[3] The application of the Le Chatelier principle is sufficient to establish that in a particular neighborhood of the uncertain variable, the value of information is a convex function of that variable.

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1. Throughout the paper, I consider only information that is perfectly accurate: that is, the recipient's expectations change from a distribution of probabilities to a single point. Information can change the distribution in other ways. For example, a bit of information may shift the mean of the recipient's expectations or it may produce a mean-preserving decrease in the variance of the recipient's expectations. Also, to use the distinction noted in Hirshleifer (1971), I explore the value of market information, not technological information.

2. Gould (1974) first explored the relation between uncertainty and the value of information. He showed by counterexample that this relation need not be positive. A positive relation can be ensured, however, if the objective function being maximized subject to uncertainty about a parameter is linear in that parameter. Hess (1982) noted that this condition is, of course, not necessary and presented a less restrictive set of conditions sufficient for the relation to be positive. Both Gould (1974) and Hess (1982) are discussed infra, pp. 6-7.

3. The principle comes from physics and, as Samuelson (1960) and Silberberg (1971, fn. 3; 1978, p. 294) point out, is purely a local property.

The local nature of the Le Chatelier principle necessarily limits this convexity to a "local" result; however, if the value of information is globally convex, a mean-preserving spread in uncertainty will increase the expected value of information.[4] Thus, a positive relation between uncertainty and the value of information holds globally under the same conditions for which the Le Chatelier principle holds globally.

The paper is divided into two sections. First, I show how uncertainty can be formulated as a constraint on maximization. The application of the Le Chatelier principle then establishes the convexity of the value of information in a particular neighborhood of the uncertain variable.[5] Second, I discuss additional conditions sufficient to ensure that this positive relation is global. These additional conditions are difficult to state exactly; instead, I use a statement in Samuelson (1960) about a global Le Chatelier principle to sketch them out. I then compare these conditions to those established by other authors as sufficient for a positive relation to exist globally.

I. Uncertainty as a Constraint on Behavior

Suppose a decision-maker must choose the levels of a vector of decision variables X to maximize a payoff function h(X, A), where A is a vector of variables parametric to the decision-maker's choice of X. Let one of the elements of A, say $\mathcal{A}_1$, be a random variable distributed with p.d.f. $f(\mathcal{A}_1)$. At some point in time, $\mathcal{A}_1$ becomes determinate. Let A_1 be the realized value of $\mathcal{A}_1$.

In general, the decision-maker is better off if A_1 is known before rather than after the choice of X is made.[6] The value a decision-maker places on this information, however, is of two distinct types. First, there is the value attached to the information after it is received, or the ex post value of information. Without the knowledge that the realization of $\mathcal{A}_1$ is A_1, a decision-maker would maximize the expected payoff given the p.d.f. $f(A_1)$. The choice of X would be "incorrect," ex post, for at least some of the elements of X. In other words, if the decision-maker knows A_1 before choosing X, the decision-maker can obtain a larger payoff than that which in fact results from choosing X without knowing A_1. The difference between the payoff with prior information on A_1 and the payoff without this information is a measure of the ex post value of the bit of information "A_1."

The second type of value is the ex ante value of information. Before the realization of the random variable, $\mathcal{A}_1$, the ex post value of information

4. See Rothschild and Stiglitz (1970).

5. This neighborhood is around a "just-binding point." See infra, p. 4, for the definition of this point.

6. This is almost always the case if two conditions are met. The first condition is that a change in the level of A_1 affects the choice of at least one of the elements of X. If not, a decision-maker would be unwilling to pay a positive amount to receive information on A_1 before making any decisons. Second, the distribution must be other than a degenerate, single-point distribution. Otherwise, there is only one possible level of A_1 and therefore prior information on A_1 (beyond some initial period) is of no value.

is itself a random variable with a distribution derived from $f(\mathcal{A}_1)$. The ex ante value of information, or the expected value of information before it is received, is therefore just the expected ex post value of information.

Formally, consider the following maximization problem where the X_i are chosen after A_1 is revealed:

$$\text{(1)} \qquad \underset{X_1, \ldots, X_n}{\text{Maximize}}\ Z = h(X_1, \ldots, X_n, A_1, \ldots, A_m).$$

Assuming a unique solution exists for each $\mathbf{A} = (A_1, \ldots, A_m)$, the optimal X_i are derived from the following system of first-order equations:

$$\text{(2)} \qquad h_i\,(X_1, \ldots, X_n, A_1, \ldots, A_m) = 0,\ i=1, \ldots, n.$$

where $h_i = dh/dx_i$. Let (h_{ij}) be the hessian matrix of cross partials of h with respect to the X_i. The sufficient second order conditions are that all principal minors $|(h_{ij})|$ of order k have sign $(-1)^k$ at the maximum point. Let the choice functions and the indirect payoff function be $X_i^*(A_1, \ldots, A_m)$ and $Z^*(A_1, \ldots, A_m)$, respectively.

Suppose instead that the realization of $\mathcal{A}_1$ occurs only after the choice of $\mathbf{X}$ is made. (The actual payoff, however, is always a function of A_1.) Suppose the p.d.f. $f(\mathcal{A}_1)$ is known and let w be a vector that summarizes the parameters associated with $f(\mathcal{A}_1)$. Suppose that for each point $(A_2, \ldots, A_m, w)$, a unique solution exists for the problem:

$$\text{(3)} \qquad \underset{X_1, \ldots, X_n}{\text{Maximize}}\ EZ = Eh(X_1, \ldots, X_n, \mathcal{A}_1, \ldots, A_m, w).$$

Here, the maximization is over the expected value (E) of the payoff function. The optimal X_i are derived from the first-order equations:

$$\text{(4)} \qquad Eh_i\,(X_1, \ldots, X_n, \mathcal{A}_1, \ldots, A_m) = 0,\ i=1, \ldots, n.$$

Let (Eh_{ij}) be the hessian matrix of cross partials of Eh with respect to the X_i ; the sufficient second order conditions are similar to those of the first problem. Let $X_i^E(A_2, \ldots, A_m, w)$ be the choice function derived from equation (4). The l.h.s of equation (4) is not a function of A_1, the realized value of $\mathcal{A}_1$; therefore, the X_i^E are not functions of A_1.

As noted above, the actual payoff is a function of A_1 and will be

(5) $$Z^E(A_1, . . ,A_m) = h(X_1^E, . . ,X_n^E,A_1, . . ,A_m).$$

Because the X_i^E are not functions of A_1, they are constant in equation (5) with respect to A_1.

The <u>ex post</u> value of information on A_1, $V(A_1, . . ,A_m)$, is the difference between the payoff levels, or

(6) $$V(A_1, . . ,A_m) = Z^*(A_1, . . ,A_m) - Z^E(A_1, . . ,A_m).$$

The <u>ex ante</u> value of information, then, is the expected value of $V(\mathcal{A}_1, . . ,A_m)$ over the p.d.f. $f(\mathcal{A}_1)$, or $EV(\mathcal{A}_1, . . ,A_m)$.

We can now examine the relation between uncertainty and the value of information by addressing the following question: will a mean-preserving spread in the distribution of $\mathcal{A}_1$ increase or decrease $EV(\mathcal{A}_1, . . ,A_m)$? As shown in Hess (1982), a sufficient condition for this relation to hold is that $V(A_1, . . ,A_m)$ is convex in A_1.[7] This condition, however, only serves to generate a further question: what conditions are sufficient to ensure that $V(A_1, . . ,A_m)$ is convex in A_1?

To address this latter question, we apply the Le Chatelier principle to show that in a particular neighborhood of A_1, $V(A_1, . . ,A_m)$ is convex in A_1 <u>given only the sufficient conditions of the original maximization problems</u>. Thus, at least in this neighborhood, no <u>additional</u> restrictions on $h(X,A)$ are necessary to establish the convexity of $V(A_1, . . ,A_m)$.

The Le Chatelier principle can be applied by viewing the presence of uncertainty in the following way: uncertainty places a <u>constraint</u> on the choices of a decision-maker. The choice of X^E must be made despite the recognition that, <u>ex post</u>, at least some of the choices will be "wrong" (almost always); in other words, $X_i^E \neq X_i^*$ for at least some i (almost always). There may be a realization of $\mathcal{A}_1$, however, such that $X^* = X^E$. Such a realization of $\mathcal{A}_1$, say. A^B[8], is called a <u>just-binding point</u>: at such a point, the constraint of uncertainty does not alter the choice of X, but in a neighborhood around the point, $X_i^* \neq X_i^E$ for at least some i.

The Le Chatelier principle implies that at a point such as A^B, the maximum value of the objective function does not change with the addition of the constraint for which A^B is a just-binding point. Hence, we have $Z^* = Z^E$, which implies that the <u>ex post</u> value of information is zero. In a neighborhood around A^B, however, the <u>ex post</u> value of information is positive. Thus, at least in a neighborhood of a just binding point, the <u>ex post</u> value of information is a convex function of A_1.

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7. This result is, of course, derived from Rothschild and Stiglitz (1970).

8. The just-binding point will, of course, be a given value of the vector A. See note 13, <u>infra</u>, for a discussion of the existence of A^B.

To demonstrate this result formally, let the index of the X_i^* be ordered in the following way. If $i \leq r$, $dX_i^*/dA_1 \neq 0$ almost everywhere; if $i > r$, $dX_i^*/dA_1 = 0$ everywhere.[9] Consider the following problem:

$$\text{(7)} \qquad \underset{X_1,..,X_n}{\text{Maximize}}\ Z = h(X_1, . . ,X_n,A_1, . . ,A_m)$$

$$\text{(8)} \qquad \text{Subject to } X_i^E(A_2, . . ,A_m,w) - X_i = 0,\ i = 1, . . ,r,\ r \leq n.$$ [10]

The optimal X_i and l_i (the Lagrange multipliers associated with the r constraints) are found by solving the following first-order equations:

$$\text{(9a)} \qquad h_i - l_i = 0,\ \ i = 1, . . ,r;$$

$$\text{(9b)} \qquad h_j = 0,\ \ j = r+1, . . ,n;$$

$$\text{(9c)} \qquad X_i^E(A_2, . . ,A_m,w) - X_i = 0,\ \ i = 1, . . ,r.$$

Let (H^r) be the bordered hessian matrix of cross partials of h with the r constraints and let H^r be its determinant. The sufficient second order conditions are examined in the Appendix. Let $X_i^r(A_1, . . ,A_m,w)$ and $Z^r(A_1, . . ,A_m,w)$ be the choice functions and the indirect objective function, respectively, and let $l_i^r(A_1, . . ,A_m,w)$ be the Lagrange multipliers of equation (9a).[11] The superscript "r" denotes the maximization problem with the r constraints of equation (8).

The indirect objective functions, Z^* and Z^r, can now be compared to establish the (local) convexity of $V = Z^* - Z^r$ in the neighborhood of a

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9. The X_i^* are the demand functions with perfect information. If $dX_i^*/dA_1 = 0$ everywhere, then X_i^* is identical to X_i^E and uncertainty produces no ex post "errors" in the choice of X_i. The set of X_i such that $dX_i^*/dA_1 \neq 0$ almost everywhere is non-empty as long as the expected value of information is non-zero. For example, if h is additively separable in A_1, then A_1 has no effect on any demand function and the ex ante value of information is zero.

10. A special case is $r = n$. In this case, any variation in the X_i is inadmissable and it makes no sense to "maximize" a constant. Nevertheless, it can be shown that the results in the text still hold for this extreme case. See Silberberg (1971), where he includes the "degenerate" case of $r = n$ in his theorem.

11. The Lagrange multiplier, l_i^r, represents the value of increasing the constrained choice of X_i, or X_i^E, for $i = 1,...,r$. This is because $dZ^r/dX_i^E = l_i^r$ by the envelope theorem. The sign of the Lagrange multiplier is the same as the direction of change in the marginal value of the X_i^* when information is revealed prior to purchase. Suppose A_1 is such that $X_i^* > X_i^E$. Then $dZ^r/dX_i^E > 0$ and l_i^r is positive; if A_1 is such that $X_i^* < X_i^E$, the opposite is the case because an increase in X_i^E is a movement away from X_i^*.

just-binding point, A^B.[12] By applying Silberberg's Generalized Envelope Theorem[13] to the functions Z^* and Z^r evaluated at A^B, we obtain:

$$(10a) \qquad Z^* = Z^r$$

$$(10b) \qquad Z^*_{A_1} = Z^r_{A_1}$$

$$(10c) \qquad Z^*_{A_1A_1} > Z^r_{A_1A_1}.$$

The last inequality is a restatement of the Le Chatelier principle: in a neighborhood of A^B, $Z^* \geq Z^r$ with the equality holding only at that point. Equation (10c) can be rearranged to show that

$$(11) \qquad V_{A_1A_1} = Z^*_{A_1A_1} - Z^r_{A_1A_1} > 0.$$

Therefore, in a neighborhood around A^B, V is convex in A_1.

The set of assumptions sufficient to demonstrate this result is simply that the function h(X,A) is capable of supporting a local maximum at the just-binding point (and of course that such a point exists). This will be the case as long as the sufficient second order conditions are satisfied for the maximization problems in equations (1) and (3). These are sufficient because the matrix (h_{ii}) will then be non-singular, which is sufficient for the derivation in equation (10), and because the matrix (Eh_{ij}) will also be non-singular, which is sufficient for the existence of the X_i^E.

The application of the Le Chatelier principle therefore involves a set of assumptions much less restrictive than those relied upon by previous authors. For example, Gould (1974) proves that if h(X,A) is linear in A_1, an increase in uncertainty increases the expected value of information. But if this is the case, the just-binding point is simply the point $(\overline{A}_1, \ldots, A_m)$, where $\overline{A}_1$ is the mean value of A_1. This is because

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12. Such a point exists if there is a point A^B that is a solution to the following system of equations:

$$X_i^E(A_2,\ldots,A_m,W) - X_i^*(A_1,\ldots,A_m) = 0, \; i = 1,\ldots,r.$$

It is likely that for any individual X_i^E there exists a set of points A^i such that

$$X_i^E(A_2^i,\ldots,A_m^i,W) = X_i^*(A_1^i,\ldots,A_m^i)$$

for each $(A_1^i,\ldots,A_m^i)$in A^i. A necessary and sufficient condition for the existence of a just binding point is that the intersection of these sets over all $i \leq r$ is non-empty. In other analyses of the Le Chatelier principle, the authors have not addressed the issue of the existence of a just-binding point. See Samuelson (1960) and Silberberg (1971).

13. See Silberberg (1971).

$$Eh(X_1, \ldots, X_n, A_1, \ldots, A_m) = h(X_1, \ldots, X_n, \bar{A}_1, \ldots, A_m)$$

and so the maximization problem represented by equation (3) is equivalent to that of equation (1) for $A_1 = \bar{A}_1$. Linearity of h in A_1, however, clearly is not a necessary condition for the application of the Le Chatelier principle to hold.

Hess (1982) states a theorem that uses a set of sufficient conditions (apparently) less restrictive than Gould's to prove that the relation between uncertainty and the expected value of information is positive. This set is, essentially, that $V(A_1)$ is everywhere convex in A_1. On the one hand, his sufficient conditions are less restrictive than the ones used here because they apply at any value of the uncertain parameter, not simply at a just binding point. On the other hand, at the just binding point, Hess' sufficient conditions are unnecessary and essentially redundant as long as uncertainty is viewed as a constraint on maximization.

This emphasizes the simplicity of applying the Le Chatelier principle to this problem: the conditions sufficient for V to be (locally) convex in A_1 are the same as the two sets of sufficient conditions for maximization with and without the constraint of uncertainty, at least in a neighborhood of a just binding point. Beyond this neighborhood, however, the Le Chatelier principle need not hold. Therefore, its application to the analysis of the value of information at first glance is limited because it is a local property and need not hold globally. The next section explores the conditions under which it does and does not hold globally.

II. Global Considerations

The set of conditions under which the Le Chatelier principle is a global property is not a settled issue. Samuelson (1960) addresses the question of when the Le Chatelier principle holds "in the large." He concludes that for most economic maximization problems, it need not in fact hold. For example, the Le Chatelier principle implies that long-run demand curves are locally more elastic than short-run demand curves.[14] But Samuelson notes the following:

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14. Briefly, let R^L and R^S be a firm's maximized long-run (i.e., when all factors are variable) and short-run (i.e., when only labor is variable) profit functions, respectively, and let w be the price of labor. The Le Chatelier principle implies that

$$d^2R^L/dw^2 > d^2R^S/dw^2.$$

By the envelope theorem, $dR^L/dw = -L^L$, the long-run demand for labor; similarly, $dR^S/dw = -L^S$, the short-run demand for labor. Therefore,

$$-dL^L/dw > -dL^S/dw,$$

which, at a just binding point ($L^L = L^S$), gives the implication cited in the text.

> When I first formulated the Le Chatelier theorem twenty years ago, I had hopes of proving the non-local property. But for many years a proof eluded my most determined efforts. Finally it dawned on me that the theorem was not true in the large. . . .
> /O/ur paradox in the large could be illustrated by the following type of example: Near the critical point where labor and land go from being substitutes to being complements, as measured by the sign of off-diagonal elements in the profit Hessian matrix $[A_{ij}]$, we could find a clever counterexample in which the long-run arc elasticity of demand for labor was more inelastic than the short-run.[15]

I believe his remarks can be applied to the problem here in the following way.

Consider the case where only one choice variable, X, is constrained by uncertainty about a single parameter, A. If uncertainty about A is a binding constraint on the choice of X and if there exists a just binding point, then $dX^*/dA \neq 0$ in a neighborhood around this point. There may exist some point A^0, however, outside of this neighborhood where dX^*/dA reverses sign. Suppose $dX^*/dA > 0$ for $A < A^0$ and $dX^*/dA < 0$ for $A > A^0$. As A increases above A^0, the level of X^* could return to the level where uncertainty is just binding: i.e., there may be two (or more) just binding points for uncertainty about A.

The effect of this on the global convexity of the value of information is shown in Figure 1. Let A^1 and A^2 be such that $X^*(A^1) = X^E = X^*(A^2)$; both A^1 and A^2 are then just binding points. Around each just binding point, the ex post value of information is convex in A. Because there are two points of tangency between Z^* and Z^r, however, there must exist a region where the ex post value of information is concave in A.[16] Therefore, there could be mean-preserving increases in uncertainty that would decrease the expected value of information.

Consider the following example, where a firm maximizes profits subject to uncertainty about a parameter, *a*, that affects the marginal productivity of the firm's only input, X. Let $g(a)\ln(X)$ be the firm's production function, where $g(a) = d - c(b - a)^2$. Finally, assume that X is available at unit cost.

If information on *a* is available before X is chosen, $X^* = pg(a)$, where p is the price of the firm's output and a is the realization of *a*; ex post profits are then $R^* = pg(a)\ln[pg(a)]$. Note that for a < b, $dX^*/da > 0$; for a = b, $dX^*/da = 0$; and for a > b, $dX^*/da < 0$. If *a* is realized only after X is chosen, $X^E = pEg(a)$ and ex post profits are $R^E = pg(a)\ln[pEg(a)]$. The value of information, then, as a function of a, is $V(a) = pg(a)\ln[g(a)/Eg(a)]$.

It is easily shown that the graph of this function is similar to that presented in Figure 1. It is then possible to specify values of the parameters

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15. Paul Samuelson, "An Extension of the Le Chatelier Principle," Econometrica, 28(2): 368-379, April 1960, p. 372 (footnote omitted).

16. This is the thrust of Hess (1982): to rule out this possibility, he assumes it away. This in many ways avoids the question addressed here: what restrictions on behavior are sufficient to produce a positive relation?

and a p.d.f. f(a) for which a mean-preserving spread decreases the expected value of information. An example is presented in Table 1:

Table 1
A Counterexample

Probability of State 1	2	3	ER^*	ER^E	Expected Value of Information[17]
.40	.20	.40	\$101.66	\$90.15	\$11.51
.45	.10	.45	69.30	62.32	6.98

d = c = b = 1 $a_1 = .1$ $a_2 = 1$ $a_3 = 1.9$ p = 100

To rule out the existence of a second just-binding point, it is sufficient to restrict X^* to be a strictly monotonic function of a; however, this apparently is not sufficient to rule out the occurence of any concave portions of V(a). Hence, it may be that, for global convexity, we must appeal to the broad sufficient conditions of Hess (1982).

There is another set of circumstances under which the Le Chatelier principle may not hold in the large. Gould (1974) presents a counterexample in which the expected value of information and uncertainty are negatively related. The example consists of a firm maximizing profits by choosing the level of a single input, X, subject to uncertainty about a discrete state variable, A. The important feature of Gould's example is a discontinuity in the profit function, perhaps due to an "escalator" clause in the firms labor contract.[18]

This discontinuity has two important effects which are more easily explored if the uncertainty in Gould's example is transformed from a discrete distribution to a continuous distribution and if the discontinuity coincides with the mean of the distribution.[19] First, the discontinuity produces the anamolous

17. Note also that the expected value of information as a percentage of ER^E, or the "rate of return to perfect information," also decreases.

18. See Gould (1974), p. 77.

19. Suppose A is uniformly distributed in the interval [1-e, 1+e]. Let the profit function be

$$F(X,A) = AX - X^2, \qquad A < 1;$$
$$= AX - (1+D)X^2, \quad A \geq 1.$$

where D is a measure of the discontinuity, $D \geq 0$, and $D = 0$ implying a continuous function. In the manner described in the text (see equation (6)), let V be the ex post value of information on A and EV be the expected (ex ante) value of information. The following propositions, discussed in the text, are presented without proof:

result that as uncertainty goes to zero around the mean, the expected value of information approaches a strictly positive value. This is because the discontinuity creates a "point" of uncertainty. In fact, the value approached is equal to the average ex post value of information at the discontinuity, calculated by taking the limit from above and below.[20]

The second important effect is to create an interval where the ex post value of information is concave. As A approaches the discontinuity from below, it goes through a just binding point, where the unconstrained choice of X, X^*, which is positively related to A, is equal to the choice of X under uncertainty, X^E. Through this point, the ex post value of information is convex. The discontinuity, however, changes the first order condition of the firm's profit maximization problem and decreases X^* (discontinuously, of course). As A continues to increase, it goes through a second just binding point where $X^* = X^E$ again.

Figure 2 illustrates this result. Strictly above or below the discontinuity, V(A) is convex. In at least part of the region between the two just binding points, A^1 and A^2, however, the ex post value of information is a concave function of A. Hence, for small levels of uncertainty (recall, the discontinuity coincides with the mean), uncertainty and the expected value of information may be negatively related. For larger levels, however, the convex portions are likely to "dominate" if the increase in uncertainty is a general spreading out of the distribution.

III. Conclusions

The application of the Le Chatelier principle shows that uncertainty and the expected value of information are positively related in a neighborhood of a point where uncertainty is a just binding constraint. With this result as a foundation, it is then possible to sketch out the sufficient conditions for this relation to be globally positive: conditions that are sufficient to ensure a global Le Chatelier principle are also sufficient for a global positive relation. Although the former set of conditions is an unsettled matter, the application of the Le Chatelier principle provides us with a new perspective on the problem.

P1: $dEV/de \gtrless 0 \qquad dEV/dD \gtrless 0$

$d^2EV/de^2 > 0 \qquad d^2EV/dD^2 < 0$

P2: $\lim_{e \to 0} EV = D^2/[8(D+1)(D+2)] > 0.$

P3: $\lim_{e \to 0} EV = (1/2) \lim_{A \uparrow 0} V \quad (1/2) \lim_{A \downarrow 0} V$

P4: $\lim_{D \to 0} (\lim_{e \to 0} EV) = 0$

20. See proposition 3, note 18.

To ensure that the Le Chatelier principle holds in the large, what types of behavior are ruled out? One such type of behavior would be a discontinuous change in the marginal benefit or cost of an action. As shown above, this can create two just binding points and result in an interval where the ex post value of information is concave. A restriction that all relevant functions be globally continuous is palatable but should be recognized as being more restrictive than the standard conditions necessary and sufficient for the existence of a local maximum.

Another type of behavior ruled out would be "reversals" of behavior. In many instances, this again would not be overly restrictive. For example, if the uncertain variable is price, there is little objection to assuming that demand curves always slope downward; somewhat more restrictive but still acceptable in many cases is the assumption that pairs of goods are always substitutes or always complements.

Other variables are harder to restrict in this fashion. For example, Leffler (1982) shows that the relation between quality and demand need not be monotonic. If this is the case, changes in uncertainty about quality can increase or decrease the expected value of information on quality. In addition, as I have stated above, restricting behavior to avoid reversals is necessary but may not be sufficient to ensure a positive relation between uncertainty and the expected value of information.

Nevertheless, the Le Chatelier principle is a valuable tool for analyzing the problem of uncertainty. Under conditions no more restrictive than those placed on a simple maximization problem, its application proves that, locally, an increase in uncertainty increases the expected value of information. But rather than asserting that this relation will hold globally, we must recognize that additional restrictions are necessary. This paper has suggested what some of these restrictions might be.

Appendix

This appendix discusses the sufficient second order conditions associated with the maximization problem of equations (7) and (8).

Let $(H_{r,r})$ be the matrix of the cross partials of the first r choice variables. These are the choice variables in the constraints of equation (8). Let $(H_{n-r,n-r})$ be the matrix of the cross partials of the last (n-r) choice variables and $(H_{r,n-r})$ be the matrix of the cross partials between the two groups. Finally, let (I_r) be the rxr identity matrix.

Consider the determinant:

$$H^r = \left| \begin{array}{c:c:c} (H_{r,r}) & (H_{r,n-r}) & (I_r) \\ \hline (H_{n-r,r}) & (H_{n-r,n-r}) & 0 \\ \hline (I_r) & 0 & 0 \end{array} \right|$$

The sufficient second order conditions for the maximization problem of equations (7) and (8) are that all kxk border-preserving principal minors of H^r have sign $(-1)^{k-r}$, $k = 2r+1, \ldots, n+r$. By evaluating the determinant on the r.h.s. along the diagonals of the two identity matrices, it can be shown that

$$\text{(A1)} \quad H^r = (-1)^{2rn+4r-r^2} \, |(h_{n-r,n-r})|.$$

The sign of $|(h_{n-r,n-r})|$ is $(-1)^{n-r}$ by the sufficient second order conditions of the unconstrained maximization problem in equation (1). Thus, we have

$$\begin{aligned} \text{sgn } H^r &= \text{sgn } [(-1)^{2rn+n+3r-r^2}] \\ &= \text{sgn } [(-1)^n] \end{aligned}$$

because $(-1)^{2rn+n+3r-r^2} = 1$.

Using similar reasoning, it can be shown that smaller, non-vanishing border preserving principal minors also have the correct sign. Some principal minors vanish, however, because the r constraints do not admit any variation in the X_i, $i = 1, \ldots, r$. There is a single point $(X_1^E, \ldots, X_n^E)$ that satisfies the constraint and therefore variation along those margins is inadmissable.

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Figure 1

Multiple Just-Binding Points

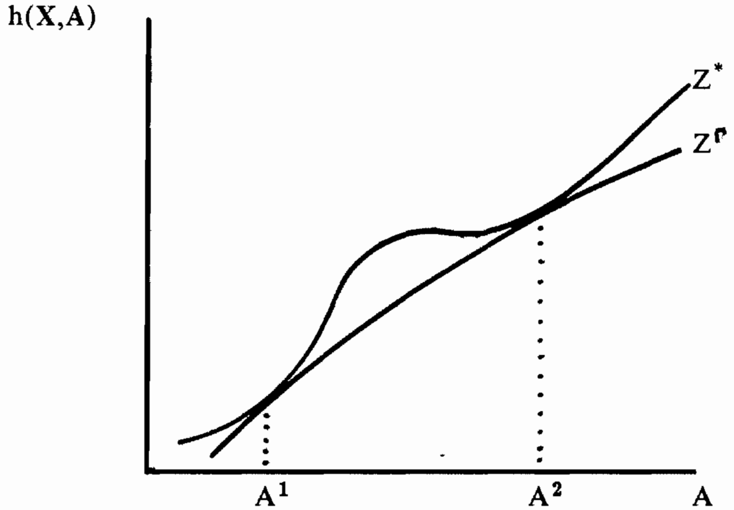

Figure 2

A Discontinuity in V(A)