Reconciling the Off-Net Cost Pricing Principle with Efficient Network Utilization

Patrick DeGraba
Federal Trade Commission
600 Pennsylvania Ave.
Washington, DC
pdegraba@ftc.gov

The off-net-cost pricing principle argues that under a broad range of environments a positive “access” charge paid by originating networks to interconnected terminating networks would cause networks to set on-net usage charges equal to off-net rates, and that these charges would fully reflect the access charge. However, other results in the literature provide reasonable conditions under which on-net usage charges will not reflect access charges, but would be set to induce the social surplus maximizing level of on-net usage. This paper harmonizes these two apparently opposing results by showing that retail usage charges depend on two effects. One is a rent seeking effect on the part of networks and the other is an efficient utilization effect. In models in which the rent seeking effect is more important, on-net usage charges will tend to equal their off-net usage charges and incorporate the access charge. In models in which the efficient utilization effect matters more, off-net usage charges will reflect access charges, but on-net usage charges will not be affected by the level of access charges, but instead will tend to be set at the levels that promote efficient on-net utilization.

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1. Introduction

Recent deregulation of telecommunications has eliminated many monopoly franchises held by incumbent local phone companies, creating competition between multiple networks in the same local calling area. Federal regulations in the United States require all competing local phone companies to interconnect, so that a customer subscribing to one network may call a customer on another network. A call between customers on different networks is referred to as an off-net call\(^1\), and federal regulations require the network serving a party that originates an off-net call to make a payment to the network serving the party that receives or terminates the call. Such payments are referred to as “access” charges or termination charges. The (per minute) rates at which such payments are made are typically set by regulators.\(^2\) Assuming regulators wish to set access rates that ultimately induce efficient network usage by customers, it is important to know how these rates affect the retail usage rates networks charge to their end user customers.

Recent literature analyzing the effects of government set access rates on retail rates has focused on the case in which each local phone company can set 4 separate per minute usage rates, an off-net origination rate, an off-net termination rate, an on-net origination rate and an on-net termination rate. The literature has produced two differing results regarding the effect of access rates on usage rates. Laffont, Marcus, Rey and Tirole (LMRT) introduce the off-net cost pricing principle (ONCPP)\(^3\) which argues that all on-net and off-net usage rates will equal the marginal cost of providing service plus (minus) access charges paid (received). DeGraba (2003) on the other hand provides analysis in which on-net usage rates depend only on (efficiently allocating) the

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\(^1\) Similarly, calls between customers on the same network are called on-net calls.

\(^2\) While the telecommunication Act of 1996 allows for networks to negotiate such rates, as a practical matter local regulators have been required to set these rates, when incumbents and entrants have failed to reach agreement.

\(^3\) See Laffont, Marcus, Rey and Tirole (2002) and (2001)
on-net costs of providing service, while the rates for off-net service depend both on the costs of
providing the service and the access fee.

The purpose of this paper is to reconcile these two seemingly opposing results. I show
that these different results depend on different assumptions regarding how usage rates affect
consumer usage, and about whether subscribers to a telephone network both initiate and receive
calls, or whether one set of customers only originates calls while a separate set only receives calls.

The analysis considers a Nash game played between two competing networks and
Customers who subscribe to the network. Competing networks will simultaneously set usage
rates. Each customer observes the rates, and subscribes to the network that offers him the
greatest surplus. Once all customers have subscribed to a network, they make their calls,
receiving benefits from each minute of conversation in which they engage, and paying their
network’s announced usage rates. Networks’ profits are the payments they receive from their
customers plus (less) net access payments they receive (make) for off-net calls, less variable
costs of providing service to customers.

In this setup, the ONCPP says that competing networks set usage rates for both
originating on-net traffic and originating off-net traffic equal to the marginal cost of providing
origination plus the access rate paid for originating off-net traffic. Similarly, networks set rates
for terminating (receiving) on-net traffic and off-net traffic equal to the marginal cost of
terminating traffic less the access rate received for terminating off-net traffic. In describing this
principle they state, “In a remarkably broad range of environments, operators set prices for their
customers as if all of their traffic is entirely off-net...” And later they state, “This principle

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turns out to be remarkably robust to generalizations of the model: mixed traffic, variable demand…”  

The intuition behind this result is that the resource costs of serving the originator of an off-net call is the same as the resource costs of terminating an off-net call. However, the network that terminates a call receives a payment, while the network that originates the call makes a payment. Thus, an interconnection regime that involves access charges endows a customer that terminates more traffic than he originates with a rent. Bertrand competition to serve such customers allows these customers to capture all of the rents, and this results in on-net usage rates equal to their off-net counterparts.

To see why this can happen, suppose initially there is a single network serving all customers, customer usage is unaffected by the per minute retail rates they are charged, all marginal costs are zero, and the access rate for off-net calls is set at 1 penny per minute. Suppose there were a customer that only received phone calls. Clearly a competing network (with zero marginal costs) would be willing to pay this customer up to 1 penny per minute for each off-net minute he terminated to subscribe to the competing network. Since the network serving all other customers would have to pay 1 penny per minute if this customer left, it too would be willing, and would have to pay him 1 penny per minute for each on-net minute he terminated to keep him on the network. Thus, competition in this example causes off-net termination and on-net termination to be priced at the same rate. (A similar argument explains why on-net origination rates must equal off-net origination rates.)

In contrast to the LMRT result, research by DeGraba (2003)⁶ presents a model in which

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⁵ See LMRT pg 2. The intuition behind this claim is that in a zero profit equilibrium, competition will cause each operator to set the price of usage for each customer equal to the opportunity cost of serving that customer’s.

⁶ See also DeGraba (2002) and (2000) and Hermalin and Katz (2002).
on-net usage rates depend only on (efficiently allocating) the on-net costs of providing service, while the rates for off-net service depend both on the costs of providing the service and the access fee. Here, DeGraba shows that on-net rates promote social surplus maximizing usage of the network. These results seem to place limits on the robustness of ONCPP, suggesting that it may not easily generalize to either mixed traffic or variable demand.

To see this intuition, consider the example above, but assume that all customers originate each minute of calling with equal probability, and that the number of minutes of usage in which customers engage depends on the usage rates they are charged for both origination and termination. Assume also that parties to a call share equally in the benefits of each minute of calling.

In this case suppose a network serving all customers charged zero for on-net usage, charged a customer 1 penny per minute for off-net origination and paid a customer 1 penny per minute for off-net termination. With all customers on the same network utilization reaches the efficient (surplus maximizing) level. Any customer that subscribes to another network will not engage in the efficient level of usage, since no customer that originates a call to this customer will consume beyond the point at which the marginal benefit of calling equals 1 penny. The network cannot profitably compensate this customer for the reduction in usage by paying him 1 penny for each minute of off-net traffic he terminates, because, since the customer originates calls with the same probability as he terminates calls, the network must charge him 1 penny for each minute of off-net traffic he originates. Thus, (roughly speaking) this customer in expectation can expect to earn no money from net access charges.

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7 With zero usage rates all customers consume minutes of conversation to the point where the marginal benefit of the last minute equals 0, which given zero marginal cost of providing service, maximizes social surplus.
8 Even though customer’s exogenous probability of originating a minute equals the exogenous probability of terminating a minute, a customer may terminate more minutes than he originates if he faces for example higher origination rates than termination rates.
This paper shows these conflicting results are the consequence of different assumptions regarding two different features of customer behavior. The first is that LMRT assume that per minute usage rates do not affect customers’ usage, while DeGraba assumes that per minute usage rates do affect customers’ usage. The second is that LMRT look primarily at a set of customers in which some customers only originate traffic while others only terminate traffic. DeGraba on the other hand looks at a set of customers in which all customers have a (sufficiently high) positive probability of originating each call.

These two different sets of assumptions suggest that there are two competing effects that determine how access rates affect retail usage rates in equilibrium. One is a rent seeking effect on the part of network operators. The other is an efficient utilization effect.

The rent seeking effect says that when originating networks pay access rates to terminating networks in excess of those necessary to efficiently allocate the cost of a call between interconnected networks, customers that receive more traffic than they originate generate rents, because they generate (in expectation) positive net access rates for the network to which they subscribe. Competition to serve such customers results in rates that enable the customer to capture these rents.

The efficient utilization effect says that networks compete for customers by setting on-net usage rates that offer customers the highest surplus. DeGraba (2003) shows this requires setting usage rates so that parties to a call bear its cost in proportion to the marginal benefit they receive.

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9 DeGraba (2003) shows the efficient access rate is the one that allocates the cost of a call between two interconnected networks so that the percentage of the cost borne by each network is equal to the percentage of the marginal benefit received by its customer.

10 A customer generates positive net access rates for its network if he receives more off-net minutes than he originates. In symmetric models in which all customers speak to all other customers, a customer that terminates more minutes than he originates will typically generate such rents in equilibrium.
from the call. Network operators have an incentive to set on-net rates to efficiently allocate the cost of a call between the two parties regardless of the access rate.

The intuition behind why ONCPP rates aren’t always sustainable in equilibrium can be seen in the following simple example of competition between two interconnected telephone networks.\(^{11}\) Suppose there are two identical networks that have a zero marginal costs of originating and terminating calls, that two customers always share equally in the benefit of each minute of calling in which they are engaged, that the benefit of a minute of conversation between two customers is a decreasing function of the number of minutes consumed between them, and that each customer is equally likely to initiate a minute of conversation as he is to be called. Suppose also the government imposes an access charge of \(a\), which the originating network must pay the terminating network for any inter-network call.\(^{12}\)

The ONCPP implies that on-net and off-net origination rates will both equal \(a\), and that on-net and off-net termination rates will both equal \(-a\).\(^{13}\) The reason these retail rates will not always be supported in an equilibrium is the following:

In the proposed ONCPP equilibrium, each network earns a zero profit. Because an originating customer is charged a usage fee of \(a\), he consumes minutes up to the point at which his marginal benefit of the last minute equals \(a\). Thus, a usage rate of \(a\) that exceeds the efficient rate creates a deadweight loss for both on-net and off-net calls. If both networks’ rates adhere to the ONCPP, one network could deviate by lowering its on-net origination rates, raising its on-net termination rates by marginally more than it lowered its origination rate, increase the surplus each customer receives from on-net calls and capture some of the additional surplus the deviation

\(^{11}\) LMRT present their result in the context of internet networks. The telephony example is a bit more convenient for exposition purposes.

\(^{12}\) An internetwork call occurs when a customer on one network calls a party on another (interconnected) network.

\(^{13}\) Negative retail rates indicate payments from the network to a customer.
creates for its customers, resulting in positive profits. Such a deviation would defeat the proposed ONCPP equilibrium.

So for example, if network 1 were to deviate from ONCPP by setting its on-net origination rates equal to \( \epsilon \), its on-net termination rate at \(-\epsilon/2\), and on-net and off-net usage rates equal to the ONCPP rates minus \( \epsilon \), it is clear that for \( \epsilon \) arbitrarily close to zero customers would be better off subscribing to network 1 (because of the additional surplus from the additional minutes of on-net calling they would consume). Since every customer unilaterally would prefer to subscribe to network 1, the only subgame perfect continuation in response to this deviation is for all customers to subscribe to network 1 and (because the on-net origination rate is greater than the absolute value of the termination rate) network 1 would earn a positive profit.\(^{14}\)

Thus, this example suggest that when customers view networks as perfect substitutes, it may be difficult for a pricing regime that creates a deadweight loss for on-net calls to persist in equilibrium if networks can price on-net and off-net calls differently. In such cases, the ONCPP, which typically results in inefficient on-net (as well as off-net) usage rates, will not likely produce sustainable rates.

To see why on-net usage rates that induce efficient on-net utilization may not be part of an equilibrium when there are rent seeking opportunities and Bertrand competition, suppose that there are 2 customers who share equally the value of each minute of calling, the marginal value of a minute is decreasing in the number of minutes, the cost of originating a call equals the cost of terminating a call = \( c/2 \), one customer originates all minutes of calling, and there is a positive access charge \( a \).

\(^{14}\) There is no subgame perfect continuation in which all customers subscribe to network 2 if network 1 keeps off-net termination at \(-a\), sets off-net origination at \( a - \delta \) for \( \delta > 0 \) along with on-net origination at \( \epsilon \) and on-net termination at \( \epsilon/2 \), then the unique subgame perfect continuation is for all customers to subscribe to network 1.
Efficient utilization would occur if all origination and all termination rates were equal to \( c/2 \). Yet a network has the incentive to offer off-net termination rates below \( c/2 \), on-net origination rates equal to infinity, serve only the customers that terminate calls and earn a positive profit. This would exceed the zero profit that would occur under Bertrand competition.

This paper shows that the structure of on-net usage rates depends on whether customer demand implies that rent seek or efficient utilization is more important. More specifically when some customers initiate a large proportion of their minutes they consume and others receive a large proportion of the minutes, rent seeking tends to be more important. When all customers initiate about half the minutes they consume, efficient utilization tends to be more important.

One condition under which one would expect on-net usage to be priced equal to off-net usage is when the access charge divides the cost of a call between interconnected networks in the same proportion as customers share in the benefit of the call.\(^{15}\) In this case, (as suggested by the ONCPP) competition results in off-net rates that cause each party to a call to bear a portion of the marginal cost equal to the portion of the benefit he receives. Similarly, as shown in DeGraba (2003) on-net usage rates that result in efficient network utilization also allocate the marginal cost of on-net calls between the parties in proportion to the benefit they receive. In other words, an access rate that efficiently allocates costs on an off-net call between the calling party and the called party creates the same pricing incentives as a network has to price on-net usage to induce efficient usage. In this circumstance LMRT’s and DeGraba’s results are identical.

In section 2 I present four examples that show how the equilibrium rates depend on the assumption regarding the effect of per minute usage by per minute retail rates and on the

\(^{15}\) DeGraba 2003 shows that if \( V(m) \) is the total value two parties receive from minute, \( m \), of calling, and that the originating party always receive \( \lambda V(m) \) of this value then, when competition causes networks to set usage rates equal to cost, the optimal access charge is the one that ensures the originator of a call incurs a cost of \( \lambda(c_o + c_t) \), which in a model is which networks set usage rates equal to the marginal usage costs would be \( a^* = (\lambda-1)c_o + \lambda c_t \).
assumptions regarding the probabilities with which customers originate traffic. In section 3 I present an example in which some customers only receive calls, some only originate calls and others are equally likely to receive as they are to initiate calls, and determine what rates would prevail in equilibrium.

2. Equilibrium rates and customer demand characteristics

In this section I present a simple example of the benchmark model presented by LMRT. I then consider four extensions to illustrate the interaction between the rent seeking and the efficient utilization effects. The two key assumptions are whether or not usage rates affect customers’ usage level, and whether each customer both originates and receives calls or whether a customer either only originates or only receives calls. Each of the extensions corresponds to one of the combinations of these assumptions.

I analyze the model in the context of voice telephony, instead of as internet usage as LMRT did, because some extensions require all customers to initiate calls. The institutional details are not especially important. The important elements of the analysis are that i) Two customers jointly consume a good (in this example called phone conversation) and share the benefits of this good; ii) To consume this good each customer must purchase an input from one firm and the cost of the input is the same regardless of whether both customers purchase from the same firm or they purchase from different firms; and iii) For each unit of consumption one customer can be identified as the “originator” and the other can be identified as the “terminator” so that origination may be priced differently from termination and access charges can be appropriately assessed.
2.a. The benchmark model

There are two customers, \( A \) and \( B \), who receive utility from speaking with each other for 2 minutes on the phone. Each customer receives a marginal benefit, \( v(m) \), from each of the 2 minutes of conversation where \( m \in \{1, 2\} \) represents the \( m^{th} \) minute. One of the customers originates (initiates) a call to the other customer, who is said to terminate (or receive) the call. Each customer remains on the call as long as his private benefit is no less than his private cost. Thus, either party can unilaterally end a call if his marginal benefit from continuing the call falls below his private marginal cost. Once a call is ended, no more calls can be made.\(^{16}\)

There are two identical and interconnected networks that provide telephone service to its customers. Because the networks are interconnected a customer of one network can talk both with other customers on the same network and customers on the other network. Networks are constrained to only charge customers for usage on a per minute basis. Each network can set 4 different usage rates, which include rates for on-net and off-net origination, and on-net and off-net termination. Networks provide service at zero marginal cost. There is an access charge of \( a \) that the originating network must pay the terminating network for internetwork calls.\(^{17}\)

Given these assumptions, I construct the following game. First, networks simultaneously announce their usage rates. Customers observe these rates and simultaneously subscribe to one of the networks. (Thus a customer’s strategy is a function that maps every possible combination of rates into a probability of subscribing to each of the two networks.) Once customers have subscribed to a network, one of them places a call, which lasts for 2 minutes or until one customer hangs up. Once this call has ended there are no more calls made. The payoff for each

\(^{16}\) An alternate assumption is that once a call has ended the called party has an opportunity to make a call. This is the assumption used in DeGraba (2003). While this affects the form of the analysis it seems to have no affect on the nature of the results.
customer is the expected benefit he receives from the minutes of calling consumed less expected payments made to or received from his network. The payoff for each network is the expected profit it earns, which includes the expected revenue received from customers less expected net access payments/receipts. I consider only subgame perfect Nash equilibria.

2.a.1. Example 1. \(v(1) = v(2) = 4, a = 3\), \(A\) always originates the call.

In this example each customer receives a benefit of 4 from each of the 2 minutes consumed. Customer \(A\) always initiates the call, and \(B\) receives it. This structure is equivalent to the benchmark case in LMRT, where \(A\) is the website and \(B\) is the customer.\(^{18}\) The access rate is set at 3.

Under these assumptions the ONCPP rates can be supported in equilibrium. Each network sets both origination rates at 3 and both termination rates at -3. At these rates, customers subscribe to each network with probability .5.

It is instructive to see why setting on-net usage rates equal to 0, off-net origination rates equal to 3 and off-net termination rates equal to -3 cannot be an equilibrium. Under these rates each network would earn a zero payoff. Customers would subscribe to each network with probability .5. Under this randomization customers would be on the same network \(\frac{1}{2}\) the time and on different networks \(\frac{1}{2}\) the time. \(A\) receives an expected surplus of \(.5(8) + .5(8-6) = 5\) and customer \(B\) receives an expected surplus of \(.5(8) + .5(8+6) = 11\).

However, given these rates (WLOG) network 1 has an incentive to deviate by setting both its on-net and off-net origination rates greater than 4 and off-net termination rates equal to

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\(^{17}\) The originating network is the network serving the party that places the call, and the terminating network is the network serving the party to whom the call is placed.

\(^{18}\) In the LMRT framework the website sends packets to customers and is therefore viewed as the originator of traffic. A customer’s request for information requires such a small use of capacity that it is ignored.
-(1.5 + \varepsilon) \text{ for } 0 > \varepsilon > 1.5. \text{ The magnitude of on-net termination is irrelevant. At these rates the only subgame perfect continuation is for } B \text{ to subscribe to network 1 and for } A \text{ to subscribe to network 2. } B \text{ receives a surplus of } 8 + (3 + 2\varepsilon) = 11 + 2\varepsilon, \text{ and } A \text{ earns a surplus of } 8 - 6 = 2. \text{ Network 1 earns a profit of } 6 - 2(1.5 + \varepsilon) = 3 - 2\varepsilon. \text{ Thus, in this deviation, network 1 earns a positive payoff.}

In this example } B \text{ generates a rent of 6. A network not serving } B \text{ is willing to pay him up to 6 to subscribe. Similarly, the network serving him would pay him 6 to stay on the network to avoid paying 6 in access fees to the other network. Thus, competition to sign up such a customer allows him to capture the entire rent, regardless of the number of other customers served by the network to which he subscribes. Since only usage rates are allowed in this analysis, the termination rate of -3 applies both to on-net and off-net termination. Similarly } A \text{ creates a rent of -6 and therefore must incur an origination rate of 3 to compensate operators for this.}

This analysis seems consistent with the evolution of rates set for dial-up internet services providers (ISP). In many countries, when an ISP’s customers used dial-up service, the customer’s phone network pays the ISP’s network an access rate for each minute during which the customer is connected. Competition to serve such ISPs to capture these payments reportedly has resulted in networks paying ISPs to be their customers. Thus, ISPs were able to capture the rents bestowed upon them by the interconnection regime.

2.a.2. Example 2. \(v(1) = 4, v(2) = 2, a = 3, A \text{ always originates the call.}\)

I now consider an example that is identical in structure in 2.a.1, except that the value of the 2nd minute of conversation to each party is 2 instead of 4. This creates a downward sloping
demand curve for each customer. Observation shows how this one change causes ONCPP rates to be unsupportable in equilibrium.

**Observation 1.** The ONCPP rates cannot be sustained in equilibrium under the assumptions, \( v(1) = 4, v(2) = 2, a = 3, \) and \( A \) initiates the call with probability 1.

**Proof:** The ONCPP says all origination rates equal 3 and all termination rates are -3. Customers subscribe to each network with probability .5. At these rates only one minute is consumed, so the surplus for \( A \) is 4-3 = 1, and the surplus for \( B \) is 4 + 3 = 7.

Suppose WLOG network 1 deviates by setting its on-net origination rate equal to \(.5 + 2\varepsilon\), its on-net termination rate to \(- (.5 + \varepsilon)\), its off-net origination to \(3 – \varepsilon\) and its off-net termination rate at -3, for \(0 < \varepsilon < 1\). The only subgame perfect response is for both customers to subscribe to network 1. It is a dominant strategy for \( A \) to subscribe to network 1 under these rates, and \( B \)'s unique best response to \( A \) subscribing to network 1 under these rates is to also subscribe to 1.

When both customers subscribe to network 1, \( A \) earns a payoff of \(6 – 2(.5 + 2\varepsilon) = 5 – 4\varepsilon\). \( B \) earns a payoff of \(6 + 2(.5 + \varepsilon) = 7 + 2\varepsilon\), and network 1 earns a payoff of \(2\varepsilon\), which is greater than 0. Thus, there is a deviation which invalidates the proposed ONCPP equilibrium. \(QED\)

The intuition behind this result was described in the introduction. Because the ONCPP rates result in inefficient network usage, a deviation exists in which the deviating network sets rates that both induce more efficient use of the network, and allow it to capture some of the surplus that results from the more efficient usage. I now show there is an equilibrium that is consistent with this intuition.
**Observation 2.** There is an equilibrium in which both networks set on-net usage rates equal to 0 and both off-net usage rates greater than 4 when \( v(1) = 4, v(2) = 2, a = 3, \) and \( A \) always initiates the call.

**Proof:** Suppose customers adopt the following decision rules as part of their strategies:

i) both subscribe to the network offering the highest joint payoff whenever, if both customers subscribe to that network, then neither customer could improve his payoff by unilaterally switching to the other network.

ii) both subscribe to network 2 whenever,

a) the joint payoff from both customers subscribing to network 2 is equal to their joint payoff from both subscribing to network 1, and

b) if both customers subscribe to network 2, then neither customer could improve his payoff by unilaterally switching to network 1.

It is a best response for each customer to play these strategies for any rates set by the networks. Under the proposed rates and the customers’ use of i) and ii) above, both customers subscribe to network 2, both networks earn a zero payoff, and the allocation is Pareto efficient.

**Lemma 1.** There is no deviation by network 1 in which both customers subscribe to network 1, jointly earn as much as they would by subscribing to network 2, and network 1 earns a positive payoff.

**Proof.** The allocation under the proposed equilibrium is Pareto efficient and each network earns 0. When both customers subscribe to network 1, network 2 earns 0. Therefore if the two customers jointly earn at least 12, then network 1’s payoff must be non-positive. ☐
Lemma 2. There is no profitable deviation by network 1 in which exactly one customer subscribes to each network.

*Proof.* If customers are on different networks there is no consumption, because both off-net usage rates on network 2 are above 4. Therefore both customers would earn a zero payoff. □

Lemma 3. There is no profitable deviation for network 2.

*Proof.* Any deviation by network 2 that would allow it to earn a positive profit would require the two customers to jointly earn less than 12. Decision rule i) implies they would both subscribe to network 1 in that case. □ QED.

While this equilibrium captures the notion that efficient network usage plays a key role in the level of equilibrium rates, it is somewhat unsatisfying. It involves each network setting both off-net origination and off-net termination rates that eliminate all inter-network calling if customers were to subscribe to different networks. In essence the rent seeking behavior in this example is defeated by the networks effectively disconnecting from each other, rather than setting on-net usage rates equal to their off-net counterparts.

Decision rules i) and ii) are essentially correlated strategies. The equilibrium behavior for the customers is to subscribe to the same network even though they are indifferent between the networks. Note that (as in the well known “battle of the sexes” game) they do not want to subscribe to each network with probability .5, because that creates a positive probability they will *ex post* subscribe to different networks and earn 0 surplus.

To see why off-net origination rates must be greater than 4, suppose each network sets off-net origination equal to $4 - \varepsilon$. Such a pricing strategy could be broken by network 1 simply
setting off-net termination to \( -(3 - \varepsilon) \), and raising its on-net origination to 4. In this case \( B \) would subscribe to network 1 and earn a surplus of \( 4 + 3 - \varepsilon = 7 - \varepsilon \), which exceeds the payoff of 6 he would receive if both customers subscribed to network 2; \( A \) would receive \( 4 - (4-\varepsilon) = \varepsilon \) by subscribing to network 2, which is greater than the 0 payoff he would receive if he subscribed to network 1; and network 1 would earn a profit of \( \varepsilon \).

2.a.3. Example 3. \( v(1) = 4, v(2) = 2, a = 3, A&B \) originate the call with probability .5

I now consider a third example that maintains all of the values from the previous example except that each customer is equally likely to originate the call. That is, once the subscription decisions are made, nature randomly chooses one customer to make the call. If that customer chooses not to make the call, then no call is made. This symmetry between customers reduces networks’ rent seeking incentives, which creates (with the help of one restriction) the possibility of a pure strategy Nash equilibrium in which the networks do not effectively eliminate internetwork calls. However, the equilibrium is still a tipping equilibrium. In this model ONCPP rates can not be supported in equilibrium.

**Observation 3.** ONCPP rates cannot be sustained in equilibrium when \( v(1) = 4, v(2) = 2, a = 3, \) and \( A \) and \( B \) originate the call with probability .5

**Proof:** See proof of Observation 1. \( QED \)

Here again the ability to deviate by offering Pareto superior on-net usage rates defeats the proposed ONCPP equilibrium. Observation 3 provides a more formal presentation of the intuition developed in the introduction.
**Observation 4.** If off-net termination rates are constrained to be no less than -3, then there is an equilibrium in which both networks set on-net usage rates equal to 0, off-net origination rates equal to 3 and off-net termination rates equal to -3. In this equilibrium both customers subscribe to the same network.

**Proof.** Assume that customers adopt decision rules i) and ii) described in the proof of Observation 2. In the proposed equilibrium each customer subscribes to network 2 and receives a payoff of 6.

**Lemma 4.** There can be no deviation in which both customers subscribe to a deviating network, jointly earn more than 12 and the deviating network earns a positive payoff.

**Proof.** The allocation in the proposed equilibrium is Pareto efficient, and each network earns 0. If both customers subscribe to one network the other network earns a 0 payoff. Thus, for any set of rates under which the customers jointly earn at least 12, the network to which they subscribe must earn a non-positive payoff. □

**Lemma 5.** There is no profitable deviation by network 1 in which exactly one customer subscribes to each network.

**Proof.** Suppose WLOG $A$ subscribed to network 1 and $B$ remained on network 2. With probability .5 $B$ will be chosen to originate the call. In this case network 1 will receive 3 in access fees, and $A$ will receive a benefit of 4 from the call. Network 1 has the option to price off-net origination so that if $A$ is chosen to originate the call, he will originate 0, 1 or 2 minutes. If origination is priced so $A$ originates 0 minutes, then the total expected joint surplus between $A$ and network 1 is $.5(4 + 3)) = 3.5$ so both customers cannot be made better off by the deviation. If origination is priced so that $A$
originates 1 minute, then expected joint surplus is only \(0.5[(4-3) + (4+3)] = 4\), so again no mutually beneficial deviation is possible. Last, if \(A\) originates 2 minutes, then the joint surplus is \(0.5[(6-6) + 0.5(3+4) = 3.5.\)

\[ \square \]

**Lemma 6.** There is no profitable deviation by network 1 in which a) if both customers subscribe to network 2 then one customer would be better off by deviating to network 1 and b) given that one customer subscribes to network 1, the other is better off by subscribing to network 1 and c) network 1 earns a positive payoff.

**Proof.** If \(A\) subscribes to network 1, then he can receive at most a surplus of 7 when \(B\) originates the call (which assumes network 1 sets its off-net termination rate at -3) so the expected surplus from this occurrence is 3.5. Thus, he must receive an expected surplus of greater than 2.5 from originating a call to be willing to unilaterally subscribe to network 1. The highest off-net origination charge that would allow \(A\) to earn an expected surplus of 2.5 from originating the call is 0.5. At this rate \(A\) originates 2 minutes, and network 1 earns a negative profit of \(2(0.5 - 3) = -5\). Network 1 would be willing to offer such off-net rates therefore, only if having one customer subscribe would cause the other customer to subscribe as well.

If \(A\) subscribed to network 1, then \(B\)’s expected surplus while staying on network 2 is \(0.5(4-3) + 0.5(6+6) = 6.5.\) \(B\) would have to receive a surplus of at least 6.5 to be willing to switch to network 1. But, since \(A\) and \(B\) are identical, any rates that allow \(B\) to earn a surplus of 6.5 would also allow \(A\) to earn 6.5. Since this market can only generate a total surplus of 12, and network 2 earns 0, network 1 would have to earn a payoff no greater than -1.

\[ \square \]

Thus, there is no profitable deviation from the proposed rates \(QED\)
The importance of Observation 4 is to show that when customers originate and terminate traffic with the same probability, there are fewer rents to seek than when one customer always receives calls. In this example, because the rents are sufficiently small, (in fact given equilibrium rates the expected rents are 0) the deadweight loss created by using usage rates to give customers the rents he generates exceeds the value of the rents, and thus do not defeat an equilibrium in which on-net usage is priced efficiently.\textsuperscript{19}

The reason for restricting off-net termination rates to be no less than -3 is illustrated in the following example. Suppose given the proposed equilibrium, network 1 deviated by setting an off-net termination rate equal to -10, an off-net origination rate greater than 4 an on-net origination rates equal to $\varepsilon$ and an on-net termination rate of 0. If both customers subscribe to network 2 they would each receive a payoff of 6. WLOG A could subscribe to network 1, and with probability .5 terminate one minute of calling from B, thereby receiving an expected payoff of $.5(4 + 10) = 7$, which is greater than the 6 he would receive by staying on 2. B’s best response to this would be to subscribe to network 1 as well, and earn a payoff of $6 - .5\varepsilon$, as opposed to the $.5(4 - 3) = .5$ he could earn by remaining on network 2.

The problem with this example is that the proposed equilibrium is broken because network 1 can offer an off-net termination rate that would never be profitable to honor, but because it induces a tipping equilibrium, it will never have to be honored. The restriction prevents network 1 from offering such non-profitable rates.\textsuperscript{20} I maintain this assumption in the remainder of section 2.

\textsuperscript{19} So for example a network could deviate by setting off-net termination equal to $-(3 - \varepsilon)$, and all other usage rates greater than 4, turning any customer into a net terminator of traffic. However, any customer that subscribed under these conditions would earn an expected surplus of only $.5(4+3) = 3.5$.

\textsuperscript{20} A more detailed justification might be that while we are modeling static equilibria that occur instantaneously, a richer model would account for the fact that a network would tend to attract customers over time. So a network that attempted to use a strategy in which it paid customers off-net termination rates in excess of the access charges would have to absorb the losses implied by such payments for a period of time, and such losses (when discounted for time) could exceed the eventual profits of the eventual tipping equilibrium.
Lemma 6 points out an interesting complication that is introduced when customers have downward sloping demand curves. Since each customer chooses the quantity of minutes to consume, and that quantity depends on usage rates, each customer’s surplus depends not only on the usage rates he faces, but on the rates the other customer’s face as well. Thus, each customer cares about the network to which the other customer subscribes. So each customer’s subscription choice must not only be optimal with respect to the rates charged by networks, but optimal with respect to the subscription choices of the other customer as well. Hence the need for decision rules i) and ii).

2.a.4. Example 4. $v(1) = v(2) = 4$, $a = 3$, $A&B$ originate the call with probability .5

I now discuss the remaining case in which both minutes have a value of 4 to each customer, and each customer is equally likely to originate the call. In this case the usage rate does not affect customer usage at the margin and neither customer generates any rents. Thus, neither rent seeking nor a downward sloping demand curves pins down equilibrium rates, resulting in:

**Observation 5.** The ONCPP rate can be supported in equilibrium. There is also (at least) a second equilibrium in which the off-net origination rate is set at 3, the off-net termination rate is set at -3 and on-net usage rates are set at 0.

**Proof: Existence of ONCPP equilibrium:**

LMRT have proven the existence of the ONCPP equilibrium.

**Existence of an efficient utilization equilibrium:**

Alternatively in a market in which there is some product differentiation a network could not expect to serve 100% of the customers. In this case the network would be required to actually pay the excessive negative termination charges it used to attract customers, and thus strategies that would be profitable in a perfectly homogeneous setting may prove to be unprofitable when a small amount of product differentiation is introduced.
Assume that customers adopt decision rules i) and ii) from the proof of observation 4. Under the proposed rates each customer earns a surplus of 8, each network earns a profit of 0, and the allocation is Pareto efficient.

**Lemma 7.** There is no deviation by network 1 in which both customers subscribe to network 1, jointly earn a surplus of at least 16, and network 1 earns a positive profit.

*Proof.* If both customers subscribe to network 1, network 2 earns a zero profit. Since the market can generate at most a surplus of 16, if the two customers earn a joint surplus of at least 16, network 1’s payoff must be non-positive. □

**Lemma 8.** There is no deviation by network 1 in which exactly one customer subscribes to network 1, and network 1 earns a positive payoff.

*Proof.* Suppose \( A \) subscribes to network 1. If off-net rates on network 1 are such that a call is always completed, then \( B \) (on network 2) receives an expected payoff of 8 and network 2 earns a profit of 0. \( A \) (on network 1) has the option of subscribing to network 2 and earning a surplus of 8. Thus, he must earn at least a payoff of 8 by subscribing to network 2. If rates are such that a call is always completed regardless of who is chosen to originate the call, then the joint payoff for \( A \) and network 1 is 8, which implies network 1 cannot earn a strictly positive payoff.

Suppose network 1 sets off-net rates so that \( A \) only terminates calls. Then the expected joint surplus between \( A \) and network 1 is \( .5(8 + 6) = 7 \). If rates are such that \( A \) only originates calls the expected joint surplus is \( .5(8 - 6) = 1 \). □

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21 E.g., if network 1 set infinite on-net usage rates, free off-net origination rates and off-net termination rates of -3.
Lemma 9. There is no deviation by network 1 in which a) if both customers subscribe to network 2 then one customer would be better off by unilaterally subscribing to network 1 and b) given that one customer subscribes to network 1, the other is better off by subscribing to network 1 as well, and c) if both customers subscribe to network 1, it earns a positive payoff.

Proof. Assuming -3 is the lowest termination rate that can be offered, the only deviation network 1 can implement that would cause one customer to unilaterally deviate requires the off-net origination rate to be less than 3. With A subscribing to network 1 with an origination rate less than 3, customer B earns a surplus of 8 by staying on network 2. Thus, he must earn at least a surplus of 8 to be willing to subscribe to network 1. But, since A and B are identical then A would also have to earn at least 8. This implies network 1’s payoff would have to be non-positive. □ \textit{QED}

It should not be surprising that the ONCPP rates do not constitute a unique equilibrium in this case. Because each customer initiates the call with probability .5, the only way for a customer to generate any rents is for him not to originate the call when nature chooses him to do so. But the rates that would induce such behavior create a deadweight loss that exceeds the value of the rents it generates. Thus, there is no profitable rent seeking behavior for a network, and any set of on-net rates (with absolute value no greater than 4) that sum to 0 can be supported in equilibrium.

3. Multiple Customer Types

Given the above analysis it is interesting to ask what rates would prevail if there were a mix of customer types. That is, if there were some pairs of customers for which each customer is equally likely to initiate a call, and others pairs for which one customer always initiates the call,
what rates could be supported in equilibrium? For example, one might think of the relationship between two residential customers as one in which each party is equally likely to initiate a call, whereas the relationship between a residential customer and, say, his dial up ISP is one in which the residential customer is likely to initiate all of the calls.

I therefore extend the model of the previous section by maintaining the basic structure of the game, and assume there are three customers, \( A, B \) and \( I \). As in 2.a.3. above, for conversations between \( A \) and \( B \), \( v(1) = 4, v(2) = 2 \), and \( A \) and \( B \) initiate the call with probability .5. For conversations between \( A \) and \( I \) or \( B \) and \( I \), \( v(1) = 4, v(2) = 2 \), \( I \) never initiates a call. Thus, in this model think of \( A \) and \( B \) as residential customers and \( I \) as their dial up ISP.

As before there are two networks, 1 and 2 and the government imposes an access charge of 3.

**Observation 6.** There is no ONCPP equilibrium.

**Proof:** Suppose each network set all origination rates at 3 and all termination rates at -3. At these rates all customers are indifferent between the networks. Each pair of customers engages in one minute of conversation. \( I \) receives a payoff of \( 2(4 + 3) = 14 \). \( A \) and \( B \) each earn an expected payoff of \((4-3) + .5(4 – 3) + .5(4 + 3) = 5 \). Each of the networks earns a payoff of 0. Suppose WLOG network 1 could set off-net origination rate at \( 3 - \varepsilon \), off-net termination at -3, on-net origination at \( 2\varepsilon \) and on-net termination at 0, for \( \varepsilon \) arbitrarily close to 0. Under this deviation \( A \) and \( B \) each have a unilateral incentive to subscribe to network 1 regardless of any other customer’s subscription decision. \( A \) earns a surplus of \((4-3) + .5(4-(3-\varepsilon)) + .4(4+3) = 5 + \varepsilon \) if he subscribes to network 1 by himself and \((4-3) + .5(6-2\varepsilon) + .5(6) = 7 – \varepsilon \) if both \( A \) and \( B \)
subscribe to network 1. Given that both residential customers are subscribing to network 1, \( I \)'s best response is to subscribe to network 2, where he earn \( 2(6+6) = 24 \).

Under this allocation of customers, network 1 earns a positive payoff of \( 6\varepsilon \) by deviating. Thus, the ONCPP rates do not constitute an equilibrium. \( QED \)

This example suggests there may be a broad set of conditions under which ONCPP rates cannot be sustained in equilibrium. The proposed ONCPP rates impose an inefficiency between residential customers that are equally likely to call each other. In this case it seems a network could always deviate from the ONCPP rates by offering on-net rates that eliminated this inefficiency for residential customers while setting off-net rates that left (essentially) unchanged the relationship between residential customers and dial up ISPs.

**Observation 7.** There exists an equilibrium in which each network sets on-net usage rates to 0, and off-net origination and termination rates greater than 4.

**Proof:** The following extensions of rules i) and ii) are best responses when played by all customers to any set of network rates:

i’’) all subscribe to the network offering the highest joint payoff whenever, if all three subscribed to that network no customer could increase his payoff by unilaterally subscribing to the other network.

ii’’) all customers subscribe to network 2 whenever

a) the joint payoff for all three customers subscribing to network 2 is equal to the joint payoff when all customers subscribe to network 1, and
b) if all three customers subscribe to network 2, no customer could increase his payoff by unilaterally subscribing to network 1.

It is clear that these two rules, if played by all three customers constitute best responses to each other for all rates set by the networks. Under these rules and the proposed equilibrium rates, all customers would subscribe to network 2. Each network would earn a payoff of 0 and each customer would earn a payoff of $2(4 + 2) = 12$.

Because network 2 sets all off-net usage rates greater than 4, any customer that unilaterally subscribes to network 1 will complete 0 minutes of calling and therefore earn a payoff of 0.

There is no deviation that network 1 can make that will both yield a positive payoff to itself and a joint payoff to the three customers of at least 36 (since the proposed equilibrium yields a Pareto efficient allocation).

Similarly any deviation by network 2 that would yield a positive payoff would have to result in the three customers earning a joint payoff less than 36 from subscribing to network 2 and therefore (under i’) subscribing to network 1.  

Observation 7 is simply the extension of observation 2, demonstrating again the potential for a tipping equilibrium, and suffering from the same problem as the equilibrium in observation 2 that pricing effectively eliminates inter-network calls. To address this problem I expand on the restriction made in the previous section that a network cannot make a payment to a customer for off-net usage in excess of revenue the network receives from other sources for providing the off-net usage. Thus, a network cannot set its off-net termination rate less than -3 and can not set its off-net origination rates less than 0. Under these restrictions there is a more palatable
equilibrium, in that networks do not set prices that effectively preclude internetwork calls. However the equilibrium is still a tipping equilibrium.

**Observation 8** There exists an equilibrium in which each network sets its off-net origination rate equal to 3, its off-net origination rate equal to -3, its on-net origination rate equal to .5, and its on-net termination rate equal to -.5.

**Proof:** Assume customers adopt the decision rules i’) and ii’) outlined in the proof of observation 7. Under the proposed equilibrium rates all customers subscribe to network 2. I receives a payoff of $2(6 + 1) = 14$, and A and B each receive an expected payoff of $(6 – 1) + .5(6 – 1) + .5(6 + 1) = 11$. Note that given the networks’ rates, no customer could unilaterally deviate (subscribe to network 1) and earn a higher payoff.

**Lemma 10.** There is no deviation by network 1 in which all customers subscribe to network 1, customers jointly earn a payoff of at least 36, and network 1 earns a positive payoff.

**Proof.** Under the proposed equilibrium the allocation is Pareto efficient and customers jointly earn a payoff of 36. When all three customers subscribe to network 1, network 2 earns a payoff of 0. Thus if the customers jointly earn a payoff of at least 36 when subscribing to network 1, network 1’s payoff must be non-positive.

**Lemma 11.** There is no deviation by network 1 in which if all three customers subscribe to network 2, I has an incentive to unilaterally deviate and subscribe to network 1.

**Proof.** With all three customers on network 2, I earns a surplus of 14. The lowest off-net termination rate network 1 could set is -3. This would result in a surplus of $2(4 + 3) = 14$ for I if
he unilaterally subscribed to 1. But ii’) along with lemma 10 implies I would need to earn more than 14 to unilaterally deviate.

Lemma 12. Any deviation by network 1 in which $A$ has a unilateral incentive to subscribe to network 1 (if all three customers are subscribing to network 2) must involve network 1 setting an off-net origination rate less than .5.

Proof. With an off-net origination rate equal to .5 the highest surplus $A$ could earn by unilaterally subscribing to network 1 is $(6 - 1) + .5(4 + 3) + .5(6 - 1) = 11$, which is the same as the 11 he could earn by subscribing to network 2. When $A$ is the only subscriber to network 1 his surplus is decreasing in its off-net origination rate and increasing in its off-net termination rate.

Lemma 13. There is no profitable deviation for network 1 in which exactly one residential customer subscribes to network 1.

Proof. With all three customers on network 2, $A$ and $B$ each earns a surplus of 11. Suppose network 1 set an off-net termination rate of -3 and an off-net origination rate of .5 - $\varepsilon$. If $A$ unilaterally subscribed to network 1, his expected surplus would be $(6-1 + 2\varepsilon) + .5(6-1 + 2\varepsilon) + .5(4+3) = 11 + 3\varepsilon$. However, network 1’s profit would be negative because it loses $3 - (.5 - \varepsilon)$ on every minute $A$ originates and earn 0 on every minute $A$ terminates.

Lemma 14. There is no profitable deviation for network 1 in which just $A$ and $B$ subscribe to network 1.
Proof. With its off-net termination rate equal to -3 and its off-net origination rate equal to .5, network 1’s maximum payoff would occur if it set both on-net usage rates equal to 2. With only $A$ and $B$ subscribing to network 1 under these rates, each customer would earn a surplus of $(6-1) + .5(6-4) + .5(6-4) = 7$, which is less than the $(6-1) + .5(4-3) + .5(6+6) = 11.5$ $B$ could earn if he subscribed to network 2 instead of network 1. If only $A$ and $B$ are subscribing to network 1 at these rates its payoff would be $2(1 - 6) + 2(2+2) = -2$. Thus, network 1 would always earn a negative payoff from such a deviation. □

Lemma 15. There is no deviation in which $I$ and exactly one residential customer subscribe to network 1, and network 1 earns a positive payoff.

Proof. If $A$ subscribes to network 1, who sets off-net origination less than .5, while $B$ and $I$ subscribe to network 2, $I$ earns a surplus of $(6+1) + (6+6) = 19$. Thus, any deviation in which $A$ and $I$ are the only customers subscribing to network 1, must give $I$ a surplus in excess of 19, must give $B$ more surplus on network 2 by himself than he could receive by subscribing to network 1, and give $A$ more surplus on 1 than if he subscribed to network 2.

Suppose network 1 set its off-net termination rate equal to -3. Then $I$ would receive a surplus of $4 + 3 = 7$ from the call from $B$, and assuming that $A$ initiates 2 minutes of calling, $I$ would need to receive at least 6 in on-net termination payments to reach a total surplus of 19. This would require network 1 to set an on-net termination rate of at least -3. But at a matching on-net origination rate of 3, $A$ would only originate 1 minute of calling to $I$, which would cause $I$’s surplus to be below 19. If $A$ initiated only 1 minute of calling to $I$, then $I$ would have to receive 8 in on-net termination payments. At a matching on-net origination rate of 8 $A$ would
Lemma 16. There is no deviation in which, if all three customers subscribe to network 1, none has a unilateral incentive to subscribe to network 2, and network 1 earns a positive profit.

Proof. With both $A$ and $B$ on network 1, (and off-net origination rate on network 1 less than .5) if $I$ remained on network 2, he would earn a surplus of $2(6 + 6) = 24$. With all three customers on network 1, assuming each residential customer originated 2 minutes worth of calling to $I$, network 1 would have to set its on-net termination rate equal to -3 for $I$ to receive a surplus of 24. To earn a positive payoff network 1 would have to set on-net origination greater than 3. But, this would cause $A$ and $B$ to originate at most only 1 minute each to $I$. If $A$ and $B$ each originated 1 minute to $I$, then $I$ would need to receive 16 in on-net termination payments. In this case network 1 would have to set its on-net origination rate equal to 8, which would cause the residential customers to originate 0 minutes. Thus, there are no rates that would prevent $I$ from unilaterally subscribing to network 2 under which network 1 could earn a positive payoff. □

QED

Observation 8 provides several interesting insights. First, $I$ receives the same surplus he would receive under ONCPP rates. This observation reflects what I believe to be the robust result of the LMRT analysis, which is that the imposition of an inefficiently high termination charge endows net terminators of traffic with a rent. When there is competition among interconnected homogenous networks, the net terminator of traffic will capture all of the rents from access charge, regardless of the network to which he subscribes. (Note that in this case he
does not receive as large a monetary payment as he would have received under ONCPP rates, rather he gets some of the benefit in increased utility from increased consumption).

Second, this observation suggest LMRT are overly optimistic regarding the ease with which access rates may be used as a regulators’ instrument of social engineering. In their model, networks are indifferent regarding the level of access charges, largely because usage rates in their model do not affect usage levels. They then argue that access rates can be used as a policy instrument to generate the efficient level of subscription to the network.22 My analysis suggests that raising access rates will inefficiently raise off-net usage rates, which will induce inefficiently low levels of off-net utilization. In observation 8 this inefficiency is eliminated, but only at the expense of all customers subscribing to a single network. This is unlikely to be a palatable result for countries attempting to foster competitive telecommunications markets.

Third, observation 8 suggests a potential generalization of the ONCPP. Specifically, when there are two types of customers, the customer that terminates more calls than he originates can earn rents equal to those he could earn if all of his calls were off-net (i.e., he were the only customer served by a network). When usage rates affect customer utilization levels, this will imply that on-net rates will be lower than their off-net counterparts.

4. Conclusion

This papers offers a reconciliation between the ONCPP which predicts that on-net usage rates will reflect any access charges that an originating network pays a terminating network, and opposing results in the literature which predict that, when customers have downward sloping demand curves, on-net usage rates will be set to maximize social surplus from on-net usage and will be set independent of access charges for off-net calls. In both analyses customers that
terminate more minutes than they originate are endowed by the interconnection regime with a positive rent, and those that originate more minutes are endowed with a negative rent.

I have shown that when it is assumed that customers’ usage levels are not affected by usage rates, then competition among non-differentiated networks results in usage rates that allow net terminators of traffic to capture all of the rents they generate, and force net originators to pay for the negative rents they generate. In this case the ONCPP prediction is born out, so on-net rates reflect off-net access charges.

However, when it is assumed that customers’ usage levels are affected by usage rates, setting usage rates to compete for rents creates inefficiently low on-net consumption. When customers’ rents are small enough, the cost that net terminators incur from consuming too few minute of on-net usage outweigh the benefits of the rents they capture. In this case equilibrium on-net usage rates are set to ensure efficient on-net usage, rather than to redistribute rents. These equilibria are tipping equilibria in which all customers join the same network. These results suggest that setting access rates that induce efficient utilization of the network may be an appropriate goal for regulators in charge of setting such rates.

References


