WORKING
PAPERS

PRODUCT VARIETY AND CONSUMER SEARCH

Jeffrey H. Fischer

WORKING PAPER NO. 201

February 1993

BUREAU OF ECONOMICS
FEDERAL TRADE COMMISSION
WASHINGTON, DC 20580

Product Variety and Consumer Search

Jeffrey H. Fischer*

January 1993

Federal Trade Commission

Abstract: Previous work on consumer search has shown that consumers facing positive search costs do not sample more than one firm; that is, no search occurs in equilibrium. This result, as well as the price charged, are independent of the magnitude of search costs. I develop a model in which consumers search for a most-preferred variety of a heterogeneous product. If products are sufficiently differentiated, consumers will sample additional firms and, consequently, search costs affect both the price charged and the probability of search.

*Bureau of Economics, Federal Trade Commission, Washington, DC, 20580. Opinions expressed in this paper are those of the author and are not necessarily those of the Federal Trade Commission. I would like to thank Joe Harrington, Bruce Hamilton, Ralph Chami, and Vandana Chandra for their contributions and the time they took from their own work, as well as participants at Johns Hopkins University and the 1991 Southeastern Economic Association meetings. As always, the usual disclaimers apply.

1 Introduction

Anyone who has shopped around for a particular type of good knows what it is like to search for the right variety at the right price. Uncertainty about the location of stores with low prices and stores with desirable products causes consumers to waste time and effort on search. If the uncertainty were only over price, wasteful search could be eliminated by a series of phone calls or through advertisements.[1] To logically explain consumer search, one must recognize that a key cause of search is product heterogeneity; in particular, heterogeneity of a type that requires visiting a store before learning the desirability of that store's product so that advertising is not a substitute for search. It is this aspect of search that is missing in most existing models. This paper corrects this omission, using the framework of optimal consumer search decisions in a heterogeneous product market.

This paper models consumer search decisions when goods and preferences are heterogeneous. The goal of the paper is to argue that heterogeneity is necessary for the equilibrium distribution of prices to be such that consumers are willing to search in a nontrivial way.[2]

In the theory of consumer search,[3] two paradoxical results are prominent: first, that consumers facing positive search costs do not sample more than one firm; and second, that this result, as well as the price firms charge, is independent of the magnitude of search costs. That is, *any* positive search cost causes consumers to behave in a manner completely opposite to behavior when search costs are zero.

To some extent, then, search theory is misnamed: consumers do not search at all in any meaningful way. By "search," I mean that some measure of consumers will go to one store,

[1] In fact, this is the way consumers often purchase some products. For example, mortgage rates are available through both newspaper articles and by phone. Airline ticket prices are advertised in newspapers, on-line services (such as EasySABRE), and are available over the phone. Some mail-order products, such as coffee, are sold on the basis of price alone. Much advertising, for products as diverse as groceries and mattresses, is devoted to convincing the consumer that a particular store has the "guaranteed lowest price." The common denominator for these products is that they are all fairly homogeneous: price is the main characteristic that distinguishes among firms.

[2] By "nontrivial" I mean that at least some consumers with positive search costs, given the expected distribution of prices and varieties, will rationally search more than one store. This definition excludes search by consumers with no search costs (Butters [1977], Stahl [1989]); search which does not maximize utility (Wilde and Schwartz [1979]); or search rules based on conjectures of behavior which does not hold in equilibrium (Carlson and McAfee [1983] and Wolinsky [1983]).

[3] This paper considers search over *horizontally* differentiated goods. Horizontal product differentiation is defined through different preference rankings across consumers of the available varieties. In contrast, vertical differentiation requires that all consumers rank the varieties the same way, though differences in an individual's demand for a variety at a given price may vary. Differences in product *quality*, rather than differences in preferences, fall under the category of vertical differentiation. I do not deal with differences in quality across firms.

observe the price and the variety sold there, and decide to sample additional stores before purchasing one of the varieties. Firms are able to exploit consumers' uncertainty about the location of low-price sellers in such a way as to make a high-price equilibrium possible: since consumers cannot observe deviations from the proposed equilibrium price without search, firm demand is inelastic to unilateral changes in price, relative to full-information models. Consumers do not expect price-undercutting behavior, so the benefit to search is nonexistent. As a result, search will not occur if search is costly. Since no search occurs, no firm wants to deviate from the proposed equilibrium by undercutting other firms.

In particular, Diamond (1971) showed that consumers facing positive search costs will not search for low prices, but will instead buy from the first firm sampled. Because consumers *expect* high prices and no deviations from the high price, firms are able to charge the monopoly price despite the presence of a large number of competitors all selling identical products. Furthermore, Diamond's results hold regardless of the magnitude of search costs.

Subsequent work has shown that Diamond's results are robust to certain changes in his basic model. Stahl (1989) introduced into the Diamond model a set of consumers for whom search costs are zero. These consumers sample all firms, buying from the lowest-priced firm, and in doing so act to restrain the prices firms can charge. As the percentage of these zero-search-cost consumers grows, prices converge to the competitive solution. Even in Stahl's model, however, no consumer with positive search costs samples more than one firm. Consequently firms with above-average prices still make some sales, even though all consumers are aware that lower prices may exist. Earlier, Butters (1977) assumed that some consumers received advertising messages from firms, in which case they would purchase the product from the firm with the lowest advertised price, while other consumers received no advertising messages and "searched" for an acceptable price. The consumers who received messages had zero search costs for the subset of firms from which they received messages. Like Stahl, consumers with positive search costs all purchased from the first firm sampled, so no search occurred in equilibrium.[4]

The aspect of the Diamond, Stahl, and Butters papers that drives their counterintuitive results is that all firms sell identical products. Since price is the only characteristic of the good that may vary across firms, firms set prices such that any differences in price across firms is insufficient to offset the expected search costs a consumer would incur in finding the low price.

I model firms' products as different varieties of a heterogeneous good. The heterogeneity is horizontal, rather than vertical: not all consumers rank the varieties in the same way. For

[4] See Stahl (1989) for a survey of other papers of consumer search.

some prices and enough heterogeneity, some consumers will not buy from the first firm sampled, preferring instead to continue search for sufficiently low search costs. If search costs are high, these consumers will not purchase the commodity at all. If search costs are sufficiently low, however, the expected benefit of search outweighs the expected costs, so additional search will occur.

In contrast, consider a model of search over a homogeneous product when all consumers have positive search costs. Price is the only object for which to search. Thus for consumers to make nontrivial searches, consumers must expect prices to differ across firms. However, a firm can improve upon any proposed equilibrium involving a distribution of prices by increasing his price by less than the lowest cost of search. Since consumers make an initial search based on the expected distribution of prices, no shopper who reaches the deviating firm will fail to buy from that firm if he would have bought under the proposed equilibrium price. Hence the proposed price distribution is not an equilibrium. With product heterogeneity as well as heterogeneity across consumers, consumers who find products sufficiently far removed from their utility-maximizing choices will incur an additional search cost in an attempt to find a closer match. Thus some kind of heterogeneity is a necessary part of a search equilibrium.[5]

In this model, two firms each produce one variety of a heterogeneous product. Each variety is a random draw from a distribution of varieties; the distribution is common knowledge to both firms and consumers. All consumers incur a common search cost to sample the second firm. The model shows that, in contrast to the established literature, (1) equilibrium search occurs for some parameter values; and (2) the average price charged by firms depends positively on search costs. The intuition is that sufficiently low search costs, relative to the degree of heterogeneity among the varieties, induces search on the part of consumers for whom the variety sold by the initial firm sampled yields little utility. Enough product heterogeneity makes each firm want to sell to those consumers who value its variety highly, rather than simply sell to those "captive" consumers who sampled that firm first, because the higher price received from these consumers more than offsets the loss in demand from consumers who do not value the firm's variety as highly. In contrast, no search occurs in the Diamond model. In addition, unlike Diamond, the price charged depends on the search cost. I obtain Diamond's results when the degree of heterogeneity is small relative to search costs, making search for a more-preferred variety an unattractive proposition. Finally, I show that increasing the degree of heterogeneity results in

[5] Introducing noise into the system, in consumers' perceptions of either the products or pricing, is a mechanism to create heterogeneity artificially. See Burdett and Judd (1983).

higher equilibrium prices as long as equilibrium does not involve serving the entire market.

2 The Model

2.1 Firms and varieties

Two firms each produce a variety x_i, $i = 1, 2$ randomly drawn from a set X, so a firm is defined by a point $x_i \in X$. I take the set X to be the points on a circle with circumference κ. Firm i knows x_i but not x_j. The space of varieties is shown in Figure 1. (See Salop [1979] or Wolinsky [1983] for other models involving circular product spaces.) One way to model increasing heterogeneity is through increases in the circumference of the circle, which is equivalent to an increase in the span of consumer valuations of varieties. This is the case in which more varieties are available, and the new varieties are refinements of the old ones. Alternatively, one can model increasing heterogeneity through an increase in the disutility a consumer receives for a given distance away from his most-preferred variety. In this case the number of varieties remains constant, but varieties become less substitutable for one another. Since the latter scenario is closer in spirit to the standard concept of heterogeneity, this is the topic I explore in Section 4.

A strategy for firm i is a price p_i. I restrict prices to the interval $[b, \bar{v}]$, where $\bar{v}$ is the maximum amount any consumer would be willing to pay for a particular variety and b is the common marginal cost; this assumption is innocuous, because profits for all prices above $\bar{v}$ are zero and are negative for all prices below b.

Firm i (correctly) believes that the location of firm j is determined solely by a draw from X, that firm j charges a price p^*, and that this price is a common expectation across firms and consumers.

Firm i chooses price p_i given his expectation that firm j has a price p^* to maximize expected profits, which are given by

$$\pi_i(p_i) = (p_i - b)d_i(p_i, p^*) \tag{1}$$

where $d_i(p_i)$ is the (expected) demand to firm i when $p_j = p^*$.

2.2 Consumers and search

A consumer is defined by a point in the set X representing his most- preferred variety. Each consumer l has a most-preferred variety x_*^l which yields a surplus of $\bar{v}$; valuations decrease linearly away from the most-preferred variety, so that, for some product x_i, the surplus to consumer l is

$$v^l(y_i) = \bar{v} - \theta|x_i - l|. \tag{2}$$

where $y_i = |x_i - l|$ is the minimum arc distance from the point on the circle denoting x_i to that denoting consumer l. y_i has support $[0, \kappa/2]$. Preferences are distributed such that the density of types l are distributed uniformly around a circle whose points are the set of varieties X; just as many consumers have x_i as their most-preferred variety as any other variety x_j. Denote by x_*^l the most-preferred variety of consumer l.

Consumers search until they find utility that satisfies their stopping rule. Consumers do not have any information *a priori* as to whether x_i or x_j is closer to its most-preferred product, so consumers pick an initial firm at random. Once at the initial firm, a consumer may buy, search a different firm, or opt out of the market. The surplus to buying is $v^l(x_i) - p_i$ for a price p_i; the surplus to opting out is normalized to zero; and the surplus to searching firm j is

$$\frac{1}{\kappa}\int_{y_j \in [0,\kappa/2]} v^l(y_j)dy - p^* - c \equiv \mathrm{ES}(p^*) - c$$

where c is the cost of searching firm j, $\mathrm{ES}(p^*)$ is the expected surplus (net of price) from search, and a consumer expects $p_j = p^*$. The integral yields an expected valuation from an additional search. The term $1/\kappa$ adjusts for the size of the circle and hence the number of available varieties: as κ increases, the probability of a search yielding a variety further away from the most-preferred variety increases, lowering the valuation expected from search.

Consumer demand is perfectly inelastic: a consumer will buy exactly one unit of the good as long as the surplus from some firm is positive. He will purchase this unit from firm i if his surplus there is positive and greater than his expected surplus from searching firm j. Thus a consumer initially at firm i will buy from firm i if and only if

$$v^l(y_i) - p_i \geq \mathrm{ES}(p^*) - c \text{ and } v^l(y_i) \geq p_i \tag{3}$$

and will search if

$$v^l(y_i) - p_i < \mathrm{ES}(p^*) - c \text{ and } \mathrm{ES}(p^*) \geq c. \tag{4}$$

The first relation in (3) indicates that the surplus at the current store exceeds the expected surplus from search, while the second ensures that the actual surplus from buying is nonnegative. Similarly, the first relation in (4) indicates that the expected surplus from search is higher than the surplus from buying from the current store, while the second requires that the expected surplus is at least as great as the search cost (so that search is optimal).

Once at firm j he observes x_j and p_j, but loses any information about x_i and p_i. A consumer either loses information about the precise combination of characteristics he previously observed, thus "forgetting" his private valuation of the object, or realizes that the product(s) offered may change while he engages in further search. I make this assumption in order to generate a smooth demand curve for each firm based on a reservation-brand/price combination. Without this assumption of no recall the model would exhibit a fundamental asymmetry between consumers who have searched all the firms and those who have searches remaining. The two consumer types that this asymmetry creates—those who have searches remaining and those who do not—respond differently to changes in prices. If firms are unable to identify members of each group and charge a price according to a consumer's remaining search opportunities, no pure-strategy price equilibrium exists. While obtaining a pure-strategy equilibrium is not essential, doing so makes the subsequent analysis cleaner and does not create the conceptual difficulties associated with mixed-strategy equilibria.

With the no-recall assumption equations (3) and (4) are independent of the number of searches a consumer has already made. The no-recall model is asymptotically the same as a model with recall: as the number of firms tends to infinity the number of consumers who have searched all the firms without buying becomes insignificant to the demand of any individual firm.[6] To avoid consumers repeatedly searching the same firm, I assume that consumers may not search the same firm consecutively.[7]

This setup generates a *reservation brand* (see Kohn and Shavell [1974]): a consumer of type l will purchase from firm i charging p^* as long as the net expected surplus from search is less than the search cost; that is, for all y_i such that

$$\frac{1}{\kappa}\int_0^{y_i}[v^l(y_j)-v^l(y_i)]dy_j \leq c. \tag{5}$$

The reservation brand for a consumer of type l is defined by a distance R from consumer l's most-preferred variety, where R is such that (5) holds with equality:

$$\frac{1}{\kappa}\int_0^{R}[v^l(y_j)-v^l(R)]dy_j = c. \tag{6}$$

Note that if $R = \kappa/2$, search costs are sufficiently low relative to the number of available varieties and the disutility parameter θ that all brands satisfy the stopping rule and no search

[6]Implicit in the model of Wolinsky (1983) is the no-recall assumption; otherwise he could not use the reservation-brand property he employs. See Kohn and Shavell (1974).

[7]This assumption is innocuous since a process of random search will generate the same expected demand to firms and the same expected utility to consumers as long as the entire search phase takes place before firms have the opportunity to changes prices or brands.

takes place. I return to this topic in Section 3, where I consider the possibility that the probability from buying after sampling an initial brand is one. Also note that

$$\int_0^R \left[v^l(y_j) - v^l(R)\right] dy_j = \int_0^R (\bar{v} - \theta y_j - \bar{v} + \theta R)\, dy_j = \theta \int_0^R (R - y)\, dy_j = \frac{\theta R^2}{2}$$

so, from (6),

$$R = \sqrt{\frac{2\kappa c}{\theta}}. \tag{7}$$

Consumer l responds to price deviations as follows: he will accept a brand priced at $p_i \neq p^*$ as long as $v^l(y_i)$ is at least as high as $v^l(r)$, where $v^l(r)$ is defined by

$$v^l(r_i) - p_i = v^l(R) - p^*. \tag{8}$$

Equation (8) says that the minimum acceptable brand must generate just enough additional surplus to offset any price increase above p^*. Simplifying (8),

$$|r_i(p_i) - l| = |R + l| - \left(\frac{p_i - p^*}{\theta}\right). \tag{9}$$

Note that $r_i(p_i)$ is a function of p_i, given p^*. Then from (9) one can see that

$$\frac{\partial r_i}{\partial p_i} = -\frac{1}{\theta} \tag{10}$$

and

$$\frac{\partial^2 r_i}{\partial p_i^2} = 0.$$

The above equations are a result of the assumption that utility decreases linearly in price.

Since an interior solution requires that some consumers do not find both brands acceptable, I restrict attention to the case where $R < \kappa/2$. Using (7), $R < \kappa/2$ if $\kappa > 8c/\theta$.

2.3 Firm demand

2.3.1 Known brands

I first consider the demand for firm i when x_j is known, then integrate over the set of possible brands to generate expected demand.

Given x_j, $d_i(p_i; x_j)$ denotes the demand for firm i at price p_i. Demand consists of the number of customers who would be willing to buy from firm i *if* they sampled firm i before buying a variety, times the probability each will buy. The number of customers willing to buy from firm i, $\{l : |l - x_i| \leq r_i(p_i)\}$, is an arc of distance $r_i(p_i)$ on either side of x_i, multiplied by

η, the density of consumers, and divided by the circumference of the circle: $2\eta r_i(p_i)/\kappa$. The probability that a consumer on this arc will buy is the inverse of the number of acceptable brands such a consumer has. Some consumers are willing to buy from either firm; some will buy from firm i but not from firm j; and some will buy from firm j but not from firm i. Define

$$\begin{aligned} L_i^1(p_i;R) &= \{l : |l-x_j| > R\} \cap \{l : |l-x_i| \le r_i(p_i)\} \\ L_i^2(p_i;R) &= \{l : |l-x_j| \le R\} \cap \{l : |l-x_i| \le r_i(p_i)\} \end{aligned}$$

and define $\mu(L_i^1)$ and $\mu(L_i^2)$ as the measure of consumers in L_i^1 and L_i^2, relative to the entire circle, so $\mu(L_i^1(p_i,\kappa/2)) = 0$ and $\mu(L_i^2(p_i,\kappa/2)) = \mu(\{l : |l-x_i| \le r_i(p_i)\})$. L_i^1 represents the set of consumers who would buy from firm i at price p_i but would not buy from firm j located at x_j at price p^*. Thus the proportion of consumers in L_i^1—$\mu(L_i^1)$—buying from firm i is 1. L_i^2 represents the set of consumers who are willing to buy from either firm i at price p_i or from firm j at price p^*; the firm the consumer actually buys from is the one the consumer shops first. For consumers in L_i^2, both x_i and x_j are similar enough to l that the pairs (x_i,p_i) and (x_j,p^*) satisfy the stopping rule. These consumers—$\mu(L_i^2)$—will buy from firm i with probability 1/2 under the assumption that consumers visit firms randomly.

Denote by $\{\underline{l},\bar{l}\}$ the set of consumers for whom $|l-x_i| = r_i(p_i)$; these consumers are indifferent between buying from firm i at p_i or continuing search. Then

$$\frac{\partial\mu(L_i^k)}{\partial p_i} = \begin{cases} -2/\theta & \text{if} \quad \underline{l},\bar{l} \in L_i^k, \\ -1/\theta & \text{if} \quad \underline{l} \in L_i^k \text{ and } \bar{l} \notin L_i^k, \\ -1/\theta & \text{if} \quad \bar{l} \in L_i^k \text{ and } \underline{l} \notin L_i^k, \\ 0 & \text{if} \quad \underline{l},\bar{l} \notin L_i^k. \end{cases} \tag{11}$$

$\partial\mu(L_i^k)/\partial p_i$ is linear and continuous everywhere except at a finite number of points where the function jumps. Hence

$$\frac{\partial^2\mu(L^k)}{\partial p_i^2} = 0$$

everywhere but at those points where the derivative does not exist.

Given any location for x_j, the consumers willing to buy there—those l for whom $|l-x_j| \le R$—are all those within a distance $2R$ of x_j. Hence the proportion of such consumers is $2R/\kappa$. Similarly, the consumers willing to buy at firm i—those for whom $|l-x_i| \le r_i(p_i)$—are all those within a distance $2r_i$ of x_i. The proportion of these consumers is $2r_i/\kappa$.

Demand to firm i is the number of consumers, η, multiplied by the proportion willing to buy at firm i, multiplied by the probability each consumer within r_i of x_i would shop at x_i before finding a suitable brand elsewhere. $\mu(L_i^1)$ of these consumers have no acceptable alternatives and will buy from firm i regardless of the order of search; $\mu(L_i^2)$ have a choice between firm i and firm j and hence will buy from firm i with probability 1/2. Then demand to firm i when x_j is known is given by

$$d_i(p_i; x_j) = \frac{\eta}{\kappa}\left[1 \cdot \mu(L_i^1(p_i; x_j, R)) + \frac{1}{2} \cdot \mu(L_i^2(p_i; x_j, R))\right] \tag{12}$$

Figure 1 shows demand to each firm.

2.3.2 Unknown brands

When firm i does not know the location of x_j, demand is given by the expectation of (12) over the possible varieties:

$$d_i(p_i) = \frac{\eta}{\kappa}\int_{x_j \in X}\left[1 \cdot \mu(L_i^1(p_i; x_j, R)) + \frac{1}{2} \cdot \mu(L_i^2(p_i; x_j, R))\right] dx_j. \tag{13}$$

That x_j is unknown smooths out expected demand so d_i is differentiable everywhere. Given a price p_i, $\mu(L_i^k)$ is differentiable everywhere except a finite number of points x_j. Since x_j is unknown and is drawn from a continuous, atomless distribution, the probability that x_j is actually at such a point is zero.

Differentiating (13) with respect to price,

$$\frac{\partial d_i(p_i)}{\partial p_i} = \frac{\eta}{\kappa}\left[\int_{x_j \in X}\frac{\partial \mu(L_i^1)}{\partial p_i}dx_j + \frac{1}{2}\int_{x_j \in X}\frac{\partial \mu(L_i^2)}{\partial p_i}dx_j\right] < 0 \tag{14}$$

and

$$\frac{\partial^2 d_i(p_i)}{\partial p_i^2} = \frac{\eta}{\kappa}\left[\int_{x_j \in X}\frac{\partial^2 \mu(L_i^1)}{\partial p_i^2}dx_j + \frac{1}{2}\int_{x_j \in X}\frac{\partial^2 \mu(L_i^2)}{\partial p_i^2}dx_j\right] = 0. \tag{15}$$

2.4 Existence of a symmetric equilibrium

From (1) we have that

$$\frac{\partial \pi_i(p_i)}{\partial p_i}|_{p_i = p^*} = (p^* - b)\frac{\partial d_i(p^*)}{\partial p_i} + d_i(p^*) = 0 \tag{16}$$

and

$$\frac{\partial^2 \pi_i(p_i)}{\partial p_i^2} = (p_i - b)\frac{\partial^2 d_i(p_i)}{\partial p_i^2} + 2\frac{\partial d_i(p_i)}{\partial p_i} < 0, \tag{17}$$

where the inequality sign in (17) comes directly from equations (14) and (15). Equations (16) and (17) represent sufficient conditions for the existence of a symmetric pure-strategy Nash equilibrium (see Friedman [1982], Ch. 2).

2.5 Equilibrium prices

For p^* to be an equilibrium price, equation (16) implies

$$(p^* - b)\frac{\eta}{\kappa}\frac{\partial q(p_i; R)}{\partial p_i} + \frac{\eta}{\kappa} q(p_i; R) = 0 \tag{18}$$

where

$$q(p_i; R) = \int_{x_j \in X} \mu(L_i^1) dx_j + \frac{1}{2}\int_{x_j \in X} \mu(L_i^2) dx_j$$

and

$$\frac{\partial q(p_i; R)}{\partial p_i} = \left[\int_{x_j \in X} \frac{\partial \mu(L_i^1)}{\partial p_i} dx_j + \frac{1}{2}\int_{x_j \in X} \frac{\partial \mu(L_i^2)}{\partial p_i} dx_j\right] < 0.$$

Substituting $p_i = p^*$ and, consequently, $r_i(p^*) = R$ into (18) we obtain

$$\frac{\eta}{\kappa}\left[(p^* - b)\frac{\partial q(p^*; R)}{\partial p_i} + q(p^*; R)\right] = 0$$

so

$$p^* = b - \frac{q(p^*)}{q'(p^*)} \tag{19}$$

where $q' = \partial q / \partial p_i$. The equilibrium price in (19) is above marginal cost since $q' \leq 0$. (19) says that the equilibrium price is a markup over marginal costs, where the amount of the markup increases with R, the distance away from the most-preferred variety of the marginal consumer.

3 Increases in Search Cost

An increase in c raises the right-hand side of equation (6), so the reservation brand R increases; that is, the marginal consumer at any firm i is further away from his most-preferred variety, so less search occurs for any symmetric price. Differentiating (7) with respect to c and substituting $R^2 = 2\kappa c/\theta$,

$$\frac{\partial R}{\partial c} = \frac{1}{2}\left(\frac{2\kappa c}{\theta}\right)^{-1/2}\left(\frac{2\kappa}{\theta}\right) = \frac{1}{2}\left(\frac{2\kappa c}{\theta}\right)^{1/2}\left(\frac{1}{c}\right) = \frac{R}{2c} > 0.$$

From (13) one may observe that demand at any price p_i increases, so the profit-maximizing price must be higher to keep the first-order condition (16) satisfied. The effect on p^* can be seen in (19): as R increases, the marginal consumer to firm i is further from firm i. Differentiating (19) with respect to c,

$$\frac{\partial p^*}{\partial c} = -\frac{\partial q}{\partial c} \cdot \frac{1}{q'} + \frac{\partial q'}{\partial c} \cdot \frac{q}{(q')^2}.$$

In equilibrium, $r_i = R$ so

$$\begin{aligned} L_i^1(R) &= \{l : |l - x_j| > R\} \cap \{|l - x_i| \leq R\} \\ L_i^2(R) &= \{l : |l - x_j| \leq R\} \cap \{|l - x_i| \leq R\}. \end{aligned} \tag{20}$$

Since an increase in c increases the reservation utility R, the set $\{l : |l - x_j| > R\}$ shrinks with c, while the sets $\{l : |l - x_j| \leq R\}$ and $\{l : |l - x_i| \leq R\}$ both grow with c.

Define $\{\underline{\underline{l}}, \bar{\bar{l}}\}$ analagously to $\{\underline{l}, \bar{l}\}$: as the set of consumers for whom $|l - x_j| = R$. The effect of R on L_i^1 and L_i^2 depends on the boundaries of each set—whether one or both boundaries of L_i^k is from the set $\{\underline{l}, \bar{l}\}$ or the set $\{\underline{\underline{l}}, \bar{\bar{l}}\}$—the location of x_j relative to x_i (since this affects the direction in which $\underline{\underline{l}}$ and $\bar{\bar{l}}$ change relative to x_i), and whether the set is contiguous or not (since this affects the number of boundaries of the set and consequently how the set changes with R).

A boundary of $\underline{l}$ or $\bar{l}$ for either L_i^1 or L_i^2 increases the set as R increases since $\underline{l}$ and $\bar{l}$ expand—that is, move further from x_i—with R. Whether L_i^1 or L_i^2 increases or decreases with R on a boundary of $\underline{\underline{l}}$ or $\bar{\bar{l}}$ depends on whether the change in R, which moves $\underline{\underline{l}}$ and $\bar{\bar{l}}$ away from x_j, moves $\underline{\underline{l}}$ and $\bar{\bar{l}}$ away from or toward x_i. This depends on the position of x_j relative to x_i. There are five basic cases: $L_i^1 = \emptyset$ and $L_i^2 = [\underline{l}, \bar{l}]$; $L_i^1 = [\underline{\underline{l}}, \bar{\bar{l}}]$ and L_i^2 is noncontiguous (either $[\underline{l}, \bar{\bar{l}}] + [\underline{\underline{l}}, \bar{l}]$ or $[\underline{l}, \underline{\underline{l}}] + [\bar{\bar{l}}, \bar{l}]$; $L_i^2 = \emptyset$ and $L_i^1 = [\underline{l}, \bar{l}]$; $L_i^2 = [\underline{\underline{l}}, \bar{\bar{l}}]$ and L_i^1 is one of the same noncontiguous sets as case 2; and $L_i^1 = [\underline{l}, \bar{\bar{l}}]$ and $L_i^2 = [\bar{\bar{l}}, \bar{l}]$. Other cases are equivalent to one of these five.

Define $\Omega \equiv |\partial\bar{l}/\partial R| = |\partial\underline{l}/\partial R| = |\partial\bar{\bar{l}}/\partial R| = |\partial\underline{\underline{l}}/\partial R|$. That the first and third equalities hold is obvious: $\underline{l}$ and $\bar{l}$ are defined in exactly the same way, as the consumers who are indifferent between buying from firm i at the equilibrium price and continuing search; the same hold for $\underline{\underline{l}}$ and $\bar{\bar{l}}$ with respect to firm j. Since x_i and x_j are fixed, the second equality must also hold:

$$|\underline{l} - x_i| = R \qquad \text{and} \qquad |\bar{\bar{l}} - x_j| = R$$

by definition so

$$\left|\frac{\partial \underline{l}}{\partial R}\right| = \left|\frac{\partial \bar{\bar{l}}}{\partial R}\right|.$$

The first possibility, shown in Figure 3.2(a), is that $L_i^1 = [\bar{\bar{l}}, \bar{l}]$ and $L_i^2 = [\underline{l}, \bar{\bar{l}}]$; both sets are contiguous. As R increases, $\bar{\bar{l}}$ moves toward x_i and $\underline{l}$ and $\bar{l}$ move toward x_j. Then

$$\frac{\partial \mu(L_i^1)}{\partial c} = \mu'(\Omega - \Omega)\frac{\partial R}{\partial c} = 0$$

and

$$\frac{\partial \mu(L_i^2)}{\partial c} = \mu'(\Omega + \Omega)\frac{\partial R}{\partial c} = \frac{2\Omega R}{2c} \cdot \mu' = \frac{\Omega R \mu'}{c} > 0$$

where $\mu' \equiv \partial\mu(L_i^k)/\partial L_i^k$ for $k = 1, 2$.

In the second case, shown in Figure 3.2(b), $L_i^1 = [\underline{l}, \bar{l}]$ while $L_i^2 = \emptyset$. Then

$$\frac{\partial \mu(L_i^1)}{\partial c} = \frac{\Omega R \mu'}{c} > 0$$

and

$$\frac{\partial \mu(L_i^2)}{\partial c} = 0.$$

In the third case, shown in Figure 3.2(c), $L_i^1 = [\bar{\bar{l}}, \underline{l}]$ while $L_i^2 = [\underline{l}, \bar{\bar{l}}] + [\bar{l}, \underline{\underline{l}}]$ so

$$\frac{\partial \mu(L_i^1)}{\partial c} = -\frac{\Omega R \mu'}{c} < 0$$

and

$$\frac{\partial \mu(L_i^2)}{\partial c} = \frac{2\Omega R \mu'}{c} > 0$$

because L_i^2 is noncontiguous and hence has four changing boundaries rather than two.

The fourth basic case (Figure 3.2(d)) has L_i^1 noncontiguous: $L_i^1 = [\underline{l}, \underline{\underline{l}}] + [\bar{l}, \bar{\bar{l}}]$ while $L_i^2 = [\underline{\underline{l}}, \bar{\bar{l}}]$, so

$$\frac{\partial \mu(L_i^1)}{\partial c} = 0$$

(since the changes in the four boundaries cancel out one another) and

$$\frac{\partial \mu(L_i^2)}{\partial c} = \frac{\Omega R \mu'}{c} > 0.$$

The fifth case (Figure 3.2(e)) is analagous to the second: $L_i^1 = \emptyset$ and $L_i^2 = [\underline{l}, \bar{l}]$, so

$$\frac{\partial \mu(L_i^1)}{\partial c} = 0$$

and

$$\frac{\partial \mu(L_i^1)}{\partial c} = \frac{\Omega R \mu'}{c} < 0.$$

The net result is that $\partial\mu(L_i^2)/\partial c \geq 0$ in each case while $\partial\mu(L_i^1)/\partial c \geq 0$ in each case but the third. In that case, shown in Figure 3.2(c), $\partial\mu(L_i^1)/\partial c = -\Omega R\mu'/c$ but $\partial\mu(L_i^2)/\partial c = 2\Omega R\mu'/c$ so, in this case,

$$\frac{\partial \mu(L_i^1)}{\partial c} + \frac{1}{2}\frac{\partial \mu(L_i^2)}{\partial c} = 0.$$

Hence

$$\frac{\partial q(R(c))}{\partial c} = \int_{x_j \in X} \left[\frac{\partial \mu(L_i^1(R(c))}{\partial c} + \frac{1}{2} \frac{\partial \mu(L_i^2(R(c))}{\partial c} \right] dx_j > 0$$

and, since, from (11), $\partial \mu(L_i^k)/\partial p_i$ is independent of c,

$$\frac{\partial \left(\frac{\partial \mu(L_i^k(R(c)))}{\partial p_i} \right)}{\partial c} = 0, \qquad k = 1, 2$$

for all but a finite number of points x_j where the function jumps (these points are at the boundaries of the regions defined in (11)). Aggregating over all $x_j \in X$,

$$\frac{\partial^2 q(R(c))}{\partial p_i \partial c} = \frac{\partial q'}{\partial c} = \int_{x_j \in X} \left[\frac{\partial \left(\frac{\partial \mu(L_i^1(R(c)))}{\partial p_i} \right)}{\partial c} + \frac{1}{2} \frac{\partial \left(\frac{\partial \mu(L_i^2(R(c)))}{\partial p_i} \right)}{\partial c} \right] dx_j = 0.$$

From this we obtain

$$\frac{\partial p^*}{\partial c} = -\frac{\partial q}{\partial c} \cdot \frac{1}{q'} + 0 \cdot \frac{q}{(q')^2} = -\frac{\partial q}{\partial c} \cdot \frac{1}{q'} > 0.$$

4 Increases in the Degree of Heterogeneity

An increase in the disutility associated with a unit move away from a consumer's most-preferred variety—that is, an increase in θ—is a decrease in the substitutability of varieties for one another, and hence an increase in the degree of heterogeneity.

As θ increases, the disutility associated with consuming a variety some fixed distance away from the consumer's most-preferred variety increases. Since for a given R the term $\frac{1}{\kappa} \int_0^R v^l(y_j) dy$ declines as θ increases, the R required for equation (6) to hold must also decline: the reservation brand comes nearer to x_i. That is, the maximum distance away from a consumer's most-preferred variety he would be willing to accept has become smaller: consumers search more. From (8) consumers also respond to price deviations by searching more.

There is a second effect, however, which goes in the opposite direction: as θ increases, consumers who are within the arc of $2r_i(p_i)$ centered around x_i are less responsive to price changes. Consumers who find good draws are less likely to abandon them in the hope of finding more attractive brand/price pairs at the second store.

These two effects—a decrease in demand at each price because of the increase in search and an increase in the responsiveness of demand to price changes (equations (13) and (14))—have opposite effects on the equilibrium price p^* as θ changes:

$$\frac{\partial p^*}{\partial \theta} = -\frac{\partial q}{\partial \theta} \cdot \frac{1}{q'} + \frac{q}{(q')^2} \cdot \frac{\partial q'}{\partial \theta} = -\frac{1}{q'} \left[\frac{\partial q}{\partial \theta} - \frac{q}{q'} \cdot \frac{\partial q'}{\partial \theta} \right]. \tag{21}$$

Differentiating (7) with respect to θ,

$$\frac{\partial R}{\partial \theta} = \frac{1}{2}\left(\frac{2\kappa c}{\theta}\right)^{-1/2}\left(-\frac{2\kappa c}{\theta^2}\right) = -\frac{1}{2}\cdot\frac{1}{\theta}\left(\frac{2\kappa c}{\theta}\right)^{1/2} = -\frac{R}{2\theta} < 0$$

so the effect of θ on R and hence $\mu(L_i^k(R))$ is exactly the opposite of the effect of c from the previous section:

$$\frac{\partial \mu(L_i^2(R(\theta)))}{\partial \theta} \leq 0$$

in each case while

$$\frac{\partial \mu(L_i^1(R(\theta)))}{\partial \theta} \leq 0$$

in each case but the one shown in Figure 3.2(c). In this case $\partial\mu(L_i^1)/\partial\theta = -\Omega R\mu'/c$ but $\partial\mu(L_i^2)/\partial\theta = 2\Omega R\mu'/c$ so the net result is that

$$\frac{\partial \mu(L_i^1(R(\theta)))}{\partial \theta} + \frac{1}{2}\frac{\partial \mu(L_i^2(R(\theta)))}{\partial \theta} = 0.$$

Combining these results,

$$\frac{\partial q(R)}{\partial \theta} = \int_{x_j\in X}\left[\frac{\partial \mu(L_i^1(R))}{\partial \theta} + \frac{1}{2}\frac{\partial \mu(L_i^2(R))}{\partial \theta}\right] dx_j < 0.$$

Using (11),

$$\frac{\partial^2 \mu(L_i^k(R))}{\partial p_i \partial \theta} \in \left\{\frac{2}{\theta^2}, \frac{1}{\theta^2}, 0\right\}$$

for almost all x_j so

$$\frac{\partial^2 q}{\partial p_i \partial \theta} = \int_{x_j\in X}\left[\frac{\partial\left(\frac{\partial \mu(L_i^1(R))}{\partial p_i}\right)}{\partial \theta} + \frac{1}{2}\frac{\partial\left(\frac{\partial \mu(L_i^2(R))}{\partial p_i}\right)}{\partial \theta}\right] dx_j > 0.$$

Consequently the sign of (21) is ambiguous.

5 Conclusions

This model presents a view of consumer search in which product heterogeneity, not price differences, induces equilibrium search. This is in contrast with Diamond (1971), in which the homogeneity of products leads to a symmetric equilibrium at the monopoly price and no search; and is in contrast with Stahl (1989), in which differences in consumer search costs lead to different intensities of search, which in turn generates a nondegenerate price distribution and equilibrium search in response to this distribution. Our model also offers a different motivation for search than the Salop and Stiglitz (1977) model of sales, in which, like Stahl, the authors derive a mixed-strategy equilibrium and consumers (may) search for low prices in the price distribution for a storable commodity if they have a sufficiently low discount rate.

The model is similar to one developed by Wolinsky (1983), who examines a model of monopolistic competition with symmetric brand locations. While Wolinsky's model has the virtue of endogenizing brand decisions, the equilibrium is not one of rational expectations: consmers believe that brand locations are each random draws from a uniform distribution and hence uncorrelated, which gives rise to symmetric (and hence correlated) location decisions. One advantage of this formulation is that the optimal consumer search problem is characterized by a reservation brand/price pair: a consumer will buy from the first firm sampled such that the net surplus from buing that firm's brand at the going price makes expected gains from the next search unprofitable. The thrust of Wolinsky's paper is to determine conditions under which firms at fixed locationswould prefer to move closer to one another; the paper is not concerned with the seemingly paradoxical results of the Diamond and Stahl models. The present paper trades off endogenous brand locations for a rational expectations equilibrium.

This paper examines a model in which firms are assigned brands at random. A consequence of this model is that consumers' beliefs that each draw is independent is correct, even in equilibrium. The reservation-brand property still holds, and the resulting equilibrium is one in which consumers have rational expectations. I assume that firms do not know the location of their rivals in product space, a characteristic either of industries with high product turnover, such as the garment or shoe industries (so firms could determine the exact composition of the products of their rivals only by incurring some cost frequently), or industries with products of many characteristics, such as the auto industry (so that firms are unable to determine which characteristics are valuable to any particular consumer).

In addition to making the somewhat severe assumption limiting the number of firms in the market to two, the model makes the assumption of a uniform density of both varieties and consumer preferences. I discuss these restrictions below.

That firms act as though consumers were distributed uniformly around the circle may be thought of as representing firms' collective uncertainty about popular varieties. The assumption of a uniform probability distribution is then equivalent to an uninformed prior.

The assumption that firms draw their varieties from a known and uniform distribution is perhaps more limiting. This formulation does not admit new varieties, created to fill perceived market niches (such as the ready-to-eat cereal industry in Schmalensee [1978]), nor does it allow firms to strategically choose products in order to, say, maximally or minimally differentiate the available varieties. To the extent that the industry is one that exhibits a high rate of turnover in its products and does no innovation to create the new varieties sold, the assumption reflects the reality that firms cannot observe the varieties of their rivals except at high cost, and have no *a priori* reason to choose one variety over another by the reasoning in the previous paragraph. Such industries might include apparel—particularly for women since the turnover rate is higher and what sells is less predictable—, hardware (while the stock is relatively fixed, the needs of shoppers at any particular time may have more variance than any one store can handle), movie theaters (because of the transient nature of each variety and the unpredictability of consumer preferences), or antique stores. One avenue of future research would be to explore the consequences of offering new varieties in new dimensions in product space, potentially changing consumer preferences over existing varieties.

Despite these limitations, the model presents a new point of view for search: that heterogeneity is not merely another dimension on which consumers search, but is a *necessary* characteristic of a particular product market for search to occur.

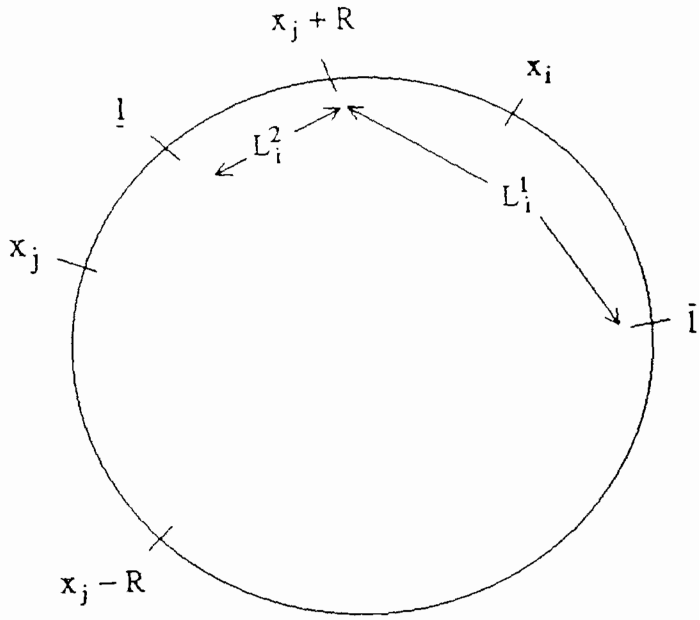

Figure 1: Circular spatial market

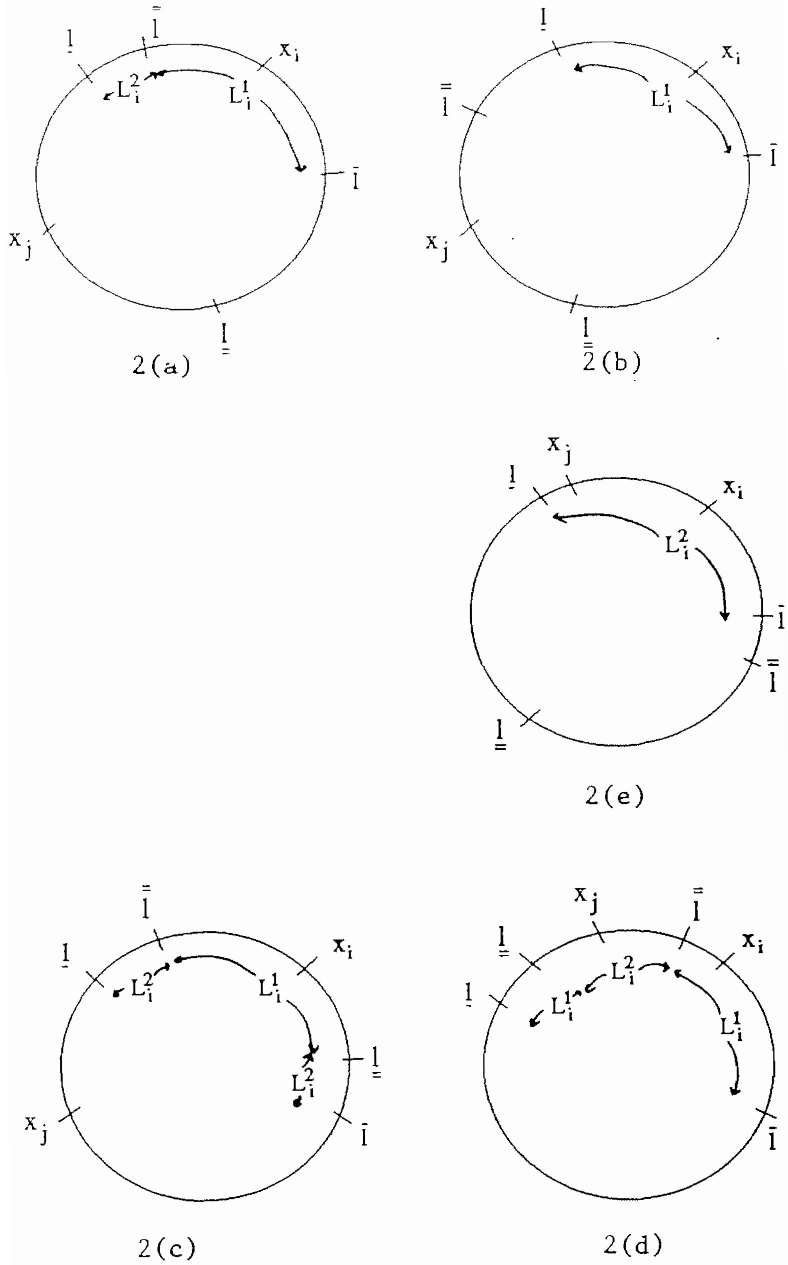

Figure 2: Possible configurations for L_i^1 and L_i^2

References

Burdett, Kenneth and Kenneth L. Judd, 1983. "Equilibrium Price Dispersion," *Econometrica* 51(4), pp. 955–70.

Butters, Gerard R., 1977. "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies* 44, pp. 465–91.

Carlson, John and R. Preston McAfee, 1983. "Discrete Equilibrium Price Dispersion," *Journal of Political Economy* 91(3), pp. 480–93.

Diamond, Peter, 1971. "A Model of Price Adjustment," *Journal of Economic Theory* 3, pp. 156–68.

Friedman, James W., 1986. *Game Theory with Applications to Economics* (New York: Oxford Univ. Press).

Kohn, Meir G., and Steven Shavell, 1974. "The Theory of Search," *Journal of Economic Theory* 9(2), pp. 93–123.

Salop, Steven C., 1979. "Monopolistic Competition with Outside Goods," *Bell Journal of Economics* 10(1), pp. 141–56.

Salop, Steven C. and Joseph E. Stiglitz, 1977. "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," *Review of Economic Studies* 44, pp. 493–510.

Schmalensee, Richard, 1978. "Entry Deterrence in the Ready-to-eat Breakfast Cereal Industry," *Bell Journal of Econ.* 9, pp. 305–27.

Stahl, Dale O. II, 1989. "Oligopolistic Pricing with Sequential Consumer Search," *American Economic Review* 79(4), pp. 700–12.

Wilde, Louis L. and Allan Schwartz, 1979. "Equilibrium Comparison Shopping," *Review of Economic Studies* 46, pp. 543–53.

Wolinsky, Asher, 1983. "Retail Trade Concentration Due to Consumers' Imperfect Information," *Bell Journal of Economics* 14, pp. 275–82.

Reproduction & Distribution Sheet

Attachment to Working Paper #201

(20) - Public Reference

(30) - Division of Economic Policy Analysis - Rm 3103 (601 Bldg.)

(03) - FTC Library - Rm 630

(00) - Ed Bush & (321) Mailing Labels Attached