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Richard S. Higgins

William F. Shughart II

Robert D. Tollison

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Richard S. Higgins

Bureau of Economics
Federal Trade Commission
Washington, D.C. 20580

William F. Shughart II
and
Robert D. Tollison

Center for Study of Public Choice
George Mason University
Fairfax, Virginia 22030

I. Introduction

The dominant firm model is normally presented as a pricing exercise in which information is complete and costless. Market demand and the marginal costs of the smaller firms are assumed to be known by the dominant firm.[1] The dominant firm is the price setter, but it docilely sets market price after deriving its residual demand function given the parametric behavior of fringe suppliers. There is no problem of price calculation in the model. All that is required is some mechanism for assuring that the followers produce the "right" quantity, namely, the quantity consistent with the leader's profit-maximizing price.[2]

By contrast, consider the dominant firm model in the following spirit. Information about demand is not complete, and forecasting is not free. The dominant firm is the price setter for the market. Fringe suppliers accept the price set by the dominant firm and maximize accordingly. Under such circumstances, the dominant firm is in the position of providing a public good for the industry. This public good derives from the leader's investment in searching for the best price. In a positive economics sense, then, the dominant firm will devote resources to the task of forecasting industry demand patterns, and this investment will impact on the level and variability of market price.

This paper presents a theory of dominant firm behavior under conditions of incomplete information. We present a model of price leadership in which more ex ante information about demand can be acquired at a cost. We derive conditions that define the price leader's optimal amount of information about demand. The major results accord well with intuition. The more information the price leader acquires, the more accurate is its pricing. That is, price changes more closely match demand changes. Both the price leader and its followers gain from additional information about demand. Furthermore, we show that the larger the market share of the price leader the greater the level of information it is efficient for the price leader to acquire, at least for market shares beyond some threshold.

The paper is organized as follows. Section II presents our dominant firm model and discusses its testable implications. The main empirical prediction of the model is that price variance depends positively on dominant firm market share. Testing this prediction requires that industry capacity be held constant since an increase in market share through internal expansion has the effect of reducing price variance. Concluding remarks are offered in Section III.

II. The Dominant Firm Model

In this section we develop our dominant firm model under conditions of incomplete information. We first consider a simplified case in which the dominant firm holds a priori information about residual demand and chooses a price to maximize expected profit. The basic model is then extended by deriving the price leader's demand for information explicitly. Finally, we show the relationship between the optimal level of information and the market share of the dominant firm, and conclude with a discussion of how one would go about testing our model.

The Basic Model

There are r plants of equal size. Entry is precluded by assumption. The fringe suppliers independently control (r-t) of the plants, and the dominant firm controls t of them. The cost of operating each plant is $(c/2)q^2$. The fringe suppliers adopt the price set by the dominant firm and equate marginal cost to the leader's profit-maximizing price. Fringe supply, S^f is $(r-t)\,p/c$. The dominant firm's cost function is $(c/2t)y^2$.

Market demand is $Q^d = a_o + \sum_{i=1}^{n} a_i X^i + b\,p + e \equiv A + b\,p + e$, where e is a random variable. The dominant firm knows the own-price coefficient, b, but has incomplete information about A + e. The dominant firm also knows S^f. Given fringe supply, the leader's residual demand is $Q^d - S^f \equiv y^D = A + [bc-(r-t)]\,p/c + e \equiv A + B\,p + e$.

The objective of the dominant firm is to maximize expected profit given a level or quantity of information about demand. Thus for each quantity of information acquired there is a maximum level of expected profit. To choose the appropriate quantity of information, the dominant firm equates the marginal increase in expected profit attributable to additional demand information with the marginal cost of acquiring such data.

We now describe the monopolist's prior information and the additional information that may be acquired. First, we make some simplifying assumptions. The variables X_i in the demand relation are random variables that are independently distributed and serially uncorrelated. Each has constant mean, $\bar{X}_i$, and variance, Var (X_i). Similarly, e is distributed independently of the X_i, has constant mean, $E(e) = 0$, and variance, Var(e), and exhibits no serial correlation.

A priori, the dominant firm is presumed to know the distributions of the X_i and of e. We also assume that the dominant firm knows the demand parameters, a_i, for all i. With prior information alone, the dominant firm has a substantial amount of information, but without observations on the X_i the leader can only form an unconditional prediction of y^D, its residual demand function. Under the postulated conditions, the marginal distribution of y^D is known to the dominant firm. Specifically, the dominant firm knows $E(y^D|p) = a_o + \sum_{i=1}^{n} a_i \bar{X}_i + B\,p = E(A) + B\,p$ and $Var(y^D|p) = Var(A) + Var(e)$. (Since all distributions are conditioned on price, we simplify notation from here on out by ignoring p.)

Now assume that by incurring a cost, d, the leader can predict y^D more precisely. In particular, the dominant firm forms the forecast, $E(y^D|X_i') = a_o + \sum_{j=1}^{n} a_j \bar{X}_j + a_i\,(X_i'-\bar{X}_i) + B\,p = E(A) + B\,p + a_i\,(X_i'-\bar{X}_i)$, by measuring X_i. (For each X_i measured, the leader must pay d.) If the dominant firm measures m of the X_i, we say the monopolist has chosen quantity m of information.[3] In this way the dominant firm acquires knowledge about the conditional distributions $\Theta(y^D|X_1',X_2',\ldots,X_m') \equiv \Theta(A+e|X^{m'})$, where $X^{m'} \equiv (X_1',X_2',\ldots,X_m')$. Specifically, $E(y^D|X_m') = E(A) + B\,p + \sum_{i=1}^{m} a_i\,(X_i' - \bar{X}_i)$ and $Var(y^D|X^{m'}) = \sum_{j=1}^{m} a_j^2\ Var\ (X_j) + Var(e)$. Var $(y^D|X^{m'})$ is simply the mean square forecast error conditioned on $X^{m'}$.

In maximizing expected profit the dominant firm adopts a particular price and output policy. We assume that the dominant firm commits to a price on this basis and supplies all customers at that price. For example, when realized demand is greater than predicted demand marginal cost may exceed price.[4] Information is valuable because the dominant firm with incomplete information produces "too much" when realized demand is high and "too little" when realized demand is low. Incomplete information is also the source of value for inventories. In a complete model the dominant firm would expand on both margins to maximize expected profit.[5]

The dominant firm's decision problem can be summarized as follows: For information level, m, and realization, $X^{m'}$, the dominant firm chooses a price to maximize expected profit. Expectations are computed based on the conditional distribution $\Theta(A+e|X^{m'})$. We denote the expected profit conditioned on $X^{m'}$ as $E(\pi|X^{m'})$. To determine the profit-maximizing level of information, the dominant firm first finds expected profit for information level m, $E(\pi|m)$. $E(\pi|m)$ is found by weighting $E(\pi|X^{m'})$ by the probability density for $X^{m'}$ and integrating over all values of X^{m}. The monopolist computes $E(\pi|m)$ for all information levels m. Finally, the optimal level of information maximizes $E(\pi|m)-d\,m$.

The Extended Model

We now turn to a more detailed treatment of the dominant firm's decision problem. We begin with decisions based on prior information alone.

The monopolist chooses price, p, to maximize

$$(1)\ E(\pi) = \iint[(A+Bp+e)p - (c/2t)(A+Bp+e)^2]\ \theta_A(A)\theta_e(e)dAde.$$

The marginal condition yields

$$(2)\ p^{o} = -E(A)(cB-t)/B(cB-2t).$$

The price in (2) is identical to the price that would be chosen were demand certain at its average value, $y^{D} = E(A) + B\,p$. Thus, the profit-maximizing price can be derived by supposing the dominant firm maximizes profit based on its demand forecast. Substituting (2) into (1) yields

$$(3)\ E(\pi^{o}) = [t/2B(cB-2t)][E(A)]^{2} - (c/2t)[Var(e) + \sum_{i=1}^{n} a_i^{2}\, Var(X_i)].$$

Inspection of (3) reveals that expected profit is the sum of two terms, the first one positive and the second one negative. The negative term is proportional to the mean square error that results from using the unconditional forecast, $E(y^{D})$, for y^{D}. Based on prior information alone, the dominant firm charges a uniform price given by (2) and, on average, gets profit $E(\pi^{o})$ given by (3). The fringe suppliers adopt p^{o}, supply $(r-t)p^{o}/c$ and earn a profit of $(p^{o})^{2}(r-t)/2c$ with certainty.[6]

We now show that the dominant firm can raise average profit above $E(\pi^{o})$ by purchasing information that allows it to make more accurate demand forecasts, not accounting for the cost of information. Suppose the monopolist measures variable X_1. For any realization, X'_1, the monopolist chooses price to maximize (1) where $\theta(A+e|X_1')$ is substituted for $\theta(A+e)$. The analog of (2) is

$$(4)\ p^{1'} = -\ E(A|X_1')(cB-t)/B(cB-2t).$$

Averaging over all possible realizations of X_1, we get

$$(5)\quad E(p^1) = -E(A)(cB-t)/B(cB-2t) = p^o.$$

On average, price is the same with or without additional information.

Substituting (4) into the profit function and taking expectations we get

$$(6)\quad E(\pi|1) = [t/2B(cB-2t)]\{[E(A)]^2 + a_1^{\,2}\,Var(X_1)\}$$
$$-(c/2t)[Var(e) + \sum_{i=2}^{n} a_i^{\,2} Var(X_i)].$$

Comparing (6) and (3), we see that $E(\pi|1)-E(\pi^o) = a_1^{\,2}$ $Var(X_1)\{[t/2B(cB-2t)]+c/2t\}>0$. Provided that $E(\pi|1)-E(\pi^o)>d$, the dominant firm will at least purchase the first level of information.

In general,

$$(7a)\quad p^{m'} = -E(A|X^{m'})(c\,B-t)/B(cB - 2t),$$

$$(7b)\quad E(\pi|m) = [t/2B(cB-2t)][E(A)]^2 + [t/2B(cB-2t)]\,[\sum_{i=1}^{m} a_i^{\,2}\, Var(X_i)] - (c/2t)\,[Var(e) + \sum_{i=m+1}^{n} a_i^{\,2} Var(X_i)],$$

$$(7c)\quad E(\pi|m)-E[\pi|(m-1)] = a_m^{\,2}Var\,(X_m)\{[t/2B(cB-2t)]+c/2t\}.$$

Inspection of (7b) reveals three terms, the first two positive and the third negative. The first positive term is independent of the amount of information acquired by the firm. The second positive term is proportional to the difference between the unconditional mean square forecast error and the mean square forecast error conditioned on information level m. This difference increases as the quantity of information increases. The negative term is proportional to the mean square forecast error for information level m. The larger the quantity of acquired information, the smaller the negative term in (7b). Thus, the higher the level of information acquired the higher is expected profit for the dominant firm.

The fringe also gains from more complete information. For a given price, fringe profit, $\pi^f(p^{m'})$, is $(p^{m'})^2(r-t)/2c$. Thus, $E[\pi^f|m] \equiv [(r-t)/2c]E[(p^m)^2]$, which given (7a) yields $E[\pi^f|m]=[(cB-t)^2/B^2(cB-2t)^2]\ \{[E(A)]^2 + \sum_{i=1}^{m} a_i^2\ Var(X_i)\}$. Since the latter expression is increasing in m, fringe profit is higher the more accurate is dominant-firm pricing.

The demand for information by the dominant firm is given by (7c). The amount the dominant firm is willing to pay for additional information is given by $E(\pi|m) - E(\pi|(m-1)]$.[7] Thus, the marginal valuation of information by the dominant firm is proportional to the difference between the mean square demand forecast errors conditioned on levels of information m and m-1, respectively. It is obvious that the marginal valuation is nonincreasing -- that the demand for information is downward sloping -- provided the X_i are ordered in the demand relation according to their contribution to error reduction, that is, according to the magnitude of $a_i^2 Var(X_i)$.

The dominant firm purchases the information level m* that maximizes $E(\pi|m)-d\,m$. The marginal condition is $E(\pi|m^*)-E[\pi|(m^*-1)]\geq d$ and $E[\pi|(m^*+1)]-E(\pi|m^*) < d$. For simplicity, we treat m as a continuous variable and write the marginal condition as $\partial E(\pi)/\partial m = d$. In Figure 1 we depict the graph of the "smoothed" marginal valuation function along with the marginal cost of information. The level of information m* maximizes expected profit net of the cost of information. At m* the residual demand mean square forecast error is $Var(e) + \sum_{i=m^*+1}^{n} a_i^2 \, Var(X_i)$. The greater is the quantity of information the smaller is the forecast error. Minimum mean square forecast error is Var(e), since e is unobservable.

Optimal Information and Firm Size

The optimal level of information, m*, depends on the size of the dominant firm. We now consider how m* changes as the dominant firm's share grows. Specifically,

$$(8)\quad \partial\{E(\pi|m)-E[\pi|(m-1)]\}/\partial t = - c(bc-r)^2[(bc-r)^2 - 3t^2]/2t^2[(bc-r)^2-t^2]^2.$$

The sign of (8) depends on the sign of $[(bc-r)^2-3t^2]$.[8] If the dominant firm's share is sufficiently large, $[(bc-r)^2-3t^2]<0$, and (8) would be positive. Intuitively, when t is small most of the marginal gain from information acquisition accrues to the fringe, which free rides in a price leadership model. We presume that for a single firm to achieve the status of dominant firm or price leader it must have substantial market share. Thus, focusing on values of t for which (8) is positive appears reasonable. When (8) is positive, growth in the dominant firm through merger increases the amount of demand information acquired by the dominant firm. In terms of Figure 1, an increase in t raises the dominant firm's marginal valuation of information at all quantities of information. Thus, the optimal quantity of information, m*, is positively related to the share of the dominant firm.

Empirical Implications

Finally, we derive an empirically testable implication of our theory. Specifically, we show that price variance increases as the dominant firm's market share increases, even if average price is held constant. First, we note that

$$(9)\ Var(p^m) = E[(p^m)^2] - [E\ (p^m)]^2$$

$$= \{E[\sum a_i(X_i-\bar{X}_i) + [E(A)]^2 - [E(A)]^2\}(cB-t)^2/B^2(cB-2t)^2$$

$$= [(cB-t)^2/B^2(cB-2t)^2][\sum_{i=1}^{m} a_i^2\ Var(X_i)].$$

The price variance is proportional to the difference between the residual demand mean square forecast errors with no information and with information level m. Since $[E(p^m)]^2 = [E(A)]^2(cB-t)^2/B^2(cB-2t)^2$, equation (9) can also be written as

$$(10)\ Var(p^m) = \{[E(p^m)]^2/[E(A)]^2]\}[\sum_{i=1}^{m} a_i^2\ Var(X_i)].$$

Assuming that the optimal level of information is chosen,

$Var(p^m)$ depends on t in two ways. First, t affects $Var(p^m)$ through its effect on $E(p^m)$; second, t affects $Var(p^m)$ through its effect on the optimal amount of information, m*. Thus, the effect of a change in dominant firm share on $Var(p^{m*})$ is

$$\text{(11)}\quad d\,Var(p^{m*})/dt = [\sum_{i=1}^{m*} a_i^2\, Var(X_i)]\ \{2E(p^{m*})/[E(A)]^2\}[\partial E(p^m)/\partial t] + \{[E(p^{m*})]^2/[E(A)]^2]\}[\sum_{i=m*(t)}^{m*(t+\Delta t)} a_i^2\, Var(X_i)]$$

The first term in (11) is positive, since $\partial E(p^m)/\partial t > 0$.[9] The second term is also positive, since $m*(t+\Delta t) \geq m*(t)$, provided t is large enough that the dominant firm buys more information the larger its share is. Thus, price variance is predicted to be positively correlated with dominant firm share.

Time-series tests of our prediction are likely to be easier to undertake than cross-section tests, since it would be difficult to control for demand volatility across product markets. In time-series tests one would have to control for industry capacity, however, because our short-run model presumes it is fixed. That is, in our model the dominant firm increases its share through merger. The model does not predict the effects on information acquisition and ultimately on price variance of internal expansion of the dominant firm.

The necessity of holding industry capacity constant to test our prediction that price variance depends positively on dominant firm share is revealed by a different <u>ceteris paribus</u> experiment. The model is altered by specifying dominant firm marginal cost as c/(t+s), where s represents plants added through internal growth.

With this specification, (7) becomes

(12a) $E(p^m) = - E(A)[cB-(t+s)]/B[cB-2(t+s)]$

(12b) $E(\pi|m) = \{(t+s)/2B[cB-2(t+s)]\}[(EA)^2 + \sum_{i=1}^{m} a_i^2 \, Var(X_i)]$

$$-[c/2(t+s)][Var(e) + \sum_{i=m+1}^{n} a_i^2 \, Var\,(X_i)]$$

(12c) $E(\pi|m)-E(\pi|m-1) = [a_m^2 \, Var(X_m)]$

$$\{[(t+s)/2B[cB-2(t+s)] + [c/2(t+s)\}.$$

Differentiating (12a) and (12c) with respect to s we find that, ceteris paribus, average price falls as s increases, and less information is purchased as s increases. The marginal value of information does not rise as dominant-firm share increases through internal expansion because the dominant firm's residual demand is not raised. Thus, price variance falls as s increases because both effects are negative.

In the context of our model with industry capacity fixed, price variance depends positively on dominant firm share. But, in a broader context, with industry capacity variable, an increase in dominant firm share does not necessarily raise price variance. As we have just illustrated, an increase in dominant firm share through internal growth reduces price variance. Thus, industry capacity must be held constant in testing our prediction.

III. Concluding Remarks

In this paper we have placed the dominant firm model in a more realistic context by considering the behavior of a price leader under conditions of incomplete information. Specifically, by devoting resources to the task of forecasting industry demand patterns, the dominant firm is in the position of providing a public good for fringe suppliers. This public good derives from the leader's investment in searching for the best price, which the smaller firms in turn accept as given as they go about solving their own optimization problems.

The major implication of the model is that price variance depends positively on dominant firm market share. This is because the larger the market share of the price leader, the greater the level of information it becomes economic for the dominant firm to acquire. The more information the price leader acquires, the more accurate are its pricing decisions. Price variance increases because additional demand data allow price changes to more closely match demand changes.

As Posner (1969, p. 585) has observed, "the monopolist has a strong incentive to determine consumers' reactions to various quality-price combinations...[The monopolist] has every incentive to be ingenious in anticipating and responding to consumers' wants." Although he is here making the point that monopolists in general are no less likely than competitive firms to explore for optimal quality-price combinations, our analysis suggests that Posner's argument can be made stronger in the sense that the dominant firm has a proportionately higher demand for information about consumers' wants. That is, the dominant firm not only economizes on the cost of searching for information about demand at various prices (with the result that a greater amount of such information is collected and used), but the dominant firm has an incentive to gather more information about all aspects of demand, including non-price aspects such as product quality. The implication is that not only will price variance be higher as dominant firm market share increases, but also that some measure of the variance of non-price aspects of the product (which, as Carlton 1986 has pointed out, are merely alternative allocation mechanisms) should also be higher. In short, a higher market share for the dominant firm increases the degree to which quality-price combinations suit consumers.

As such, our analysis identifies a welfare-enhancing aspect of merger heretofore neglected in the literature. Specifically, a normative implication of the model is that increased price variance should be counted as one of the possible efficiency-creating effects of horizontal mergers that raise the market share of a dominant firm.

FOOTNOTES

[*]We benefitted from comments by Don Boudreaux. Remaining errors are our own.

[1]See Stigler (1965) or any standard textbook treatment (Koutsoyiannis 1979, pp. 246-247, for instance) of the subject.

[2]See Williamson (1975, pp. 208-218) for a discussion of the antitrust treatment of the dominant firm.

[3]The level of information can be characterized by an integer correspondence because there is a natural ordering of the X_i by informativeness. We suppose the X_i are ordered accordingly in the demand function.

[4]Alternatively, we could have assumed that buyers queue when realized quantity demanded exceeds quantity supplied at marginal cost equals price. In this way consumers would bear more of the cost of short supply. Our positive analysis would not be affected much by adopting this alternative output policy.

[5]Our model in effect assumes that the cost of holding inventories is prohibitive.

[6]Strictly speaking, our model is incomplete. Implicitly we assume that the quantity supplied by the fringe is never greater than realized quantity demanded at the dominant firm's price. For stochastic demand this assumption is very unrealistic when the dominant firm is small. However, since we are not really interested in the model's implications when t is small, our assumption reasonably approximates reality.

[7]This is just a specific case of Stigler's (1961) general point concerning the incremental value of information.

[8]Note that $(bc-r)^2-t^2>0$ since $t \leq r$ and $|bc-r| > r$.

[9]$\partial E(p^m)/\partial t = 4t[E(A)]^2(cB-t)^2/cB^3(cB-2t)^3 > 0$.

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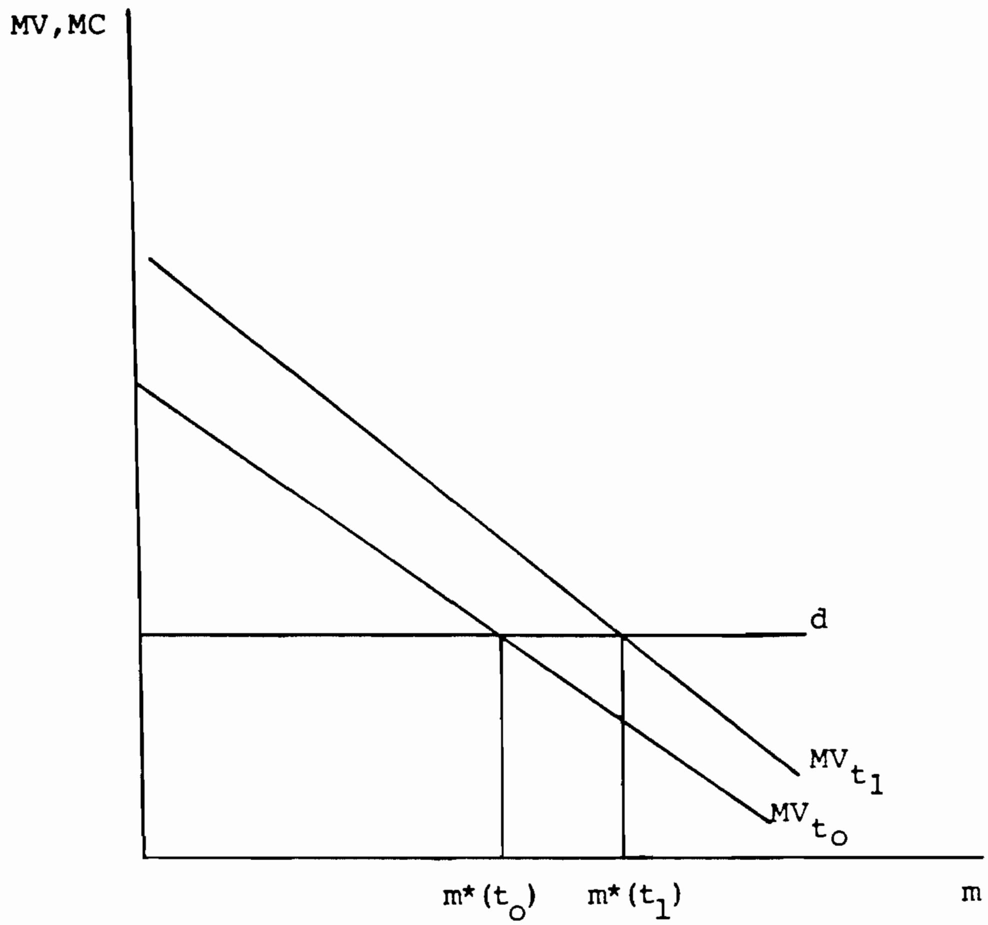

FIGURE 1