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ABSTRACT: Based on weakly separable utility, this paper develops a condition that should be useful in obtaining and evaluating estimates of cross-price elasticities of demand.

I. INTRODUCTION

Reliable econometric estimates of cross-elasticities of demand are notoriously difficult to obtain.[2] For this reason researchers have developed methods of calculating the required elasticities when only limited information is available. This note is in that tradition. The Armington (1969a) model is perhaps the best known example of this type of research.[3] As footnote nine explains, however, the Armington model has limitations in some contexts.

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[2]Richardson (1976) describes the difficulties in obtaining cross price elasticities in international trade from time series data.

[3]Other efforts to provide methods for calculating cross-elasticities, when only limited information is available include: Frisch (1959); Rousslang and Parker (1984); and Rousslang and Suomela (1985, Appendix E).

Intuition varies considerably on what constitutes a plausible set of cross-elasticity estimates. For example, Grossman (1982) plausibly argued that US imports from developing and developed countries of products such as leather goods and tires and tubes are complements rather than substitutes. Moreover, some have criticized the methodology developed by Rousslang and Parker (1984), because it appears to lead to "small" estimates of cross-elasticities of demand. Thus, if a researcher has obtained a set of elasticity estimates, which are to be used in an applied problem, it would be useful to have available a result from economic theory that would help determine if the elasticities are plausible.

This note develops such a condition. It is based on the fact that when researchers estimate disaggregated demand functions, they invariably exclude from the analysis the prices of goods not closely related to the product of interest. This means that they are either implicitly or explicitly assuming that the utility function from which the demand functions are derived is "weakly separable." Given that this assumption is implicitly being imposed in empirical analysis, I assume it from the start, and derive a theoretical condition that I call the modified Cournot aggregation condition. This condition has proved useful in assessing the plausibility of a set of own and cross-elasticity of demand estimates.

Obviously, the modified Cournot aggregation condition should not substitute for well established statistical methods, such as testing hypotheses regarding the parameters or defining confidence intervals. In the examples discussed below, however, I show how the modified Cournot aggregation condition has been employed to calculate cross-elasticities, when the econometric estimates failed to pass statistical tests.

In section II, the condition is derived for a general specification of the (branch) utility function. I then discuss how it may be utilized to calculate elasticities of demand. In section III., the condition is illustrated for the constant elasticity of substitution utiltity function. Finally, the condition is applied to the problem of estimating the effects of the US and EC extensive system of voluntary export restraints on world steel trade. Through the use of the modified Cournot aggregation condition, perverse results are avoided.

III. THE MODIFIED COURNOT AGGREGATION CONDITION

Weakly Separable Utility

A utility function $u = u(x_1,...,x_n)$ is said to be weakly separable or a "tree" if: there exists a partition of the n products into m subsets; m functions $u^r(x^r)$; and a function F such that

$$u(x_1,...,x_n) = F[u^1(x^1),...,u^m(x^m)] \quad (1)$$

where $m \geq 2$ and x^r is the vector of the products in the r-th subset. The m subsets, which correspnd to the product groupings, such as food or clothing, are referred to as branches of the utility tree. It is convenient to utilize double subscripts to denote products. Let x_{rj} equal the j-th product in the r-th branch. Also define p_{rj} as the price of x_{rj}; y as aggregate income available to consumers and y_r as income allocated to the r-th branch of products. It is well known that a necessary and sufficient condition for the demand functions of products within a group to depend only on prices within the group and income allocated to the group, is that the utility fuction be weakly separable, i.e., a utility tree.[4] Thus, with a weakly separable utility function, prices outside the r-th branch affect the purchases of products within the branch only through their impact on y_r; and the demand functions for x_{rj} can be written without explicit reference to prices outside the branch:

$$x_{rj} = f_{rj}(p^r, y_r) \qquad \text{for all } r \text{ and } j \qquad (2)$$

where p^r is the vector of prices on the r-th branch.

Modified Cournot Aggregation

With a weakly separable utility function the consumer's decision problem can be viewed as a two-stage maximization procedure. The consumer first optimally allocates expenditure among the broad commodity groups corresponding to the m braches of the utility function. This determines the budget allotment for each branch, where $\sum_{r=1}^{m} y_r = y$. The consumer then maximizes a branch utility function subject to income allocated to the branch.[5] The budget constraint for the r-th branch is:

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[4] See Goldman and Uzawa (1964) for proof. Strotz (1957) apparently coined the term utility tree. If the good in question is an intermediate product, then a weakly separable production function for final output is assumed.

[5] See Green (1964), Blackorby, Primont and Russell (1978), Phlips (1984) and Pollack (1971) for derivations of the results of weakly separable utility.

$$\sum_j p_{rj} x_{rj} = y_r \qquad r = 1,\ldots,m.^{6} \qquad (3)$$

Partially differentiate both sides with respect to p_{rk} yielding:

$$\sum_j p_{rj} \frac{\partial x_{rj}}{\partial p_{rk}} + x_{rk} = \frac{\partial y_r}{\partial p_{rk}} \qquad \begin{array}{l} r = 1,\ldots,m \\ k = 1,\ldots,n_r \end{array} \qquad (4)$$

Multiply each term in the sum on the left by one in the form of x_{rj}/x_{rj} and multiply both sides of (4) by p_{rk}/y_r obtaining the **"modified Cournot aggregation condition"**:

$$\sum_j S_{rj} e^r_{jk} + S_{rk} = \frac{\partial y_r}{\partial p_{rk}} \frac{p_{rk}}{y_r} \qquad \begin{array}{l} r = 1,\ldots,m \\ k = 1,\ldots,n_r \end{array} \qquad (5)$$

where $e^r_{jk} = \frac{\partial x_{rj}}{\partial p_{rk}} \frac{p_{rk}}{x_{rj}}$ and $S_{rj} = \frac{p_{rj}\, x_{rj}}{y_r}$

are, respectively, the elasticity of the j-th product in the r-th branch with respect to a change in the price of the k-th product in the r-th branch; and the share of income allocated to the r-th branch spent on the j-th product in the r-th branch.

The left hand side of (5) is directly analogous to the condition Frisch (1959) has called the Cournot aggregation condition. The latter condition applies when a consumer maximizes utility over all goods, not just over goods within the branch, and is independent of weak separability. The summation on

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[6] Denote by n_j the number of products within each branch. Then: $\sum_{j=1}^{m} n_j = n$. We use $\sum_j$ in expressions like (3) to denote $\sum_{j=1}^{n_j}$.

the left side of (5) is a weighted average of the own and cross-price elasticities of demand for products within the branch, where the weights are the respective shares of expenditure on all products in branch r. On the left side, the share of the product whose price has changed is added to the weighted average of the elasticities.

Unlike the analogous Cournot aggregation condition, the right hand side is not equal to zero, because a change in the price of a product within a branch has an effect on income allocated to the branch. The right hand side is the elasticity of income allocated to the r-th branch with respect to a change in the price of the k-th product within the branch. If we think of cross-price elasticities as being nonnegative, the smaller the elasticity of income allocated to the branch with respect to a change in the price of the k-th product within the branch, the smaller the cross-elasticities (with respect to a change in the k-th price) must be in relation to the own elasticity.

In general, the sign of the right hand side of (5) is unknown. Casual empiricism would suggest, however, that regardless of which price within the branch that is changing, these income elasticities should possess the same sign; moreover, one would not expect them to vary widely within a branch by more than a proportionality factor reflecting the share of branch income spent on the good.[7] That is, if the branch is clothing, and the products are men's

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[7] It is shown below, that for the constant elasticity of substitution utility function, the income elasticities must have the same sign and all are equal to the quantity one minus the common aggregate demand elasticity for the branch good times their respective share of branch expenditure. For some products, however, such as imported automobiles from West Germany versus imported automobiles from Yugoslavia, an asymmetric treatment of the income elasticities may be plausible.

suits and men's shirts, then we would expect the ratio of the elasticities of income spent on clothing with respect to a change in the price of men's suits or men's shirts, would approximately equal the ratio of the shares of branch income spent on these two goods, respectively. If the elasticities, when plugged into the left hand side of (5), imply very different elasticities of income allocated to clothing, depending on whether the price of men's suits or shirts is changing, this should be a cause of considerable skepticism regarding the elasticities. In such cases, the researcher has a number of alternatives. One is to examine the statistical tests of the price elasticities. If one price elasticity is particularly doubtful, one can accept the implied income elasticity from the product whose price elasticities pass statistical tests, as applying to the other products in the branch as well; given this elasticity of income with respect to a price change, for any other product within the branch, equation (5) will enable solving for a single price elasticity.[8] Alternatively, the Slutsky symmetry condition may be utilized, along with equation (5) and available statistical tests, to guide the reasearcher toward an appropriate substitution of elasticities. I show below, how the modified Cournot aggregation condition was combined with statistical tests and the Slutsky symmetry condition, to avoid perverse results in an applied model of world steel trade.

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[8] In a model employed to provide estimates to the US International Trade Commission, regarding the gains to consumers and the economy of removing quotas on stainless steel bars and rods and alloy tool steel, I treated imports and domestic products as differentiated (Tarr, 1987b). That is, stainless steel bars and rods and alloy tool steel are assumed to be a branch of the utility (production) function of the representative US buyer of steel, and import and domestic versions are products within the branch. Taking econometric estimates of the elasticities from the literature (Crandall, 1981), one of the two estimated cross-elasticities was not significantly different from zero. Since experts from the industry argue that the products are close, if not perfect, substitutes, this estimate was not regarded as plausible. Utilizing the procedure mentioned in the text, however, enabled calculation of this cross-elasticity within the short time deadline imposed by the hearing.

III. APPLICATIONS

Constant Elasticity of Substituion (CES) Utility Functions

The CES function is the basis of the often appied Armington model.[9] The elasticities e^r_{jk} have been derived by Armington (1969a, equation 26) for the CES utility function case. They are:

$$e^r_{jk} = S_{rk}(\sigma_r - \eta_r) \qquad \text{for } j \neq k, \quad r = 1,\ldots,m \\ k = 1,\ldots n_r \tag{6}$$

and

$$e^r_{jj} = -[(1 - S_{rj})\,\sigma_r + S_{rj}\,\eta_r] \qquad j = 1,\ldots,n_r. \\ r = 1,\ldots,m \tag{7}$$

Where

$$\sigma_r = \frac{\partial(x_{rj}/x_{rk})}{\partial(p_{rj}/p_{rk})} \frac{p_{rj}/p_{rk}}{x_{rj}/x_{rk}}$$

is the common elasticity of substitution between products in the CES function, and

$$\eta_r = -\frac{\partial x_r}{\partial p_r}\frac{p_r}{x_r}$$

is the elasticity of demand (defined to be positive) for the r-th branch of goods, and p_r is the price of the aggregate good r.

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[9] The CES function imposes certain restrictions that are not necessary for two stage budgeting. In particular, the CES function is itself separable, implying that the demand for imports is independent of home prices; and the CES function is homothetic, implying that the shares will not change unless relative prices do. Utilizing data for UK manufactures, Winters (1984) finds neither homotheticity nor separability of the branch utiltity functions acceptable.

If we substitute from (6) and (7) into (5) we get:

$$\sum_j S_{rj}\, e^r_{jk} + S_{rk} = S_{rk}\,(1 - \eta_r) \qquad \begin{array}{l} r = 1,\ldots,m \, . \\ k = 1,\ldots,n_r \end{array} \qquad (8)$$

Thus, with the CES utility function, the sign of the modified Cournot aggregation condition is determined entirely by η_r, the elasticity of demand for the aggregate of the goods in the branch r. If the branch good, say clothing, is elastic, then η_r exceeds one, (8) is negative and weighted cross-elasticities cannot be large in relation to own elasticities. Contrary to the general case, the sign of (8) cannot change with the product k whose price is changing. Thus, even if an estimate of η_r is unavailable, if we have estimates of the values on the left hand side of (8) for all goods in the branch, the sign of the summation on the left must be the same for all goods in the branch. (In fact, we may solve for η_r from the shares and the elasticities on the left for any given price change.)

A Model of World Steel Trade

Consider a model of steel trade flows in which, for the purpose of modeling a restriction on imports, we group the world into three regions: South Korea (K); the regions that are restraining imports, the United States and the European Community (U); and the rest of the world (R). Following the Armington assumption we assume that consumers regard products from different regions as differentiated. Then let the r-th branch of the utility function (1) be steel, and the products within the branch be the products from K, U and R.

Thus, each region has three demand functions for steel in the form of equation (2). Each demand function depends on three prices: the price of steel from each region. Thus, each region has nine elasticities of demand: three own and six cross.

The market shares of the three regions are listed in table 1. For the regions K and R, I use the Armington elasticities. For the region U, however, a set of estimates may be based on the work of Grossman (1982). I use Grossman's "iron and steel shapes, angles and sections" as a proxy for steel mill products, and let developing countries, non-US developed countries and the US in Grossman be proxies for Korea, the rest of the world and the US-EC, respectively, in what follows. The elasticities for the US-EC, which are adapted from Grossman, are presented in table 2.[10]

Since Grossman only estimated import demand equations, the middle row of table 2 is not available from Grossman. We may use the Slutsky equation, however, to obtain approximations for the missing elasticities. Ignoring separability for the moment, consider the case where the consumer maximizes utility over n goods subject to a budget y. The Slutsky equation in elasticity form is:

$$e_{ij} = z_{ij} - S_j E_i \qquad i,j = 1,\ldots,n \qquad (9)$$

where e_{ij} is the Marshallian elasticity of demand of the i-th good with respect to a change in the price of the j-th good; z_{ij} is the analogous income compensated elasticity of demand; S_j is the share of the consumer's income

[10] Grossman employed the "double logarithmic" or constant elasticity specification for the estimating equations. It is known (Brown and Deaton, 1972, pp. 1176-1178) that such a specification for all products in the system (as opposed to just those within one branch) will likely lead to a violation of the "adding up" or "Engel aggregation" condition that the share weighted sum of the income elasticities equals unity.

Table 1: Shares in the U.S. Steel Market of Developed Countries, Developing Countries and the U.S., 1984 and Supporting Data

Region	tons (1000)	$ per ton	value of shipments (in $1000)	Shares
LDC	6583	300	1974900	.0405
US	72759	532	38707788	.7946
DC	19580	410	8027800	.1648
Total	98922		48710488	1

Source: American Iron and Steel Institute, Annual Statistical Report, 1984 for quantity data; World Steel Dynamics, The Steel Strategist, September 1985 for DC and U.S. prices and World Steel Dynamics, POSCO: Korea's Emerging Steel Giant, 1985 for the LDC price.

Table 2: Price Elasticities of Demand for Steel in the US-EC and Modified Cournot Aggregation (Grossman Adapted).

	Given Change in Price of Steel of:		
Elasticity of US Demand for Steel from:	K	U	R
K	-4.512	.08	4.503
U	.004	-1.702	.327
R	2.309	1.575	-3.388
Modified Cournot Aggregation (equation 5)	.197	-.295	.049

Source: Adapted from Grossman (1982) as explained in text; table 1; and equation (5).

spent on j; and E_i is the elasticity of x_i with respect to income. Define: $Z_{ij} = z_{ij}q_i/p_j$. $Z_{ij} = Z_{ji}$ from symmetry of the pure cross-substitution effects in the Slutsky equation. It follows that:

$$z_{ji} = S_i z_{ij}/S_j. \tag{10}$$

From (9) we have

$$e_{ji} = z_{ji} - S_i E_j. \tag{11}$$

Substitute for z_{ji} from (10) into (11), and solve for z_{ij} in both (9) and (11) to get:

$$e_{ji}\, S_j/S_i + S_j E_j = e_{ij} + S_j E_i = z_{ij}. \tag{12}$$

Rearrange to get:

$$e_{ij} - e_{ji} S_j/S_i = S_j(E_j - E_i). \tag{13}$$

Since the share S_j is the share of expenditure on the j-th product in the total consumer's budget, not just the share allocated to steel, the share of any particular product such as Korean or US-EC steel is small. Moreover, income elasticities of steel products from differrent countries are not likely to be significantly different. Thus, we may closely approximate the right hand side of (13) by zero and obtain:[11]

$$e_{ij} = e_{ji} S_j/S_i. \tag{14}$$

We seek e_{UK} and e_{UR}, where we have estimates of e_{KU} and e_{RU} from Grossman.

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[11]In the US, the share for sales of US steel is less than .02, which is greater than the other steel shares. For a product such as oil, the share may be large enough that we may not be able to approximate (13) by (14).

These are easily obtained from (14).[12]

From table 2, we observe that the calculated value of the modified Cournot aggregation condition is negative if the price of steel fron the region U or the region R changes; but it is equal to .2 for a change in the Korean price. The positive value of the modified Cournot aggregation condition in the case of a Korean price change means that more income is allocated to steel purchases when the price of Korean steel increases in the US-EC; when the price of steel from the region R or U increases, however, no additional income is allocated to steel purchases. In effect, the elasticities, with respect to a change in the Korean price of steel in the US-EC, are not constrained by the original budget allocation to steel, but the other elasticities are. This asymmetric treatment of the consumer's budget allocation to steel products, depending on which nation's steel price changes, I believe should cause one to be very skeptical of the estimated elasticities.

If we assume that the value of $e^U_{KR} = 4.503$ is correct, then from equation (14) the value of e^U_{RK} should approximate .28 rather than 2.039 from table 2. Moreover, Grossman's standard error for e^U_{RK} is high in relation to his other sandard errors, so tat a ninety percent confidence interval for this variable would include negative values. Thus, it seems more reasonable to take $e^U_{RK} = .28$; while the modified Cournot aggregation condition, for a change in the Korean price, remains positive, it is reduced to close to zero at .0075.

[12]For the value e_{UU}, I use the Armington elasticity. The elasticity of subsituion and elasticity of demand necessary to calculate Armington elasticities are taken from Shiells, Deardorff and Stern (1986) and Hekman(1978), respectively.

This means that the income elasticities, with respect to a change in the price of steel from the three regions, will be much closer to each other. That is, the modified Cournot aggregation condition indicates that at least one of the elasticities with respect to a change in the price of Korean steel in the US-EC is not accurate. Statistical tests identify a problem with e^U_{RK}, and the Slutsky symmetry condition is used to obtain an alternate estimate.

Failure to recognize this inconsistency in the elasticity estimates would lead to perverse results in the model for which these elasticities were developed. In particular, these elasticities were employed in a three region model of world trade in steel (Tarr, 1987). The impact of the US and EC extensive system of voluntary export restraints, on world trade in steel was simulated. With the elasticities of table 2, the US-EC consumed more steel after the imposition of the voluntary export restraints and the resulting price increases. This apparently perverse result is a consequence of the significant high income elasticity of demand with respect to a change in the price of Korean steel, and is corrected when $e^U_{RK} = 2.039$ is replaced with the more reasonable value of .28.

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