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MERGERS AND FREE RIDERS

IN

SPATIAL MARKETS

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Mergers and Free Riders in Spatial Markets

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Abstract: Prior analyses have found little incentive for merger in the absence of efficiency gains. Either merger is unprofitable, or outside firms earn higher profits than the merged parties. In both cases, firms have incentives to free ride on the merger activity of rivals. We examine merger in a model with differentiated consumers, and find that mergers are profitable. Moreover, the free-rider problem is largely eliminated under uniform pricing; it is completely eliminated under discriminatory pricing.

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Mergers and Free Riders in Spatial Markets

1. Introduction

Past theoretical analyses find limited gains to merging purely for anticompetitive purposes. Salant, Switzer and Reynolds (1983) show that, in a quantity-setting game with homogeneous goods and constant marginal costs, a merger will generally lower the combined profits of the merged parties. On the other hand, Deneckere and Davidson (1985) discover that the formation of a coalition increases the profits of the combined parties in a price-setting game with differentiated products.[1] However, all outside firms earn higher profits than the merged parties. Firms therefore have an incentive to refrain from merger themselves, and instead free ride on merger activity by their rivals.[2] This paper focuses on the free-rider problem by examining merger in a market with differentiated consumers (i.e., a spatial market). When firms participate in a spatial market, they have considerable incentive to merge even in the absence of efficiency gains (see Farrell and Shapiro (1990a,1990b) for mergers that generate efficiencies).[3] The free-rider problem is largely eliminated under uniform

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[1] A coalition may differ from a merger. A coalition is formed for mutual gain, but can always disband into individual noncooperative players with separate profit-maximizing goals. A merged entity maximizes combined profits, unless broken into separate players through the sale of a portion of its assets. Thus, a merged firm cannot credibly act as separate players, except in spatial models when the merger chooses nonneighboring locations (see below).

[2] Deneckere and Davidson (1984) find similar results when examining collusive behavior in a model with homogeneous goods and capacity constraints.

[3] Recent literature [e.g., Perry and Porter (1985), Chang and Harrington (1988), McAfee and Williams (1988), and Farrell and Shapiro (1990a,1990b)] finds adequate incentives to merge when capital can be transferred between firms. In these models, mergers generate efficiency

pricing; and, it ~~is~~ completely eliminated in the case of discriminatory pricing.

The paper is organized as follows: section 2 describes the basic model, section 3 presents a two-firm merger with uniform pricing, section 4 considers a multifirm coalition with uniform pricing, section 5 examines discriminatory pricing, and section 6 offers concluding remarks including a discussion of relocation and entry. In the section on multifirm coalitions, we find that the coalition desires to use a "sequential limit pricing" strategy that bears similarity to a basing-point system. However, the tendency to use this strategy may preclude the possibility of a pure-strategy equilibrium in certain cases.

2. The Model

In our model, N firms and a continuum of consumers are located around a circle. Distances are measured in units of 2π radians, implying that the circle is of unit circumference. The location of a given firm or consumer is identified by its equivalent arc measure in 2π radians. Firms are evenly-spaced, located at positions, 1/N, 2/N, ..., N/N. For reference purposes, firm i denotes the producer located at position i/N. Consumers are spread uniformly around the circle, and w denotes the buyer located at position w/N. By definition, d(i,j) expresses the shortest arc distance

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gains due to improved rationalization of resources, or from synergies related to scale advantages or a learning process. Although price may increase after merger in these models, anticompetitive effects must be untangled from merger-specific efficiencies in making welfare assessments.

between any two locations, i/N and j/N.[4]

Each consumer may purchase a bundle containing a variable amount of a homogeneous good and one unit of a differentiated good. If consumer w's bundle contains m units of the homogeneous good and a single unit of the differentiated good offered by firm i, then its utility is expressed as follows:

$U(m,i,w) = m + (a - td(i,w))$, where t is unit transport cost.[5]

In the above function, m represents the subutility derived from the homogeneous product, and $(a - td(i,w))$ represents the subutility derived from variety i. Without loss of generality, we assume that the price of the homogeneous good equals one, and the price of variety i equals p^i. Letting $\hat{i}$ represent $\text{argmin}_i\ p^i + td(i,w)$, a utility-maximizing consumer would purchase one unit of variety $\hat{i}$ whenever $a - (p\hat{i} + td(\hat{i},w)) > 0$. In the equilibria considered below, we assume that this inequality holds for all consumers. If all N firms produce positive output levels, then this restriction implies that the boundaries of each firm's market must touch those of its closest rivals. Hence, there is direct price competition among firms to secure customers.

Each firm has a cost function $C(x) = cx + f$, where x is output, c is marginal cost, and f is fixed cost. A portion of the fixed cost is assumed

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4 Hence, the distance function can be expressed as follows:

$$d(i,j) = (1/N)|i-j|, \quad \text{when } (1/N)|i-j| \leq 1/2$$
$$= 1 - [(1/N)|i-j|], \quad \text{when } (1/N)|i-j| > 1/2.$$

5 Distance can represent either physical distance or a measure of the difference between the product characteristics offered and the consumer's "ideal" product. Notice that t represents the unit "transport" cost to the consumer, which may differ from the "transport" cost to a given producer. This distinction is important when examining firm behavior under price discrimination. In that case, the "transport" cost to the producer largely determines observed differences in "delivered" prices.

to be sunk to location (e.g., due to the establishment of immobile physical assets or the maintenance of reputation for a given brand location). We further assume that diseconomies preclude a firm from establishing multiple brands, except through the acquisition of another firm's capital.

Each firm located on the circle produces a positive level of output in equilibrium and, therefore, competes only with its closest rival in either direction. With respect to firm i, we let i+n (i-n) identify the firm located at a distance equal to n/N units in a (counter)clockwise direction. The profits of firm i, denoted by π^i, can be expressed as follows:

$$\pi^i = (p^i - c)(x^{i+} + x^{i-}) - f, \tag{1}$$

where $x^{i+}(x^{i-}) \equiv$ the length of the market segment in a (counter)clockwise direction.

By simple manipulation, the market segments in each direction are:

$$x^{i+} = (p^{i+1} - p^i + t/N)/2t$$

$$x^{i-} = (p^{i-1} - p^i + t/N)/2t \tag{2}$$

By substituting into equation (1) and differentiating with respect to price, we obtain the following first-order condition:

$$p^i = (1/4)(p^{i+1} + p^{i-1}) + (1/2)Z \tag{3}$$

where $Z \equiv c + (t/N)$

All firms face a similar first-order condition, implying that a symmetric Bertrand-Nash equilibrium occurs where $p^i = p^*$ and $\pi^i = \pi^*$ for all i. The pre-merger equilibrium is described as follows:[6]

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[6] Salop (1979) obtains these results in solving for a symmetric zero-profit equilibrium. Whether evenly-spaced firms are representative of a true locational equilibrium depends on the type of price equilibrium observed when a given firm locates in close proximity to one of its rivals. D'Aspremont, Gabszewicz, and Thisse (1979) have shown that, at close

$$p^* = Z; \quad \pi^* = t/N^2 - f. \tag{4}$$

The equilibrium is sustainable under threat of entry if $t/(N+1)^2 - f < 0$.

3. The Merger of Two Firms with Uniform Pricing

Consider a merger of two firms, i' and i'+1. The merger generates no efficiency gains; marginal and fixed costs are unaffected. However, the acquisition of firm-specific capital through merger implies that the merged parties can and will produce two brands. We further assume that substantial location-specific investment makes relocation a prohibitively costly strategy during the period under consideration. Entry is also precluded. The implications of these assumptions are discussed in the concluding section.

We first solve the profit-maximization problem for the merged firm, which can be expressed in the following manner:

$$\max_{p^{i'},p^{i'+1}} \pi^M \equiv \pi^{i'} + \pi^{i'+1}$$
$$= (p^{i'} - c)[(p^{i'-1} + p^{i'+1} - 2p^{i'} + 2t/N)/2t]$$
$$+ (p^{i'+1} - c)[(p^{i'} + p^{i'+2} - 2p^{i'+1} + 2t/N)/2t] - 2f. \tag{5}$$

The first-order conditions yield the following results:

$$p^{i'} = (1/4)(p^{i'-1} + 2p^{i'+1} + Z + t/N) \tag{6}$$

$$p^{i'+1} = (1/4)(p^{i'+2} + 2p^{i'} + Z + t/N), \tag{7}$$

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distances, no pure-strategy Bertrand-Nash equilibrium will exist when transport costs are linear. Thus, some other type of equilibrium becomes relevant in assessing prices.

Evenly-spaced firms are representative of a locational equilibrium when firms face quadratic transport costs. If transport costs are expressed as $td^2(l^i,w)$, then the equilibrium price is $p^* \equiv z = c + t/N^2$. Later, we discuss both similarities and differences in those results obtained from linear versus quadratic cost specifications.

If $p^{i'-1} = p^{i'+2}$, then equations (6) and (7) are symmetric. The merged parties would charge the same price. All outside firms face a first-order condition expressed by equation (3), and that equation is symmetric with respect to the prices of neighboring firms. By recursive application of these first-order conditions, it can be shown that whenever $p^{i'} = p^{i'+1}$, firms equidistant from the merger would charge the same price (see Appendix A). Hence, an equilibrium does exist where pairs of firms act symmetrically.

We now introduce notation to distinguish firms based on their location relative to the merged parties. Let P^0 represent the price charged by a merged firm, and p^k represent the price charged by an outside firm that lies at distance k/N from the closest merged party. Under symmetry, the merged parties use the following reaction function:

$$P^0 = (1/2)(p^1 + Z + t/N). \tag{8}$$

When all nonmerging firms satisfy their first-order conditions, firm k's reaction function can be expressed as follows (see Appendix A):

$$p^k = (B^{K-k}/B^K)P^0 + ((B^K - B^{K-k})/B^K)Z, \tag{9}$$

where $K \equiv (N-M)/2$ $((N-M+1)/2)$ if N-M is even(odd)

$M \equiv$ number of merging firms,

$B^n \equiv 4B^{n-1} - B^{n-2}$, $B^0 = 1$, $B^{-1} = 1(2)$ if N-M is even(odd).

Letting $p^k = p^1$ in the above equation, we can now solve (8) and (9) simultaneously to obtain equilibrium values for P^0 and p^1. By recursive application, we can then obtain the equilibrium values for all prices:[7]

$$P^0 = Z + (B^K/L)(t/N) \tag{10}$$

$$p^k = Z + (B^{K-k}/L)(t/N) \tag{11}$$

where $L \equiv 2B^K - B^{K-1}$.

Given the initial conditions and the defined behavior of B^n, we can assert that $B^K > B^{K-1} >\ldots> B^0 > 0$.

The profits of the merged parties can be described as follows:[8]

$$\pi^0 \equiv \pi^M/M = [(P^0 - c)^2/2t] - f$$

$$= [1 + (B^K/L)]^2(t/2N^2) - f. \tag{12}$$

For any outside firm, profits can be represented in the following manner:[9]

$$\pi^k = [p^k - c)^2/t] - f \tag{13a}$$

$$= [1 + (B^{K-k}/L)]^2(t/N^2) - f. \tag{13b}$$

It can be shown that $\pi^1 > \pi^0 > \pi^2$.[10] Using the subsequent definition,

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[7] In spatial models with linear transportation costs, a firm must consider a market-appropriation strategy where it sets a sufficiently low price to capture its neighbor's entire market. (This can be accomplished by setting price $t/N + \epsilon$ below that of the neighboring firm.) If a given firm finds that market appropriation dominates market sharing at the price vector representing the fixed-point solution under market-sharing strategies, then a pure-strategy Bertrand-Nash equilibrium will not typically exist. In the case of 2-firm mergers, firms do prefer to share the market at the fixed-point solution described below (as shown in Appendix B for $N \geq 4$).

[8] Under symmetric pricing, first-order conditions are not satisfied unless the total market equals $(P^0 - c)/2t$ for each of the merged parties.

[9] For any outside firm, the first-order condition for optimal pricing implies that $p^i - c = t(x^{i+} + x^{i-})$. Hence, the total market for firm k equals $(p^k - c)/t$.

[10] The following relationship holds:

$$D \equiv \pi^0 - \pi^1 = (t/2L^2N^2)[(B^K)^2 - 6B^{K-1}B^K + (B^{K-1})^2]$$

Treating the above expression as a quadratic equation with respect to B^K, we see that $dD/dB^K = 2B^K - 6B^{K-1} < 0$ if $B^K < 3B^{K-1}$.

Setting $D(B^K) = 0$, we find that $B^K = (3 \pm 2\sqrt{2})B^{K-1}$. Thus, $D < 0$

we summarize our findings concerning post-merger prices and profits.[11]

Definition:

A given pair of firms are considered "neighbors" if there exists a nonempty set of consumers where these firms represent the two closest producers.

Proposition 1:

If two neighbors merge, then post-merger price behavior implies that $P^0 > p^1 >\ldots> p^K > p^*$. All outside firms charge lower prices than the merged firms, and prices decline as the distance from the merger increases. Further, $\pi^1 > \pi^0 > \pi^2 >\ldots> \pi^K > \pi^*$. Each of the merged firms earns larger profits than any outside firm except those that are neighbors to the merger.

It can easily be shown that the above proposition holds when transportation costs are quadratic.[12] Thus, the free-rider problem suggested by

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whenever $(3 - 2\sqrt{2})B^{K-1} < B^K < (3 + 2\sqrt{2})B^{K-1}$. The solution to the difference equation for B^n satisfies this condition (since $2B^{n-1} \leq B^n < 4B^{n-1}$).

Further, the following relationship holds:
$$d \equiv \pi^0 - \pi^2 = (t/2L^2N^2)[23(B^{K-1})^2 - 38B^{K-2}B^{K-1} + 7(B^{K-2})^2]$$

Using reasoning similar to the prior example, $d > 0$ if $B^{K-1} > [(19 + 10\sqrt{2})/23]B^{K-2}$. The solution to the difference equation for B^n satisfies this condition also.

[11] Braid (1986) obtains similar merger results in a model where an infinite number of firms are located on an infinite line. The results below apply whenever $N \geq 4$.

[12] Notice that our conclusions are independent of the number of firms, N. If distance costs are represented by $td()^2$ instead of $td()$, equilibrium prices are merely expressed in terms of t/N^2 instead of t/N. Thus, our basic results are unaffected. Since both demand functions and

Deneckere and Davidson (1985) appears limited. A firm would only avoid merger when close rivals are expected to merge within a given time period, and some constraint limits the number of viable mergers in the market.

By reexamining the profit-maximization problem shown in equation (5), it is easily shown that firm i' only gains from merging with either of two firms: i'+1 or i'-1. If firm i' merges with any other firm, the first-order condition for either of the merged parties is still represented by equation (3). Hence, conditions are unchanged from the pre-merger equilibrium. In a Bertrand-Nash equilibrium with uniform pricing, mergers would only occur between neighboring firms in the absence of efficiency gains.

Of course, incentives may exist for merging with nonneighboring firms under more strategic interaction (see Braid (1986) and Levy and Reitzes (1989)). For instance, consider a Stackelberg leader-follower game. A merger among nonneighboring firms produces no benefits from coordination if the merged parties act as followers (i.e., price takers). Instead, let the merged parties act as leaders. If one merged party raises its price, then the followers choose a higher price level. As a consequence, the profits of the other merged party would rise. A merger among nonneighboring firms raises the payoffs to leading, but does not affect the payoffs from following. Thus, the act of merging may serve to "credibly" commit a pair of firms to a more active leadership role. In this manner, the free-rider problems inherent in price-leadership games may be either reduced or eliminated.

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reaction functions are continuous in the case of quadratic transport costs, the existence of a pure-strategy equilibrium is guaranteed (see D'Aspremont, et. al. (1979) and Caplin and Nalebuff (1989)).

4. Multifirm Coalitions

If more than two firms belong to a coalition of neighboring firms,[13] only the "border" firms face direct competition from outside firms. Other coalition members only compete directly among themselves. Significant market power potentially exists in serving consumers that are "internal" to the merger. However, with uniform pricing, no pure-strategy Bertrand-Nash equilibrium occurs when multifirm coalitions are formed. While the coalition reacts to keep its prices just low enough to discourage consumers from going to outside firms, neighbors of the coalition respond by altering their prices in order to appropriate part of the coalition's "internal" market. This combination of strategies removes the possibility of reaching a pure-strategy equilibrium when transportation costs are linear.[14]

We initially examine the profit-maximization problem facing an M-firm coalition of neighboring firms. For brevity, we only consider the case where M is an even number. Hence, $M = 2S$, where S represents the number of pairs of firms that belong to the coalition. If $M = 2S+1$, the exposition would be quite similar.

To discuss the coalition's profit-maximization problem, we identify the location of firms by their distances from the "border" firms of the coalition. In our spatial model, the "border" firms are the two coalition members that are each "neighbors" with an outside firm. We let s'(s")

[13] These multifirm coalitions may arise as a result of collusion, or through multiple mergers. In this section, we presume that the multifirm coalition maximizes joint profits. Hence, we assume that either the coalition arises through merger, or that the cartel operates efficiently.

[14] Equilibrium can be obtained for the case of quadratic transport costs. Later, we describe the results obtained under this cost assumption.

denote a coalition member located s/N from the closest border, when that border lies in a (counter)clockwise direction. That coalition member charges a price, $P^{s\prime}(P^{s\prime\prime})$. Further, $D^{s\prime}(D^{s\prime\prime}) \equiv P^{s\prime}-P^{s-1\prime}$ $(P^{s\prime\prime}-P^{s-1\prime\prime})$ when $0 < s \leq S-1$. Two other definitions are needed. First, $D^{0\prime}(D^{0\prime\prime}) \equiv P^{0\prime}-p^{1\prime}$ $(P^{0\prime\prime}-p^{1\prime\prime})$, where $P^{0\prime}(P^{0\prime\prime})$ represents the price set by the border firm, and $p^{1\prime}(p^{1\prime\prime})$ represents the price set by the neighboring outside firm. Second, the term, $D^{SS} \equiv (p^{1\prime\prime} + \Sigma_{n=0}^{S-1} D^{n\prime\prime}) - (p^{1\prime} + \Sigma_{n=0}^{S-1} D^{n\prime})$, represents the difference in prices set by the two most-insulated coalition members. The profit-maximization problem of a multifirm coalition can be expressed as follows:[15]

$$\max_{D^{s\prime},D^{s\prime\prime}} \text{ (for } s = 0,1,\ldots S-1)$$

$$\begin{aligned}\pi^M &= \Sigma_{s=0}^{S-2} (p^{1\prime} + \Sigma_{n=0}^{s} D^{n\prime} - c)[(-D^{s\prime} + D^{s+1\prime} + 2t/N)/2t] \\ &+ \Sigma_{s=0}^{S-2} (p^{1\prime\prime} + \Sigma_{n=0}^{s} D^{n\prime\prime} - c)[(-D^{s\prime\prime} + D^{s+1\prime\prime} + 2t/N)/2t] \\ &+ (p^{1\prime} + \Sigma_{n=0}^{S-1} D^{n\prime} - c)[(-D^{S-1\prime} + D^{SS} + 2t/N)/2t] \\ &+ (p^{1\prime\prime} + \Sigma_{n=0}^{S-1} D^{n\prime\prime} - c)[(-D^{S-1\prime\prime} - D^{SS} + 2t/N)/2t] \quad - Mf\end{aligned}$$

subject to

$$D^s \leq t/N; \ -D^{SS} \leq t/N. \tag{14}$$

The following first-order conditions may be derived:

$$d\pi^M/dD^{0\prime} = (1/t)[-D^{0\prime} + S(t/N) - (1/2)(p^{1\prime}-c) + D^{SS}] + \lambda^{0\prime}. \tag{15a}$$

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[15] The constraints are needed to preclude one coalition member from appropriating its neighbor's entire market. Thus, the price differential between two neighboring firms can never exceed t/N. By assumption, a consumer buys from the closer firm when two firms set mill prices that result in equivalent prices inclusive of transportation costs. This represents the limiting case of a situation where, in order to avoid market appropriation, a firm would never set its price more than $(t/N - \epsilon)$ above that of a neighboring rival. Letting $\epsilon \Rightarrow 0^+$, we obtain the above result.

$$d\pi^M/dD^{s'} = (1/t)[-D^{s'} + (S-s)(t/N) + D^{SS}] + \lambda^{s'} \text{ for all } s \neq 0 \qquad (15b)$$

Comparable first-order conditions exist with respect to $D^{0''}$ and $D^{s''}$.

If outside neighbors behave symmetrically, then $p^{1'} = p^{1''} = p^1$. We can now describe the solution to the above first-order conditions, based solely on the distance that a coalition member lies from its closest border:

$$D^0 = (t/N) \qquad (p^1-c) \leq 2(S-1)(t/N) = (M-2)(t/N)$$

$$= S(t/N) - (1/2)(p^1-c); \quad (p^1-c) > 2(S-1)(t/N) = (M-2)(t/N) \qquad (16a)$$

$$D^s = t/N. \qquad (16b)$$

When $p^1 \leq c + (M-2)(t/N)$, the strategy of the coalition can be described as "sequential limit pricing":[16]

$$P^0 = p^1 + t/N,$$

$$P^s = P^0 + s(t/N) \text{ for all } s \neq 0. \qquad (17)$$

When $p^1 > c + (M-2)(t/N)$, the border firms do not set limit prices. Nonetheless, given the price charged at the border, all "internal" members of the coalition continue to set limit prices (i.e., $P^s = P^0 + s(t/N)$). The coalition's pricing strategy in both cases implies that each member serves customers between its own location and that of the member next furthest from the border.[17,18] Due to the above mill-pricing strategy,

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[16] Of course, if $P^s(P^{s+1}) > a - t/2N$, the coalition would lose customers. Although the coalition's price eventually reaches a maximum, the essence of the forthcoming analysis is unaltered.

[17] This result holds for all coalition members except the two most distant firms from the "border." Those firms set equal prices; each serves a set of "internal" customers that lies between (S-1)/N and (S-(1/2))/N from the closest border.

prices inclusive of transportation costs are identical to those of a "basing-point" system, where a base lies at each border. A basing-point system may thus indicate the operation of an effective coalition.

To show nonexistence of a pure-strategy Bertrand-Nash equilibrium, note that the coalition engages in sequential limit pricing if $p^1 \leq c + (M-2)(t/N)$. Given this reaction, a neighbor of the coalition can earn higher profits by reducing its price by ϵ and appropriating part of the coalition's market. If $p^1 > c + (M-2)(t/N)$, then the coalition does not set a limit price at the border. However, the coalition's best response implies that $P^0 > c + (M-1)(t/N)$ and $P^s = P^0 + s(t/N)$. If the multifirm coalition responds in this manner (when $M > 2$), an outside neighbor would still earn higher profits by lowering its price enough to appropriate the coalition's internal market [see Appendix C]. Regardless of the price set by an outside neighbor, the coalition's response always induces a change in the neighbor's behavior. From the prior discussion, Proposition 2 follows:

<u>Proposition 2:</u>

No pure-strategy Bertrand-Nash equilibrium exists for a multifirm

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[18] With linear transport costs, it can be easily shown that the coalition maximizes its profits by retaining all of its premerger locations. This result does not necessarily apply to other cost specifications, though. With quadratic transport costs, it can be shown that the coalition continues to charge progressively higher prices as the distance increases from the nearest border. The coalition's profit-maximizing strategy requires that some locations be dropped in order to gain more price freedom in dealing with insulated customers. If insulated customers have to travel farther to reach an alternative supply source, then a nearby firm can charge a higher price (that increases quadratically with the distance to the alternate supplier). Some locations near outside firms will necessarily be eliminated. A pure-strategy Bertrand-Nash equilibrium continues to exist under quadratic transport costs due to the continuity of the demand functions (and the associated upper hemicontinuity of the reaction functions, see Caplin and Nalebuff (1989)). In equilibrium, some firms do not serve their closest customers.

coalition, $M \geq 4$, where M is an even number.[19]

Any mixed-strategy Bertrand-Nash equilibrium would undoubtedly be complex;[20] further, internal coalition members do not set limit prices under a mixed-strategy equilibrium (Appendix D). Nonetheless, we would still expect the coalition to set progressively higher prices as members become more distant from the "border." This behavior occurs because an outside firm has to overcome larger transport costs in order to appropriate a more distant market. With prices increasing internally, we may often find that coalition members earn higher profits than outside firms.

When a coalition forms among a continuum of firms, a pure-strategy Bertrand-Nash equilibrium <u>does</u> exist where the coalition uses <u>sequential limit pricing</u>. Assume that a continuum of firms lies along the circle, and let i, j, and w denote either firms or consumers located at positions i, j, and w (instead of i/N, j/N, and w/N). We redefine the distance function so that $d^*(i,j) = |j - i|$. Let p(w) denote the price at any location w. In a symmetric pre-merger Bertrand-Nash equilibrium, $p(w) = c$ at all w. Otherwise, if $p(w) > c$, a firm would earn positive profits (and obtain a significant market) by undercutting a nearby firm.

Consider the formation of a coalition M, where $M = \{w: i' < w < j'\}$. As a result, the symmetric pure-strategy Bertrand-Nash equilibrium changes

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19 Through a similar technique, the above proposition can be extended to a multifirm coalition, $M \geq 3$, where M is an odd number.

20 The existence of a mixed-strategy equilibrium is guaranteed because the set of actions that lead to discontinuities in the profit function, and the behavior of the payoff functions themselves, conform to conditions sufficient for equilibrium in Dasgupta and Maskin (1986).

in the following manner. If $w \notin M$, then $p = c$. If $w \in M$, then $p(w) = c + t\{\min [(d^*(i',w), d^*(j',w)] \}$. The coalition maximizes its profits by using sequential limit pricing; this strategy is actually <u>basing-point</u> pricing where the bases are located at i' and j'. Based on the above results, the following proposition is established:

<u>Proposition 3:</u>

Assume that firms form a continuum. Let i(i'), j(j'), and w denote locations along the circle, where $i \leq j$. The profits of all firms between i and j can be defined as $\pi(i,j) = \int_i^j (p(w)-c)dw$. In a symmetric pre-merger Bertrand-Nash equilibrium, $\pi(i,j) = 0$ for any i,j. Consider the formation of a coalition set, $M = \{w: i' < w < j'\}$. In the subsequent Bertrand-Nash equilibrium, $\pi^M = \pi(i',j') = td^*(i',j')/4 > 0$. Further, $\pi(i,j) = 0$ whenever $i,j \leq i'$ or $i,j \geq j'$. The formation of a coalition raises profits within the coalition; however, the profits outside the coalition are unchanged.

Since customers are served individually when there is a continuum of firms, we might expect that coalition formation would yield similar results to those achieved under pure price discrimination (involving a finite number of firms). This possibility is examined in the next section.

5. Coalition Forming with Discriminatory Pricing

Price discrimination becomes possible when firms can identify the location of their customers and set prices accordingly. Then, firms potentially offer identical "delivered" products to a given consumer. Thisse and Vives (1988) have shown that, in many spatial models, discriminatory pricing is the preferred practice when firms choose their pricing policy in a noncooperative fashion.

Since firms offer identical products and act as Bertrand competitors, any firm will undercut the price of its rivals unless this strategy leads to a price below marginal cost. Let $p^i(w)$ represent the delivered price offered by a firm at i/N to a consumer at w/N. Firm i will set its price in the range where $p^i(w) \geq c + td(i,w)$.[21]

A Bertrand-Nash equilibrium exists where, with the exception of the low-cost producer, all firms set price equal to marginal cost. The low-cost producer sets its price equal to the cost of the next-most-efficient firm. In that manner, the low-cost firm still obtains the customer while charging the highest possible price.[22] Moreover, other firms have no incentive to engage in further price reductions. Let $i,j \in I = \{1,2,...N\}$, where I denotes the set of firms in the industry. Further, let $j^*(i,w) \equiv \text{argmin}_{j \neq i}\ d(j,w)$. Hence, $j^*(i,w)$ represents the low-cost firm in serving customer w with the possible exception of firm i. We can now express a given firm's strategy in a Bertrand-Nash equilibrium with pure price discrimination:[23]

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[21] For analytical convenience, we assume that the transport cost for producers is identical to that for consumers. We comment on this assumption later (see footnote 23).

[22] We continue to assume that the consumer buys from the closer firm when two firms offer equal prices inclusive of transportation costs.

[23] Let $b \neq t$, where b(t) represents the transport cost to the producer(consumer). Under pure price discrimination, b determines the differences in prices across locations. The ability of a given firm to charge higher prices to more insulated consumers depends on the transport costs facing rivals. This situation contrasts with that of the previous section, where price differentials across locations depended on t (such as in the case of "sequential limit pricing"). These differences reflected the added transportation cost to the consumer of finding an alternative

$$p^i(w) = c + td(j^*(i,w),w) \quad \text{if } d(i,w) < d(j^*(i,w),w) \qquad (18a)$$

$$ = c + td(i,w) \quad \text{if } d(i,w) \geq d(j^*(i,w),w) \qquad (18b)$$

In the future, we let $j^*(i^1,i^2,\ldots,i^n,w)$ represent $\operatorname{argmin}_j d(j,w)$ for $j \neq i^1,i^2,\ldots,i^n$.

Consider a merger between two firms, i' and $i'+n$, where $I' = (i',i'+n)$. When either merged firm has a cost advantage over all outside firms, the best strategy of the merger is to merely outcompete the low-cost outside rival. Let $i^*(I',w) = \operatorname{argmin}_{i=i',i'+n} d(i,w)$. Of the two merged parties, $i^*(I',w)$ denotes the low-cost producer for serving customer w. If outside firms follow the strategy described in equation (18), the following price schedule represents the best response for the merged parties:

$$p^{i^*}(w) = c + td(j^*(I',w),w) \text{ if } d(i^*(I',w),w) < d(j^*(I',w),w) \qquad (19a)$$

$$\phantom{p^{i^*}(w)} = c + td(i^*(I',w),w) \text{ if } d(i^*(I',w),w) \geq d(j^*(I',w),w). \qquad (19b)$$

When firms use the above strategies, any consumers that are closest to the two merged parties would face a higher post-merger price. Consider a set of consumers, $W^1 = \{w: d(i',w) \leq d(i'+n,w) < d(j^*(I',w),w)\}$. For $w^1 \in W^1$, firm i' sets the following price prior to merger:

$$p^{i'}(w^1) = c + td(j^*(i',w^1),w^1) = c + td(i'+n,w^1). \qquad (20)$$

After merger, the price becomes:

$$p^{i^*}(w^1) = c + td(j^*(I',w^1),w^1)$$

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supply source.

Note that in the context of discriminatory pricing, the profits of firms increase as b rises. Hence, even if $b > t$, firms would still be tempted to offer "delivered" prices in order to relax competition. Of course, the ability to price discriminate would be constrained if consumers can arbitrage.

$$= c + td(j^*(i',i'+n,w^1),w^1) > c + td(i'+n,w^1). \qquad (21)$$

In serving consumers that belong to W^1, the revenue of firm i' increases subsequent to the merger. By analagous reasoning, the revenue of firm i'+n would also increase over any set of consumers, $W^2 = \{d(i'+n,w) < d(i',w) < d(j^*(I',w),w)\}$. When $w \in W^1 \cup W^2$, the merged parties still outcompete their outside rivals even if those rivals charge their marginal delivered cost. With respect to this set of consumers, outside firms are unaffected by the merger. Note that, unless firms i' and i'+n are neighboring firms, (i.e., i'+n = i'+1 or i'-1), W^1 and W^2 are both empty sets.

Consider next a set of consumers that are closest to one of the merging parties, but the next-closest firm is outside the merger. For instance, let $w^3 \in W^3 = \{w\colon d(i',w) < d(j^*(I',w),w) \leq d(i'+n,w)\}$. Merger between firms i' and i'+n would not affect the price for any consumer in W^3 because $j^*(i',w^3) = j^*(I',w^3)$. Gains from merger only occur when firms i' and i'+n are the two firms closest to a given consumer. Hence, there are no benefits to merger for nonneighboring firms. Notice that, with respect to consumers in W^3, outside firms are again unaffected by merger.[24]

From the prior discussion, outside firms receive no free-rider benefits from merger when they do not have a cost advantage. It is also apparent that no free-rider benefits occur when an outside firm does possess a cost advantage over the merged parties. Consider a consumer, $w^5 \in W^5 = \{w\colon d(j^*(I',w),w) \leq d(i^*(I',w),w)\}$. An outside firm, $j^*(I',w^5)$, is the low-cost producer. Given the strategies used by its rivals, firm

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[24] A consumer belonging to W^3 is still served by firm i' after the merger. Analagous reasoning shows that the merger yields no gains for the merging parties (or outside firms) with respect to consumers in W^4, where $W^4 = \{w\colon d(i'+n,w) < d(j^*(I',w),w) \leq d(i',w)\}$.

$j^*(I',w^5)$ cannot obtain customer w^5 unless it offers the following price:

$$p^{j^*}(w^5) = c + t\ \{\min\ [d(j^*(j^*(I',w^5),I',w^5),w^5),d(i^*(I',w^5),w^5)]\ \}. \quad (22)$$

which simplifies to:

$$p^{j^*}(w^5) = c + t[d(j^*(j^*(I',w^5),w^5),w^5)]. \quad (23)$$

Equation (23) is identical to equation (18a), where $i = j^*(I',w^5)$. An outside firm with a cost advantage must set the same price before and after merger.

Based on the above discussion, each outside firm retains the same price schedule after merger. Further, this schedule still represents profit-maximizing behavior since a low-cost outside firm sets the highest possible price that still allows it to serve a given customer. Regarding consumers located where it does not possess a cost advantage, an outside firm charges its marginal delivered cost and the consumer purchases elsewhere. Thus, the following proposition holds:[25]

<u>Proposition 4:</u>

In a Bertrand-Nash equilibrium with pure price discrimination, no free-rider problem exists. Outside firms experience no post-merger change in profits, and their pricing strategy is unaffected by merger. Moreover, only neighboring firms can increase their profits through merger.

The above results also apply to price-leadership behavior, and to coalitions that contain more than two firms. Even if price discrimination

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[25] Using prior reasoning, Proposition 4 can be extended to an N-dimensional representation of consumer characteristics and firm location with the distance function redefined to conform with Euclidean norms or other appropriate metrics. The conclusions in this proposition also apply to models of quadratic transportation costs, and to models where multiple purchases occur at each location.

can only occur in a more limited fashion, the free-rider benefits from merger may be small or nonexistent.

Proposition 4 has powerful implications. Since outside firms face unchanged competitive conditions after the merger, they are unlikely to relocate. For a similar reason, merger creates no additional incentive to enter the industry. Firms may thus receive significant long-term benefits from merger in markets characterized by price discrimination. From the consumer's perspective, merger may cause a sustained adverse impact on competition in those markets where customers contract individually with firms (including markets where a customer seeks bids for the provision of services).

6. Concluding Remarks

In contrast to previous literature, we find that the free-rider problems associated with merger are often absent from spatial markets. Market power is largely concentrated among firms that are closest in their brand offerings (or in their geographic locations) to the actual brand preferences (or locations) of a given subset of consumers. Consequently, firms desire to merge in order to isolate specific consumer groups that lie in close proximity. When a coalition becomes more successful in separating consumers from outside competitors, its profits will typically increase but the profits of rivals may not be affected. For example, the ability to isolate consumers is enhanced by the ability to price discriminate. When firms engage in pure price discrimination, direct competitors can substantively increase their profits through merger. Their rivals gain nothing, however. In this type of environment, direct competitors have no incentive to <u>refrain</u> from merger.

Even with uniform pricing, coalitions of adequate size tend to internalize much of the benefits from coordinated behavior. The incentive to refrain from merger applies mainly to the potential formation of small coalitions under uniform pricing. Even then, firms would only hold out if they expected their rivals to merge, and some constraint limited the number of viable mergers within the market.

In spatial markets, the formation of a coalition may create two potential sources of welfare losses. First, certain consumers may be forced to buy from more distant firms. Second, if consumers can make multiple purchases (or if multiple consumers with different reservation prices are associated with each location), the associated price increases would cause a loss of allocative efficiency.[26] Coalition formation in spatial markets would often result in adverse welfare effects in the absence of any merger-specific efficiencies. However, the formation of a coalition may create welfare gains, even when the merger generates no internal efficiencies. If some consumers do not purchase from the closest source of supply, then the formation of a coalition may improve efficiency by inducing them to switch to a closer firm.

The anticompetitive aspects of merger may be tempered by the ability of firms to eventually relocate. Depending on the nature of price competition, a number of post-merger Nash locational equilibria are possible. In the uniform-pricing case, outside firms may compete to occupy locations neighboring the merger. Hence, they would attempt to reposition

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[26] See Anderson and De Palma (1988), Thisse and Vives (1988), and Norman (1989) for spatial models where multiple purchases occur at each location.

closer to the merged parties (where prices are highest), thereby dissipating some of the gains from merger. One refinement to the set of potential locational equilibria may be to identify an equilibrium consistent with equal profits for all outside firms. In this equilibrium, firms far from the merger would be separated by greater distances than firms nearer to the merger.[27]

In the uniform-pricing case, the ability of the merged parties to maintain their locations might also be limited by entry. Judd (1985) finds that an incumbent cannot credibly deter entry by choosing to operate from multiple locations. An entrant can supplant an incumbent at a given location, since the reduction in price from two firms competing at the same location adversely affects profits at the incumbent's other locations. Nevertheless, as Judd observes, entry may be deterred if the incumbent faces substantial exit costs. In addition, entry may sometimes be precluded when there are limitations on the scale of entry (i.e., number of locations), and the incumbent occupies a sufficiently small market area. Consider a situation where, in an equilibrium with nonnegative profits, each firm would locate at least 1/N from its neighbors. A merged firm can then choose two (or possibly more) neighboring locations so that each is

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[27] A post-merger locational equilibrium may not always exist. The merged parties have incentives to expand their "internal" market area by moving toward nearby rivals. By moving too close, the possibility of a pure-strategy Bertrand-Nash equilibrium may be eliminated because neighboring firms desire to appropriate the merged firm's market.

If there is no possibility for entry, the merging parties may consider dropping a location in order to reduce fixed costs. However, this behavior is unlikely if the initial establishment of a location requires substantive sunk investment. With possible entry, the merged parties would not typically operate from a single location, but may consider operating from two nonneighboring locations so that the merger could credibly act like two distinct players.

less than 1/N from any other entity. If entry occurs at a point coincident with one of the merged parties (or nearby), the best strategy of the merger may be to drop only one location. Then, entry is not profitable.[28]

When firms can price discriminate, merger creates no incentive to enter the market. Merger does not affect the competitive conditions facing outside firms, and the anticipated equilibrium for prospective entrants is also unchanged.

[28] This strategy would be more likely if considerable sunk costs are initially required to establish a brand, but fixed costs are small each period. Note also that, in the case of a two-firm merger, entry would disrupt the collusive aspect of the merger, thereby eliminating the higher profits arising from the merger.

Appendix

A. Establishing Equation (9).

Consider the firms outside the merger. Let k'(k") denote an outside firm that must travel k/N in a (counter)clockwise direction to reach the closest of the merged parties. When N-M is even, we can partition the set of all outside firms into two subsets, K' and K". Each subset consists of K members, {1'(1"),2'(2"),...,K'(K")}, where K = (N-M)/2. Then, it is easy to show that if k' ∈ K', then there exists k" ∈ K". [When N-M is odd, this proof is slightly modified to consider the single firm that is equidistant from both merged firms.]

Note that, when evenly spaced, all outside firms face symmetric first-order conditions with respect to the prices of neighboring firms [as described by equation (3)]. Let $P^{0'}(P^{0''})$ denote the price charged by the closest merged party, when that party is reached by (counter)clockwise travel. By recursive application of equation (3), we can express $p^{k'}(p^{k''})$ in terms of $P^{0'}(P^{0''})$ and $p^{k+1'}(p^{k+1''})$ [relative to firm k, $p^{k+1'}(p^{k+1''})$ represents the price charged by the firm that is located at distance 1/N in a counterclockwise(clockwise) direction]. Since they involve the same recursive pattern, we can assert that $p^{k'} = f^k(P^{0'},p^{k+1'})$ and $p^{k''} = f^k(P^{0''},p^{k+1''})$ are symmetric with respect to their arguments. Hence, $p^{K'} = f^K(P^{0'},p^{K''})$ and $p^{K''} = f^K(P^{0''},p^{K'})$ are symmetric [where $p^{K''}(p^{K'}) = p^{K+1'}(p^{K+1''})$].

By inspection, the functional relationships described above can be expressed:

$$p^{k'} = (1/\Sigma_{n=0}^{k} B^n)\ [P^{0'} + (\Sigma_{n=0}^{k-1} B^n)p^{k+1'} + (B^k-1)Z] \qquad \text{(A.1)}$$

$$p^{k''} = (1/\Sigma_{n=0}^{k} B^n)\ [P^{0''} + (\Sigma_{n=0}^{k-1} B^n)p^{k+1''} + (B^k-1)Z], \qquad (A.2)$$

where $B^n \equiv 4B^{n-1} - B^{n-2}$, $B^0 = 1$, $B^{-1} = 1(2)$ if N-M is even(odd).

When $k = K$ and $P^{0'} = P^{0''} \equiv P^0$, we can solve equations (A.1) and (A.2) simultaneously to derive the following result:

$$\begin{aligned} p^{K'} = p^{K''} &= (1/B^K)[P^0 + (B^K - 1)Z] \\ &= (1/B^K)[B^0P^0 + (B^K - B^0)Z] \end{aligned} \qquad (A.3)$$

Given that $p^{K'} = p^{K''}$ whenever $P^{0'} = P^{0''}$, we can recurse backwards using equations (A.1) and (A.2) to establish that $p^{k'} = p^{k''}$. For instance, let $k = K-1$. By substituting (A.3) into (A.1), we derive the following:

$$\begin{aligned} p^{K-1'} = (1/B^KG(K-1))\{\ &[(B^K-B^{K-1}B^0) + B^0G(K-1)]P^0 \\ &+ [(B^K-B^0)G(K-1) - (B^K-B^{K-1}B^0)]Z\ \} \end{aligned} \qquad (A.4)$$

where $G(k) \equiv \Sigma_{n=0}^{k} B^n$

Under the behavior of B^n defined above, we can show that $G(k) = (B^K-B^kB^{K-k-1})/(B^{K-k}-B^{K-k-1})$ for any $k \leq K$. Alternatively, this can be expressed as $(B^K-B^kB^{K-k-1}) = (B^{K-k}-B^{K-k-1})G(k)$. When $k = K-1$, we obtain the result, $B^K-B^{K-1}B^0 = (B^1-B^0)G(K-1)$. This expression can be substituted into equation (A.4), which yields the following:

$$p^{K-1'} = (1/B^K)[B^1P^0 + (B^K - B^1)Z]. \qquad (A.5)$$

An expression identical to (A.5) is obtained for $p^{K-1''}$. Thus, equations (A.3) and (A.5) conform to the specification in equation (9). Using an analagous technique, it can be shown that equation (9) holds for all p^k.

B. **Proof that a Pure-Strategy Equilibrium Occurs in a 2-Firm Merger**

When two firms merge, the equilibrium prices resulting from market-sharing strategies are described by equations (10) and (11). Accordingly, the profits of an outside neighbor are described by equation (13b):

$$\pi^{1b} = [1 + (B^{K-1}/L)]^2(t/N^2) - f$$

$$= 4(B^K)^2(t/L^2N^2) - f \tag{B.1}$$

where $L = 2B^K - B^{K-1}$

If a neighbor instead pursues a market-appropriation strategy, it sets $p^1 \approx P^0-(t/N) = Z + [(B^{K-1}-B^K)/L](t/N)$. Further, the neighbor's market segments equal 3/2N in the direction of the merger, and $(B^K+B^{K-1})/NL$ in the other direction. The profits from a market-appropriating strategy can thus be described as:

$$\pi^{1a} = [4(B^K)^2 - (1/2)B^KB^{K-1}](t/L^2N^2) - f \tag{B.2}$$

Since $\pi^{1a} < \pi^{1b}$, a neighboring firm prefers a market-sharing strategy [except when $N = 3$ and the lone neighbor can appropriate the merger's entire market].

Given the equilibrium prices in equations (10) and (11), market sharing remains the preferred strategy for any other outside firm. Consider a strategy where an outside firm only appropriates the market of the neighbor nearer to the merger. Based on rival prices, its total market never exceeds 3/N. However, to appropriate that neighbor's market, p^k must be set below $c+(B^{K-1}/L)(t/N) < c+(1/3)(t/N)$ [since $2B^K \leq B^{K-1} < 4B^K$]. Thus, $\pi^{ka} < (t/N)^2 - f$. Referring to equation (13b), the market-sharing strategy yields higher profits. A similar argument can be used to show that an outside firm would not gain from undercutting its other neighbor.

Moreover, an outside firm would need to set a price below cost in order to undercut any nonneighboring firm.

C. Proof that Market Appropriation Necessarily Occurs When $P^0 > c + (M-1)(t/N)$ and $P^s = P^0 + s(t/N)$.

Without loss of generality, let the border firms charge $P^0 \equiv Z + \alpha(t/N)$, and other coalition members charge $P^s = P^0 + s(t/N)$. For a given level of P^0, consider the two possible strategies of an outside neighbor. With a market-sharing strategy, the prices of all outside firms are described by p^k from equation (9). Hence, $p^1 = p^{1b} \equiv Z + (B^{K-1}/B^K)\alpha(t/N)$ and $p^k = Z + (B^{K-k}/B^K)\alpha(t/N)$. Instead of sharing the market, an outside neighbor can appropriate half of the coalition's market by setting its price at $p^1 = p^{1a} \equiv P^0 - t/N - \epsilon \approx Z + (\alpha-1)(t/N)$. To compare these strategies, let $\Gamma \equiv \pi^{1a} - \pi^{1b}$, where $\pi^{1a}(\pi^{1b})$ represents the profits from a market-appropriating(market-sharing) strategy. We obtain the following expression for Γ:

$$\Gamma = (B^{K-1})^2\{-(c(K)-1)^2\alpha^2 + [((M+3)/2)c^2(K)-2c(K)]\alpha - c^2(K)\} \qquad \text{(C1)}$$

where $c(n) \equiv B^n/B^{n-1}$.

Equation (C1) can be set equal to zero. The upper-bound solution to the quadratic is irrelevant. Using that solution, the neighbor's optimal response under market sharing would imply that $p^{1b} < P^0 - t/N$. Hence, market appropriation would necessarily represent a dominant strategy at this price level. The lower-bound solution to (C1), referred to as α^T, represents the maximum "border" price that would still result in a market-sharing strategy:

$$\alpha^T = (1/2(c(K)-1)^2)\ [((M+3)/2)c^2(K) - 2c(K)]$$

$$- (1/2(c(K)-1)^2)\{[((M+3)/2)c^2(K) - 2c(K)]^2$$

$$- 4(c(K)-1)^2c^2(K))\}^{(1/2)}. \qquad (C2)$$

By solving the differential equation that describes B^n [refer to (9)], we can show that $2 \leq c(n) < 4$. Using this result, $0 < \alpha^T < 1/2$ for $M \geq 4$. [Since α^T reaches a maximum at $c(K) = 2$ and $M = 4$, we can plug these values into equation (C2). From this result, $\alpha^T < 1/2$.] Hence, for any $M \geq 4$, market appropriation necessarily occurs when $P^0 > c + (M-1)(t/N) = Z + (M-2)(t/N)$.

D. Proof that No Mixed-Strategy Equilibrium is Compatible with Sequential Limit Pricing Within the Coalition (i.e., $P^s = P^0 + s(t/N)$)

Assume that the coalition uses sequential limit pricing within its borders [i.e., $P^s = P^0 + s(t/N)$]. Consider the mixed-strategy used by an outside neighbor, expressed as a (Borel) probability measure on $A = \{p^1 : c \leq p^1 \leq \alpha - t/2N\}$. Let p^{1u} represent the upper-bound of the support of this measure. Further, let $F(p^1)$ represent the corresponding cumulative density function. In response to the outside firm, the coalition chooses a probability measure with a corresponding cumulative density function, $F(P^0)$. Moreover, the upper-bound of the support of the probability measure, P^{0u}, cannot be an optimal choice unless $P^{0u} \leq p^{1u} + (t/N)$. Otherwise, market appropriation would necessarily occur at P^{0u}.

Given this reply by the coalition, we need to show that a neighboring firm prefers a different strategy than $F(p^1)$. If $p^{1u} \geq P^{0u} - t/N$, then p^{1u} could only be potentially optimal if it represented the single best response under a market-sharing strategy [because $d^2E(\pi^1)/d(p^1)^2$ is defined and negative under market-sharing behavior]. Hence, optimal behavior would also require that $p^1 < P^{0u} - (t/N)$ at any other p^1 where the cumulative density is increasing. To prove nonexistence of equilibrium (under sequential limit pricing), we will show that $F(p^1)$ does not conform to these required conditions.

First, assume that an atom existed at p^{1u} [i.e., $F(p^1)$ was discontinuous at p^{1u}]. Let $p^1 < P^{0u} - (t/N)$ at any other p^1 where $F(p^1)$ increases. If $F(p^1)$ met these conditions and the coalition had chosen $F(P^0)$ optimally, then an atom must exist at P^{0u}. However, we have a contradiction. If an atom exists at P^{0u}, then p^{1u} would now be suboptimal. The neighbor can increase its profits by lowering p^{1u} marginally below $P^{0u} - (t/N)$ [see Digression]. Second, if no atom had existed at p^{1u}, then $F(p^{1u})$ would be lower semicontinuous at p^{1u}. As an optimal reply, the coalition would set P^{0u} strictly less than $p^{1u} + (t/N)$ [since there is no mass at p^{1u}]. Hence, given that $F(p^{1u})$ was continuous, there would now exist $p^1 = p^{1'}$ such that $P^{0u} - (t/N) \leq p^{1'} < p^{1u}$, and $F(p^{1u}) > F(p^{1'})$. Hence, if p^{1u} represents $\text{argmax}_{p^1}\ E(\pi^1(p^1, F(P^0))$ under market-sharing behavior, then $p^{1'}$ is a suboptimal response. This second contradiction excludes the possibility that $F(p^1)$ is now an optimal response. Hence, no equlibrium exists when the coalition uses "sequential limit pricing".

[Digression: Assume that the coalition prices sequentially within its borders. Let $P^0 = Z + \alpha^T(t/N)$ represent the maximum border price that still induces a neighboring firm to use a market-sharing strategy; we have previously shown that $0 < \alpha^T < 1/2$ when $M \geq 4$. Under sequential pricing, no mixed-strategy equilibrium could be possible unless $P^{0u} > Z + \alpha^T(t/N)$. Suppose not. Then, the neighbor always wishes to share the market. Based on equation (3), the neighboring firm would always set $p^{1l} \geq c + 1/2(t/N)$ [where p^{1l} is the lower-bound of the support of the probability measure]. Given this behavior, the coalition would necessarily prefer to set $P^{0u} > Z + (1/2)(t/N) > Z + \alpha^T(t/N)$.

Assume that $P^{0u} > Z + \alpha^T(t/N)$, and let an atom occur at P^{0u}. With the exception of p^{1u}, let $P^{0u} > p^1 - (t/N)$ at all p^1 where $F(p^1)$ is increasing. Hence, to constitute an optimal reply, P^{0u} must represent a profit-maximizing response to p^{1u}. Let $P^{0u} = p^{1u} + (t/N)$ represent this optimal reply. Then, a neighboring firm would increase its profits by lowering p^{1u} by ϵ. Further, the coalition would only set $P^{0u} < p^{1u} + (t/N)$ as an optimal reply if p^{1u} had assumed an extremely high value [refer to equation (16(a))]. Letting P^{0u} represent the optimal response to p^{1u}, and given that $P^0 < P^{0u}$ elsewhere, it can be shown that the upper-bound of (the support of) the neighbor's optimal reply must be less than p^{1u}.

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