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INCENTIVES TO COMPLY WITH UNCERTAIN LEGAL STANDARDS

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1. Introduction

Economic models of legal rules often assume that the behavior required by the rule is known in advance by all parties. In practice, though, this is rarely the case. In regulatory fields such as antitrust or securities fraud, or in common law subjects such as torts or contracts, legal rules are often defined in vague terms like "reasonable" or "substantial." At the time the parties must choose their behavior, they may have only a very rough idea of how a court will apply those standards to any particular set of facts.

We analyze the effects of such uncertainty on the compliance decisions of profit-maximizing risk-neutral parties. In general, we find that uncertainty may lead the parties subject to the rule either to overcomply or to undercomply. Uncertainty can create a positive probability that an offender will not be held liable, thereby reducing his incentives to comply, a result that is familiar from the criminal deterrence literature. However, uncertainty usually also means that each increment of increased compliance marginally increases the chance that the defendant will not be held liable, thus increasing the chance that the social costs of his behavior will be borne by someone else. This tends to create an incentive to overcomply.

As a result, many traditional recommendations of the law-and-economics literature must be abandoned or modified when legal standards are uncertain. For example, the recommendation that penalties should be increased by one over the probability of punishment, in order to prevent underdeterrence, remains valid only in a special set of cases. In other contexts, where incentives to overcomply are present, the multiplier should be much smaller -- possibly even less than one. The traditional recommendation that negligence standards (for example) should be set at the cost-effective level of care must also be modified to take account of potential under- or overcompliance.

Sections 2 and 3 develop the basic model of the behavior that is being controlled and the legal institutions used to control it. Section 4 introduces uncertainty into the legal system, and presents the basic results respecting over- and undercompliance. Sections 5 and 6 discuss two methods of correcting those problems, either by changing the damage rules or by changing the nominal legal standard. Finally, Section 7 discusses some complications that arise when the behavior of both plaintiffs and defendants must be controlled, and Section 8 states the conclusions and possible extensions. Additional legal interpretations and applications of these results are discussed in Calfee & Craswell (1984).

2. Behavioral Variables

We assume that the behavior of parties subject to the legal rule (referred to here as "defendants") can be measured by a single, continuous variable $\underline{x}$ ($\underline{x}>0$). Higher values of $\underline{x}$ benefit a defendant, but impose costs on other members of society. The following notation will be used:

B(x) The expected benefits accruing to the defendant at each level of $\underline{x}$, $\underline{B}(\underline{x})>0$. We assume $\underline{B}'(\underline{x})>0$ and $\underline{B}''(\underline{x})<0$, reflecting diminishing marginal returns to $\underline{x}$.

L(x) The expected costs imposed on other members of society at each level of $\underline{x}$, $\underline{L}(\underline{x})>0$. We assume $L'(x)>0$ and $L''(x)>0$, implying diminishing social returns to reductions in $\underline{x}$.[1]

For example, $\underline{x}$ could represent the risk that a railroad's sparks would set fire to neighboring fields, with $\underline{B}(\underline{x})$ representing the gain to the railroad from increasing that risk by running more frequent trains or spending less on spark arresters. We refer to "expected" costs and benefits because it is usually impossible to know how many accidents will in fact result at any level of $\underline{x}$. However, our analysis would also apply to contexts in which neither the costs nor the benefits had any stochastic element -- e.g., if *x* represented the amount of pollution emitted by a factory, and any given amount of pollution always caused the same amount of damages. We assume that all parties are risk-neutral, and that utility functions are additive, so that the parties' welfare can be expressed in expected value terms.

On these assumptions, the socially optimal level of behavior x^* is that which maximizes net social benefits $B(x)-L(x)$. Thus, the following first-order condition defines x^*:

$$B'(x^*) = L'(x^*) \tag{1}$$

If other variables also affected the total social costs, but these variables could not be controlled by the legal system, then some value of x other than x^* might be welfare-maximizing in a second-best-sense. For example, if the legal system controls the railroad's investment in spark prevention but not the number of trains it runs, Shavell (1980a) shows that welfare could be improved by requiring more investment in spark prevention than would otherwise be optimal (in the first-best sense), as this would indirectly reduce the number of trains.[2] However, we ignore those issues here, and ask only about how the value of x chosen by defendants under various legal rules compares with the first-best value x^*. We also assume that all defendants have identical B and L functions, thus abstracting from any difficulties (discussed by Diamond (1974), Cooter (1982), and others) caused by trying to apply a single legal standard to defendants with different cost or benefit functions.

3. Legal Institutions

The class of legal rules we model are those that define a legally required value of *x*, which we will call *z*, and inflict punishment on a defendants whose value of *x* exceeds the legally permissible value. Thus, *z* could correspond to the level of carelessness permitted under the "reasonable care" standard of tort law, or to the permissible level of pollution defined by an EPA regulation.

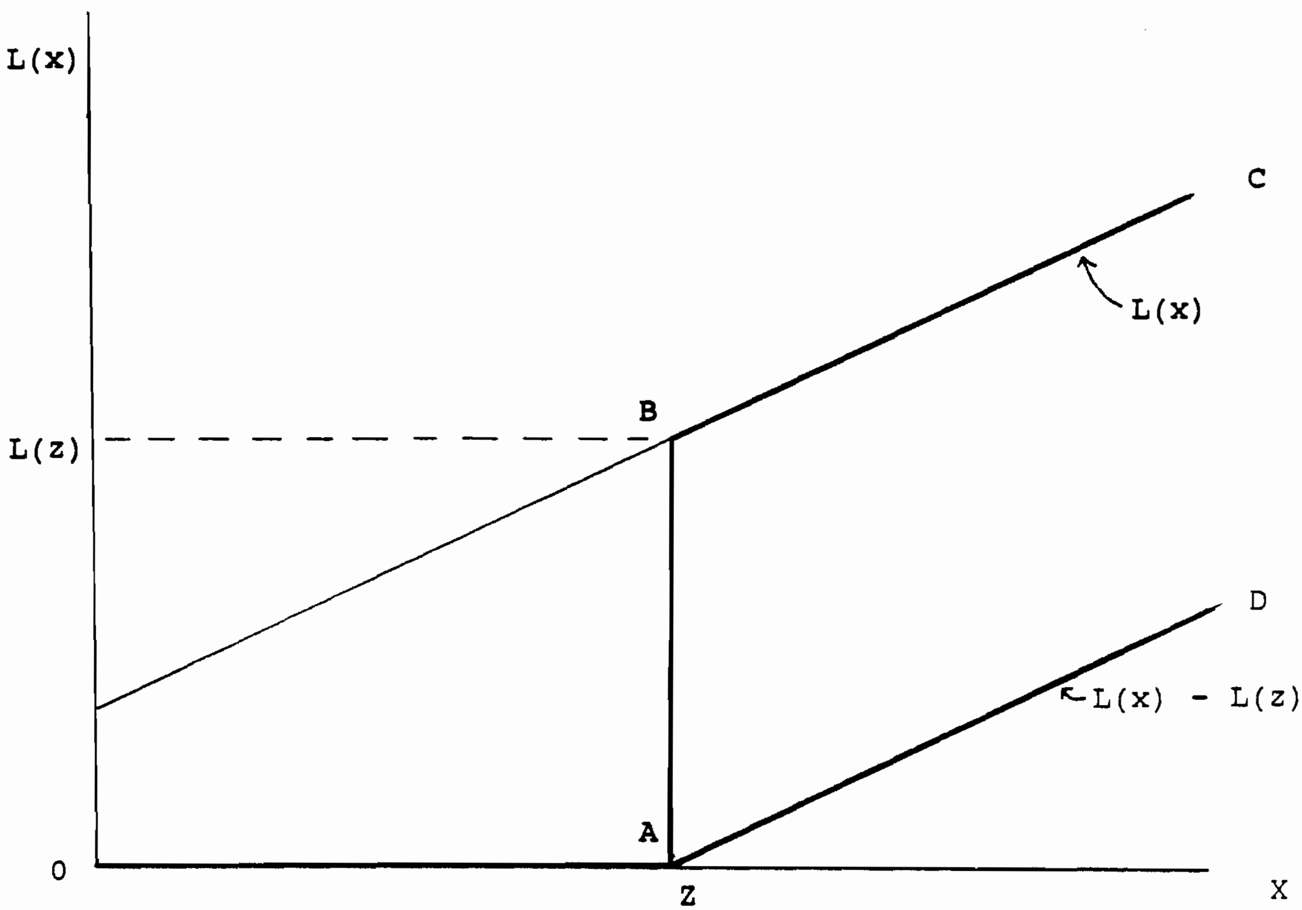
Initially, we will consider two possible damage rules that define the schedule of penalties.[3] Both rules are summarized in Table 1. Under a *full damage rule*, defendants who have violated the legal standard pay the full social costs associated with their chosen value of *x*, while those who have complied with the legal standard (by choosing a value of $x \leq z$) pay nothing. This creates a discontinuity at *z* in the schedule of fines, as shown in Figure 1 by the kinked line connecting points OABC.

[INSERT TABLE 1 AND FIGURE 1 HERE]

Table 1

Legal Rule	Payment Schedule	Geometric Representation
Full damages	0 if $x \leq z$ $L(x)$ if $x > z$	OABC
Marginal damages	0 if $x \leq z$ $L(x)-L(z)$ if $x > z$	OAD

Figure 1



By contrast, under an <u>incremental damage rule</u> defendants who have violated the legal standard pay only the difference between the social costs associated with their chosen level of $\underline{x}$, and the social costs associated with the maximum level of $\underline{x}$ permitted by the legal standard (that is, the social costs associated with $\underline{z}$). Geometrically, this is represented by the bent line connecting points OAD. At common law, incremental damage rules are often used when it is known exactly what social costs would have been inflicted if the defendant had complied with the legal standard, as the defendant can then claim that only the costs over and above that amount were actually <u>caused</u> by his violation. When those costs can only be stated in expected value terms, however -- e.g., when each level of $\underline{x}$ represents a different probability of a costly accident -- the common law sometimes applies a full damage rule, and holds negligent defendants liable for all resulting accidents.[4]

If the legal standard $\underline{z}$ is set equal to the optimal behavior $\underline{x}^*$, and both the location of that standard and the applicable damage rule are known to all defendants, it is easy to show that defendants maximize their private benefits by choosing $\underline{x}^*$ and exactly complying with the legal standard. Defendants prefer $\underline{x}^*$ to all lower values of $\underline{x}$ because at all lower values they have no liability for damages, and will therefore choose the highest value of $\underline{x}$ they can (subject to $\underline{x} \leq \underline{z}$) in order to maximize $\underline{B}(\underline{x})$. Defendants will also prefer $\underline{x}^*$ to all higher values of $\underline{x}$ because at higher values they will be in violation of the legal standard and will have to pay damages, and consequently will prefer the value of $\underline{x}$ that maximizes $\underline{B}(\underline{x})-\underline{L}(\underline{x})$ (under a full damage rule) or $\underline{B}(\underline{x})-[\underline{L}(\underline{x})-\underline{L}(\underline{z})]$ (under an incremental damage rule). Simple differentiation shows that $\underline{x}^*$ is the value that maximizes either of these expressions.[5] The intuitive explanation is that either rule forces the defendant to bear the full social costs of any <u>increase</u> in $\underline{x}$ above $\underline{x}^*$, as illustrated by the identical slope of the two lines in Figure 1. They differ by the amount of a constant equal to the "inframarginal" costs, or those costs ($\underline{L}(\underline{z})$) that would have been expected even if the defendant had complied with the legal standard. However, this constant has no effect on the defendant's marginal incentives.

4. Uncertainty About Legal Standards

These results no longer hold if defendants are uncertain about the legal rule, even if defendants are risk-neutral and other regularity conditions apply. We model this uncertainty by assuming that defendants do not know the exact location of the legal standard $\underline{z}$ until after they have chosen their value of $\underline{x}$. <u>Ex ante</u>, defendants only know the distribution of possible legal standards. The following notation will be used to describe this distribution:

$f(z)$ The probability density function of possible values of $\underline{z}$.

$F(x)$ The cumulative distribution function,

$F(x) \equiv \int_0^x f(z)dz$.

$F(x)$ thus represents the probability that the legal standard will be set below any given value of x. Since defendants must pay damages if their value of x exceeds the legal standard, $F(x)$ also represents the probability that a defendant choosing that value of x will be "found guilty" or "held liable."

Uncertainty about the legal standard produces a marked effect on defendants' incentives. Under a full damage rule, defendants may be unsure whether they will have to pay damages or not, as this will depend on where the court sets the legal standard. Their expected private benefits, $P(x)$, must therefore be written as follows:

$$P(x) = B(x) - \int_0^x L(x)f(z)dz \tag{2}$$

The integral is evaluated only from zero to x because if the legal standard is set above the defendant's level of x then the defendant will not be liable and will pay no damages at all.

Rearranging the terms of the integral, and substituting for the definition of $F(x)$, yields the following:

$$P(x) = B(x) - F(x)L(x) \tag{3}$$

Differentiation then yields:

$$dp/dx = B'(x) - F(x)L'(x) - F'(x)L(x) \qquad (4)$$

Evaluating this expression at the optimal level of care, x^*, we can substitute for $B'(x^*)=L'(x^*)$ from Equation (1) and rearrange the terms to get the following:

$$dP/dx|_{x^*} = [1-F(x^*)]L'(x^*) - F'(x^*)L(x^*) \qquad (5)$$

If this expression is negative, defendants will have an incentive to reduce x below the optimal level, or to overcomply with the legal standard by restraining their behavior "too much." If the expression is positive, defendants' incentives will be to undercomply, and only if it equals zero will they have an incentive to choose the socially optimal value of x.[6]

The intuition behind Equation (5) is simple. The first term reflects the gains to the defendant from reducing x and thereby reducing the damages he will have to pay ($L(x)$) if he is found liable. However, this gain is discounted by the chance that he will not be found liable at all ($1-F(x)$) and therefore will not benefit from this reduction. This is analogous to the observation of Becker (1968) and others that the chance that a criminal will not be punished tends to reduce the deterrent impact of any punishment. Indeed, this model becomes formally identical to a model of criminal deterrence if the damage rule is changed to one of absolute liability, as if the activity in question were a crime and all defendants caught engaging in it were made to pay the full social costs $\underline{L}(\underline{x})$. $\underline{F}(\underline{x})$ would then be redefined as the probability that a defendant choosing any particular level of $\underline{x}$ won't be detected.

If this were the only factor present, uncertainty would always lead to undercompliance, as the first term of Equation (5) is unambiguously positive. However, the second term of Equation (5) reflects a second gain to the defendant from reducing $\underline{x}$, a gain which comes not from reducing the total damage caused by his behavior but from increasing the chance that he won't have to pay for that damage. This creates an incentive to overcomply: The second term of Equation (5) is unambiguously negative. Traditional deterrence models typically overlooked this, apparently on the assumption that the probability of punishment varied only with the amount of law enforcement activity and not with the egregiousness of the defendant's behavior.[7] If reduced egregiousness reduces the chance the defendant will be found liable, though, this creates an incentive that (taken alone) tends to push defendants toward overcompliance.

The effect of these two factors becomes clearer when the full damage rule is compared with the incremental damage rule decribed in the previous section. Under this rule, defendants found to have violated the <u>ex post</u> legal standard pay only $\underline{L}(\underline{x})-\underline{L}(\underline{z})$, so their <u>ex ante</u> expected benefits are as follows:

$$P(x) = B(x) - \int_0^x [L(x)-L(z)]f(z)dz \tag{6}$$

The integral in Equation (6) can be rewritten to yield the following:

$$P(x) = B(x) - F(x)L(x) + \int_0^x f(z)L(z)dz \tag{7}$$

Differentiation then yields:

$$dP/dx = B'(x) - F(x)L'(x) \tag{8}$$

Substituting again for $\underline{B}'(\underline{x}^*)=\underline{L}'(\underline{x}^*)$ shows that this expression can never be negative when evaluated at $\underline{x}^*$:

$$dP/dx|_{x^*} = [1-F(x^*)]L'(x^*) \tag{9}$$

Thus, while defendants under a full damage rule might either undercomply or overcomply, defendants under an incremental damage rule will only undercomply.[8]

A comparison reveals that Equation (9) is simply Equation (5) without the latter's second term. The second term drops out of Equation (9) because under an incremental damage rule the defendant no longer risks being held liable for any "inframarginal" damages. The damage payments for a slight violation of the legal standard, though positive under a full damage rule, approach zero under an incremental damage rule.

Thus, there is no pay-off to the defendant from reducing his chance of having to bear those costs, so the $F'(x^*)L(x^*)$ term drops out. All that remains is the incentive to undercomply stemming from the fact that there is still a chance that the defendant "won't get caught" and won't have to pay any damages whatsoever. (If this chance could be completely eliminated, thus raising $F(x)$ to one, Equation (9) shows that defendants would then have an incentive to choose exactly the optimal value of x.)

5. Alternate Damage Rules

5.1 Changing the Threshold Damage Payment

The preceding analysis implies that one factor bearing on the likelihood of over- or undercompliance is the absolute size of the penalty for a slight violation of the legal standard. Under a full damage rule, this penalty equals the expected social costs at the optimal level of behavior ($L(x^*)$). The incentive to over- comply stems from the fact that reductions in x reduce the chance that the defendant will have to pay those costs, so that incentive will be strongest when $L(x^*)$ is greatest. Equation (5) confirms this: All else equal, a sufficiently large value of $L(x^*)$ will make dP/dx negative at x^*.

More generally, this analysis also suggests that one way to correct the incentives to over- or undercomply is by raising to correct the incentives to over- or undercomply is by raising or lowering the penalty for a slight violation while leaving everything else constant. The "full damage" and "incremental damage" rules are only two of a family of possible rules illustrated in Figure 2. In each case, defendants who comply with the legal standard pay nothing, while those found guilty pay a fine that increases with their chosen value of x at a rate equal to $L'(x)$. The only difference is in the starting point for this increase, or the fine charged a defendant who has just barely violated the legal standard. The incremental damage rule sets this threshold fine equal to zero while the full damage rule sets it equal to $L(x^*)$, but any other value could also be chosen.

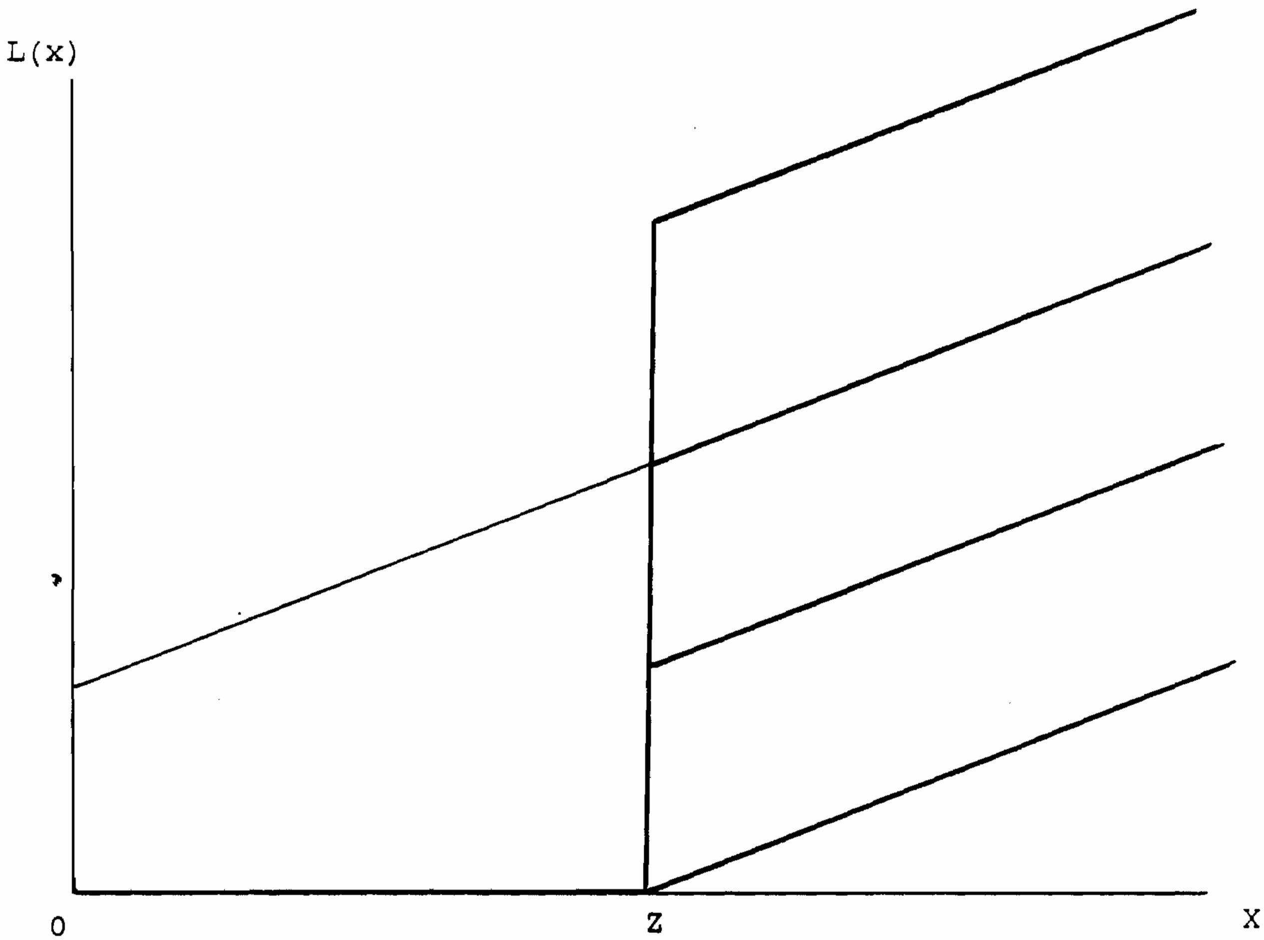
[INSERT FIGURE 2 ABOUT HERE]

The optimal value of this starting point is easily calculated. If D represents the starting point, so that a defendant who is found liable must pay $D+L(x)-L(z)$, his *ex ante* expected benefits are:

$$P(x) = B(x) - \int_0^x [D+L(x)-L(z)]f(z)dz \qquad (10)$$

Differentiating, and following steps analogous to those taken in Equations (3) through (5), yields the following:

Figure 2



$$dP/dx\big|_{x^*} = [1-F(x^*)]L'(x^*) - F'(x^*)D \tag{11}$$

Thus, the value of D that will set dP/dx equal to zero at x^* is:

$$D^* = \frac{1-F(x^*)}{F'(x^*)} L'(x^*) = \frac{1-F(x^*)}{f(x^*)} L'(x^*) \tag{12}$$

A comparison with Equation (5) reveals, not surprisingly, that this threshold should be lower than $L(x^*)$ when the incentives under a full damage rule would be to overcomply, but higher than $L(x^*)$ when the full damage rule would lead to undercompliance.

5.2 Changing the Slope of the Payment Schedule

The incentives to over- or undercomply can also be affected by changing the rate at which the fines or damage payments increase (for defendants who are found liable) with the defendant's choice of x. Equation (5) indicates that the incentive to undercomply will be weakest, and the incentive to overcomply strongest (all else equal), when the social costs of the defendant's activity do not rise very rapidly with x (i.e., when $L'(x)$ is small). The rate at which actual social costs rise with x is of course determined technologically, by the nature of the defendant's activity -- but the rate at which the defendant's liability payments rise can always be changed by adjusting the damage rules.

This is most easily seen in connection with the incremental damage rule. Multiplying the incremental damages ($L(x)-L(z)$) by a constant multiplier M means that damages still rise incrementally from zero, but they will rise at a rate that is M times the actual rate of increase in social costs ($L'(x)$). Under this rule, the defendant's expected private benefits are given by the following:

$$P(x) = B(x) - \int_0^x M[L(x)-L(z)]f(z)dz \tag{13}$$

This is equivalent to:

$$P(x) = B(x) - MF(x)L(x) - M\int_0^x L(z)f(z)dz$$

The optimal multiplier can then be derived by calculating dP/dx and evaluating it at x^*:

$$dP/dx|_{x^*} = [1-MF(x^*)]L'(x^*) \tag{14}$$

The value of M that makes this expression equal zero, thus giving defendants an incentive to choose the optimal value of x, is:

$$M^* = 1/F(x^*) \tag{15}$$

This is the familiar recommendation that the punishment should be multiplied by the reciprocal of the probability of being punished (evaluated at $\underline{x}^*$).[9] However, notice that this only leads to the optimal results if the penalty to which the multiplier is applied is calculated according to what we have called the "incremental damage rule." In particular, the same multiplier does not produce optimal results if applied to the full damage rule that is often used by the common law.[10] Applied to a full damage rule, a constant multiplier $\underline{M}$ gives defendants the following expected private benefits:

$$P(x) = B(x) - \int_0^x ML(x)f(z)dz \tag{16}$$

The integral can be rewritten to yield the following, equivalent expression:

$$P(x) = B(x) - MF(x)L(x)$$

Under this rule, defendants will have an incentive to choose the optimal value of $\underline{x}$ only if the following expression equals zero:

$$dP/dx|_{x*} = [1-MF(x*)]L'(x*) - MF'(x*)L(x*) \tag{17}$$

The optimal value of $\underline{M}$ under a full damage rule is therefore the following:

$$M^* = \frac{L'(x^*)}{F(x^*)L'(x^*) + F'(x^*)L(x^*)} \tag{18}$$

Simple arithmetic shows that this will always be less than $1/\underline{F}(\underline{x}^*)$, the multiplier recommended by the traditional deterrence literature. In fact, a comparison with Equation (5) shows, again not surprisingly, that the optimal constant multiplier would actually be less than one (indicating that damages should be <u>reduced</u>) in all cases where a full damage rule would otherwise lead to overcompliance.

The traditional multiplier of $1/\underline{F}(\underline{x}^*)$ is also incorrect in any system using a constant fine, where all defendants who are found liable pay the same amount $\underline{K}$. Under such a system, defendants' expected private benefits will simply be

$$P(x) = B(x) - K \int_0^x f(z)dz \tag{19}$$

The optimal constant fine is that which sets the following expression equal to zero:

$$dP/dx|_{x^*} = L'(x^*) - KF'(x^*) \tag{20}$$

Thus, the optimal fine, rather than multiplying the social costs by one over the probability of punishment, should multiply the <u>marginal</u> social costs by one over the <u>marginal</u> probability of punishment (again evaluated at $\underline{x}^*$):

$$K^* = L'(x^*)/F'(x^*) \tag{21}$$

This is equivalent to a fine of $L(x^*)/F(x^*)$ (the recommendation of the traditional deterrence literature) only in the special case where the defendant has only two choices, rather than choosing from a continuous range. If the defendant's only choice is, e.g., to murder or not to murder, then the absolute harm caused by the murder is also the marginal or incremental harm from the defendant's behavior, and the absolute chance of being punished for the murder is also the incremental change in the probability of punishment.[11]

5.3 Other Optimal Damage Rules

Two other optimal damage rules (optimal in the sense that they eliminate any incentive to under- or overcomply) should briefly be noted. The first involves multiplying the full social costs of each defendant's behavior ($L(x)$) by one over *that defendant's* probability of being punished -- i.e., by $1/F(x)$ evaluated at that defendant's chosen level of x, rather than by the constant $1/F(x^*)$. This gives defendants the following expected private benefits:

$$P(x) = B(x) - \int_0^x [L(x)/F(x)]f(z)dz \tag{22}$$

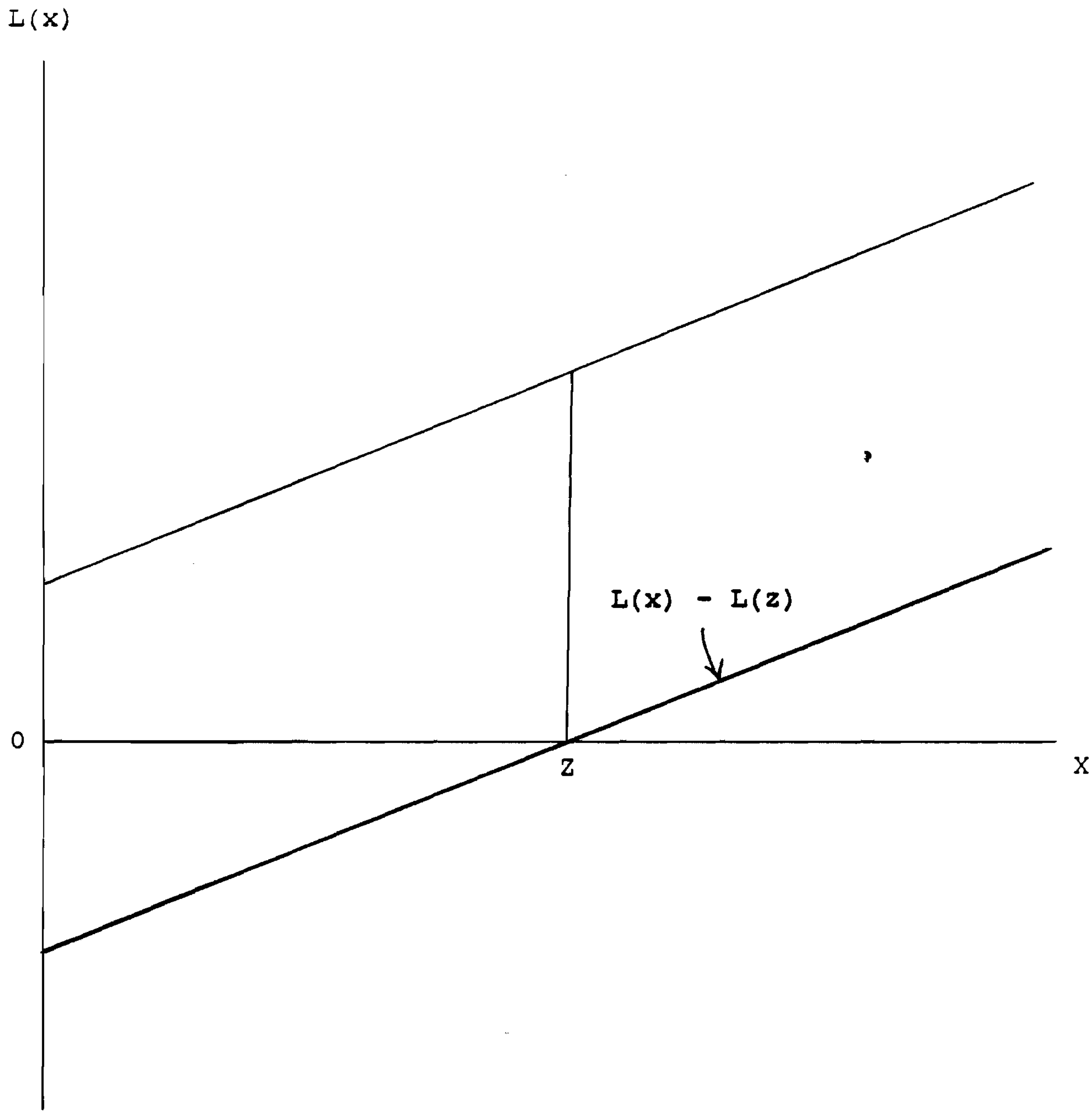
As $L(x)/F(x)$ can be factored out of the integral, and the remaining portion of the integral is simply the definition of $\underline{P}(\underline{x})$, this reduces to $\underline{P}(\underline{x}) = \underline{B}(\underline{x})-\underline{L}(\underline{x})$. Thus, this rule (if strictly applied) would give defendants every incentive to choose the value of $\underline{x}$ that maximizes total social welfare.[12]

One reason this rule is not actually used in the legal system may be the difficulty of calculating each defendant's probability of punishment (based on that defendant's chosen value of $\underline{x}$). An additional reason is that this rule can produce the incongruous result of penalties being <u>inversely</u> related to the seriousness of the offense. If the most egregious offenders were more likely to be caught and convicted than were the marginal offenders, then the egregious violators would receive the smallest damage multipliers, which might conflict with common notions of retributive justice.

A second way of eliminating any distortions caused by uncertainty involves paying compensation to defendants whose level of $\underline{x}$ is below the level permitted by the <u>ex post</u> legal standard. This is most easily seen in connection with an incremental damage rule, under which defendants who violated the legal standard must pay $\underline{L}(\underline{x})-\underline{L}(\underline{z})$ while defendants who comply with the <u>ex post</u> standard are paid $\underline{L}(\underline{z})-\underline{L}(\underline{x})$. Figure 3 gives a geometric representation of this rule, where negative "damage" awards reflect payments made to (rather than by) the defendant.

[INSERT FIGURE 3 ABOUT HERE]

Figure 3



Under this rule, defendants can expect $\underline{L}(\underline{x})-\underline{L}(\underline{z})$ from the legal system wherever the standard $\underline{z}$ is set, as this only affects the sign (or the direction) of the payment. A defendant's expected private benefits are therefore:

$$P(x) = B(x) - \int_0^\infty [L(x)-L(z)]f(z)dz \tag{23}$$

This can be rewritten as follows:

$$P(x) = B(x) - L(x) + \int_0^\infty L(z)f(z)dz \tag{24}$$

Differentiation shows that defendants will maximize their private benefits by choosing the value of $\underline{x}$ that sets the following expression equal to zero:

$$dP/dx = B'(x) - L'(x) \tag{25}$$

Again, this also defines the socially optimal level of $\underline{x}$, that maximizes net social benefits as well.

Uncertainty has no effect on defendants' incentives under this rule because there is no kink in the damage schedule, and the <u>slope</u> of the schedule remains the same regardless of where the <u>ex post</u> legal standard is set. In effect, this rule is equivalent (in expected value terms) to a system of strict liability under which the defendant always pays the full social costs of his behavior, but also receives the lump sum transfer reflected by the final term of Equation (24). As this lump sum has no effect on the defendant's marginal incentives, it can be raised or lowered without changing the incentive-preserving characteristics of this rule. In terms of the graph in Figure (3), the line that now intercepts the $\underline{x}$-axis at $\underline{z}$ could be shifted vertically by any amount without affecting defendants' incentive to choose the optimal value of $\underline{x}$.[13]

This rule also has the advantage of not requiring courts to calculate the probability that a defendant will in fact be punished, or any of the other factors necessary to determine an optimal multiplier. Its only drawback appears to be that it would require <u>every</u> defendant to be brought to court to receive or make a payment. In most common law contexts, the judicial machinery is invoked only for defendants suspected of violating the legal standard, or (if a violation only creates a <u>risk</u> of social costs, as in an accident context) only when an accident actually takes place. These administrative costs may partly explain why such a rule has not been adopted by the legal system.[14]

5.4 <u>Other Damage Multipliers</u>

As a final point, this model can also be used to analyze other multipliers applied to a "full damage" rule, even in the absence of any uncertainty about the legal standard. For example, "comparative fault" systems typically hold defendants liable for only a fraction of the total costs, with that fraction increasing from zero to one as the egregiousness of the defendant's behavior increases (relative to the behavior of other tortfeasors, or of the victim). As long as the defendant takes the behavior of others as given, the $\underline{F}(\underline{x})$ factor in Equation (3) can be reinterpreted as the fraction of the social costs assigned to the defendant, as $\underline{F}(\underline{x})$ also varies from zero to one as $\underline{x}$ increases. Thus, this form of comparative fault system can also lead to under- or overcompliance, for the same reasons (and under the same conditions) that were discussed earlier in connection with Equation (5).[15]

Our analysis is also consistent with Shavell's (1983) analysis of the "more likely than not" causation rule. Under this rule, the fraction of the social costs that even negligent defendants must pay drops abruptly from 100% to 0% as the defendant's level of $\underline{x}$ falls below some value $\underline{\hat{x}}$ (typically, the point at which the risk created by the defendant is sufficiently high to be deemed a legal "cause" of any resulting losses). Shavell shows that when $\underline{\hat{x}}$ is known, the ability to escape all liability by choosing a value of $\underline{x}<\underline{\hat{x}}$ can easily lead to overcompliance. If only the distribution of possible $\underline{\hat{x}}$'s is known, but the defendant is liable for all social costs whenever he chooses a value of $\underline{x}$ greater than the value of $\underline{\hat{x}}$ ultimately chosen by the courts, this rule becomes identical to the full damage rule with uncertainty about the legal standard, as modeled above in Equation (2).

5. Changing the Probability Density Function

The previous section discussed ways that changes in the damage formula might restore defendants' incentives to choose the optimal value of *x*. However, in some cases it will also be possible to restore those incentives by changing the shape of the probability density function *f*(*x*), as this function is also affected by institutional features of the legal system. Interpreted broadly as the distribution determining the probability that a defendant choosing any particular level of *x* will be caught and punished, *f*(*x*) could depend on such factors as the level of public enforcement, the costs of bringing suit, or the rules of evidence and burdens of proof applied in trials.[16] Our focus will initially be more narrow, as we examine the effect on the *f*(*x*) function of changes in the nominal legal standard.

Whenever a legal standard is defined in vague terms -- e.g., defendants should exercise "all reasonable care" -- the distribution of actual *ex post* standards can be thought of as a nominal standard plus or minus some error term. For example, courts might attempt to define a reasonable level of care as that which equates marginal costs and benefits,[17] but might err in identifying that level in any particular case. Raising or lowering the nominal legal standard might not affect the distribution of errors, but it should shift the entire distribution to a higher or lower level.

Formally, we will limit our analysis to distributions that are single-peaked at some modal value *m*, which we will call the nominal legal standard. We assume throughout this section that shifts in *m* affect the location of the distribution but not its shape, as illustrated in Figure 4.[18] A convenient way to represent this is to redefine the *f* and *F* functions as functions of two variables which satisfy the following conditions:

$$f(x,m) = f(x+a,\ m+a) \qquad \text{for all } a$$

$$F(x,m) = F(x+a,\ m+a) \qquad \text{for all } a$$

This implies the following relationships among partial derivatives (note that the *f* function is still defined as the derivative of *F* with respect to *x*, so $f(x,m) = F_x(x,m)$):

$$F_x(x,m) = -F_m(x,m) \tag{26}$$

$$F_{xx}(x,m) = -F_{mx}(x,m) = F_{mm}(x,m) \tag{27}$$

These simply say that, for any distribution, a shift of *x* in one direction produces the same effects as an equal shift of *m* in the other direction. Finally, since $F_x(x,m)$ is a probability density function in *x* that is single-peaked at *m*, we know that:

$$F_x(x,m) \geq 0 \qquad \text{for all } x \tag{28}$$

$$F_{xx}(x,m) \gtrless 0 \qquad \text{for } x \lessgtr m \tag{29}$$

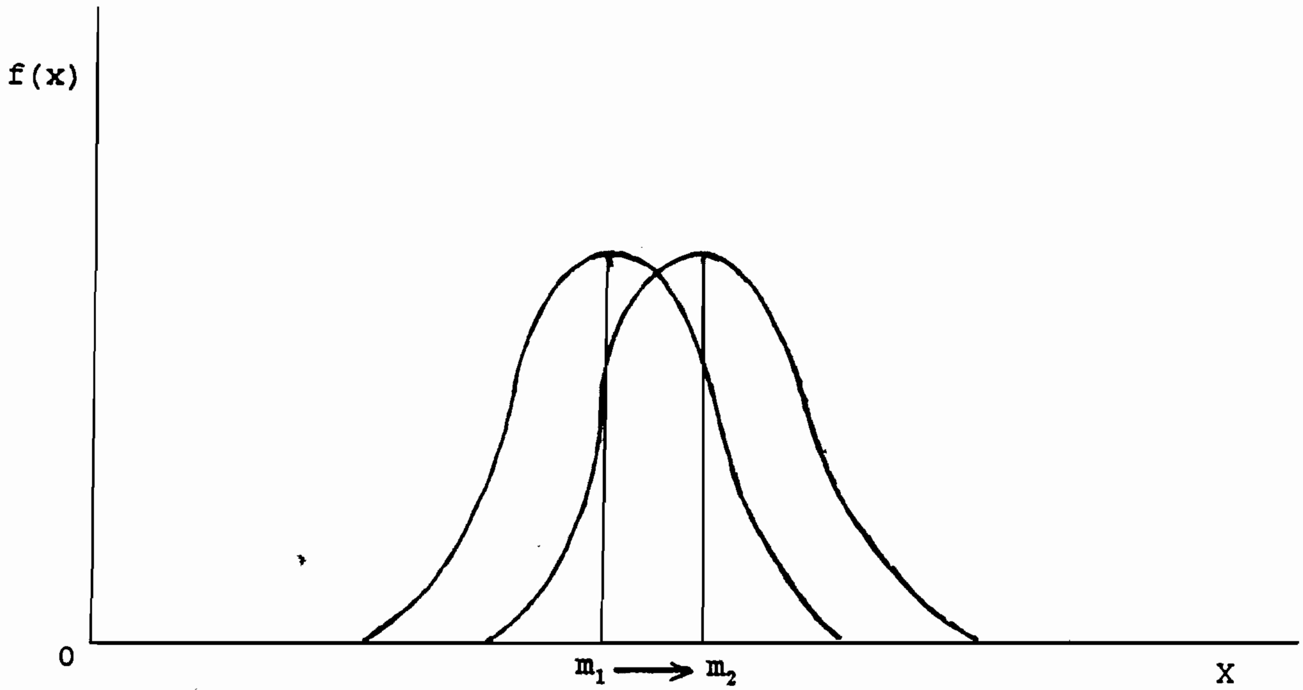
From Equations (26) and (27), we know that the inequalities would be reversed for F_m and F_{xm}, respectively.

[INSERT FIGURE 4 ABOUT HERE]

The general results derived in Sections 4 and 5 hold regardless of where the nominal legal standard is located, as we made no assumptions about the shape of the f function in those sections. For example, all of the results derived there hold even if the nominal standard is set at the optimal level of behavior x^*, and even if the chance of error is distributed in an unbiased way on either side of x^*. Indeed, Appendix A presents a plausible case where overcompliance is quite likely even though the distribution of possible legal standards is perfectly symmetric and is centered at x^*. This contradicts the apparent conclusions of earlier writers analyzing the effects of legal errors in a less rigorous way.[19]

However, it remains true that the actual location of the nominal legal standard will have an important effect on the level of over- or undercompliance. Reference to Equations (4) and (8) will show that the defendant's compliance decision, in addition to depending on the damage rule, also depends on the values of $F(x^*)$ and $F'(x^*)$ -- which, in turn, will depend on the

Figure 4



location of the nominal standard. Intuitively, one would suspect that relaxing the nominal standard would always increase the tendency toward undercompliance, while making it more severe would increase the incentive to overcomply.

This intuition is easy to confirm in the case of defendants operating under an incremental damage rule. If Equation (8) is rewritten using the notation of this section, and set equal to zero, it defines the level of $\underline{x}$ that maximizes the defendant's expected private benefits as an implicit function of $\underline{m}$:

$$\partial P/\partial x = B'(x) - F(x,m)L'(x) = 0 \tag{30}$$

Implicit differentiation then yield the following:

$$\left[\frac{B''(x) - F_x(x,m)L'(x) - F(x,m)L''(x)}{F_m(x,m)L'(x)}\right]\frac{\partial x}{\partial m} = 1 \tag{31}$$

The fact that $\underline{F}_x=-\underline{F}_m$ lets us reduce this as follows:

$$\left[\frac{B''(x) - F(x,m)L''(x)}{F_m(x,m)L'(x)} + 1\right]\frac{\partial x}{\partial m} = 1 \tag{32}$$

As the fraction on the left is positive for all values of $\underline{x}$ and $\underline{m}$, this implies that $0 < \partial\underline{x}/\partial\underline{m} < 1$. In other words, changes in the nominal standard $\underline{m}$ will shift the defendant's choice of $\underline{x}$ in the same direction, though generally by a lesser amount.

The relation is slightly more complex under a full damage rule. The analogous implicit function, derived from Equation (4), is the following:

$$\partial P/\partial x - B'(x) - F(x,m)L'(x) - F_x(x,m)L(x) = 0 \qquad (33)$$

The same process of implicit differentiation and canceling out of equivalent but opposite-signed terms yields the following characterization of $\partial \underline{x}/\partial \underline{m}$:

$$\left[\frac{B''(x)-F(x,m)L''(x)-F_x(x,m)L'(x)}{F_m(x,m)L'(x) + F_{mx}(x,m)L(x)} + 1\right]\frac{\partial x}{\partial m} = 1 \qquad (34)$$

This yields the same conclusion as before -- i.e., $0 < \partial \underline{x}/\partial \underline{m} < 1$ -- whenever $\underline{x} \leq \underline{m}$. That is, whenever defendants have "overcomplied" by choosing a value of $\underline{x}$ even lower than that required by the nominal standard, increases in the nominal standard should always correct this by leading defendants to increase their value of $\underline{x}$. However, when defendants have "undercomplied" with a level of $\underline{x}$ above that permitted by the nominal standard, $\partial \underline{x}/\partial \underline{m}$ will only be positive if the $\underline{f}$ distriubtion does not fall too steeply at values of $\underline{x}$ above the point at which it peaks. A sufficient condition is:

$$-f_x(x,m) \leq f(x,m)[L'(x)/L(x)] \qquad (35)$$

As long as this condition holds for all $\underline{x}$ and $\underline{m}$, the fraction on the left in Equation (34) will always be positive.

The slope of the density function matters because, under a full damage rule, the incentive to undercomply is at least partially checked by the fact that reductions in x (i.e., increased compliance) reduce the chance that defendants will have to pay any of the damages. If the f distribution is very steep, though, a slight shift in that distribution could significantly reduce this check, by significantly reducing the value of f at x (which is also the value of F_x). Thus, if the resulting reduction in the incentive to overcomply is sufficiently large, then a slight tightening of the nominal legal standard could nonetheless have the paradoxical effect of increasing the defendants' undercompliance. The condition in Equation (35) eliminates this possibility.

If this condition is satisfied, though, there will always be the potential at least to reduce any over- or undercompliance problems by raising or lowering the nominal legal standard. In fact, if the f distribution is truncated (so that $f(x,m)=0$ for values of x sufficiently far from m), then in theory there will always be a value of m sufficiently high or low to induce exactly the optimal level of compliance. Overcompliance can be corrected because defendants will always increase their value of x at least as far as the point at which some liability begins to attach, so sufficiently large increases in m can increase defendants' chosen value of x without any limit. Undercompliance can be corrected by choosing so low a value of m

that liability becomes 100% certain over the entire relevant range, so that defendants choose the optimal value of $\underline{x}$ just as they would under strict liability with no uncertainty.

In any real institutional setting, though, such extreme shifts in $\underline{m}$ may not be possible (at least, not without simultaneously changing the shape of the distribution). Some variables have natural limits -- e.g., a standard defining the acceptable risk of an accident must always lie between zero and one -- and as the nominal standard approaches either limit the distribution of errors on one side of that standard must inevitably be compressed. More generally, very little is known about how changes in nominal legal standards (which are often operationalized through the instructions given to a jury) actually affect the resulting probability density function. Thus, it may be premature to say that it will always be possible to completely eliminate any over- or undercompliance problems simply by adjusting the nominal legal standard.

However, our qualitative conclusion seems likely to remain valid: Any shift in the nominal legal standard (in the appropriate direction) should at least reduce the extent of over- or undercompliance. If this is so, then one traditional recommendation of the law-and-economics literature -- that negligence or other legal rules should be defined in terms of the optimal level of behavior $\underline{x}^*$ -- will not necessarily hold when the legal standard is uncertain. Setting the nominal standard at $\underline{x}^*$ could just as easily induce over- or undercompliance. A better course might then be to set the nominal standard somewhat above or below *x**, in order to counteract those tendencies.

More generally, we conclude that incentives to over-comply or undercomply can be corrected *either* by modifying the damage rules (as discussed in Section 5) or by modifying the nominal legal standard (as discussed here). Both sorts of corrections involve administrative difficulties (e.g., how large should the correction be?), and the choice between the two may well turn on the ease of implementing either correction within existing legal institutions. This is much more of an empirical question, raising a host of administrative issues that lie beyond the limits of this paper.

7. Bilateral Accidents

As a final issue, we examine the effect of uncertain legal standards when both the injurer's and the victim's behavior affect social costs. We will return to the notation of Sections 2 through 5, and suppress the nominal legal standard m as one argument of the f and F functions. However, the following notation will be added in this section:

y	Some behavioral variable controlled by potential victims ($y>0$).
A(y)	The victim's expected benefits (or costs saved) at each level of y, analogous to $B(x)$ ($A'(y)>0$, $A''(y)<0$).

$L(x,y)$ Expected costs inflicted on the victim, as a function of both $\underline{x}$ and $\underline{y}$. We assume $\underline{L}_x$ and $\underline{L}_y > 0$, $\underline{L}_{xx}$ and $\underline{L}_{yy} > 0$, and $\underline{L}_{xy} < 0$ (indicating that cost-reducing behavior by one party can substitute for cost-reducing behavior by the other).

Following Brown (1973) and Assaf (1984), we make the important assumption that both the injurers and victims take each other's level of behavior as given.

Under these assumptions, the socially optimal levels of $\underline{x}$ (given $\bar{\underline{y}}$) and $\underline{y}$ (given $\bar{\underline{x}}$) are defined by the following equations:

$$A'(y^*) = L_y(\bar{x}, y^*) \tag{36a}$$

$$B'(x^*) = L_x(x^*, \bar{y}) \tag{36b}$$

The first-best optimum is realized when both victims and defendants choose $\underline{x}^*$ and $\underline{y}^*$ as their respective levels of behavior.

Until now, we have not had to specify whether any damages assessed against the injurer were paid to the victim or to the state. If the payments do go to the victim (as is often the case under common law), this creates a moral hazard problem that may lead victims to choose too high a value of $\underline{y}$. Under a full damage rule, for example,[20] if there is no legal rule constraining the victim's behavior then the victim will have to bear the losses if and only if the injurer is not held liable. As the probability that the injurer will be held liable is still given by $\underline{F}(\underline{x})$,[21] the victim's expected private benefits, $\underline{V}(\underline{y})$, are as follows:

$$V(y) = A(y) - [1-F(\bar{x})]\ L(\bar{x},y) \tag{37}$$

These will be maximized when the following condition is satisfied:

$$dV/dy = A'(y) - [1-F(\bar{x})]\ L_y(\bar{x},y) = 0 \tag{38}$$

A comparison with Equation (36a) shows that as long as $\underline{F}(\bar{\underline{x}})$ is greater than zero, $\underline{dV}/\underline{dy}$ will be positive at $\underline{y}$*, implying that victims will have an incentive to choose a higher value of $\underline{y}$.

As Shavell (1980, p. 18) and others have noted, if there were no uncertainty in the legal standard applied to injurers then this distortion of victims' incentives would disappear. As discussed in Section 3, if there is no uncertainty then defendants will always exactly comply with a legal standard set at $\underline{x}$*, and will therefore never have to pay damages. Victims, knowing that they will then have to bear all resulting costs, should therefore adjust their own behavior optimally, as Equation (38) itself indicates when $\underline{F}(\bar{\underline{x}})$ is set equal to zero. Thus, it is only under conditions of uncertainty about defendants' liability that there is any need to introduce legal rules governing victims' behavior.[22]

Perhaps because legal standards obviously are uncertain, though, the legal system often does apply legal constraints to victims' behavior as well as to injurers'. The most familiar of these is the contributory negligence rule, under which victims are compensated only if (a) the injurer is found to have violated the legal standard governing injurers' behavior, and (b) the victim is found not to have violated the legal standard governing victims' behavior. Using $\underline{g}(\underline{y})$ and $\underline{G}(\underline{y})$ to represent the density and cumulative distribution of legal standards applied to victims (analogous to $\underline{f}(\underline{x})$ and $\underline{F}(\underline{x})$), and assuming that the two distributions are independent, a victim's expected benefits under this rule must be expressed as follows:

$$V(y) = A(y) - [1 - F(\bar{x})(1-G(y))]\, L(\bar{x},y) \tag{39}$$

Victims can maximize these benefits by satisfying the following condition:

$$dV/dy = A'(y) - [1-F(\bar{x})(1-G(y))]L_y(\bar{x},y) - F(\bar{x})G'(y)L(\bar{x},y) = 0 \tag{40}$$

Substituting for $\underline{A}'(\underline{y}^*) = \underline{L}_y(\underline{\bar{x}},\underline{y}^*)$ from Equation (16b), and rearranging the terms, shows that defendants will now have an incentive to choose too high or too low a level of $\underline{y}$ depending on whether the following condition is positive or negative:

$$dV/dy|_{y*} = \left\{[1-G(y*)]L_y(\bar{x},y*) - G'(y*)L(\bar{x},y*)\right\}F(\bar{x}) \qquad (41)$$

Apart from differences in notation, and the addition of the $\underline{F}(\bar{x})$ multiplier (which does not affect the sign), this is the exact equivalent of Equation (5), which described whether injurers would over- or undercomply under a simple negligence standard. In short, the same conditions determine whether victims are likely to over- or undercomply under a contributory negligence standard. As a result, the corrective techniques discussed in Sections 5 and 6 are also available to correct the effects of the contributory negligence standard -- i.e., adjusting the damage awards, or changing the nominal legal standard. Notice, though, that any change in the damage award would affect injurer's incentives in exactly the opposite direction, so it may not be possible to use this technique to optimize both parties' incentives.

It only remains to consider the effect of a contributory negligence defense on the injurer's incentive. With such a defense, the injurer must pay only if he violates his legal standard <u>and</u> the victim does not violate the standard applied to victims. The injurer's expected private benefits are therefore the following:

$$P(x) = B(x) - F(x)[1-G(\bar{y})]L(x,\bar{y}) \qquad (42)$$

Differentiating, and substituting for $\underline{B}'(\underline{x}^*)=\underline{L}_x(\underline{x}^*,\bar{\underline{y}})$, shows that injurers will have an incentive either to over- or under-comply depending on whether the following expression is negative or positive when evaluated at $\underline{x}^*$:

$$dP/dx|_{x*} = [1-F(x)+G(\bar{y})]L(x,\bar{y}) - [1-G(\bar{y})]F'(x)L_x(x,\bar{y}) \quad (43)$$

A comparison with Equation (5), the analogous condition without any contributory negligence defense, shows that the introduction of $\underline{G}(\bar{\underline{y}})>0$ increases the incentives favoring undercompliance. This should not be surprising: From the injurer's standpoint, the contributory negligence defense is simply an exogenous factor that increases the probability that the injurer will not "get caught" and will not have to pay damages. Since it reduces that probability in a way that is not affected by the injurer's own level of behavior, it does not give rise to any counteracting incentive to overcomply.

However, this complicates the problem of optimally correcting both parties' incentives. While every change in $\underline{G}(\underline{y})$ (to correct victims' incentives) will also affect injurers' incentives as well, the same is not true of changes in $\underline{F}(\underline{x})$ (to correct any remaining distortion of injurers' incentives). As Equation (41) indicates, $\underline{F}(\underline{x})$ drops out as a factor affecting victim's incentives once their incentives are otherwise optimally adjusted (i.e., if the large bracketed factor in Equation (41) is made to equal zero). Thus, apart from the practical difficulties noted in Section 6, it should at least be theoretically possible to optimize the victims' incentives by adjusting the legal standard that determines $\underline{G}(\underline{y})$, and then to optimize injurers' incentives by adjusting the standard determining $\underline{F}(\underline{x})$. As a practical matter, though, the administrative difficulties of making such simultaneous adjustments should be obvious.

8. Conclusions

One purpose of this analysis has been to demonstrate formally that which has long been known to practicing lawyers -- i.e., that uncertainty about the legal standard does indeed make a difference. Propositions that seemed easy to prove when the legal standard was clear turn out not to be robust with respect to the introduction of legal uncertainty. For example, it is simply not true (when legal rules are uncertain) that negligence standards should necessarily be set at the socially optimal level of care, that equal chances of error in either direction will have no net effect on defendants' incentives, or that penalties should generally be increased by a factor reflecting the probability of not getting caught.

Our analysis thus joins a growing body of work assessing the effects of legal uncertainty in other contexts. For example, the "suit vs. settlement" literature discusses litigation strategy when neither party is certain about the outcome of a trial,[23] and recent work has begun addressing the effects of these strategies on parties' prior compliance decisions regarding behavior that may give rise to litigation.[24] Uncertain legal rules have also been invoked as one factor affecting the evolution of common law precedents, although to date these models have not taken account of the effects of this uncertainty on the behavior governed by the precedents.[25]

Other forms of uncertainty bear even more closely on the general model presented here. For example, Diamond (1974a,b) modeled a system in which the legal standard was certain, but there were random errors in courts' measurement of defendants' chosen value of $\underline{x}$ (or the legal standard was defined in terms of an outcome variable that was only stochastically related to the defendants' choice variable $\underline{x}$). This produces a distribution centered around the defendants' choice of $\underline{x}$, rather than around the nominal legal standard (as in our model), but the marginal effects of a change in $\underline{x}$ on the likelihood of liability are the same in either case. Golding (1982) addressed similar uncertainties in the context of multiple tortfeasors, with results very similar to ours.[26] More recently, Polinsky (1984) addressed the effect on compliance decisions of uncertainty about the amount of damages that will be awarded.

Thus, there is obviously much more to be learned in this area, not only about the theoretical effects of different kinds of uncertainty, but also about the links between these various uncertainties and real-world legal institutions. What seems certain is that these inquiries are worth pursuing.

APPENDIX A

In this Appendix we assume that the density function of possible legal standards is symmetric, single-peaked and centered at the optimal level of care x^*. Even under these conditions, with an equal chance of an "error" in either direction, the uncertainty may still lead defendants to over- or undercomply.

We show this first for a specific form of the $L(x)$ function. Assume that x represents the probability of an accident, and all accidents cause L dollars of damage, so that $L(x) = xL$. Under a full damage rule, defendants who are found negligent must pay the cost of the accident (L). The condition for overcompliance under the full damage rule (Equation (5) of the text) can then be reduced to the following:

$$[1-F(x^*)] - x^*F'(x^*) \gtrless 0$$

If the left side is less than zero, defendants will overcomply; if it is greater than zero, they will undercomply.

Figure A1 shows that the left side will be negative (indicating overcompliance) whenever x^* is sufficiently large. The cross-hatched rectangle has a base equal to x^* and a height equal to $F'(x^*)$; its area thus equals $x^*F'(x^*)$. An area equal to $1-F(x^*)$ (the chance that the defendant will not be found liable) is represented by the shaded area under the curve to the right of $\underline{x}^*$. Thus, the condition for overcompliance will always be satisfied when the cross-hatched rectangle is greater than the shaded area. Since the cross-hatched rectangle is greater than the area under the curve to the left of $\underline{x}^*$, it must also be greater than the area under the curve to the right of $\underline{x}^*$, as we assumed that the curve was symmetric about $\underline{x}^*$.

[INSERT FIGURES A1 AND A2 ABOUT HERE]

The cross-hatched rectangle could only be less than half of the area under the curve (implying that defendants should undercomply) when $\underline{x}^*$ was very close to zero, as shown in Figure A2. In other words, undercompliance is most likely when the optimal level of care involves reducing the risk of an accident almost all the way to zero, so that the expected accident costs are small at the optimal level of care.

This can be generalized to other forms of the $\underline{L}(\underline{x})$ function. Inspection of Equation (4) of the text shows that a sufficiently large value of $\underline{L}(\underline{x}^*)$ will always induce overcompliance (all else equal), while a sufficiently small value will always induce undercompliance. Thus, even when the chance of an "incorrect" legal standard is completely unbiased, defendants may still over- or undercomply.

Figure A1

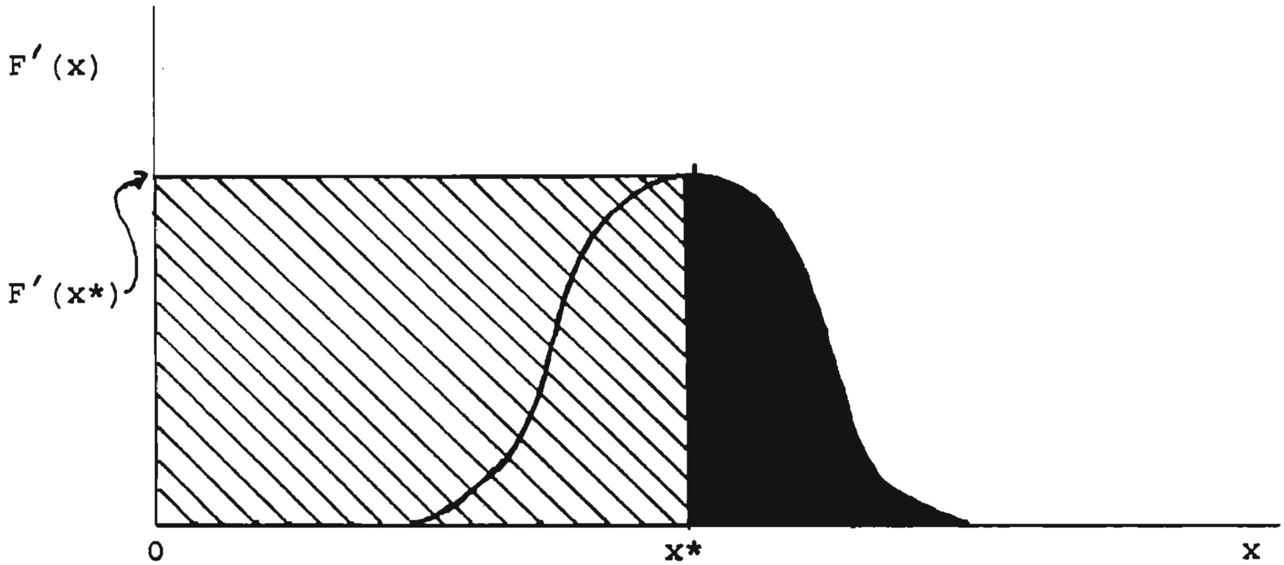
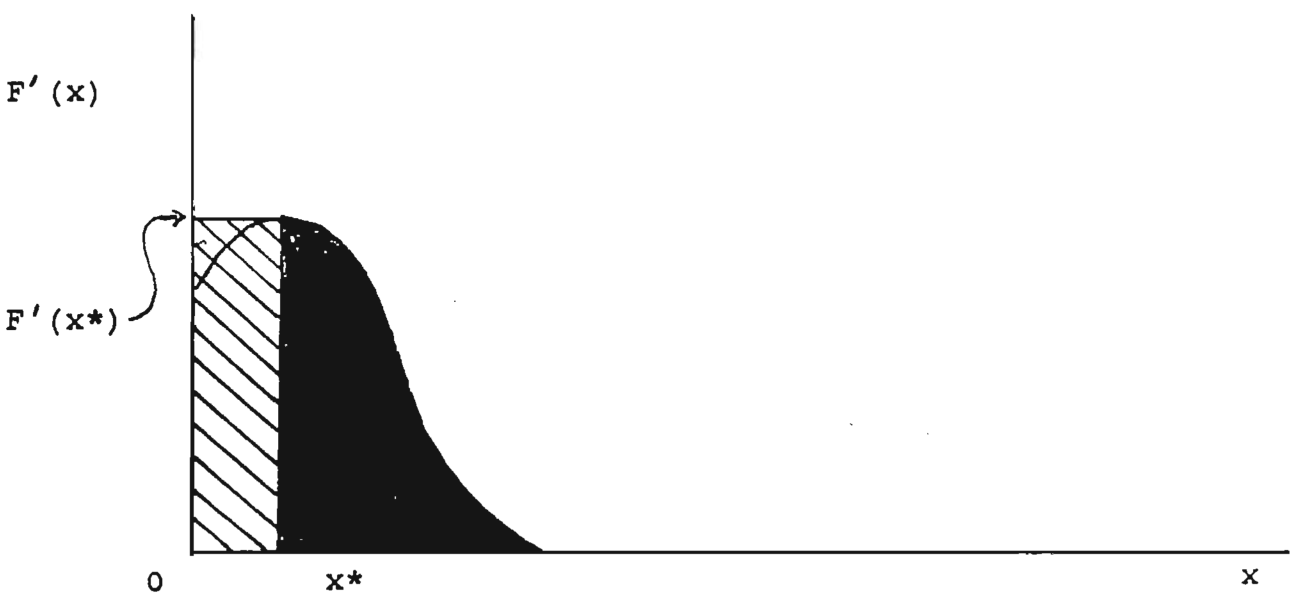


Figure A2



APPENDIX B

In this Appendix, we show that risk-aversion on the part of defendants strengthens their incentives to overcomply under a full damage rule. Intuitively, this result should be expected, as risk-averse defendants should be willing to pay even more (in certainty-equivalent terms) than risk-neutral defendants would in order to reduce by any given increment the chance of being held liable. Greater expenditures on care -- i.e., increased levels of compliance -- are one way to do this.

Formally, if defendants are risk-averse than we must give them an explicit utility function $U(w)$, with $U'(w)>0$ and $U''(w)<0$. We assume either that society is risk-neutral with respect to accident losses, or that the $L(x)$ function has been defined in a way that takes society's (or victims') risk-aversion into account. The socially optimal level of care (x^*) is the level that maximizes the net social benefits, or $U[B(x)] - L(x)$. This is defined by the following first-order condition:

$$U'[B(x^*)]B'(x^*) - L'(x^*) = 0$$

As the defendant will have to pay $L(x)$ only if he is found liable, his expected private benefits are:

$$P(x) = U[B(x)] - F(x)U[L(x)]$$

Differentiating with respect to $\underline{x}$, and evaluating the result at $\underline{x}^*$, shows that defendants' incentives will be to overcomply if the following expression is negative and to undercomply if it is positive:

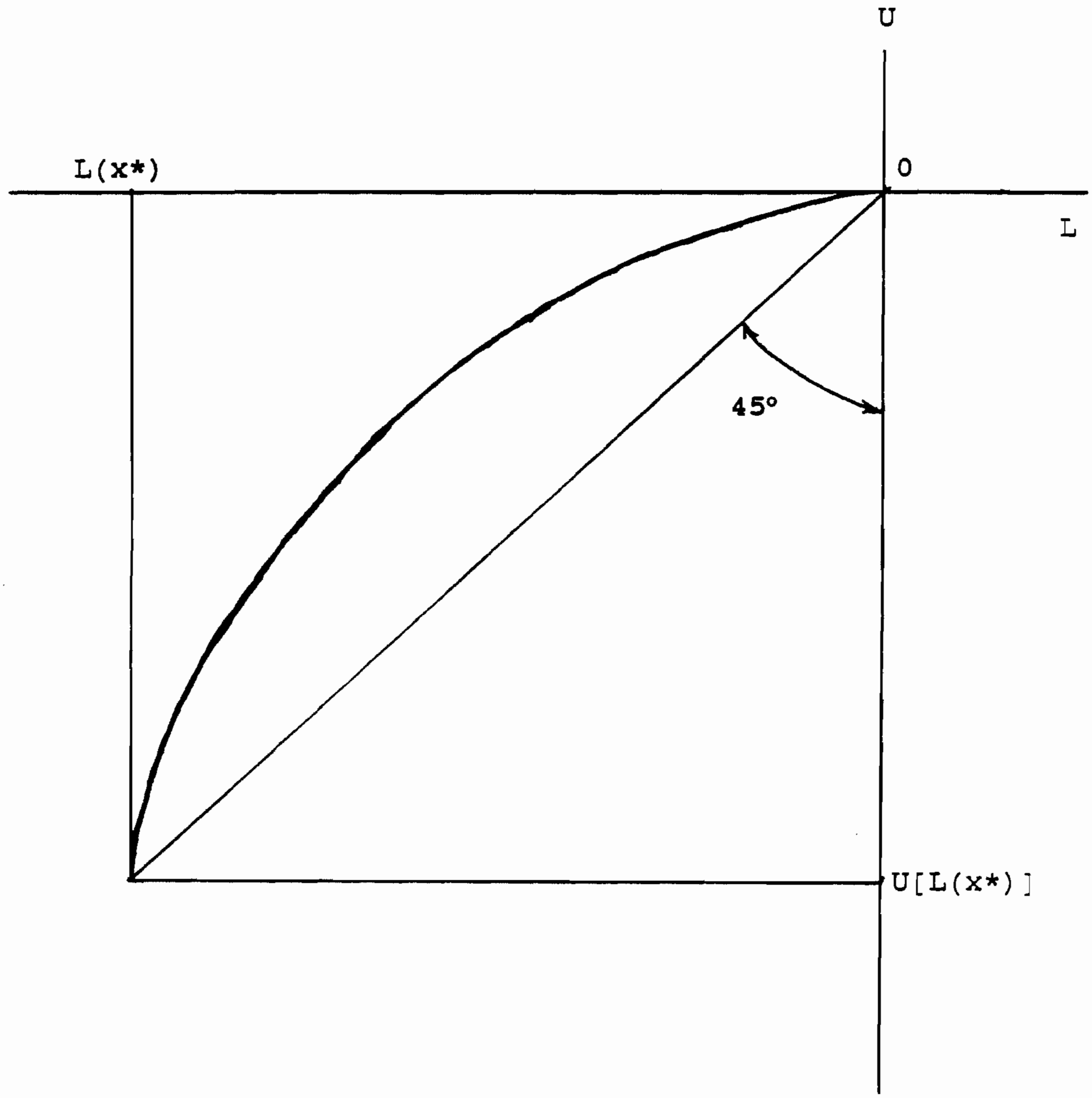
$$dP/dx|_{x}* = [1-U'[L(x^*)]F(x^*)]L'(x^*) - F'(x^*)U[L(x^*)]$$

This is identical to Equation (5) in the text (the analogous condition for risk-neutral defendants), except for the introduction of a $\underline{U}'[\underline{L}(\underline{x}^*)]$ factor in the first term, and the substitution of $\underline{U}[\underline{L}(\underline{x}^*)]$ for $\underline{L}(\underline{x}^*)$ in the second term.

Without loss of generality, we can define the $\underline{U}(\underline{w})$ function at any two values of $\underline{w}$, as this only works a linear transformation of the utility function. Thus, define $\underline{U}(\underline{w})$ so that $\underline{U}[\underline{L}(\underline{x}^*)] = \underline{L}(\underline{x}^*)$, and $\underline{U}[o] = 0$. The first of these makes the second term of $\underline{dP}/\underline{dx}$ identical to the second term of Equation (5). The second guarantees that the slope of the $\underline{U}$ function at $\underline{L}(\underline{x}^*)$ will be greater than one, as the $\underline{U}$ function must be concave downwards for risk-averse defendants (see Figure B1). Simple inspection shows that the $\underline{U}'[\underline{L}(\underline{x}^*)]$ factor makes the first term of $\underline{dP}/\underline{dx}$ smaller when $\underline{U}'[\underline{L}(\underline{x}^*)]>1$. Thus, risk-aversion reduces the value of the entire expression, thereby strengthening the incentive to overcomply or weakening the incentive to uncercomply.

[INSERT FIGURE B1 ABOUT HERE]

Figure B1



FOOTNOTES

1. The assumptions about the second derivatives are added only to satisfy various second-order conditions. In general, they are sufficient but not necessary assumptions.

2. Other second-best considerations are discussed in Polinsky (1980) and Polinsky & Rogerson (1983). The case where social costs are affected by the _victim's_ behavior (a variable which often _is_ controlled by the legal system) is discussed in Section 7 below.

3. We refer to "fines" and "damage payments" interchangeably. From the defendant's point of view, it is irrelevant whether these payments are made to the victims who suffered as a result of his behavior, or to the public treasury. (The effect on the victim's behavior will be taken up in Section 7.)

4. The legal doctrines surrounding causation are a good deal more complex than this brief description indicates. For some interesting discussions of these issues as they arise in tort law, see Shavell (1980b, 1983), Landes & Posner (1983), and Grady (1984). See also Section 5.4, below.

5. That $\underline{x}^*$ maximizes defendants' profits under an

optimally designed full damage rule has long been recognized in the literature -- see, e.g., Brown (1973).

6. The second-order condition is:

$$B''(x) - F(x)L''(x) - 2f(x)L'(x) - f'(x)L(x) < 0$$

This will be satisfied as long as $f'(x)$ never takes on a large negative value, as the sum of the first three terms is unambiguously negative. If the density function is single-peaked, this condition is equivalent to assuming that it does not fall away extremely rapidly at values of $\underline{x}$ above the value at which it peaks.

7. Those models also typically assumed that defendants chose from only two discrete options (e.g., to murder or not to murder), so that overcompliance was meaningless. Stigler (1970) relaxed this assumption, but limited his analysis to crimes like theft which could vary in egregiousness but were <u>always</u> undesirable, so that overcompliance was never a concern.

8. The second-order condition here is:

$$B''(x) - F'(x)L'(x) - F(x)L''(x) < 0$$

Given our assumptions abut $\underline{B}''(\underline{x})$ and $\underline{L}''(\underline{x})$, this condition will always be satisfied.

9. As Becker (1968) notes, this principle dates back at least as far as Jeremy Bentham's observation: "The more deficient in certainty a punishment is, the severer it should be."

10. Polinsky & Shavell (forthcoming) note that $1/\underline{F}(\underline{x}^*)$ is also not the appropriate multiplier if administrative costs can be saved by reducing the probability of punishment (e.g., by cutting back on enforcement resources). What we show is that $1/\underline{F}(\underline{x}^*)$ may not be the appropriate multiplier even if $\underline{F}(\underline{x}^*)$ is fixed, or even if changes in the probability of punishment are reflected by changes in the legal rules (as discussed in Section 6) that do not produce any administrative savings.

11. Equation (21) also generalizes P'ng's (1983a) result that, if there is a positive probability that a defendant who chooses not to murder will nonetheless be punished, the denominator of the traditional multiplier should be the <u>difference</u> between this probability and the probability that a guilty defendant will be punished.

12. This multiplier is actually one of a family of optimal multipliers of the form $\underline{A}/\underline{F}(\underline{x})\underline{L}(\underline{x}) + \underline{B}/\underline{F}(\underline{x})$, where $\underline{A}$ and $\underline{B}$ are arbitrary constants. (The example in the text sets $\underline{A}=0$

and $\underline{B}$=1.) This can be derived by rewriting Equation (3) as $\underline{P}(\underline{x}) = \underline{B}(\underline{x}) - \underline{F}(\underline{x})\underline{M}(\underline{x})\underline{L}(\underline{x})$ (where $\underline{M}(\underline{x})$ is the multiplier), then setting $\underline{dP}/\underline{dx}$ equal to zero when evaluated at $\underline{x}^*$, and solving the resulting differential equation in $\underline{M}(\underline{x})$.

13. Changes in the damage rule would, however, affect the second-best issues discussed earlier at note 2.

14. Wittman (1984) explores these issues in more detail.

15. Golding (1982) analyzes in more detail the effect of various damage-sharing rules under conditions of uncertainty.

16. Posner (1973) and Ehrlich & Posner (1974) discuss some of the factors affecting the level of uncertainty surrounding legal rules, and the likelihood of error in applying any particular rule.

17. This was the standard endorsed in United States v. Carroll Towing Co., 159 F.2d 169, 173 (2d Cir. 1947). See Brown (1973) for a discussion.

18. Changes in the legal system that might change the variance of the distribution, thus raising or lowering the absolute level of uncertainty, are discussed in Calfee &

Craswell (1984).

19. See, e.g., Posner (1977, pp. 430-32), suggesting that errors in setting the legal standard will affect defendants' incentives only when those errors are biased.

20. The results are qualitatively similar (i.e., victims will still choose too high a value of $\underline{y}$) under an incremental damage rule.

21. The victim's care may also affect the standard of care to which the defendant is held. That is, in the notation of Section 6, the probability of the defendant being held liable might be better expressed as the function $\underline{F}(\underline{x},\underline{m}(\underline{y}))$. However, as we assume that each party takes the other's behavior as given, we can suppress the second argument of this $\underline{F}$ function. This assumption is closely related to the assumption we make below, that the $\underline{F}$ and the $\underline{G}$ distributions are independent.

22. A contributory negligence defense is of course much more important under a rule of strict liability, where (absent contributory negligence) the defendant must pay the full costs $\underline{L}(\underline{x})$ regardless of his choice of $\underline{x}$. However, in such a system the victim's incentives are exactly analogous to the defendant's incentives under a simple negligence standard -- that is, the victim will have to bear the accident costs if (and only if) his

behavior fails to comply with some legal standard. Thus, our analysis of defendants' incentives to comply under a simple negligence standard applies with equal force to victims' incentives under a strict-liability-cum-contributory-negligence regime.

23. For a recent discussion, see Shavell (1983).

24. E.g., Ordover (1981), Simon (1981), P'ng (1983b).

25. E.g., Priest (1977, 1980), Rubin (1977). Uncertainty and probability theory have also been used to model rules of evidence (see Kaye (1979) for a survey) -- but, again, without considering the effects of that uncertainty on defendants' incentives to comply with the underlying legal rules.

26. In addition, Cooter (1982, pp. 100-01) sketches a model very similar to that used here to analyze the use of punitive damages when defendants differ in their cost and benefit functions. Goetz (1984, pp. 299-302) and Grady (1984, pp. 403-09) discuss uncertain negligence standards in a less technical way.

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