Who Benefits From Online Privacy?

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Abstract

When a firm is able to recognize its previous customers, it may use information about their purchase histories to price discriminate. We analyze a model with a monopolist and a continuum of heterogeneous consumers, where consumers are able to maintain their anonymity and avoid being identified as past customers, possibly at an (exogenous) cost. When consumers can costlessly maintain their anonymity, they all individually choose to do so, which paradoxically results in the highest profit for the firm. Increasing the cost of anonymity can benefit consumers, but only up to a point; at that point, the effect is reversed.

Keywords: Privacy, anonymity, price discrimination, electronic commerce

JEL Classifications: L1, D8

1 Introduction

Perhaps the most important factor contributing to concerns about personal privacy is the potential for discrimination. In an effort to avoid differential treatment, individuals are typically reluctant to disclose sensitive personal information such as income, family status, ethnicity, race, or lifestyle. In recent years, revolutionary developments in information technology regarding collection, storage, and retrieval of personal data have brought privacy to the forefront of public awareness and debate. This paper addresses a key component of the emergent concerns regarding electronic privacy, namely, the ability of firms to track individual purchasing patterns and to use this information to practice behavior-based price discrimination (Armstrong, 2006; Fudenberg and Villas-Boas, 2006).

Records containing the sequence of web sites visited and the online purchases made by individuals provide valuable clues about their personal information, clues that can be used to target tailor-made offers to them (Chen and Zhang, 2008; Wathieu, 2006; Pancras and Sudhir, 2007; Chen, 2006). Such behavior-based advertising and price discrimination are already ubiquitous in electronic commerce (Odlyzko, 2003; Hann et al., 2007). Nevertheless, the economic impact of these practices is not fully understood. Presently, privacy practices in electronic commerce are dictated largely by voluntary compliance with industry standards, recommendations by regulatory agencies, and consumer concerns (FTC, 2007).

Although technology has allowed sellers to store and process consumers' online activities with relative ease, consumers do have some control over allowing sellers to record their individual activities. For instance,

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they can exert effort to understand sellers' privacy disclosures and take actions to circumvent being tracked. Such actions can include erasing or blocking browser cookies, using a temporary email address, making payments using a gift card acquired for cash in a brick-and-mortar store, and renting a postal box.

The current set of guiding standards and recommendations (FTC, 2007) takes into account a large variety of concerns, but appears to have little basis in formal economic theory or empirical evidence. This paper provides a theoretical analysis of the economic impacts of privacy regulation, focusing specifically on consumer profiling and behavior-based price discrimination. We consider a monopolist who is able to track consumers' purchases from the firm. Consumers, however, are able to avoid being identified as past customers (or to "opt out"), possibly at a cost. We find that when consumers can costlessly maintain anonymity, they all individually choose to do so, which paradoxically results in the highest profit for the seller. We show that increasing the cost of anonymity can benefit consumers, but only up to a point; at that point, the effect is reversed.

The intuition for this somewhat startling finding is closely related to the celebrated Coase Conjecture (Coase, 1972) and runs as follows. When the cost of maintaining anonymity is high, the seller is better able to price discriminate against past customers. Thus, consumers hesitate to make an initial purchase, knowing this will cause them to pay a premium on future purchases. Anticipating this reluctance by consumers, the seller is forced to offer a lower initial price, and this effect actually dominates the increase in profits arising from price discrimination in future periods. In other words, the seller would prefer to commit itself not to price discriminate based on prior purchases. When the cost of maintaining anonymity is low, consumers – in effect – give the seller this commitment power when they each rationally choose to keep their purchases private.

This paper is also related to work in the literatures on intertemporal price discrimination, consumer recognition, and online privacy. Research on intertemporal price discrimination and the "ratchet" effect, where the firm sets higher prices for consumers who signaled higher willingness to pay, dates back to the late 1970's. Stokey (1979) and Salant (1989) show that intertemporal price discrimination is never optimal for a monopolist who can commit to future prices. This is analogous to the fact, mentioned above, that in our model, the monopolist obtains its highest profit when anonymity is costless. Villas-Boas (2004) shows that committing to future prices can also help in a model with overlapping generations of consumers. Skreta (2006) shows that without commitment, for a seller who is selling a single object under a finite time deadline, posting a price in each period is a revenue-maximizing allocation mechanism.

A relatively small literature on consumer recognition and online privacy has begun to develop over the past several years. Early contributions by Chen (1997), Fudenberg and Tirole (1998), Fudenberg and Tirole (2000), Villas-Boas (1999), Shaffer and Zhang (2000), Taylor (2003), and Chen and Zhang (2008) introduced the notion of consumer recognition and personalized pricing into economic theory, but did not explicitly consider privacy issues in online environments. Fudenberg and Tirole (1998) explore what happens when the ability to identify consumers varies across goods. They consider a model in which consumers can be anonymous or "semi-anonymous," depending on the good bought. Villas-Boas (1999) and Fudenberg and Tirole (2000) analyze a duopoly model in which consumers have a choice between remaining loyal to a firm and defecting to the competitor, a phenomenon they refer to as "consumer poaching." Chen and Zhang (2008) analyze a "price for information" strategy, where firms price less aggressively in order to learn more about their customers.[1]

Closest to our work is an emerging literature on optimal online privacy policies. These were first studied by Taylor (2004), Acquisti and Varian (2005), Hermalin and Katz (2006), and Calzolari and Pavan (2006). Fudenberg and Villas-Boas (2006) offer a survey of this literature. These papers provide important insights regarding the fundamental economic tensions between consumer privacy and price discrimination. This paper considers a richer environment, in which a firm's customers can choose to remain anonymous at some cost. We study how this cost affects equilibrium behavior and welfare, which has significant implications for privacy policy and regulation.

2 The model

2.1 The consumers

There is a continuum of consumers with total mass normalized to one. All consumers are risk-neutral, possess a common discount factor $\delta \in (0, 1]$, and maximize their present expected utilities. Each consumer demands at most one unit of a non-durable, indivisible good in each of two periods. Consumer i's valuation for the good is the same in each period and is determined by the realization of a random variable v_i with support normalized to be the unit interval. Consumer valuations are independently and identically distributed according to a cumulative distribution function $F(v)$ with density $f(v)$, which is strictly positive on $[0, 1]$. Consumer i's valuation v_i is initially private information.

2.2 The firm

There is a monopolist that produces and sells the good in each period. The firm's production cost is normalized to zero, it possesses discount factor δ, and maximizes its discounted expected profit. It does not observe consumer valuations directly but maintains a database containing purchasing histories. Each consumer is either *anonymous* or *identifiable*. If a consumer is anonymous, then there is no record of her prior purchase; i.e., she is not in the database. If she is identifiable, then in the second period the firm knows the purchasing decision that she made in the first period. We emphasize that the firm has *no commitment power*, i.e., the firm is unable to set and commit to second-period prices in the first period. Because there is a continuum of

[1] For a general discussion of price discrimination, see Stole (2007). For an early review of the consumer switching literature, see Klemperer (1995). For an economic analysis of privacy with respect to lawful search and seizure, see Mialon and Mialon (2008).

consumers, each of them realizes that her first-period purchasing decision alone does not affect the prices charged by the firm in the next period.[2]

2.3 THE GAME

All aspects of the environment, including the distribution of valuations $F(v)$, are common knowledge. At the beginning of the game all consumers are anonymous. Hence, the firm offers the same first-period price p_1 to all of them. Next, each consumer decides whether to buy the good in the first period, $q_1^i = 1$, or not to buy it, $q_1^i = 0$. Consumers who elect to buy the good also decide whether to let the firm keep a record of the transaction ($r^i = 1$) or to *opt out* and maintain anonymity by deleting the record of the sale ($r^i = 0$). The cost to any consumer who opts out is $c \geq 0$,[3] and we without loss of generality assume that this cost is expended in the second period (i.e., it is discounted by δ). This cost represents the time, effort, and any monetary expense of maintaining anonymity. A consumer who does not purchase the good continues to be anonymous and is thus pooled with the buyers who opted out, from the firm's perspective. At the beginning of period two, the firm posts a price p_2^0 to the unidentified (anonymous) consumers and a price p_2^1 to the identified ones.[4] Consumers can buy the good only at the price offered to them, $q_2^i \in \{0, 1\}$; i.e., no arbitrage is possible. Hence, a consumer i with valuation v_i who purchases in both periods has (present discounted) utility $v_i - p_1 + \delta(v_i - p_2^1)$ if he does not opt out, and utility $v_i - p_1 + \delta(v_i - p_2^0 - c)$ if he does. Figure 1 summarizes the timeline of the game.

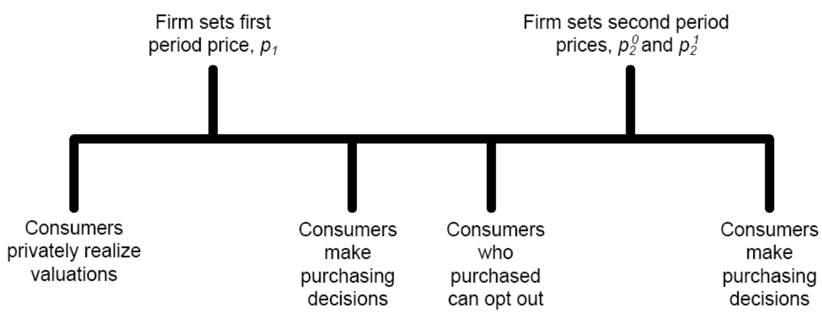


Figure 1: Timeline of the game.

The game we analyze is isomorphic to one between a firm and a single consumer whose type is dis-

[2]The results go through when there is a finite number of consumers, provided that we add the following assumption: the firm cannot update its beliefs over how many consumers opted out based on an inventory count. This could happen, for example, if the firm is unable to conduct an inventory count because inventory is controlled by a third party. Without this assumption, the firm could infer how many consumers opted out based on the inventory count. With a continuum of (massless) consumers, inventory expectations on the equilibrium path are confirmed even if a single agent deviates.

[3]The qualitative nature of the results still holds under certain conditions when the cost of maintaining anonymity varies across consumers, or when it is correlated with their valuations. In order to simplify the analysis, we assume that consumers incur the same cost of maintaining their anonymity. See Taylor (2004) for a model with correlated valuations, and Acquisti and Varian (2005) for a model with varying levels of consumer sophistication.

[4]If the firm sets second period prices *before* consumers decide on whether or not to opt out, then it can be shown that no consumer would opt out in equilibrium, for any $c > 0$, which is both not realistic and not interesting.

tributed according to F, with one additional assumption: in the second period, the firm cannot distinguish between the situation where the consumer did not purchase in the first period, and the situation where the consumer did purchase in the first period and opted out. (For example, the firm cannot check its inventory to find out if a purchase was made. This assumption is unnecessary when there is a continuum of consumers because the firm's behavior would not change if a single, massless consumer deviated.) Since the firm sets the first period price when it only knows the prior distribution F, the game is unaffected if consumers realize their types (according to the same F) after the firm sets p_1; it will be conceptually easier to think of p_1 being set first. This results in the extensive-form game in Figure 2.

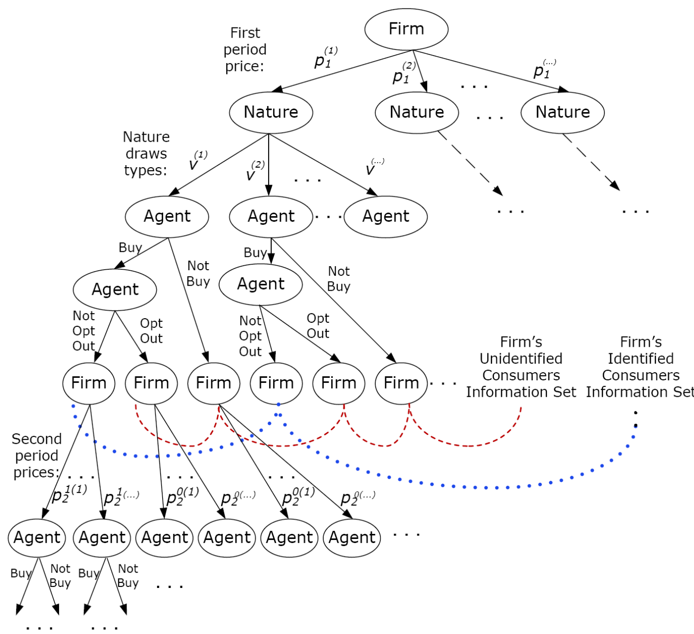


Figure 2: Sketch of the extensive form of the isomorphic game.

The solution concept we use is Perfect Bayesian Nash Equilibrium (PBE). A PBE here consists of the firm's strategy (composed of first-period price p_1 and second-period prices p_2^0 and p_2^1, corresponding to the firm's two information sets in the second period[5]); the consumer's strategy (composed of the first-period purchasing decision q_1 and opting-out decision $r \in \{0, 1\}$ as a function of p_1 and v, and second-period purchasing decision q_2 as a function of v and p_2^r.[6]); and the firm's beliefs about consumers' valuations given their identification status (F^1 and F^0 for identified and anonymous consumers, respectively[7]). These constitute a PBE if all strategies are sequentially rational given the beliefs and the beliefs are consistent given the strategies.

[5]Technically, the strategy should also specify what the firm would have charged in the second period if it had made a different pricing decision in the first period, but we omit this for notational simplicity.

[6]In principle, the second-period decision can also directly depend on the first-period price, but in equilibrium it will only depend on v and p_2^r.

[7]The firm will also have beliefs about what actions an anonymous agent took in the first period (did the agent purchase and opt out or not purchase at all), but this will not affect the analysis.

We assume F is thrice differentiable, and:

Assumption 1. ***(i)*** *$p(1-F(p))$ is strictly quasiconcave, with $p^\star = \arg\max_p p(1-F(p))$.*
(ii) *For $h(v) = F(v) + vf(v)$ and $h'(v) = 2f(v) + vf'(v)$, $h(v) + h'(v)$ is nondecreasing in v.*

Assumption 1 is satisfied, for example, by $F(v) = v^x$ for all $x > 0$.

3 BENCHMARKS

3.1 NO RECOGNITION

First, consider as a benchmark the case where there is no consumer recognition, so that the firm cannot price discriminate in the second period between consumers that bought and did not buy in the first period. Since the firm does not price discriminate based on purchasing history, consumers whose valuation exceeds the price would always buy the good. Thus, the firm sets the same price in each period, $p^\star = \arg\max_p p(1 - F(p))$, generating a per-period profit of $p^\star(1 - F(p^\star))$. Consumer surplus in each period is given by $\int_{p^\star}^1 (v - p^\star) dF(v)$. (We let $p^\star = \arg\max_p p(1 - F(p))$ throughout the paper.)

Example 1. When valuations are uniformly distributed, $p^\star = 0.5$. Equilibrium present-discounted profit is given by $\frac{1+\delta}{4}$, (present-discounted) consumer surplus is $\frac{1+\delta}{8}$, and social surplus is $\frac{3(1+\delta)}{8}$.

3.2 FULL RECOGNITION

Consider now the polar-opposite case, in which the firm is able to recognize its previous customers and consumers are unable to maintain their anonymity at any cost (as in Hart and Tirole (1988), Schmidt (1993), Villas-Boas (2004), Taylor (2004), and Fudenberg and Villas-Boas (2006)). In this setting, the firm can discriminate between two different groups of consumers in the second period: identified consumers who purchased in the first period, and unidentified consumers who did not. The firm consequently sets two different prices in the second period, p_2^1 to identified consumers and p_2^0 to unidentified consumers. (We emphasize again that the firm has no commitment power.)

Proposition 1 (Fudenberg and Villas-Boas 2006)**.** *In the full-recognition equilibrium, for some $\tilde{v}$,*
(i) Consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods; consumers with valuations $v \in [p_2^0, \tilde{v}]$ purchase only in the second period. The cutoff type $\tilde{v}$ satisfies $\tilde{v} = (1-\delta)^{-1}(p_1 - p_2^0)$ and $\tilde{v} \geq p^\star$.
(ii) The firm sets $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$ and p_2^0 to satisfy $F(p_2^0) + f(p_2^0)p_2^0 = F(\tilde{v})$. The firm's first-period price p_1 is determined from the first-order condition:

$$1 - F(\tilde{v}) - p_1 f(\tilde{v})\tilde{v}' + \delta\tilde{v}'(1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + f(\tilde{v})p_2^0) = 0$$

We first consider the monopolist's pricing strategy towards identified consumers in the second period. If the cutoff type for identified consumers (those who purchase in the first period) $\tilde{v}$ satisfies $\tilde{v} \geq p^\star$, then the monopolist sets $p_2^1 = \tilde{v}$. If, on the other hand, $\tilde{v} < p^\star$, the monopolist sets $p_2^1 = p^\star$. That is, $p_2^1 = \max\{\tilde{v}, p^\star\}$. From Proposition 1, since $\tilde{v} \geq p^\star$ holds on the path of play of the full-recognition equilibrium, $p_2^1 = \max\{\tilde{v}, p^\star\} = \tilde{v}$. Hence, the marginal consumer who buys in the first period—the one with valuation $\tilde{v}$—gets no surplus in the second period. This is the *ratchet effect* of consumers who reveal their types (Freixas et al., 1985; Laffont and Tirole, 1988). Paradoxically, the full-recognition case can result in higher consumer surplus than the no-recognition case, because the firm will need to set p_1 lower to attract consumers in the first period. Correspondingly, in the model with opting out, we will show that a low cost of opting out can lead to a Prisoner's Dilemma situation where consumers maintain anonymity but collectively suffer as a result, in comparison to the situation where anonymity is prohibitively costly and consumers face the ratchet effect.

By definition, a consumer with valuation $\tilde{v}$ is indifferent between purchasing in both periods and purchasing only in the second period. It follows that the indifference condition that characterizes $\tilde{v}$ is given by $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. Hence, $p_1 = \tilde{v} - \delta(\tilde{v} - p_2^0)$. Using $p_1 = \tilde{v} - \delta(\tilde{v} - p_2^0)$ and $p_2^1 = \tilde{v}$, one can simplify the firm's present discounted profit to obtain

$$\tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$$

For $\delta > 0$, since $p^\star$ uniquely maximizes $p(1 - F(p))$ and $\tilde{v} \geq p^\star$, we have

$$(1 + \delta)p^\star(1 - F(p^\star)) \geq \tilde{v}(1 - F(\tilde{v})) + \delta p_2^0(1 - F(p_2^0))$$

Thus, the firm's (expected) present-discounted profit under full recognition is lower than its profit under no recognition. The intuition is that some consumers refrain from purchasing in the first period because they anticipate a lower price in the next, and the firm is unable to fully recoup the loss in first-period profit by price discriminating in the second period (notably, the firm can always obtain the no-recognition profit in the second period by setting $p_2^0 = p_2^1 = p^\star$). Hart and Tirole (1988) and Fudenberg and Villas-Boas (2006) show that if the firm is able to commit to second-period prices in the first period, it would set $p_2^1 = p_2^0 = p^\star$, a result which our Proposition 5 below extends to the general model where consumers can opt out. Hence, the firm's profit under commitment coincides with its profit in the no-recognition equilibrium.

Example 2. When valuations are uniformly distributed, $\tilde{v} = \frac{2+\delta}{4+\delta}$ and $p_2^0(\tilde{v}) = \frac{\tilde{v}}{2}$. Hence, a fraction $\frac{2+\delta}{4+\delta}$ of consumers buys in both periods, while a fraction $\frac{2+\delta}{8+2\delta}$ of consumers only buys in the second period. The number of consumers buying in both periods decreases in the discount factor δ, as more consumers prefer to wait for future deals. Equilibrium present-discounted profit is given by $\frac{\delta}{4} + \frac{1}{4+\delta}$, lower than the no-recognition profit of $\frac{1+\delta}{4}$. Consumer surplus and social surplus are given by $\frac{1+\delta}{8}+\frac{3\delta}{8(4+\delta)}$ and by $\frac{3(1+\delta)}{8}+\frac{\delta}{32+8\delta}$, each greater than its no-recognition counterpart, $\frac{1+\delta}{8}$ and $\frac{3(1+\delta)}{8}$, respectively.

4 Opting out and partial recognition

We now consider the setting in which consumers who purchase in the first period can opt out and preserve anonymity at a cost of c. Consumers who purchase in the first period and do not opt out are identified by the firm in the second period (the firm recognizes that they purchased in period 1 at a price p_1) and will be offered price p_2^1 in period 2. All other consumers are offered p_2^0 in period 2. As above, let $\tilde{v}$ denote the lowest consumer type to purchase in the first period. We note that, given that a consumer with valuation $\tilde{v}$ prefers to buy in the first period, i.e., $\tilde{v}-p_1+\delta\max\{\tilde{v}-p_2^1,\tilde{v}-p_2^0-c,0\}\geq\delta\max\{\tilde{v}-p_2^0,0\}$, then all consumers with valuations $v\geq\tilde{v}$ do as well. Denote by $\alpha(v)$ the probability that a consumer of type $v\in[\tilde{v},1]$ maintains her anonymity after purchasing. Then the distribution of valuations among anonymous consumers is

$$F^0(v)=\begin{cases}\frac{F(v)}{F(\tilde{v})+\int_{\tilde{v}}^1\alpha(x)f(x)\,dx} & \text{if } v\leq\tilde{v}\\ \\ \frac{F(\tilde{v})+\int_{\tilde{v}}^v\alpha(x)f(x)\,dx}{F(\tilde{v})+\int_{\tilde{v}}^1\alpha(x)f(x)\,dx} & \text{if } v>\tilde{v}\end{cases}$$

and the distribution of valuations among identifiable consumers (for $v\geq\tilde{v}$) is given by

$$F^1(v)=\frac{\int_{\tilde{v}}^v(1-\alpha(x))f(x)\,dx}{\int_{\tilde{v}}^1(1-\alpha(x))f(x)\,dx}$$

4.1 Costless anonymity: equilibrium characterization

As a starting point, we first consider the case where $c=0$. Here, we show that the equilibrium is effectively unique and corresponds to the no-recognition benchmark.

Proposition 2. *When anonymity is costless ($c=0$), for $p^\star=\arg\max_p p(1-F(p))$, every*[8] *PBE satisfies (and a PBE exists that satisfies):*
(i) The firm sets $p_1=p_2^0=p^\star$ and $p_2^1\geq p^\star$.
(ii) Consumers with valuations $v\in[p^\star,1]$ purchase in both periods and opt out.
(iii) The no-recognition benchmark outcome is obtained.

Proof: For any p_1, since it is costless to opt out, all consumers with valuations $v\geq p_1$ will purchase the good in the first period. If $p_2^1<p_2^0$, no purchasing consumers will opt out. In this case, however, since the firm sets period 2 prices after consumers decide whether or not to maintain their anonymity, p_2^1 targets identified

[8] This is excluding the possibility of a PBE in which a measure zero subset of the consumers uses a different strategy.

consumers in $[\underline{v}, 1]$, where $\underline{v} \leq p_1$ (some range of additional consumers $[\underline{v}, p_1]$ will decide to purchase in the first period in order to obtain the discount in the second period). On the other hand, p_2^0 targets the anonymous consumers in $[0, \underline{v}]$ (and there will be at least some consumers in this interval, because f is positive on $[0, 1]$). Hence, setting $p_2^1 < p_2^0$ cannot be a best response for the firm. Thus, in every equilibrium, $p_2^1 \geq p_2^0$.

Since consumers anticipate that $p_2^1 \geq p_2^0$, it is a best response for consumers who purchased in the first period to opt out. We now show that there is a PBE where all of them use this best response; moreover, we characterize all the PBEs in which this is the case. If all of them opt out, then all consumers are anonymous in the second period, and the firm sets $p_2^0 = p^\star$ to maximize period 2 profit. Moreover, given that in the second period everyone will be anonymous, only consumers with valuations $v \in [p_1, 1]$ purchase in the first period, so that $\tilde{v} = p_1$. Hence, the firm's first-period problem is to choose p_1 to maximize $(1 - F(p_1))p_1 + \delta(1 - F(p^\star))p^\star$, which results in $p_1 = p^\star$. Since no consumer is identified in period 2, the firm's beliefs about identified consumers' valuations are off equilibrium. Consistent off-equilibrium beliefs here are, for example, for the firm to believe identified consumers' valuations to be at least $\tilde{v} = p_1 = p^\star$, so that setting $p_2^1 \geq p^\star$ is a best response.

Now, suppose, for the sake of contradiction, that there are equilibria in which some consumers do *not* opt out on the path of play. For this to be the case in equilibrium, since $p_2^1 \geq p_2^0$, we must have $p_2^1 = p_2^0$, otherwise no consumer would choose to stay identified. Let $p_2^1 = p_2^0 = \tilde{p}$, and assume that $\tilde{p} \neq p^\star$. Then in period 2, only consumers with valuations in $[\tilde{p}, 1]$ purchase, and the firm's period 2 profit is given by $\tilde{p}(1 - F(\tilde{p}))$. However, if the firm sets $p_2^1 = p_2^0 = p^\star$ in the second period, consumers with valuations in $[p^\star, 1]$ would buy, resulting in period 2 profit of $p^\star(1 - F(p^\star))$ — which is strictly higher since $p^\star$ uniquely maximizes $p(1 - F(p))$. Hence, $p_2^1 = p_2^0 = p^\star$ holds in this case, and the firm's period 2 profit is given by $p^\star(1 - F(p^\star))$ for any p_1. Thus, it is profit maximizing for the firm to set $p_1 = p_2^0 = p_2^1 = p^\star$.

Assume now that some positive mass of consumers who purchased in period 1 stays identified. Let G denote the distribution of anonymous consumers in period 2, so that $\int_0^1 dG(p) < 1$. $G(p)$ coincides with $F(p)$ up to $p = p^\star$ because identified consumers can only be in $[p_1, 1] = [p^\star, 1]$. Thus, $F(p^\star) = G(p^\star)$. Let $g^-(p^\star) = \lim_{p \to p^\star} dG(p)/dp$. $g^-(p^\star)$ exists and satisfies $g^-(p^\star) = f(p^\star)$ because F is twice differentiable.[9] In period 2, the firm sets prices optimally to each group of consumers, and from our above observations, this has to result in $p_2^0 = p^\star$ in a PBE. Since $p(1-F(p))$ is strictly quasiconcave, the first-order condition that gives $p^\star$ is $1 - F(p^\star) - p^\star f(p^\star) = 0$. However, the first-order condition of the firm's problem in pricing towards anonymous consumers, evaluated at $p_2^0 = p^\star$, gives $\int_0^1 dG(p) - G(p^\star) - p^\star g^-(p^\star) = \int_0^1 dG(p) - F(p^\star) - p^\star f(p^\star) < 0$ since $\int_0^1 dG(p) < 1$. Hence, for some $\epsilon > 0$, the seller is better off setting $p_2^0 = p^\star - \epsilon$ than setting $p_2^0 = p^\star$, resulting in the desired contradiction. ■

This result says that if the cost of maintaining anonymity is nil, then it is in the best interest of every

[9] We note that if f is discontinuous, then PBEs in which some consumers stay identified do exist.

individual who purchases the good in the first period to maintain her anonymity, effectively resulting in the no-recognition outcome from Subsection 3.1. However, as indicated in Subsection 3.2, this turns out to be exactly what the firm wants.

From the perspective of consumers, in Subsection 4.4, we show that this outcome is a Prisoner's Dilemma situation: individually, each consumer chooses to maintain her anonymity; collectively, however, all consumers are worse off as a result of there being no price discrimination. In other words, by opting out, consumers impose a negative externality on other consumers in the anonymous pool. Below we study how firm profit and consumer surplus are affected by the cost c of maintaining anonymity.

4.2 Costly anonymity: equilibrium characterization

We now move on to the general case in which there is some cost $c \geq 0$. We will restrict our attention to PBEs in which the following holds: all consumers who purchase the good in the first period opt out with the same probability α. This restriction is motivated by the fact that all consumers who purchased in the first period face the same tradeoff when deciding to opt out: either pay p_2^1, or pay $p_2^0 + c$. (In equilibrium, all first-period buyers will buy again in the second period.) We refer to such an equilibrium as a *pooling equilibrium.*[10]

The firm's second-period beliefs over valuations in a pooling equilibrium are given by

$$F^0(v) = \begin{cases} \frac{F(v)}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v \leq \tilde{v} \\ \\ \frac{F(\tilde{v})+\alpha(F(v)-F(\tilde{v}))}{F(\tilde{v})+\alpha(1-F(\tilde{v}))} & \text{if } v > \tilde{v} \end{cases} \tag{1}$$

and

$$F^1(v) = \frac{F(v) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{2}$$

In the second period, the firm chooses its prices to maximize profit according to

$$\max_{p_2^r} \, (1 - F^r(p_2^r))p_2^r \;\text{ for }\; r = 0, 1 \tag{3}$$

The following lemma shows that when $c > 0$, there does not exist a PBE in which all of the consumers who purchased in the first period opt out. The intuition is that when $c > 0$, it is optimal for the firm to lower the first period price in order to attract more customers in the first period. For some of these customers,

[10] The restriction to pooling equilibria can also be justified using a purification argument. Suppose that, instead of all agents facing the same cost of opting out, each agent's opt-out cost is drawn from a commonly known distribution. Furthermore suppose these costs are drawn i.i.d. across agents, and are independent of the agent's valuation. Let d_i, $i \in \mathbb{N}$, denote a sequence of continuous distributions over the opt-out cost that an individual agent faces such that $\lim_{i\to\infty} d_i$ is the degenerate distribution on c (where c is the cost of the original game G). Let G^{d_i} denote the cost-perturbed game where each consumer's cost of opting out is realized after the first-period purchasing decisions according to d_i. It can be shown that the pooling equilibrium we characterize is the unique equilibrium that results from taking the limit of the equilibria of G^{d_i} when $i \to \infty$.

opting out is not be a best response unless some other consumers purchase and stay identified.

Lemma 1. *For $c > 0$, there does not exist a PBE in which all first-period customers opt out.*

Proof: Assume on the contrary that there does exist such an equilibrium. Since all consumers would be anonymous in the second period, the seller sets $p_2^0 = p^\star$. For opting out to be a best response, we need $p_2^1 \geq p_2^0 + c = p^\star + c$, along with appropriate beliefs for the consumers and the firm. In addition, we need $\tilde{v} \geq p^\star + c$, or a positive measure of consumers would prefer to abstain from purchasing altogether in the second period. The resulting second period profit for the firm is $p^\star(1 - F(p^\star))$. Notably, the firm can always obtain this profit in the second period by setting $p_2^0 = p_2^1 = p^\star$. From the indifference condition for the $\tilde{v}$, $\tilde{v} - p_1 + \delta(\tilde{v} - c - p^\star) = \delta(\tilde{v} - p^\star)$, we obtain $p_1 = \tilde{v} - \delta c$. Combined with $\tilde{v} \geq p^\star + c$, we have $p_1 \geq p^\star + (1 - \delta)c$. If it is indeed optimal for the firm set p_1 such that $p_1 \geq p^\star + (1 - \delta)c$ (whereby all consumers with $v \in [\tilde{v}, 1]$ purchase and opt out), p_1 would be set to maximize the firm's profit, $(1 - F(p_1 + \delta c))p_1 + \delta p^\star(1 - F(p^\star))$. The first-order condition gives $p_1 = \frac{1-F(p_1+\delta c)}{f(p_1+\delta c)}$. However, by strict quasiconcavity (Assumption 1), $\frac{1-F(p)}{f(p)}$ is strictly decreasing in p at $p = p^\star$. Thus, $p_1 = \frac{1-F(p_1+\delta c)}{f(p_1+\delta c)} \leq \frac{1-F(p^\star+c)}{f(p^\star+c)} < \frac{1-F(p^\star)}{f(p^\star)} = p^\star$. Therefore, $p_1 \geq p^\star + (1 - \delta)c$ is violated — a contradiction. ■

The next lemma provides a useful ordering of the equilibrium prices and cutoff type.

Lemma 2. *For $c > 0$, if $\tilde{v} \geq p^\star$, $p_2^0 \leq p_1 \leq \tilde{v} = p_2^1$ holds on the path of play of every pooling equilibrium.*

Proof: It is straightforward to see that due to Lemma 1, as in the previous section, $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$. As before, if $p_2^1 < p_2^0$, no purchasing consumers will opt out. In this case, however, since the firm sets period 2 prices after consumers decide whether or not to maintain their anonymity, p_2^1 targets identified consumers in $[\underline{v}, 1]$, where $\underline{v} \leq p_1$ (some range of consumers $[\underline{v}, p_1]$ will decide to purchase in the first period to obtain the discount in the second period). On the other hand, p_2^0 targets the anonymous consumers in $[0, \underline{v}]$ (and there will be at least some consumers in this interval, because f is positive on $[0, 1]$). Hence, setting $p_2^1 < p_2^0$ cannot be a best response for the firm. Consequently, $p_1 \leq \tilde{v}$ must hold for the marginal consumer with valuation $\tilde{v}$ to be willing to purchase in the first period.

It remains to show that $p_2^0 \leq p_1$. Assume on the contrary that $p_2^0 > p_1$. Then no consumer waits to purchase in the second period, so that all consumers with valuations $v \geq p_1$ purchase in the first period. Hence, $\tilde{v} = p_1$. If $p_2^0 > p_1 = \tilde{v}$, the firm has a profitable deviation in the second period: by setting $p_2^0 = \tilde{v} - \epsilon$, for some $\epsilon \in (0, \tilde{v})$, consumers with valuations $v \in [\tilde{v} - \epsilon, \tilde{v}]$ would purchase, strictly increasing the firm's profit. Hence, $p_2^0 \leq p_1$. ■

In the remainder of the paper, we make the following technical assumption:

Assumption 2. *Let $p^\star = \arg\max_p p(1 - F(p))$, and let w satisfy $F(w) + f(w)w = F(p^\star)$. For all $v \in [w, p^\star)$, we have $f(p^\star) + (p^\star - v)(3f'(v) + vf''(v)) > 0$.*

Assumption 2 is satisfied, for example, by $F(v) = v^x$ for all $x > 0$.

The following proposition characterizes the pooling equilibrium for sufficiently small values of c. (Proposition 4, which follows, gives the relevant range on c.)

Proposition 3 (Pooling equilibrium)**.** *For sufficiently small $c > 0$, every pooling equilibrium satisfies:*
(i) Consumers with valuations $v \in [\tilde{v}, 1]$ purchase in both periods and maintain anonymity with probability α; consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ purchase only in the second period; and $\tilde{v} \geq p^{\star}$.
(ii) For $h(v) = F(v) + vf(v)$ and $h'(v) = 2f(v) + vf'(v)$, the cutoff type $\tilde{v}$ and opting out probability α are determined by:

$$\tilde{v} = \delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta \frac{1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)}{f(\tilde{v})} \tag{4}$$

$$\alpha = \frac{h(\tilde{v} - c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{5}$$

(iii) Prices satisfy $p_1 = \tilde{v} - \delta c$, $p_2^1 = \tilde{v}$, and $p_2^0 = \tilde{v} - c$. The firm's beliefs about anonymous and identified consumers' valuations are given by (1) and (2).

Excluding deviations of measure 0, this uniquely determines the behavior on the path of play. The resulting equilibrium, with prices $p_1 = \tilde{v} - \delta c$, $p_2^0 = \tilde{v} - c$, and $p_2^1 = \tilde{v}$, has the following properties. A consumer with valuation at least $\tilde{v}$ will purchase in the first period as well as in the second period, and be indifferent between opting out and staying identified. A consumer with valuation $\tilde{v}$ will be indifferent among only buying in the first period, only buying in the second period, and buying in both periods. A consumer with valuation at most $\tilde{v}$ will not purchase in the first period, and purchase in the second period if and only if her valuation is at least $\tilde{v} - c$. Essentially, the firm offers anonymous customers "introductory" prices in each period. The proof is in the appendix.

We now move on to general values of c (not necessarily small). Let $\alpha(c)$ denote the probability that a consumer who purchased in the first period maintains anonymity, when the cost of doing so is c. (In a pooling equilibrium, by definition, this probability is the same for all agents who purchase in the first period.) Proposition 4 characterizes the pooling equilibrium across different values of c, and establishes existence.

Proposition 4. *Let $p_2^{1,FR}$ and $p_2^{0,FR}$ denote the full-recognition benchmark period 2 prices, and let $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$. A pooling equilibrium exists, and any pooling equilibrium satisfies the following properties on the path of play:*

1. *There exists $\hat{c} \in (0, \bar{c})$ such that $\alpha(c) > 0$ for all $c \in [0, \hat{c})$ and $\alpha(c) = 0$ for all $c \geq \hat{c}$.*

2. *For $c = 0$, the unique pooling equilibrium outcome coincides with the no-recognition equilibrium outcome and is characterized by Proposition 2.*

3. *For $c \in (0, \hat{c}]$, the unique pooling equilibrium outcome is characterized by Proposition 3.*

4. *For $c \in (\hat{c}, \bar{c})$, let $\bar{v}$ be defined by $F(\bar{v} - c) + f(\bar{v} - c)(\bar{v} - c) = F(\bar{v})$. The firm sets $\tilde{v}$ (equivalently, p_1) to maximize $\tilde{v}(1 - F(\tilde{v}))(1 + \delta) + \delta p_2^0(F(\tilde{v}) - F(p_2^0))$, subject to $F(p_2^0) + f(p_2^0)p_2^0 = F(\tilde{v})$ and $\tilde{v} \leq \bar{v}$.*

5. *For $c \geq \bar{c}$, the outcome from any pooling equilibrium coincides with the full-recognition benchmark outcome.*

6. *The firm's profit is non-decreasing in c over $[\hat{c}, \bar{c}]$ and is strictly higher under $c \geq \bar{c}$ than under $c = \hat{c}$. Consumer and social surplus are non-increasing over $[\hat{c}, \bar{c}]$ and strictly lower under $c \geq \bar{c}$ than $c = \hat{c}$.*

Proof: See appendix. ■

Figure 3 shows how the probability of opting out is affected by the cost of maintaining anonymity, c. The interval $[\hat{c}, \bar{c}]$ is of particular interest, because as parts (1) and (6) of Proposition 4 indicate, consumer and social surplus (weakly) decrease in this range, the firm's profit (weakly) increases, but the probability of opting out is fixed at 0. These intervals can be explained as follows. First, the firm loses profit when con-

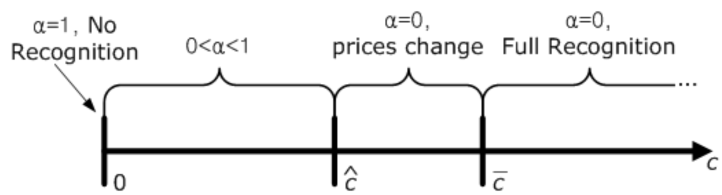


Figure 3: Equilibrium probability α of opting out as a function of the cost of opting out, c, on the path of play.

sumers opt out. When a consumer chooses to opt out, because the firm's second-period price for anonymous consumers is c lower than for identified consumers (to keep consumers indifferent between opting out and staying identified), this effectively costs the firm c. That is, the cost of opting out is passed on to the firm. Second, since some consumers opt out, the second-period price to anonymous consumers, p_2^0, targets both first-time buyers and repeat customers who opted out. Hence, (anonymous) repeat customers are interfering with the firm's ability to capture more first-time buyers in the second period, lowering the firm's profit. On the other hand, because consumers can opt out, more consumers decide to buy in the first period, which helps the firm's profit. This latter effect dominates when c is low, but is overcome by the former two effects as c grows larger – to the point (at $c = \hat{c}$) where it pays off for the firm to lower the first price sufficiently so that no consumer opts out. Once the cost reaches $\hat{c}$, nobody will opt out, allowing the firm to more easily price-discriminate as c increases.

4.3 FIRM PROFIT

For $c \in [0, \hat{c}]$ and for $c = \bar{c}$, the (present value of) the firm's profit is given by

$$(\tilde{v}(c) - \delta\alpha(c)c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0)) \tag{6}$$

From part (6) of Proposition 4, the firm's profit under $c \in (\hat{c}, \bar{c})$ is lower than its profit under $c = \bar{c}$. When $c \geq \bar{c}$, the firm obtains the full-recognition equilibrium profit. However, if $c = 0$, then the firm obtains the no-recognition equilibrium profit, which is greater than the full-recognition equilibrium profit (and, therefore, greater than the firm's equilibrium profit for any value $c \in (\hat{c}, \bar{c})$). This indicates that the firm's profit is non-monotonic in the cost of opting out. We recall that in the full-recognition model (with no opting out), if the firm is able to commit, then the outcome coincides with the no-recognition benchmark. It turns out that this remains true in the model with (costly) opting out. The following proposition summarizes the above observations.

Proposition 5 (Firm profit)**.** *For $\delta > 0$, the firm's profit is highest at $c = 0$ and is non-monotonic in c. If the firm is able to commit to second period prices in the first period, the firm would set $p_1 = p_2^0 = p_2^1 = p^\star$.*

Proof: For a given cost of opting out, c, the firm's profit is at most its profit when it collects this cost c. This profit is given by adding $\delta\alpha(c)c(1 - F(\tilde{v}))$ to (6). Then, an upper bound on the firm's profit is given by:[11]

$$\tilde{v}(c)(1 - F(\tilde{v}(c))) + \delta p_2^0(1 - F(p_2^0)) \tag{7}$$

The expression in (7) is uniquely maximized when $\tilde{v}(c) = p_2^0 = p^\star$, which then gives the firm's profit in the no-recognition outcome, also obtained, by Proposition 2, when $c = 0$. However, $\tilde{v} > p^\star$ for all $c > 0$ and $\delta > 0$. Thus, the firm's profit is highest under $c = 0$. Non-monotonicity follows from part (6) of Proposition 4: the firm's profit is higher under $c = \bar{c}$ than under $c = \hat{c}$, where $\hat{c} < \bar{c}$.

If the firm were able to commit to second period prices, it would set p_2^0 and p_2^1 such that no consumer maintains her anonymity. To see this, suppose otherwise. Then $p_2^1 - p_2^0 \geq c$. Consider first the case where $p_2^1 - p_2^0 > c$, so that all consumers who purchase in period 1 opt out. Then, the firm can strictly increase its profit by setting $p_2^1 \in (p_2^0, p_2^0 + c)$, whereby no consumer opts out, more consumers potentially purchase in the first period, and identified consumers pay strictly more in the second period. If, on the other hand, $p_2^1 - p_2^0 = c$, and a positive mass of consumers opts out, the firm can infinitesimally decrease p_2^1, whereby no consumer would opt out, leading to an increase in profit. Hence, if the firm were able to commit to second period prices, it would set them so that $p_2^1 - p_2^0 \leq c$ and no consumer opts out.

[11]For $c \in (\hat{c}, \bar{c})$, this profit is bounded above by the profit under $c = \bar{c}$, for which Equality (7) is satisfied. For $c > \bar{c}$, profit is the same as under $c = \bar{c}$. Hence, Equality (7) provides an upper bound on profit at any given c.

Thus, the firm's commitment problem in the partial-recognition game is a constrained version of the firm's commitment problem in the full-recognition game. In addition, the firm's best reply and corresponding outcome in the full-recognition game with commitment (setting $p_1 = p_2^0 = p_2^1 = p^\star$) is feasible in the constrained problem. It follows that under commitment, the firm sets $p_1 = p_2^0 = p_2^1 = p^\star$ for any $c \geq 0$. ■

4.4 DO CONSUMERS BENEFIT FROM MORE PRIVACY?

We now turn to consumer surplus. Proposition 5 shows that the firm obtains its highest profit when consumers can costlessly maintain their anonymity. However, this does not immediately imply that consumer surplus is at its lowest in this case, because the total surplus may vary depending on the cost of opting out. Specifically, since there is no cost to production, the efficient outcome in this model – the first best – would be for every consumer to obtain the good in each period. Hence, the efficient outcome is not obtained for any $c \geq 0$, since some consumers do not purchase.

For $c \in [0, \hat{c}]$ and $c = \bar{c}$, consumer surplus is given by[12]

$$\underbrace{\int_{\tilde{v}}^{1} vf(v)dv - (1-F(\tilde{v}))(\tilde{v}-\delta c)}_{(\star)} + \underbrace{\delta\Big(\int_{\tilde{v}-c}^{1} vf(v)dv + (F(\tilde{v})-F(\tilde{v}-c))c - (1-F(\tilde{v}-c))\tilde{v}\Big)}_{(\star\star)} \quad (8)$$

In (8), $(\star)$ is consumer surplus from period 1 transactions: consumers with valuations $v \in [\tilde{v}, 1]$ purchase the good and pay a price $\tilde{v} - \delta c$. $(\star\star)$ is consumer surplus from period 2 transactions: consumers with valuations $v \in [\tilde{v}, 1]$ are repeat customers and end up expending $\tilde{v}$ (factoring in the cost of opting out), and consumers with valuations $v \in [\tilde{v} - c, \tilde{v}]$ are first-time customers, receiving a price discount of c.

When valuations are uniformly distributed, Proposition 4 can be used to derive equilibrium firm profit and consumer surplus as a function of the cost parameter c . We characterize these in Proposition 6. Figures 4(a)-(d) show comparative statics for the case of $\delta = 1$. We also consider social surplus, which can be interpreted in two different ways, depending on whether the cost of opting out is deadweight loss, or collected as a fee by a third party (for example, one can rent an anonymous postal box for a fee). In the former case, social surplus equals firm profit plus consumer surplus; in the latter, social surplus is higher than this sum if consumers opt out at positive costs.

Proposition 6. *With uniformly distributed valuations, the pooling equilibrium satisfies the following:*

[12] The expression in (8), evaluated at $c = \bar{c}$, gives a lower bound on consumer surplus for $c \in (\hat{c}, \bar{c})$, by part (6) of Proposition 4. Consumer surplus for $c \geq \bar{c}$ equals consumer surplus at $c = \bar{c}$, since the equilibrium outcome is unchanged.

- *Consumers with valuations in* $[\tilde{v}, 1]$ *purchase in the first period and opt out with probability* α*, where*

$$\tilde{v} = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} + \delta c & \text{if} \quad c \leq \hat{c} \\ \\ \min\{2c, 2\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{9}$$

$$\alpha = \begin{cases} \frac{1+\delta-(4+3\delta)c}{1+\delta(1-c)} & \text{if} \quad c \leq \hat{c} \\ \\ 0 & \text{if} \quad c > \hat{c}, \end{cases} \tag{10}$$

Here, $\hat{c} = \frac{1+\delta}{4+3\delta}$*, and* $\bar{c} = \frac{2+\delta}{8+2\delta}$*.*

- *The firm sets prices*

$$p_1 = \begin{cases} \frac{1+\delta-\delta c(1+2\delta)}{2(1+\delta)} & \text{if} \quad c \leq \hat{c} \\ \\ \min\{(2-\delta)c, (2-\delta)\bar{c}\} & \text{if} \quad c > \hat{c}, \end{cases} \tag{11}$$

$p_2^0 = \frac{1}{2}(\tilde{v} + \alpha(1-\tilde{v}))$*, and* $p_2^1 = \tilde{v}$*. The firm's beliefs about anonymous and identified consumers' valuations are given by (1) and (2).*

In addition, the following properties are satisfied:

1. $\tilde{v}$ *and* p_2^1 *are strictly increasing in c for* $c \in [0, \bar{c})$*.*

2. p_1*,* p_2^0*, and* α *are strictly decreasing in c for* $c \in [0, \hat{c})$*; and* p_1 *and* p_2^0 *are strictly increasing in c for* $c \in (\hat{c}, \bar{c})$*.*

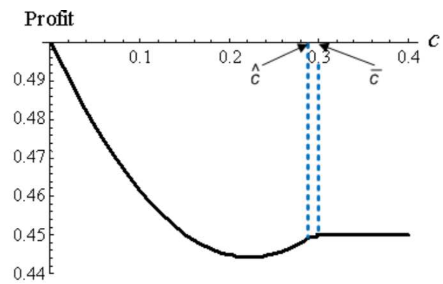
3. *Consumer surplus is non-monotonic in c, increasing (and every consumer's (expected) utility is weakly increasing) in c over* $[0, \hat{c})$*, decreasing over* $[\hat{c}, \bar{c})$*, higher for* $c \geq \hat{c}$ *than at* $c = 0$*, and highest at* $c = \hat{c}$*.*

4. *The firm's profit is non-monotonic in c, decreasing in c over* $[0, \frac{1+\delta}{4+5\delta})$*, increasing over* $[\frac{1+\delta}{4+5\delta}, \bar{c})$*, and highest at* $c = 0$*.*

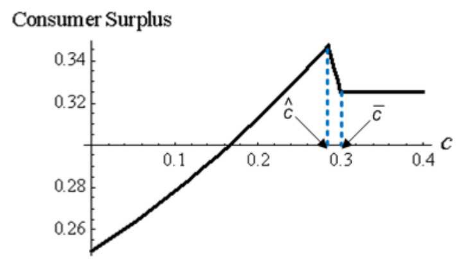
5. *When the cost of opting out is deadweight loss, social surplus is non-monotonic in c, decreasing in c over* $[0, \frac{1+\delta}{12+11\delta})$*, increasing over* $[\frac{1+\delta}{12+11\delta}, \hat{c})$*, decreasing over* $[\hat{c}, \bar{c})$*, higher for* $c \geq \hat{c}$ *(where no consumer opts out) than at* $c = 0$ *(all purchasing consumers opt out), and highest at* $c = \hat{c}$*.*

6. *When the cost of opting out is collected (i.e., not wasted), social surplus is non-monotonic in c, increasing in c over* $[0, \hat{c})$*, decreasing over* $[\hat{c}, \bar{c})$*, higher for* $c \geq \hat{c}$ *than at* $c = 0$*, and highest at* $c = \hat{c}$*.*

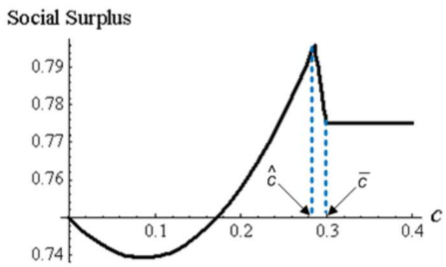
The intuition is as follows: when $c = 0$, all consumers who purchase in the first period choose to maintain their anonymity. As c begins to rise, consumers must pay a non-trivial resource cost in order to opt



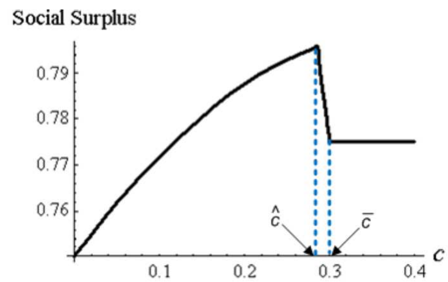
(a) Firm profit as a function of opting out cost c



(b) Consumer surplus as a function of c



(c) Social surplus, c is deadweight loss



(d) Social surplus, c is a fee

Figure 4: Comparative statics when valuations are uniformly distributed.

out. The firm resorts to reducing the first period price to counteract the negative effect on consumers' buying incentives: consumers are reluctant to purchase in the first period because of the cost they will incur in the second period either from opting out or from paying a high price. This results in a lower profit for the firm than it would have had with a lower c, but in higher consumer surplus. In the case where c is deadweight loss, this also results in lower social surplus because many consumers opt out in this region of cost. LEO

As c approaches $\hat{c}$, fewer consumers opt out. This gives the firm more flexibility in setting second period prices, allowing it to better price discriminate which leads to a slight increase in profit. The firm continues to depress the first period price over this range; additionally, better price discrimination allows the firm to target more low valuation consumers in the second period. This results in higher consumer surplus. Hence, as c approaches $\hat{c}$, both profit and consumer surplus are increasing so that social surplus is increasing when the cost c is deadweight loss. When it is not deadweight loss, social surplus is increasing but not as steeply because fewer consumers opt out (we recall that in this case, social surplus equals the sum of profit, consumer surplus, and the total cost of opting out). When c is in $[\hat{c}, \bar{c}]$, no consumer opts out. The firm increases prices in this range in order to better price discriminate in the second period, which results in fewer consumers purchasing and lower surplus over all.

5 Conclusions and extensions

We studied a model in which a firm is able to recognize and price discriminate against its previous customers, while consumers can maintain their anonymity at some cost. We showed that the firm obtains its highest profit when consumers can costlessly maintain their anonymity, but consumers can be better off when maintaining anonymity is costly, up to a point.

This paper suggests that certain aspects of online consumer privacy may be misjudged by policymakers and consumer advocacy groups. In particular, facilitating opting out can decrease consumer and social surplus when the cost of opting out is already low, although the opposite (or a neutral) effect takes place at higher costs. Of course, in practice, many other considerations need to be taken into account that are not in the scope of our model, such as the intrinsic value of privacy, as well as the (possibly accidental) release of sensitive information and corresponding spillover effects—for example, the release of an individual's medical records to her employer.

Our model can, in principle, be extended to take certain other economic considerations into account. An obvious direction is to study a setting with competition. Indeed, we have some initial findings in a similar model with two firms selling differentiated products; these findings suggest that phenomena similar to those identified above continue to occur. Another direction for further study is to make the cost of opting out endogenous. For example, one can consider a setting where a third party—a privacy gatekeeper—can provide anonymity to consumers for a fee.[13] One of our preliminary findings in this extension of the model is that the privacy gatekeeper would prefer to bargain with the firm and actually set the cost to consumers of opting out to zero. One can also consider settings where consumers obtain some benefit from being identified, such as smaller search costs or better technical support. Our preliminary finding here is that the results from the basic model cary through, except the relevant range of costs grows larger.[14] Another variant is to consider an opt-*in* policy. For instance, the firm could pay consumers to be identified (as in the case of membership programs that offer discounts). Finally, it would be interesting to study the steady-state equilibrium of an infinite-horizon version of our model with overlapping generations of consumers.

There are, in fact, a multitude of questions concerning issues of on-line privacy that are both interesting and potentially important. A primary message of this paper is that the answers to such questions may not be as obvious as they first appear. Indeed — as we have illustrated — it is often necessary to parse carefully the underlying economic forces at work before one can make correct policy recommendations.

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[13]For instance, a consumer could rent a postal box from UPS so as not to disclose her home address. However, UPS could make such an "anonymizing service" much simpler: it could provide customers with individualized one-time codes. Consumers would give these codes to sellers instead of their home addresses. When sellers ship purchases via UPS, they would print the corresponding code on the label, and UPS would use this code to determine the customer's address.

[14]Notably, in such environments, consumers could benefit from having multiple accounts: one account to use for obtaining these benefits, and another account to use for potential access to lower prices.

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Appendix

Proof of Proposition 3

Proof: Assume $\tilde{v} \geq p^\star$ holds on the path of play, so that by Lemma 1, $p_2^1 = \max\{p^\star, \tilde{v}\} = \tilde{v}$. (We show below that $\tilde{v} \geq p^\star$ is indeed satisfied.) From Lemma 2, the (relevant) solution to the firm's period 2 problem for anonymous consumers satisfies the first-order condition of (3) for $r = 0$:

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v})(1-\alpha) + \alpha \tag{12}$$

If $\alpha > 0$, in order for consumers who mix between opting out and not opting out to be indifferent, $p_2^0 + c = p_2^1$ must hold. Since $p_2^1 = \tilde{v}$, we therefore have $p_2^0 = \tilde{v} - c$. Combining the observation that $p_2^1 = \tilde{v} = p_2^0 + c$ together with (12), we obtain:

$$\alpha = \frac{(\tilde{v}-c)f(\tilde{v}-c) + F(\tilde{v}-c) - F(\tilde{v})}{1 - F(\tilde{v})} \tag{13}$$

Using $h(v) = F(v) + vf(v)$ in the above, (5) is obtained.

Since $p_2^1 = p_2^0 + c = \tilde{v}$, a consumer with valuation $\tilde{v}$ receives zero payoff in period 2. Moreover, since a consumer with valuation $\tilde{v}$ is indifferent between purchasing now and possibly later and purchasing only later, she is overall indifferent between purchasing only now, purchasing only later, and purchasing now and later (with and without opting out). Since this consumer receives zero payoff in period 2, the following equality holds: $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$. Substituting for $p_2^0 = \tilde{v} - c$, we obtain $\tilde{v} = p_1 + \delta c$. Hence, if $\tilde{v} \geq p^\star$, the firm's first-period problem of choosing p_1 is equivalent to choosing $\tilde{v}$ such that $p_1 = \tilde{v} - \delta c$ and $p_2^0 = \tilde{v} - c$.

The firm's first-period problem is to choose p_1 to maximize its present-discounted profit:

$$\max_{p_1} (1 - F(\tilde{v}(p_1)))p_1 + \delta\big((1-\alpha)(1 - F(\tilde{v}(p_1)))\tilde{v}(p_1) +$$

$$+ (F(\tilde{v}(p_1)) + \alpha(1 - F(\tilde{v}(p_1))) - F(p_2^0(\tilde{v}(p_1))))p_2^0(\tilde{v}(p_1))\big)$$

Using the above observations, the firm's period 1 problem reduces to:

$$\max_{\tilde{v}} (\tilde{v} - \alpha\delta c)(1 - F(\tilde{v})) + \delta(\tilde{v} - c)(1 - F(\tilde{v} - c)) \tag{14}$$

Substituting for α using (13) in the above, we obtain

$$\max_{\tilde{v}} \ \tilde{v}(1 - F(\tilde{v})) + \delta(\tilde{v} - c)(1 - F(\tilde{v} - c))) - \delta c(F(\tilde{v} - c) + (\tilde{v} - c)f(\tilde{v} - c) - F(\tilde{v}))$$

Letting $h(v) = F(v) + vf(v)$ and $h'(v) = \partial h(v)/\partial v = 2f(v) + vf'(v)$, the first-order condition is

$$1 - F(\tilde{v}) - f(\tilde{v})\tilde{v} + \delta(1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)) + \delta c f(\tilde{v}) = 0 \tag{15}$$

Rearranging (15), we obtain

$$\delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \delta\frac{1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)}{f(\tilde{v})} - \tilde{v} = 0 \tag{16}$$

which gives (4). Given a sufficiently small c, (16) can be used to solve for $\tilde{v}$.

To show that $\tilde{v} \geq p^\star$ is indeed satisfied, substitute $\tilde{v} = p^\star$ into (15). Using the fact that $1 - F(p^\star) - p^\star f(p^\star) = 0$ and dividing by δ gives $1 - h(p^\star - c) - c(h'(p^\star - c) - f(p^\star))$. First, it is easy to see that this expression equals 0 when $c = 0$, since $h(p^\star) = 1$. Now, from quasi-concavity of $p(1 - F(p))$, we have $h'(v) \geq 0$ for $v \in [0, p^\star]$. Differentiating this expression with respect to c gives $f(p^\star) + ch''(p^\star - c) \geq 0$, where the inequality follows from Assumption (2), and is strict for $c > 0$. Hence, the first-order condition in (16) evaluated at $\tilde{v} = p^\star$ is nonnegative and strictly positive for $c > 0$ (recall $\delta > 0$ is assumed throughout). Therefore, $\tilde{v} \geq p^\star$ holds, with strict inequality for $c > 0$. To see that the first-order condition yields a unique solution, rearrange (16) to give

$$1 - F(\tilde{v}) + \delta(1 - h(\tilde{v} - c) - ch'(\tilde{v} - c)) = (\tilde{v} - \delta c)f(\tilde{v}) \tag{17}$$

It follows from the fact that F is a CDF and from Assumption 1 (ii) that the left side of (17) is decreasing in $\tilde{v}$ for $\tilde{v} \geq p^\star$. In addition, it follows from Assumption 1 (i) that the right side of (17) is increasing in $\tilde{v}$. Hence, a unique $\tilde{v}$ satisfies the first-order condition, which entails unique behavior in a pooling PBE on the path of play. However, behavior is not unique off path: if the firm sets $p_1 = 0$, there exists an equilibrium of the continuation game in which all consumers buy. If no consumer opts out, the firm does not learn anything about identified consumers. In this case, the firm sets $p_2^1 = p^\star$, while any $p_2^0 \geq p^\star$ can be sustained since beliefs about anonymous consumers are off path. Similarly, if p_1 is set sufficiently high (as in the proof of Lemma 1), it may be an equilibrium of the off-path continuation game for all consumers to opt out. Further discussion of off-equilibrium considerations is relegated to the proof of Proposition 4.

Existence of a solution follows from the fact that the firm's problem is defined over a compact interval, and profit is clearly not maximized at $\tilde{v} = 0$ or $\tilde{v} = 1$. Since the firm's period 1 and period 2 problems are well defined given $\tilde{v} \geq p^\star$, and since the solution to the firm's period 1 problem is interior (where it was shown, $\tilde{v} \geq p^\star$), the premise of $\tilde{v} \geq p^\star$ is satisfied, and the pooling equilibrium, as characterized, exists. ■

PROOF OF PROPOSITION 4

Proof: We have shown in Proposition 2 that the pooling equilibrium outcome coincides with the no-recognition equilibrium outcome when $c = 0$. Let p_1^{FR}, $p_2^{0,FR}$, and $p_2^{1,FR}$ denote the first and second period prices from the full-recognition outcome, respectively. Let $\hat{c}$ denote the smallest $c > 0$ such that $\alpha(\hat{c}) = 0$ (recall $\alpha(0) = 1$). Consider $\bar{c} = p_2^{1,FR} - p_2^{0,FR}$. In the partial-recognition game, given $c = \bar{c}$, the full-recognition outcome is obtainable by the seller. To see this, note that if the seller sets $p_1 = p_1^{FR}$, then $p_2^1 = p_2^{1,FR}$ and $p_2^0 = p_2^{0,FR}$ would be set optimally given $\alpha = 0$, and since consumers are indifferent about maintaining anonymity, this is indeed an equilibrium of the continuation game. Now, assume on the contrary the firm possesses some profitable deviation by setting a different first period price such that $\alpha > 0$ (if $\alpha = 0$, the same deviation would have been possible in the full-recognition setting). Since the marginal type, $\tilde{v}$, gets no payoff from purchasing in period 2 and since $p_2^1 - p_2^0 = \bar{c}$ must hold for $\alpha > 0$, the same deviation in first period price (with the same resulting $\tilde{v}$ and period 2 prices) is possible under full recognition (with no opting out). However, in the full-recognition model, $\alpha = 0$, resulting in a higher profit for the firm. Hence, the firm would have a profitable deviation in the full-recognition model, contradicting p_1^{FR} being set optimally. An analogous argument can be made for any $c > \bar{c}$, and so it follows that when $c \geq \bar{c}$, the pooling equilibrium outcome coincides with the full-recognition outcome. Thus, there exists $\hat{c} \in (0, \bar{c}]$ such that $\alpha(\hat{c}) = 0$.

Now consider $\hat{c}$. Since $\hat{c}$ is the smallest cost where no consumer opts out, continuity implies that the first-order condition of the firm's first period problem in (4) is satisfied at $\hat{c}$. By Proposition 3 then, $p_2^1 - p_2^0 = \tilde{v} - p_2^0 = \hat{c}$. Using $\alpha(\hat{c}) = 0$ in the firm's second period problem towards anonymous consumers, we obtain

$$F(\tilde{v} - \hat{c}) + f(\tilde{v} - \hat{c})(\tilde{v} - \hat{c}) = F(\tilde{v}) \tag{18}$$

Deriving the expression for α in (5) with respect to $\tilde{v}$ and simplifying using (18), we have at $c = \hat{c}$:

$$\alpha'(\hat{c}) = \left.\frac{\partial \alpha}{\partial \tilde{v}}\right|_{c=\hat{c}} = \frac{2f(\tilde{v} - \hat{c}) + f'(\tilde{v} - \hat{c})(\tilde{v} - \hat{c}) - f(\tilde{v})}{1 - F(\tilde{v})}$$

Taking $\alpha = 0$ and substituting these observations in the first-order condition, we have

$$\tilde{v} = \frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1 - \alpha'(\hat{c})\hat{c})) \tag{19}$$

From the firm's first-order condition in the *full-recognition game*, we have

$$\tilde{v}^{FR} = \frac{1 - F(\tilde{v}^{FR})}{f(\tilde{v}^{FR})}(1 + \delta p_2^{0'}) \tag{20}$$

and from the firm's second period problem in the full-recognition game,

$$F(\tilde{v}^{FR} - \hat{c}) + f(\tilde{v}^{FR} - \hat{c})(\tilde{v}^{FR} - \hat{c}) = F(\tilde{v}^{FR}) \tag{21}$$

where $p_2^{0'} = f(\tilde{v}^{FR})/\big(2f(\tilde{v}^{FR} - \bar{c}) + (\tilde{v}^{FR} - \bar{c})f'(\tilde{v}^{FR} - \bar{c})\big)$. Assume on the contrary that $\hat{c} = \bar{c}$, so that $\tilde{v}(\hat{c}) = \tilde{v}^{FR}$ (by the argument in the beginning of the proof). For (19) and (20) to yield the same solution, we must have $p_2^{0'} = 1 - \alpha'(\hat{c})\hat{c}$. It can be checked that this holds if only if $f(\tilde{v}) = 2f(\tilde{v} - \bar{c}) + (\tilde{v} - \bar{c})f'(\tilde{v} - \bar{c})$, whereby $\alpha'(\hat{c}) = 0$ and $p_2^{0'} = 1$. However, from (5), if $\tilde{v}$ were to increase, this would entail $\alpha(\hat{c}) > 0$, violating $\alpha'(\hat{c}) = 0$. Thus, $\hat{c} < \bar{c}$. In addition, from (18) and (21) it follows that $\tilde{v}(\hat{c}) \neq \tilde{v}^{FR}$. Since the equilibrium outcome under $\hat{c}$ is obtainable by the firm when $c = \bar{c}$ (and in the full-recognition game), it follows that the firm is worse off under $\hat{c}$ than under $\bar{c}$.

To see that $\alpha(c) = 0$ holds on the path of play for any $c \in (\hat{c}, \bar{c})$, note that when $\alpha > 0$, from Proposition 3 we have $p_2^0 = \tilde{v} - c$, $p_2^1(c) = \tilde{v}$, and $p_1 = \tilde{v} - \delta c$. The first-order condition of the firm's first period maximization problem with respect to $\tilde{v}$ can be rearranged to give

$$\tilde{v} = \alpha\delta c + \frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1 - \alpha - \alpha' c)) \tag{22}$$

where $\alpha' = \partial\alpha/\partial\tilde{v}$ as before. The first-order condition in (22) at $c = \hat{c}$ where $\alpha(\hat{c}) = 0$ reduces to $\tilde{v} = (1 - F(\tilde{v}))(1 + \delta(1 - \alpha'\hat{c}))/f(\tilde{v})$. Using the fact that $\alpha = 0$ in α', we can obtain

$$\alpha'(\hat{c}) = \frac{2f(\tilde{v} - \hat{c}) + f'(\tilde{v} - \hat{c})(\tilde{v} - \hat{c})}{1 - F(\tilde{v})} - \frac{f(\tilde{v})}{1 - F(\tilde{v})}$$

Substituting this into the simplified first-order condition, we have

$$\frac{1 - F(\tilde{v})}{f(\tilde{v})}(1 + \delta(1 - \frac{2f(\tilde{v} - \hat{c}) + f'(\tilde{v} - \hat{c})(\tilde{v} - \hat{c})}{1 - F(\tilde{v})}\hat{c})) - (\tilde{v} - \delta\hat{c}) = 0 \tag{23}$$

Assume on the contrary that there exists some $k' \in (\hat{c}, \bar{c})$ such that $\alpha(k') > 0$. Since $\alpha(\bar{c}) = 0$, there must exist $k > k'$ such that $\alpha(k) = 0$ and the first-order condition (23) is satisfied at $c = k$. From Proposition 3, we have $p_2^0(k) = \tilde{v}(k) - k$. Substituting this into the firm's second period problem, we obtain

$$(\tilde{v}(k) - k)f(\tilde{v}(k) - k) + F(\tilde{v}(k) - k) = F(\tilde{v}(k)) \tag{24}$$

Moreover, since $\alpha(\hat{c}) = 0$, $(\tilde{v}(\hat{c}) - \hat{c})f(\tilde{v} - \hat{c}) + F(\tilde{v}(\hat{c}) - \hat{c}) = F(\tilde{v}(\hat{c}))$. From Assumption 1, the latter expression together with (24) can both hold only if $\tilde{v}(k) - \tilde{v}(\hat{c}) > k - \hat{c}$ (implying $\tilde{v}(k) > \tilde{v}(\hat{c})$). Furthermore, (24) implies that $p_2^0(k) = \tilde{v}(k) - k \leq p^\star$ since $\tilde{v}(k) \leq 1$. Since $\alpha(k) = 0$, the first-order condition at k similarly gives:

$$\underbrace{\frac{1 - F(\tilde{v}(k))}{f(\tilde{v}(k))}(1 + \delta(\overbrace{\frac{1 - F(\tilde{v}(k)) - k\big(2f(\tilde{v}(k) - k) + f'(\tilde{v}(k) - k)(\tilde{v}(k) - k)\big)}{1 - F(\tilde{v}(k))}}^{(\star\star)}))}_{(\star\star\star)} \underbrace{-(\tilde{v}(k) - \delta k)}_{(\star)} \tag{25}$$

However, this expression is negative. To see this, note that from the above, $\tilde{v}(k) - \tilde{v}(\hat{c}) > k - \hat{c} \geq \delta(k - \hat{c})$, so that $\tilde{v}(k) - \delta k > \tilde{v}(\hat{c}) - \delta\hat{c}$. Thus, $(\star)$ is lower than its corresponding term in (23). Additionally, Assumption 1 of quasiconcavity along with $\tilde{v}(k) - k \leq p^\star$ entail a decreasing inverse hazard rate. Also, since $\tilde{v}(k) > \tilde{v}(\hat{c})$, $1 - F(\tilde{v}(k)) < 1 - F(\tilde{v}(\hat{c}))$. It remains to show that $(\star\star)$ is smaller. Since $\alpha(k) = 0$, it follows from (5) that $(\star\star)$ reduces to $1 - (h(\tilde{v}(k) - k) + kh'(\tilde{v}(k) - k))$, where $h(v) = F(v) + vf(v)$ and $h'(v) = \partial h(v)/\partial v = 2f(v) + vf'(v)$. By part (ii) of Assumption 1, $(\star\star)$ is thus lower under $c = k$ than its corresponding term under $c = \hat{c}$, which together with the previous observation implies that $(\star\star\star)$ is lower under $c = k$. But then the first-order condition is violated at $c = k$ — a contradiction. Therefore, for all $c \in [\hat{c}, \bar{c}]$, $\alpha(c) = 0$.

For $c \in [\hat{c}, \bar{c})$, the equilibrium first-period price p_1 is low enough to satisfy $\alpha(c) = 0$. From the firm's second period problem, when $\alpha = 0$, p_2^0 is derived from

$$F(p_2^0(\tilde{v})) + f(p_2^0(\tilde{v}))p_2^0(\tilde{v}) = F(\tilde{v}) \tag{26}$$

Let $\bar{v}(c)$ satisfy

$$F(\bar{v} - c) + f(\bar{v} - c)(\bar{v} - c) = F(\bar{v}) \tag{27}$$

From (5), given $c \in [\hat{c}, \bar{c})$, $\bar{v}(c)$ denotes the highest cutoff type such that $\alpha = 0$. Note that $\tilde{v}(c) \geq \tilde{v}(\hat{c})$ for $c \in [\hat{c}, \bar{c})$; if not, the firm would possess a profitable deviation under $\hat{c}$. Thus, $\tilde{v}(c) \geq \tilde{v}(\hat{c}) \geq p^\star$ and

$p_2^1(c) = \tilde{v}(c)$. The firm's first-period problem reduces to choosing $\tilde{v}$ (or equivalently, p_1) to maximize

$$\max_{\tilde{v}(c)} \tilde{v}(c)(1 - F(\tilde{v}(c)))(1 + \delta) + \delta p_2^0(F(\tilde{v}(c)) - F(p_2^0)) \tag{28}$$

Since $\partial\bar{v}(c)/\partial c > 0$ by Assumption 1, it follows that the firm is less constrained as c increases, and that $\partial\tilde{v}(c)/\partial c \geq 0$. (If $\tilde{v}(c)$ were to decrease at some c in this range, the firm would possess a profitable deviation at a lower c.) Moreover, since $\tilde{v}(\hat{c}) \neq \tilde{v}(\bar{c})$, it follows that $\tilde{v}(c)$ is strictly decreasing for some c in this range. Similarly, (26) combined with $p_2^0 \leq p^\star$ (since $\tilde{v}(c) \leq 1$) and $p_2^1 = \tilde{v}$ imply that p_2^0 and p_2^1 are non-decreasing in c over this range (and strictly increasing for some c). Finally, from the equality that defines the cutoff type $\tilde{v}$, we have $\tilde{v} - p_1 = \delta(\tilde{v} - p_2^0)$, or $p_1 = \tilde{v}(1-\delta) + \delta p_2^0$, which entails that p_1 also is non-decreasing over $[\hat{c}, \bar{c}]$ (and strictly decreasing for some c in this range).

Now, because no consumer opts out under $c \in [\hat{c}, \bar{c}]$, social surplus is simply a function of how many consumers purchase in equilibrium. Since prices and the cutoff type are non-decreasing, it follows that social surplus decreases in c over this range (and is strictly lower under $\bar{c}$ than under $\hat{c}$). The same holds for consumer surplus since all prices go up and fewer consumers purchase.

Existence follows from the fact that the firm's profit is defined on a compact interval, and its profit is strictly higher for $\tilde{v} \in (0, 1)$ than for $\tilde{v} = 0$ or $\tilde{v} = 1$. Hence, the firm obtains its maximum profit in this compact interval, at a point where the appropriate first-order condition is satisfied.

In regards to off-equilibrium path situations, we note that for all $c > 0$, per Lemma 1, there is a positive mass of both identified and anonymous consumers on the path of play (when $c \geq \hat{c}$, anonymous consumers are only those who did not purchase in the first period). Aside for the case of $c = 0$, addressed in Proposition 2, and the case of $p_1 \geq p^\star + (1-\delta)c$, addressed in Lemma 1, there are no other non-trivial[15] off-equilibrium beliefs on the continuation game that follows the first period. ■

[15]There are off-path situations in which consumers behave in a non-utility maximizing way, but such individual behavior does not affect the prices offered by the firm or the firm's beliefs. Similarly, off-path situations following $p_1 = 1$ (no consumer purchases in the first period) or $p_1 = 0$ (all consumers purchase in the first period) are obvious. These prices cannot be sustained in equilibrium: the firm possesses a profitable deviation in the first period by setting $p_1 \in (0, 1)$, whereas it is still able to obtain the same second period profit by setting $p_2^0 = p_2^1 = p^\star$.