AN ESTIMABLE DEMAND SYSTEM FOR A LARGE AUCTION PLATFORM MARKET

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MOTIVATION

- Auction mechanisms are used to allocate goods in many large and important markets
  - Online Marketplaces (eBay, Taobao.com)
  - Online Advertising
  - Procurement
  - Indian tea auctions, used car auctions etc
- Characteristics of these markets
  - Repeated auctions, often sequential
  - Infinite horizon
  - Persistent bidders
  - Heterogeneous goods, preferences

APPLICATIONS I

- With good models (both theory and empirics) there are interesting questions to be answered
- How much consumer surplus is generated by online auction markets?
  - Useful number for analyzing value of e-commerce
- How should we define ``markets'' when allocation is via auctions?
  - Want to evaluate which group of products are close substitutes, can get this from a demand system
  - May be useful for antitrust

APPLICATIONS II

- How should a seller dispose of a block of products?
  - Products compete with each other, but delay costly
  - Need a demand system to evaluate trade off
  - Practical problem: Hertz and expiring leased car fleet
- How much should a seller forecast a new product will sell for?
  - Analogous to discrete choice, if can project down to characteristics, can forecast bids on new product
  - May be useful for planning in public procurement

APPLICATIONS III

- How should a platform optimally set fees?
  - Two-sided market, fees cause dynamic changes in participation
  - Too costly to experiment
- How should we think about mergers between major suppliers?
  - To the extent that we think search keywords on Yahoo and Microsoft are substitutes, what effects do we think their merger should have?
  - What does ``exert market power'' even mean in an auctions context?

MOTIVATION

- Currently we lack good models to analyze these auction markets
- Theory
  - Huge literature on static auction mechanisms
  - Little on dynamic marketplaces, sequential auctions
  - Classic model is Milgrom and Weber (1982 / 2000)
  - Sequential auctions of k homogenous goods to n bidders
  - Turns out to be static!
  - Problem 1: Don't know how to think about multi-product systems
  - Problem 2: Dynamics matter for accurate measurement

CURRENT EMPIRICAL VIEW OF THE DATA

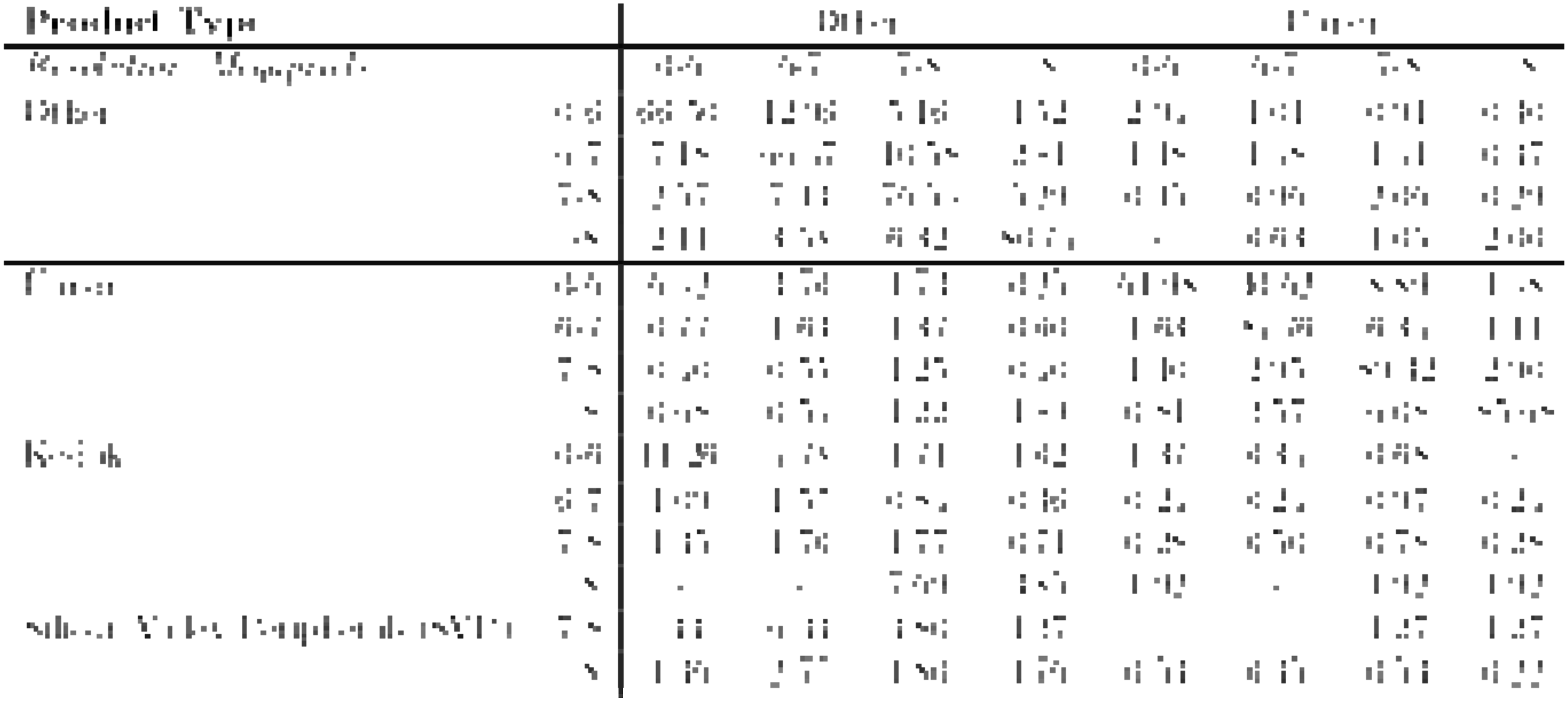
WHAT DATA OFTEN LOOKS LIKE

Good X → Good Y → Good Z

MOTIVATION

- Structural auctions literature designed for estimation with cross-sectional data
  - Auction observations are IID
  - Different population draw in each auction
  - Identical products, or idiosyncratic differences for all products (only the error term varies)
- Data is generally a panel
  - Observe same bidders participate in multiple auctions
  - Pattern of participation reflects preferences, says something about which goods are substitutues

SUBSTITUTION MATRIX



Digital Camera Auctions on eBay: pattern of participation (first vs second auction they bid on)

WHAT WE DO

1. Develop a stylized model of a large auction market
   - Sequential second price sealed-bid auctions
   - Many persistent buyers, dynamic entry and exit
   - Exogenous supply
   - Multiple products, unit demand (*)
   - Multidimensional private valuations

2. Characterize long-run equilibrium
   - Define equilibrium concept appropriate for large anonymous markets with finite buyer/seller ratio
   - Characterize strategies, show existence

WHAT WE DO

3. Analyze resulting demand system
    - Show demand is non-parametrically identified
    - Provide non-parametric and semi-parametric estimation procedures
    - Show how to estimate when valuations are projected onto characteristics
    - Perform Monte Carlo experiments to show it works well in finite samples

- Paper is deliberately abstract: trying to walk a fine line between worrying about practical estimation issues and theoretical tractability

RELATED LITERATURE

- Theory
  - Sequential Auctions: Milgrom and Weber (1982/2000)
  - Long-run market equilibrium: Hopenhayn (1992), Ericson and Pakes (1996)
  - Equilibrium in large auction markets: Wolinsky (1988), Jovanovic and Rosenthal (1988), Satterthwaite and Shneyerov (2007)
  - Dynamic mechanisms for sequential auctions: Said (2008)
  - Alternate equilibrium concepts: Krusell and Smith (1998), Weintraub, Benkard and van Roy (2008), Fershtman and Pakes (2009)
- Demand systems for discrete choice
  - Berry, Levinsohn and Pakes (1995)

RELATED LITERATURE

- Estimation
  - Static Auctions: Guerre, Perrigne and Vuong (2000)
  - Dynamics: Pesendorfer and Jofre-Benet (2003)
  - Dynamics on eBay: Zeithammer (2006), Sailer (2007), Ingster (2008)
  - Simultaneous Auctions: Adams (2009)
  - Dynamic Games: Aguirregabiria and Mira (2007), Bajari, Benkard and Levin (2007), Pakes, Ostrovsky and Berry (2007), Pesendorfer and Schmidt-Dengler (2003)

ROADMAP

1. Model setup
2. Analysis of bidder behavior and equilibrium
3. Identification
4. Estimation
5. Monte Carlo Results

MODEL

- Bidders and Payoffs:
  - Have private valuations X defined over a finite set of J goods, distribution F has continuous density
  - Risk-neutral with unit demand (*), discount future at rate δ
- Market:
  - Operates in discrete time
  - Each period an auction is held. Winning bidder exits certainly; losing bidders exit randomly at rate ρ
  - Losing bidder payoff is normalized to zero
  - New bidders then enter (# of entrant depends on how many already in market), draw valuation from F.
  - Last, seller posts a new item to be sold m periods in future

MODEL

- Auctions
  - Second-price sealed bid auctions
  - Bidders can either bid, or not participate (*)
- Bidder Information
  - Bidders observe an anonymized history of the game for the last k periods
  - Together with the foresight over m upcoming auctions, have a window [t-k, t+m] that is public
  - Also know their private valuation
- Bidding Strategies
  - A bid strategy β(I) is a map from information set to their decision as to what to bid (or not participate)
  - Assume symmetric strategies

BELIEFS

- Bayes-Nash equilibrium requires bidders form beliefs about the opposing set of types
  - Relevant object is a high dimensional vector of J vectors of valuations
  - Solve a filtration problem given initial prior and observed history
  - Implausibly complicated, so we simplify
- Assumption 1: Bidders condition beliefs on finite “state", coarser than full history
  - State variable could be the range of transaction prices in last 7 days; # of upcoming auctions in next 7
  - Believe they face a draw from long-run (stationary) distribution of types in that state

EQUILIBRIUM

- Assumption 2: Bidders believe state transitions are exogenous and first-order Markov
  - Bidders do not account for how their bids affect state
  - Reasonable approximation in large market
- Let ``coarsening function'' T partition information sets into states
- Competitive Markov Equilibrium with respect to T
  - Bidders use symmetric Markovian strategies that depend only on valuation and state
  - Take state transitions as exogenous, and correctly anticipate transition matrix
  - Have correct beliefs about the distribution of opposing types conditional on state
  - Choose strategies that maximize payoffs given these beliefs

CHARACTERIZATION

- Fix an equilibrium. Look at value function:

$$v(x,s) = \max_b \underbrace{G_1(b|s)}_{\text{P(Win)}} \underbrace{\left(x_t - E[B^1|B^1 < b]\right)}_{\text{expected payoff if win}}$$

$$+ \delta \underbrace{(1 - G_1(b|s))(1-\rho)}_{\text{P(Lose \& not exit)}} \underbrace{\sum_{s'} v(x,s')Q_{ss'}}_{\text{E[continuation value]}}$$

  - Where G_1 is the distribution of highest opposing bid given state
  - Q is transition matrix across states

CHARACTERIZATION

- Take a first order condition to get optimal strategies

$$\beta(x,s) = x_t - \delta(1-\rho)E[v(x,s')|s]$$

- Bid valuation less discounted continuation value
- Intuition:
  - Like a second-price auction where winners get object, but losers get their continuation value
  - Turn it into a static SPA by re-normalizing prizes
  - Get "prize" worth object valuation less continuation value if win, nothing if lose
  - Optimal strategy to bid value of prize

THE LONG RUN

- Buried in that expression is the long-run
  - How do bidders evaluate their continuation value?
  - Geometric series, but need to have beliefs about equilibrium distribution of $G_1(b|s)$
- *Lemma 2: Fix any CME. Given any initial measure on the type space, the market converges at geometric rate to a unique invariant measure*
  - Long-run makes some sense: wherever we start, we'll end up at the same set of types in market
  - Notice that in the end, the informational demands on bidders are not that strong!

EXISTENCE

- *Theorem 1: For any T, a CME exists. If there is only one product, the equilibirum is unique.*
- Proof Sketch (1-product case):
  - Restrict to increasing strategies; then any two strategies produce same ergodic distribution
  - So can fix ergodic distribution, and look for optimal strategies
  - Policy iteration works out here
  - So e.g. start with all bidders bidding type: $\beta(x) = x_t$
  - Simulate economy forward, and update everyone's continuation value $v(x, s)$
  - Update according to $\Gamma(\beta) = x - v(x, s)$
  - Show Γ a contraction mapping
  - Apply Banach fixed point theorem ! done!

DEMAND

- Have equilibrium, return to demand estimation
- Remember: demand is willingness to pay = distribution of valuations
- But which distribution of valuations: the entry distribution F, or the steady-state F^* ?
- Show that both are identified from panel data
- Data
  - Observe a sequence of bids for each bidder
  - Observation = [auction, product, bidder, bid]
  - Assume econometrician knows how to classify public history into states, so state known as well
  - Assume discount rate known or can be calibrated

DEMAND

- Game is to get willingness to pay x from bids b
- Sketch identification with 1 product / 1 state
  - Bidder bids according to:

$$\beta(x) = x - v(x)$$

  - Where we have:

$$v(x) = G_1(b)\left(x - E[B^1|B^1 < b]\right) \\ + \delta(1-\rho)(1-G_1(b))v(x)$$

DEMAND

- Substitute in from bidding function to eliminate x:

$$v(x) = G_1(b)\left(b + v(x) - E[B^1 | B^1 < b]\right) + \delta(1-\rho)(1 - G_1(b))v(x)$$

- Re

$$v(x) = \frac{G_1(b)\left(b - E[B^1 | B^1 < b]\right)}{(1 - G_1(b))(1 - \delta(1-\rho))}$$

- The F
- Also gives us x, since we can just add v(x) to bid

DEMAND

- This identifies stationary distribution F* pointwise : for each bid, ``invert'' to get valuation
- This gives us demand
- Result extends to more products and more states: turns out to be a linear system
- Data requirements are stronger though: can only do the inversion on “complete observations”
- Complete observation = a bid in every state by the same bidder

THE SELECTION PROBLEM

- If observations were IID, we could call it a day
- Treat as cross section: take each bidder and get back their valuation gives us F*
- Treat as panel data: must account for the fact that same bidders may show up multiple times
- If we count each guy only once (on entry), get F
- Correcting for this sort of “selection problem” gets more difficult as we have more products and more states (can’t just restrict to bid on entry)
- Can only use complete bid observations, must re-weight resulting valuations to account for selection

ESTIMATION

- Three cases:
- Case 1: Few producsts / states (relative to data)
  - Follow identification argument to nonparametric estimator
- Case 2: Moderate number of products / states
  - Need complete observations for nonparametric approach to work well; this is a tough data requirement
  - Instead show that for any type, can solve for optimal bidding function based on ``first-stage'' estimates
  - Given parametric model, can simulate bid distributions and match simulated and observed distributions

ESTIMATION

- Case 3: Large number of products / states
  - Project down to characteristics space
  - Assume linear relationship between characteristics z and valuations x

$$x_{it} = z_t \alpha_i + \gamma_i$$

  - Type is now a random coefficient on characteristics z
  - Show that in this case, distribution of types is estimable by OLS!
  - Intution: data is much better than discrete choice
- Show approaches work via Monte Carlo simulations (N = 500 auctions, 2 products)

CONCLUSIONS

- Paper has focused on long-run equilibrium and demand estimation in auction markets
- Theory side: tractable equilibrium concept, intutive characterization of bidder strategies
- Empirics: identification, relatively simple estimation strategies that work in finite samples
- Plan to extend the model to allow for small suppliers, participation fees charged by platform
- Although stylized, hope this framework will be useful for economists analyzing these markets