Patent Pools and Product Development: Perfect Complements Revisited*

Thomas D. Jeitschko[†] Nanyun Zhang[‡]

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Abstract

The conventional wisdom is that the formation of patent pools is welfare enhancing when patents are complementary, since the pool avoids a double-marginalization problem associated with independent licensing. The focus of this paper is on (downstream) product development and commercialization on the basis of perfectly complementary patents. We consider development technologies that entail spillovers between rivals, and assume that final demand products are imperfect substitutes. If pool formation either increases spillovers in development or decreases the degree of product differentiation, patent pools can actually adversely affect overall welfare by reducing incentives towards product development and product market competition—even with perfectly complementary patents. This significantly modifies and possibly even negates the conventional wisdom for many settings. The paper suggests insights into why patent pools are uncommon in science-based industries such as biotech, despite there being strong policy advocacy for them.

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[†]Economic Analysis Group, Antitrust Division, U.S. Department of Justice.

[‡]Corresponding author: Department of Economics, 103 Stephens Hall, Towson University, Towson, MD 21252, nzhang@towson.edu.

1 Patent Pools and the Structure of Innovation

In many important industries, prominently so in electronics, computer software, telecommunications, pharmaceuticals and biotechnology, innovative progress is for some time alleged to have become stifled by a so-called patent-thicket, "a dense web of overlapping intellectual property rights that a company must hack its way through in order to actually commercialize new technology" (Shapiro, 2001, p. 120). This has prompted an active debate among academics and policymakers concerning the reform of patents and patent law, with arguments ranging from the abolition of intellectual property (IP) rights altogether (*e.g.*, Boldrin and Levine, 2009) or limiting patents in certain affected areas (*e.g.*, Heller and Eisenberg, 1998), to the deliberate bundling of related patents in so-called patent pools (*e.g.*, Clark *et al.*, 2000). In this paper we consider the latter suggestion by exploring the interplay of the initial pooling of IP and the subsequent development and commercialization of final products.

A patent pool is an arrangement in which patent holders bundle distinct patents so as to collectively license these. The first such combination in the United States was the formation of a patent pool covering patents related to sewing machines in 1856. After initially being considered as fully protected under the doctrine of freedom of contract,[1] in 1912 the U.S. Supreme Court ruled that patent pools were subject to antitrust scrutiny.[2] Since then a nuanced view of patent pooling has emerged in which "blocking" (complementary) and "competitive" (substitutable) patents are distinguished.

In this context, Shapiro (2001) notes that when the patents that are included in the pool are perfect complements, a pool should generally be viewed benignly. The insight rests upon applying Cournot's (1838) analysis of independent monopolies providing perfectly complementary inputs to a downstream producer, in which neither of the upstream suppliers incorporate the negative externality that their pricing decision has on the other.[3] The implied (horizontal) double-marginalization then results in lower producer and consumer

[1] See, *e.g.*, *E. Bement & Sons v. National Harrow Co.*, 186 U.S. 70 (1902).

[2] *Standard Sanitary Manufacturing Co. v. U.S.*, 226 U.S. 20. For a brief synopsis of the historical development see Miller and Almeling (2007) or Gilbert (2004).

[3] Cournot illustrates his point by considering the pricing decisions of a monopolist for copper and a monopolist for zinc who are providing the necessary inputs to a downstream producer of brass.

surplus. The analysis was further refined in a general model by Lerner and Tirole (2004), who also conclude that the more complementary the patents in the pool are, the greater are the welfare benefits associated with the formation of a pool.[4] Quite in line with these theoretical findings, contemporary antitrust recommendations and practice in Europe and the U.S. hold that pooling complementary patents is generally not anti-competitive.[5]

Good illustrations of this can be found in consumer electronics with the DVD6C patent pool that was formed by nine leading home entertainment companies to facilitate technology related to digital versatile discs; or in the software industry with the several MPEG patent pools that govern video and audio compression and transmission. In contrast to these and many more examples in electronics and software, however, patent pools in pharmaceuticals and biotechnology are rare, despite active advocacy for them.[6] Indeed, there are many instances in which patent pools were advocated, but did not form. For example, the several entities that had sequenced parts or the whole of the severe acute respiratory syndrome associated coronavirus (SARS-CoV) proved unable to form a pool to facilitate the development of an effective vaccine. Similarly, the development of a DNA Microarray to arrange 300 cancer-associated genes would greatly facilitate the diagnosis and possible treatment of many cancers; yet such a DNA chip would require the pooling of widely dispersed patents, which has not happened.

In this paper we develop a better understanding of the determinants and implications of pool formation for complementary patents. We do so by departing from the previous literature on patent pooling, which generally abstracts from the subsequent development that takes place on the basis of the relevant IP.[7] Instead, we expressly consider a product

[4]It should be noted that they recognize that, in the context discussed, the notions of complementarity and substitutability are not actually as clear-cut as it might seem, but a meaningful distinction is nonetheless possible on the basis of changes in patentees' willingness to pay for additional patents.

[5]*Cf.* the Guideline on the Application of Art. 81 of the European Commission Treaty to Technology Transfer Agreements (2004/C 101/02), and Chapter 3 of USDOJ/FTC (2007).

[6]See, *e.g.*, USTPO's white paper on the subject: Clark *et al.* (2000).

[7]There is actually little formal study of patent pools in the economics literature. Kim (2004), extending Shapiro's (2001) model, finds that vertical integration can alleviate double-marginalization problems, thus reinforcing Shapiro's argument. Brenner (2009), building upon Lerner and Tirole's (2004) framework, looks at rules that govern the formation of pools to discern how welfare-enhancing pools can be made stable and welfare-reducing pools can be destabilized. Choi's (2010) focus is altogether different, as he considers the welfare implications of patent uncertainty and possible litigation as reasons for pool formation.

development stage, in which development efforts entail spillovers across firms and affect the degree of product differentiation in the final demand market.

We conjecture that the formation of a pool may impact the development process and, thus, our main concern is that models that have ignored product development and spillovers have not necessarily drawn the correct conclusions. For instance, for many scientific discoveries, the pioneer inventions owned by the patent holders are not complete and need further innovation to be embodied in a final product that is to be marketed. Consequently, discovering scientists are sometimes closely involved in the subsequent innovation. The nature of this knowledge transfer within and across developing firms can depend on whether it occurs on an independent basis with each IP holder individually, or is undertaken jointly by members of a pool. Thus, patent pools may facilitate information exchange, potentially affecting both spillovers in development and the resulting degree of product differentiation, either of which can impact firms' anticipated market profits and, hence, their incentives to apply effort at the development stage.

We find that the conditions that are particularly favorable for pooling prevail in industries in which the practice is indeed widespread. For instance, in consumer electronics, spillovers in development and product differentiation play important roles. However, these specific factors are generally not affected by whether IP is pooled or not, since there is often no additional knowledge transfer tied to the IP licensing—the requisite knowledge is frequently fully embodied in the IP. Indeed, much of the product development in consumer electronics and software actually takes place before (but in anticipation of) the acquisition of IP rights.

In contrast, in biotechnology, for example, many discoveries are characterized by 'tacit knowledge.' In particular, "[t]o the extent that the knowledge is both scarce and tacit, it constitutes intellectual human capital retained by the discovering scientist," (Zucker *et al.*, 2001, p. 153); so the product innovation based on those scientific discoveries must start with the transformation from tacit to codified knowledge, which requires the interaction of the patent holders with the developing firms. In these cases, spillovers and the degree of product differentiation depend on the extent of coordination among patent holders, with pool formation potentially having adverse effects on downstream firms' development incentives.[8]

[8]The aforementioned USPTO white paper on patent pools in biotechnology (see footnote 6) apparently

Should this be the case, patent holders prefer to remain independent, despite the otherwise recognized advantages of pool-formation. Indeed, it is even possible that overall welfare is reduced due to pooling, calling into question the unqualified policy recommendations made concerning pool formation of complementary patents.

The importance of the innovation structure on product development and downstream competition has been studied elsewhere in the literature, notably so in the context of research joint ventures (RJVs). Since the seminal papers by Katz (1986) and d'Aspremont and Jacquemin (1988) implications of spillovers in product development have been studied extensively.[9] However, the focus is generally on cooperation between rivals in the development process, frequently in order to internalize spillovers, avoid cost-duplications and generally coordinate development efforts. This is in contrast to the potential effect of patent pooling on development with spillovers. In particular, the decision to pool is made by IP holders, rather than the developing firms; and the existence of a pool does not induce any cooperation or coordination among the competing downstream developing firms.

An exception to the majority of the literature on coordination and spillovers in RJVs is the notion of research sharing joint ventures (RSJVs) in which firms agree to share the results of their research effort, but do not otherwise coordinate efforts (Greenlee, 2005). Kamien *et al.* (1992) consider an extreme version of this where industry-wide joint ventures yield complete spillovers. Going beyond this, Greenlee (2005) considers endogenous joint venture sizes and allows the degree of spillovers to vary. Indeed, citing many such joint ventures over the past decades, Greenlee suggests that RSJVs (rather than cooperative RJVs) are actually the prevalent form of joint ventures in the development of products.[10] From a modeling standpoint, Greenlee's variations in the degree of spillovers is akin to our notion of spillovers tied to patent pool formations, suggesting that the analysis of our model may—to a degree—carry over to RSJVs. Nevertheless, as mentioned above, an important distinction between

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does not recognize this possibility as it cites information sharing specifically as an advantage of pool formation (*id.* p. 10).

[9]See, *e.g.*, Matsui (1989), Kamien *et al.* (1992), Motta (1992), Freshtman and Gandal (1994), Brod and Shivakumar (1999), Fraja and Silipo (2002), Amir *et al.* (2003), or Moltó, Georgantzís, and Orts (2005), to name just a few.

[10]In a similar vein Erkal and Minehard (2010) present a dynamic model of research exchange among rivals and consider the endogenous timing of sharing of information.

our model of patent pooling and RSJVs is that the pooling decision does not lie in the hands of the firms that undertake the commercialization and then compete in the product market, but rather, it depends on the incentives and interests of the upstream patent holders.

Closely related to spillovers at the development stage, we further contend that patent pooling may affect the downstream product market competition. The more tightly aligned are the research paths that are pursued, the smaller is the degree of horizontal product differentiation that result from research efforts that are undertaken to develop and commercialize final products. Hence, pooling is likely to lead to less-differentiated products than when firms develop on a more independent basis, due to the congruence inherent in research trajectories that are closely interrelated.[11]

The effect of the degree of product differentiation on development efforts has also been examined elsewhere, with some models specifically examining endogenous product differentiation. A precursor to this literature is Choi (1993) who examines the private and social incentives of research collaboration in anticipation of its effect on product market profits. However, he considers generic profits, rather than derived profits in a closed form model. Similarly, Amir *et al.* (2003) also use generic profit functions and consider differences between cooperative and non-cooperative R&D. As for the interplay of effort and spillovers in development, Moltó *et al.* (2005) have a closed-form model with a result that is similar to one of ours (albeit in a very different set-up) in that the social planner may wish to limit the extent of spillovers in development, as these lead to under-performance due to free-riding. Bourreau and Doğan (2010) allow for cost sharing in development and study how increased collaboration in development leads to diminished product differentiation. However, effort is not part of the development process. Ghosh and Morita (2008) also study possible trade-offs concerning development collaboration and product differentiation, using a circular city model with a focus on how insiders differ from outsiders.[12]

The remainder of the paper is structured as follows. In Section 2 the model is presented

[11] Indeed, the compatibility of product lines across firms is precisely the main rationale for standard-setting patent pools in computing and electronics.

[12] In contrast to these approaches, that postulate a positive relationship between collaboration and product similarity, Lin and Saggi (2002) actually consider the case where firms coordinate in order to increase product differentiation.

and the continuation equilibrium for product development and competition is derived. Section 3 gives a benchmark in which it is assumed that the pooling decision has no direct bearing on the development process or the subsequent product market competition, and it is shown that the conventional wisdom regarding the effect of pooling of perfectly complementary patents holds in more general settings. In Section 4 we lay the groundwork to extend this by considering the impact of pooling in an extended model. Specifically, we examine the impact of marginal changes in spillovers in development and marginal changes in the degree of product differentiation on the payoffs of market participants; *viz.*, patent holders, downstream firms, and consumers. On the basis of this, the welfare implications of patent pools are more fully evaluated in Section 5, where we distinguish between royalty contracts and up-front fees. Some concluding remarks are presented in Section 6, and two Appendices contain the detailed derivations and the proofs.

2 The Model and the Downstream Equilibrium

Our model of product development and commercialization consists of three stages. The first stage takes place after the foundations research has already been completed. Specifically, prior to the first stage two fundamental discoveries have been made that resulted in two patents being awarded to two distinct patent holders. In Stage I of the model these patent holders decide whether they form a patent pool or remain independent. In Stage II downstream firms acquire access to the relevant IP and undertake costly efforts to each develop a differentiated product. Finally, in Stage III, the developing firms engage in Bertrand price competition against each other in the downstream market.

In this section the three stages of the model are first characterized in greater detail and the equilibrium actions of the downstream firms are derived. Throughout we refer to the upstream providers of the perfectly complementary IP inputs as ‘patent holders,’ and the downstream developers and competitors of imperfectly substitutable goods are the ‘firms.’

To avoid potential confusion, it bears repeating and emphasizing that the patents involved are perfect complements (in production); whereas the final goods produced are imperfect substitutes (in consumption).

2.1 The Basic Framework

Stage I—Pool Formation Stage I begins after foundation research has already been completed and two patents have been awarded to two distinct patent holders, k and l. The two patents are both deemed essential in the further development and commercialization of a final product. That is, the patents constitute perfectly complementary inputs. Patent holders can either license their patents independently to downstream developing/retailing firms, or they can form a pool and license both patents jointly.

There are two possible types of licensing contracts between patent holders and the developing/retailing firms that we consider. Following Shapiro (2001) and Lerner and Tirole (2004), the first are per-unit-of-output royalty rates, denoted by R. This is the standard contractual structure that underlies the Cournot-Shapiro double-marginalization result, and is also the prevalent type of contract found in pools (Serafino, 2007; Gilbert, 2010). Absent a pool, each patent holder independently (non-cooperatively) sets a royalty rate for each of the developing/retailing firms, whereas a uniform royalty rate for the downstream firms is agreed upon between the patent holders when they have formed a pool.

As the double marginalization caused by independently set royalty payments provides a central rationale for pool formation of perfectly complementary inputs, we also consider non-distortionary licensing arrangements for comparison purposes. Thus, the second form of contract is an upfront fixed fee F that firms pay to access the patent rights. Because the fee constitutes a fixed cost for the firms, it does not distort downstream actions. In particular, it does not affect the firms' marginal costs of production in Stage III and, because the firm is the residual claimant of all market profit, it also does not distort efforts applied in product development in Stage II.

Since our focus is on the welfare implications of pool formation in light of its effects on development and product market competition, we preclude the possibility of strategic foreclosure (*e.g.*, the deliberate creation of monopoly in the final-demand market by excluding all but one downstream firm from access to the patents). Indeed, foreclosure would be the subject of independent antitrust concerns, and in both European and U.S. jurisprudence patent pools are subject to non-discrimination rules.

Independent of the contract form that governs the IP transfer, the pooling decision has the potential to affect both subsequent stages. Thus, if there are knowledge-spillovers between the downstream firms in the development stage (Stage II), then the formation of a pool may increase these, as the pool may serve a conduit for knowledge transfer. As for Stage III, should a pool be formed, then the products that are sold in the final demand market may be more similar to one another, that is, the degree of horizontal product differentiation may become diminished and product homogeneity increases.

Stages II and III—Product Development and Commercialization Much of the literature on IP-licensing in general and on patent pools in particular assumes either a monopoly or a perfectly competitive downstream market. Both of these polar extremes obscure important aspects of downstream activities. The former fails to recognize important trade-offs that exist in the degree of product differentiation among rival downstream firms, while the latter fails to appreciate how downstream market interactions affect development efforts of the firms. We capture both of these important aspects by assuming that there are two imperfectly competitive downstream firms i and j poised to develop and market a product based on the two patents.

Following the modeling framework of Singh and Vives (1984), the firms engage in (non-cooperative) differentiated goods price competition in the final demand market in Stage III. Inverse demand for each firm's product is linear and is given by:

$$P_i = A_i - Q_i - \gamma Q_j, \qquad i, j = 1, 2;\ i \neq j, \tag{1}$$

where $\gamma \in \{\gamma_p, \gamma_n\}$ is the degree of product differentiation, with $1 > \gamma_p \geq \gamma_n > 0$ where p denotes the case that a pool has been formed, and n that patent holders operate independently.

Departing from Singh and Vives (1984), A_i is the base demand, or market size, for firm i's product. Its size depends on efforts e expended in development (Stage II) prior to product market competition. In principle this effort may be jointly applied by the patentholder/scientist and the researchers/developers in the firm. However, since the outside patent holders in such instances are generally fully compensated for their efforts, the cost of effort only enters the firms' objective function.[13] In particular,

$$A_i = a + e_i + \beta e_j, \qquad i, j = 1, 2;\ i \neq j, \tag{2}$$

where $\beta \in \{\beta_p, \beta_n\}$, with $1 \geq \beta_p \geq \beta_n \geq 0$ denotes the degree of spillovers in development, measuring how much of firm j's effort is captured and appropriated by firm i in order to augment firm i's base demand.

Firms face a quadratic cost of effort in development and for simplicity we assume that the only production costs are associated with acquiring the requisite IP. Thus, the marginal cost is given by any royalty rates the firms pay, R, and any upfront license fees, F, constitute the firms' (sole) fixed costs.

The sequence of events characterizing the structure of innovation and competition is depicted in Figure 1.

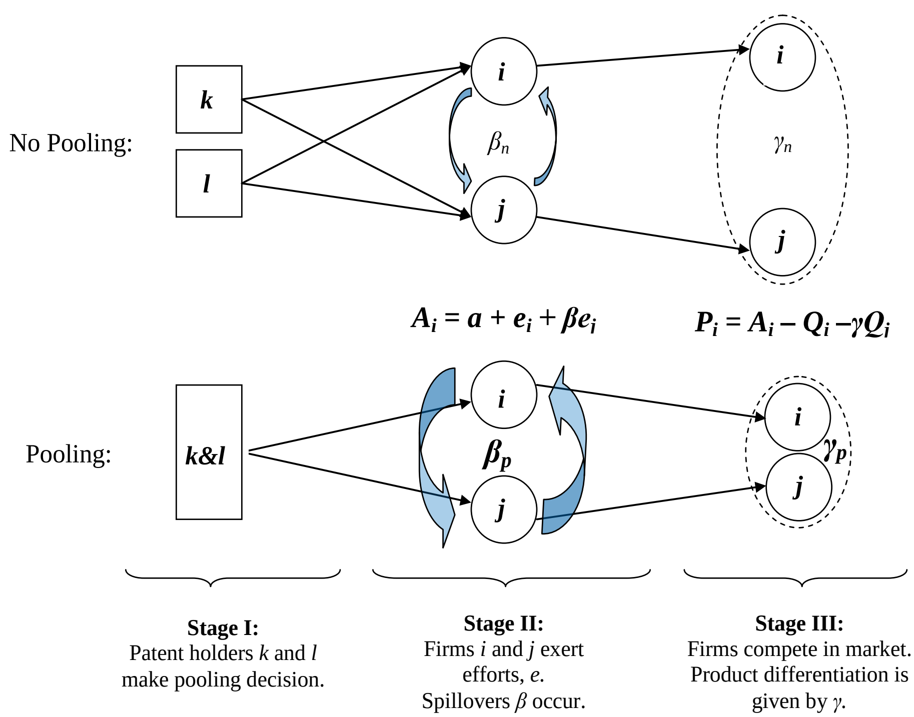

Figure 1: The Structure of Innovation and Competition

[13]See Zucker *et al.* (2001), esp. p. 167.

2.2 The Continuation Equilibrium

We seek the subgame perfect Nash equilibrium and solve the model through backward induction. We first consider the product market competition for a generic degree of product homogeneity $\gamma \in \{\gamma_n, \gamma_p\}$, arbitrary demand intercepts, A_i and A_j, and arbitrary licensing (royalty/fee) structures. Thereafter we analyze the optimal development efforts for generic spillovers $\beta \in \{\beta_n, \beta_p\}$. The analysis is conducted from firm i's point of view, which is without loss of generality as firms are symmetric.

The firms' inverse demand functions, given in (1), are solved for the firms' demands as functions of the strategic variables, namely the prices P_i and P_j:

$$Q_i = \frac{(A_i - P_i) - \gamma(A_j - P_j)}{1-\gamma^2}. \tag{3}$$

While all production costs apart from licensing expenses are normalized to zero, firms may face (per unit) royalty rates R. Moreover, for the case of fixed fees, firms make an upfront payment to patent holders of F. Letting $\mathbb{I} \in \{0, 1\}$ be an indicator denoting the type of the licensing arrangement, with 1 designating the case of royalties and 0 the case of fixed fees, firm i's objective is to choose a price to maximize

$$\pi_i = (P_i - \mathbb{I}R)Q_i - (1-\mathbb{I})F = (P_i - \mathbb{I}R)\frac{(A_i - P_i) - \gamma(A_j - P_j)}{1-\gamma^2} - (1-\mathbb{I})F. \tag{4}$$

Detailed derivations of the model are found in Appendix A, where it is shown that the Bertrand-Nash equilibrium of this game yields,

$$Q_i^* = \frac{(A_i - P_i^*) - \gamma(A_j - P_j^*)}{(1-\gamma)(1+\gamma)} = \frac{\frac{(2-\gamma^2)A_i - \gamma A_j}{2-\gamma^2-\gamma} - \mathbb{I}R}{(2-\gamma)(1+\gamma)}. \tag{5}$$

with

$$\pi_i^*(A_i, A_j) = \frac{(1-\gamma)\left(\frac{(2-\gamma^2)A_i - \gamma A_j}{2-\gamma^2-\gamma} - \mathbb{I}R\right)^2}{(2-\gamma)^2(1+\gamma)} - (1-\mathbb{I})F. \tag{6}$$

Consider now the equilibrium effort exerted in the development stage. Equation (6) gives equilibrium market profits as a function of the demand intercepts A_i and A_j. In accordance with (2), these depend on the firms' effort levels, *viz.* $A_i = a + e_i + \beta e_j$. Thus, given quadratic effort costs of e_i^2, the firm's objective is given by

$$\max_{\{e_i\}} \Pi_i(e_i, e_j) = \frac{(1-\gamma)\left(a - \mathbb{I}R + \frac{(2-\gamma^2-\gamma\beta)e_i + (2\beta-\gamma^2\beta-\gamma)e_j}{2-\gamma^2-\gamma}\right)^2}{(2-\gamma)^2(1+\gamma)} - (1-\mathbb{I})F - e_i^2, \tag{7}$$

with first-order condition[14]

$$e_i^* = \frac{a - \mathbb{I}R + \frac{\left(2-\gamma^2-\gamma\beta\right)e_i^* + \left(2\beta-\gamma^2\beta-\gamma\right)e_j}{2-\gamma^2-\gamma}}{(2-\gamma)^2(1+\gamma)} \frac{2-\gamma^2-\gamma\beta}{2+\gamma}. \tag{8}$$

This yields a best response function of

$$e_i^*(e_j) = \left(a - \mathbb{I}R + \frac{(2\beta-\gamma^2\beta-\gamma)\, e_j}{2-\gamma^2-\gamma}\right) \frac{(2-\gamma^2-\gamma\beta)(2-\gamma^2-\gamma)}{(2-\gamma)^2(1-\gamma^2)(2+\gamma)^2-(2-\gamma^2-\gamma\beta)^2}. \tag{9}$$

Given symmetry, the equilibrium effort choices are

$$e^* = (a - \mathbb{I}R)\frac{2-\gamma^2-\gamma\beta}{(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)}. \tag{10}$$

Thus far the firms' equilibrium behaviors for the general set-up of the development process and the downstream market competition. We now consider the implications of patent pooling for this general setting, proceeding first with the conventional analysis that abstracts from any possible effects that pooling may have on the subsequent development and commercialization. This is followed by a discussion of the impact of marginal changes in spillovers or product differentiation on welfare independent of the pooling structure. On the basis of this we then examine the welfare implication and potential pitfalls of patent pooling in Section 5, where we differentiate between license fees and royalties.

3 Benchmark Analysis

Given the equilibrium effort and pricing decisions of the firms, we now consider the patent holders' incentives concerning the formation of a pool and analyze how welfare is affected by the pooling structure.

While we extend the existing literature on patent pooling by explicitly modeling the costly development of differentiated products in an imperfectly competitive market, in our benchmark analysis we remain in line with the received literature by initially supposing that the formation of a pool has no effect on the parameters governing the interaction between the downstream firms. That is, we assume that possible spillovers in the development process

[14] The first order conditions are sufficient and yield an interior solution (*i.e.*, positive equilibrium effort) provided that $\gamma \lesssim 0.9325$—an assumption that we henceforth maintain.

are unaffected by the pooling decision so that $\beta_n = \beta_p = \beta$; and pooling also does not affect the degree of horizontal product differentiation so that $\gamma_n = \gamma_p = \gamma$.

We first derive contracts for the case of per-unit-of-output royalties. Then we consider the case of upfront fixed fee schemes. Afterwards we compare welfare between the pooling structure and the equilibrium absent a pool for the two cases. Here we re-establish the fundamental Cournot-Shapiro argument in our more general setting that includes development and horizontal product differentiation and we show an indifference result that obtains for the case when licensing is governed by fixed fees and therefore does not introduce any distortions in the output market. Lastly we remark upon the circumstances under which patent holders may prefer royalty contracts over fee arrangements even though the former induce downstream distortions.

3.1 Licensing

Let R denote the per-unit-of-output royalty rate that firms are charged. The patent holder's objective is to maximize the revenue obtained from the firms. We first consider the case of a pool in which the patent holders jointly set a royalty rate R_p for the firms.

The patent holders' objective is to choose a royalty rate R_p that maximizes $V := R_p \times 2Q^*$, while recognizing that the firm's equilibrium output is a function of the royalty rate, *i.e.*, $Q^* = Q^*(R_p)$. The firm's output is given in (5) and the equilibrium value is derived in Appendix A and given by (56), from whence if follows that the patent holders' objective is

$$\max_{\{R_p\}} \; R_p 2(a - R_p) \frac{4-\gamma^2}{(2-\gamma)^2(1+\gamma)(2+\gamma) - (1+\beta)(2-\gamma^2-\gamma\beta)}, \tag{11}$$

with solution

$$R_p^* = \frac{a}{2}. \tag{12}$$

Absent a pool, both patent holders independently choose a royalty rate (r_k and r_l, respectively) that they will charge to each firm for each unit of output sold. Hence, patent holder k's objective is for given r_l to choose a royalty rate r_k that maximizes the revenues obtained from the two downstream firms, $v_k := r_k \left(Q_i^*(r_k, r_l) + Q_j^*(r_k, r_l)\right)$.

Since the patents are essential (*i.e.*, perfectly complementary) each of the downstream firms contract with and pay royalties to both patent holders so that their unit costs are given by

$$R_n := r_k + r_l. \tag{13}$$

Using (56) once again yields

$$Q_i(r_k|r_l) + Q_j(r_k|r_l) = 2\,(a - r_k - r_l)\,\frac{4-\gamma^2}{(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)}; \tag{14}$$

and the patent holder's objective is

$$\max_{\{r_k\}}\; r_k 2(a - r_k - r_l)\frac{4-\gamma^2}{(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)}, \tag{15}$$

yielding the best response function of

$$r_k^*(r_l) = \frac{a - r_l}{2}. \tag{16}$$

The symmetric equilibrium royalty rates are, thus,

$$r^* = \frac{a}{3} \tag{17}$$

so that

$$R_n^* = 2r^* = \frac{2}{3}a. \tag{18}$$

Consider now the case of upfront fixed fees. Because our primary focus is on overall welfare implications of patent pooling, we abstract from explicitly modeling how upfront fees are set and we have therefore treated them simply as a fixed cost from the firms' perspectives. An implication of this is that the fee itself is welfare-neutral as it is merely a transfer from firms to patent holders.

Noting that a firm's willingness to pay for access to the required IP is increasing in the profits that it obtains by using the IP, we assume that the patent holder's ability to extract rents is also increasing in the firms' market profit. Therefore it is in the interest of the patent holders to base their pooling decision on whichever format (pool or no-pool) generates the greater profit for the firms they are dealing with, as this allows them to charge a higher upfront fee.

3.2 Welfare

We now derive welfare in the benchmark and revisit the conventional wisdom concerning the pooling of perfectly complementary patents for the case in which subsequent product development is explicitly modeled and the resulting products are differentiated. We let x denote the given pooling structure (pool or no pool) with $x \in \{p, n\}$, but continue to maintain for this section that $\gamma_p = \gamma_n\,(= \gamma_x)$ and $\beta_p = \beta_n\,(= \beta_x)$. Moreover, we remind the reader that $\mathbb{I} \in \{0, 1\}$ is an indicator designating the IP license arrangement with $\mathbb{I} = 1$ indicating royalties and $\mathbb{I} = 0$ fixed fees. With this indexation, market profit Π_x and consumer surplus CS_x, which are derived in Appendix A as a result of equilibrium effort given in (10) are

$$\Pi_x = (a - \mathbb{I}R_x)^2 \frac{(2-\gamma_x)^2(1-\gamma_x^2)(2+\gamma_x)^2 - (2-\gamma_x^2-\gamma_x\beta_x)^2}{[(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)]^2} - (1-\mathbb{I})F_x. \quad (19)$$

and

$$CS_x = (a - \mathbb{I}R_x)^2 \frac{(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x)^2}{[(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)]^2}. \quad (20)$$

Note that for the case of royalty payments patent holder payoffs are given by (11) for the case of a pool, whereas for the case without a pool they are given by twice (15). For the case of fixed fees, total patent holders' payoffs are generically $2F_x$ for a given pooling structure. Thus, patent holders' total payoffs are

$$V_x = \mathbb{I}R_x 2(a - \mathbb{I}R_x) \frac{(2-\gamma_x)(2+\gamma_x)}{(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)} + 2(1-\mathbb{I})F_x. \quad (21)$$

Because we do not establish an algebraic value of F_x, profit and patent holders' payoffs are not further detailed for the case of upfront fees. However, since F_x is merely a transfer payment between firms and patent holders, the magnitude of F_x has no total welfare implication, so total welfare is given by

$$\begin{aligned} TW_x :&= CS_x + 2\Pi_x + V_x \\ &= (a - \mathbb{I}R_x)^2 \frac{(3-2\gamma_x)(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x)^2 - 2(2-\gamma_x^2-\gamma_x\beta_x)^2}{[(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)]^2} + \\ &\quad (a - \mathbb{I}R_x) 2\mathbb{I}R_x \frac{(2-\gamma_x)(2+\gamma_x)}{(2-\gamma_x)^2(1+\gamma_x)(2+\gamma_x) - (1+\beta_x)(2-\gamma_x^2-\gamma_x\beta_x)}. \end{aligned} \quad (22)$$

We can now reiterate the conventional wisdom concerning the pooling of perfectly complementary patents for the case in which subsequent product development is explicitly modeled and the resulting products are differentiated. Formally,

Theorem 1 (Generalized Conventional Wisdom) *Pooling increases all measures of welfare when there are royalty contracts, even when products are differentiated and there are are spillovers in development;*

$$W_p > W_n, \qquad \forall \gamma, \beta \text{ and } W \in \{CS, \Pi, V, TW\} \text{ and } \mathbb{I} = 1 \text{ (i.e., royalties).} \tag{23}$$

Thus, regardless of the size of spillovers in subsequent product development and independent of the degree of the resulting product differentiation, consumer surplus, market profit, licensing revenue, and hence total welfare are all strictly greater with pooling when there are royalties. The insight follows readily: royalty revenue, given by (21), is by design maximized at R_p^*, so patent holder payoffs are lower when there is no pool compared to the case where there is a pool. Notice from Equations (19) and (20) that firm profit and consumer surplus are decreasing in R. Hence, since $R_n^* = \frac{2}{3}a > \frac{1}{2}a = R_p^*$, firm profit and consumer surplus, in addition to patent holder payoffs, are all higher under pool-formation when compared to the absence of a pool. This is the double-marginalization, or 'royalty stacking,' argument that reflects the negative externalities that are not accounted for when royalty rates are set independently across perfectly complementary inputs.

For the case of fees, in contrast to Theorem 1, note that since we have assumed that pooling does not effect either the level of spillovers, or the degree of product differentiation, it is clear that the pooling decision also does not affect equilibrium effort levels of the firms, and ultimately also has no effect on profits or consumer welfare. We conclude for the case of license fees that the formation of a patent pool has no overall welfare effects.

Note that absent any distortions total welfare is also greater with fees than under royalties. However, this does not imply that patent holders would generally prefer the non-distortionary fee arrangement over royalties. Indeed, if products are sufficiently homogenous so that downstream firms have little market power, then royalties serve as a means for reducing output and extracting consumer surplus. In contrast, if products are more differentiated, then firms' market power generates enough profit that can be extracted by means of a fee whereas royalties would introduce a vertical double marginalization.

The more general point here is that pool formation of complementary IP in itself should not raise antitrust concerns, even when one considers more general frameworks of competition with differentiated products that first require further development. Indeed, for the case of royalties pool formation is strictly preferred over independent licensing. Furthermore, if transactions costs of contractual agreements between licensees and licensors are lower in the pool structure (an argument that is sometimes made, but goes beyond our stylized model), then pooling is also strictly preferred to a situation without a pool in the case of upfront fixed fees.

We now consider how marginal changes in spillovers and the degree of product differentiation impact the analysis.

4 Spillover- and Differentiation-Effects

To lay the groundwork for a discussion of how the interactions between pooling, development efforts, and product differentiation play out, this section deals with how marginal changes in spillovers and product differentiation affect payoffs assuming a given pooling structure.

Where the academic literature on patent pools addresses efficiency, total welfare is generally used as the standard for assessing the best structure for licensing patents. In the benchmark case in Theorem 1 any further differentiation between welfare measures leads to the same insights as an exclusive focus on total welfare, so any separate evaluation of payoffs to producers or patent holders or consumers does not lead to any additional insight regarding the desirability of pooling. However, in the presence of spillover and differentiation effects this is no longer necessarily the case and it needs to be determined when disparate measures of welfare are in congruence and when they are in conflict when it comes to evaluating the formation of patent pools. Thus, in addition to deriving total welfare, we continue to include in our analysis other measures of welfare, as these may result in distinct evaluations and insights, given the specifics of spillover and differentiation effects across industries.

A direct consideration of patent holder payoffs indicates when the formation of pools might be initiated by patent holders. Industry profit is relevant in this context as this will indicate in which circumstances the industry would lobby for or against policies that facilitate the formation of pools. Consumer surplus is also pertinent for our analysis, since, in contrast with much academic literature, antitrust practice often views consumer welfare as the guiding criterion that is to be considered when evaluating a given policy.[15]

4.1 Spillover Effects

We first consider the impact of changes in the amount of spillovers in development. Specifically, assuming a given licensing contract (either royalties or fees), we determine the marginal payoff implications of changes in spillovers for arbitrary constellations of inherent spillovers and fixed levels of product differentiation.

Ceteris paribus, increasing the spillover effect increases welfare by generating a greater demand base A. Hence, all else equal, patent holders view increased spillovers favorably. However, *ceteris non paribus*: When considering the impact that spillovers in the development process have on optimal effort choices, the degree of product differentiation plays a critical role. Thus,

Lemma 1 *Equilibrium effort at the development stage is increasing in the amount of spillovers if products are strongly differentiated, but decreasing if products are similar. Specifically, there exists a function $\mathcal{S}_e$ such that*

$$\frac{de^*}{d\beta} \gtreqless 0 \quad \Longleftrightarrow \quad \gamma \lesseqgtr \mathcal{S}_e. \tag{24}$$

The intuition here is straightforward. Strong spillovers bestow a positive externality on a firm's rival. If the rival is a close competitor, *i.e.*, $\gamma > \mathcal{S}_e$, then firms recognize a larger negative impact on their profits from the rival's strength, that is, π_i^*, given in (6), is decreasing in γA_j. As a result, for γ sufficiently large, firms reduce the amount of effort applied to the development process if spillovers increase.

This negative effect can be sufficiently strong so that increases in the spillovers in development actually have negative effects on equilibrium base demands as measured by the demand intercept A^*. Thus,

Lemma 2 *When products are sufficiently homogenous, increased spillovers reduce the market size. Specifically, there exists a function $\mathcal{S}_A$ with $\mathcal{S}_A > \mathcal{S}_e$ such that*

$$\frac{dA^*}{d\beta} \gtreqless 0 \quad \Longleftrightarrow \quad \gamma \lesseqgtr \mathcal{S}_A. \tag{25}$$

[15] See, *e.g.*, Farrell and Katz (2006), or the contrasting positions in Heyer (2006) and Pittman (2007); but also Lyons (2002).

The two critical thresholds for the degree of product differentiation are depicted in Figure 2, with positive relations between the variables and changes in spillovers occurring to the left of the lines.

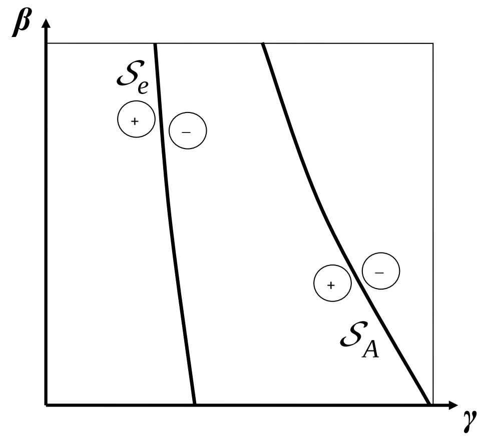

Figure 2: Impact of the Spillover Effect on Effort and Market Size

For the case that there is a license fee arrangement in place, we postulate that the incentives of the patent holders are aligned with those of the firms. As a result of Lemma 2, patent holders therefore prefer increased spillovers, for a given licensing contract, only if the degree of product homogeneity is sufficiently small. Thus,

Proposition 1 *Unless inherent spillovers are very large and products are close substitutes, increased spillovers are beneficial for fee-charging patent holders and firms. That is, there exists a function* $\mathcal{S}_{V_0,\Pi}$ *such that*

$$\frac{dV^*_{\mathbb{I}=0}}{d\beta}, \frac{d\Pi^*}{d\beta} \gtreqless 0 \quad \Longleftrightarrow \quad \beta \lesseqgtr \mathcal{S}_{V_0,\Pi}. \tag{26}$$

with $\mathcal{S}_{V_0,\Pi} > 0.85$.

Figure 3 illustrates the combinations of product differentiation and inherent spillovers referenced in the proposition, with increased spillovers being beneficial below the line identified as $\mathcal{S}_{V_0,\Pi}$.

If instead of a fee arrangement there are per-unit royalty payments then this divorces the patent holders' incentives from those of the firms. In particular, while the firms' objectives are profits, the patent holders' interests are tied to the level of output.

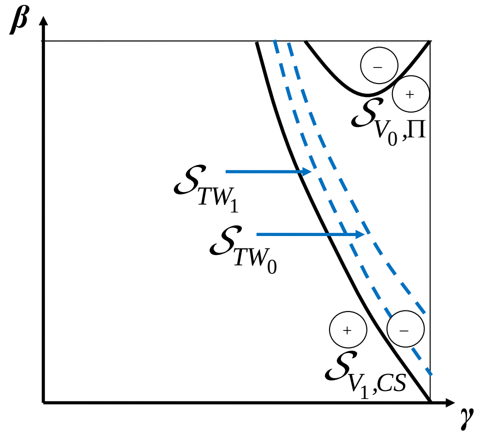

Figure 3: Impact of the Spillover Effect on Payoffs

In this case the constellations for which increased spillovers are beneficial to the patent holders is smaller—that is, firms benefit more on the margin from increased spillovers compared to patent holders under royalties. This is due to the fact that firms benefit from the reduced effort costs associated with lower efforts when goods are more substitutable (Lemma 1); whereas the patent holders benefit from increased output associated with increases in the base market size; that is, Q, given in (5), is increasing in A.

The same intuition for preferring increased spillovers also applies to consumers. In fact, when a given royalty contract is in place, patent holders' interests in terms of increased spillovers are perfectly aligned with those of consumers and are characterized by Lemma 2. Thus,

Proposition 2 *Unless products are close substitutes an increase in spillovers is beneficial for a given royalty rate for patent holders and also for consumers. In particular, there exists a function $\mathcal{S}_{V_1,CS}$, with $\mathcal{S}_{V_1,CS} = \mathcal{S}_A$ such that*

$$\frac{dV^*_{\mathbb{I}=1}}{d\beta}, \frac{dCS^*}{d\beta} \gtreqless 0 \quad \Longleftrightarrow \quad \gamma \lesseqgtr \mathcal{S}_{V_1,CS}. \tag{27}$$

The critical dividing line is again depicted in Figure 3, with increased spillovers being beneficial below and to the left of the line $\mathcal{S}_{V_1,CS}$.

Given the discrepancy between firms' and consumers' interests, with the patent holders' interests aligning with those of the former for the case of fees and with the latter for the case of royalties, it is instructive which interests weigh more when looking at total welfare. As one would expect, the effect of changed spillovers on total welfare lies (necessarily) between those of firms and consumers, being closer to consumers in the case of royalties.

Proposition 3 *Unless products are close substitutes, the spillover effect makes pooling more attractive from a total welfare perspective. That is, there exists functions $\mathcal{S}_{TW_1}$ and $\mathcal{S}_{TW_0}$ with $\mathcal{S}_{V_1,CS} < \mathcal{S}_{TW_1} < \mathcal{S}_{TW_0} < \mathcal{S}_{V_0,\Pi}$, such that*

$$\frac{dTW_{\mathbb{I}}^*}{d\beta} \gtreqless 0 \quad \Longleftrightarrow \quad \gamma \lesseqgtr \mathcal{S}_{TW_{\mathbb{I}}}, \qquad \mathbb{I} \in \{0,1\}. \tag{28}$$

The overall conclusion from this discussion is that in isolation, that is, absent differentiation effects and for a given licensing contract, spillover effects tend to be beneficial when products are sufficiently differentiated.

4.2 Differentiation Effects

We now consider the impact of marginal decreases in product differentiation for given licensing contracts and given degrees of spillovers in development. Again, a critical feature in understanding distinct welfare effects of changes in product differentiation is to understand firms' incentives to provide effort at the development stage.

In contrast to changes in spillovers, the effect of marginal changes in the degree of product differentiation on equilibrium development effort is unambiguous, and therefore also results in an unambiguous effect on the products' base market size reflected in A. In particular:

Lemma 3 *Equilibrium effort, and hence equilibrium base market size, is decreasing in the degree of product homogeneity,* i.e.,

$$\frac{de^*}{d\gamma} < 0 \qquad \Longrightarrow \qquad \frac{dA^*}{d\gamma} < 0, \qquad \forall \beta, \gamma. \tag{29}$$

The intuition is straightforward. As γ increases products become more similar and product market competition becomes more fierce, which decreases the returns on investment in development efforts. This, in turn, reduces the firm's market size, which adversely affects the firm's profit and, for the case of licensing fees, also directly affects the patent holders' interests. Formally,

Proposition 4 *Increases in the degree of product homogeneity adversely affect fee-charging patent-holders' and firms' interests. That is,*

$$\frac{dV^*_{\mathbb{I}=0}}{d\gamma}, \frac{d\Pi^*}{d\gamma} \leq 0, \qquad \forall \beta, \gamma. \tag{30}$$

As discussed, Proposition 4 reflects that increases in γ translate into fiercer product market competition. However, while firms and fee-charging patent holders eschew fiercer competition, if this translates into increased output then per-unit royalty-charging patent holders may actually benefit from decreases in product differentiation. Similarly, consumers also might benefit from increased competition. Indeed, this may, but need not be the case. Thus, Figure 4 depicts the regions in which decreased differentiation is beneficial, which is formalized in the following proposition.

Proposition 5 *When spillovers are sufficiently large, a decrease in differentiation is undesirable from both the royalty-charging patent holders' and the consumers' viewpoints. Specifically, there exist functions $\mathcal{D}_{V_1}$ and $\mathcal{D}_{CS}$ with $\mathcal{D}_{V_1} < \mathcal{D}_{CS}$ such that*

$$\frac{dV^*_{\mathbb{I}=1}}{d\gamma} \lesseqqgtr 0 \iff \beta \gtreqqless \mathcal{D}_{V_1}, \tag{31}$$

$$\frac{dCS^*}{d\gamma} \lesseqqgtr 0 \iff \beta \gtreqqless \mathcal{D}_{CS}. \tag{32}$$

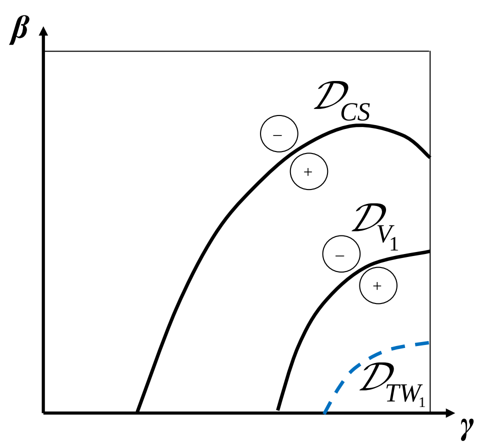

Figure 4: Impact of the Differentiation Effect on Royalty Revenue and Consumer Surplus

The reason that royalty-charging patent holders and consumers may not find the differentiation effect desirable is because of the equilibrium incentives to exert effort in the development process. Due to Lemma 3, if spillovers in the development process are large then the adverse effect of diminished effort results in a reduction in equilibrium output Q^*, which negatively impacts consumers' and patent holders' interests. Otherwise, if spillovers are sufficiently small (provided γ is not too small), royalty-charging patent holders and consumers benefit from the differentiation effect.

This raises the question of what the overall welfare implications of the differentiation effect is, which, it turns out, is unambiguous for the case of fees, but depends on intrinsic differentiation not being too large and spillovers not being too small for the case of royalties.

Proposition 6 *A decrease in the degree of differentiation decreases total welfare unambiguously under fees and does so for royalties if spillovers are sufficiently small whenever goods are fairly homogenous to begin with. Thus, there exists $\mathcal{D}_{TW_1} < \mathcal{D}_{V_1}$ with*

$$\frac{dTW^*_{\mathbb{I}}}{d\gamma} \begin{cases} < 0 & \forall \beta, \ \mathbb{I} = 0, \\ \lesseqgtr 0 \iff & \beta \gtreqless \mathcal{D}_{TW_1}, \mathbb{I} = 1. \end{cases} \tag{33}$$

Thus, despite the fact that consumers may benefit from the increased competition brought about by reduced differentiation, this is more than offset by reductions in profits. That is, once one accounts for the effort incentives in development, total welfare is unambiguously increasing in product differentiation for the case of fees and also so for the case of royalties provided intrinsic differentiation is not too large and spillovers not too small.

We now turn to how spillover and differentiation effects affect the incentives to form patent pools and determine what the implications of patent pooling is for welfare.

5 Welfare Effects of Patent Pools

Having studied the marginal impact of spillover and differentiation effects for a given contract structure, we are now in a position to evaluate the overall incentives to pool and derive the welfare implications of patent pooling. We first consider the case of upfront licensing fees, since for this case some insights can directly be gleaned from the analysis of the previous section. In contrast, when it comes to pool formation with (per-unit) royalties, the avoidance of double-marginalization and royalty-stacking adds another distinct element to consider when contemplating pools.

5.1 Fees

In the case of upfront fixed fees, the incentives implied by the spillover and differentiation effects carry over and can directly be applied to the analysis of pool formation. However, because spillover effects and differentiation effects do not paint a consistent picture across interests and generally depend on the magnitude of intrinsic spillovers and the inherent degree of product differentiation, there are few immediate and straightforward results. Nevertheless some patterns emerge and some noteworthy constellations exist, which we discuss in greater detail now.

Of the three market participants—patent holders, firms, and consumers—the direction of marginal welfare effects are most sensitive to intrinsic spillovers and inherent product differentiation when it comes to consumers and least so when it comes to firms, with patent holders being in between. That is, whether consumers benefit or suffer on the margin from either of the effects generally depends on the degree of spillovers and the degree of product differentiation, whereas for firms most constellations of parameters have the same implications concerning the marginal impact of the effects. In particular, firms and fee-charging patent holders largely benefit from increases in spillovers (*cf.* Prop. 1) and decreases in product homogeneity (Prop. 4).

However, while it may generally be easy to evaluate the marginal effects for firms and hence also for fee-charging patent holders, this does not mean that the incentive to form a pool is straightforward. Notice, thus, from Propositions 1 and 4 and the accompanying Figures 3 and 4 that from the fee-charging patent holders' perspective the two effects almost always operate in opposite directions so that any definitive evaluation of the desire to pool must account for the magnitude of the two effects. In general, whenever the differentiation effect increases, to keep the incentives for pooling the same, there must also be an increase in the spillover effects.

The only exception to the fee-charging patent holders' two incentives moving in opposite directions is the case characterized in Proposition 1. Indeed, since here the patent holders' critical threshold on the parameter values is entirely encased by that of consumers, this also yields the only unambiguous prediction concerning the desirability of pooling that can be drawn on the basis of the previous section.

Theorem 2 *If the intrinsic degree of product differentiation is small and spillovers in development are sufficiently high, then firms, consumers, and patent holders are all worse off by the formation of a pool. Formally,*

$$W_p < W_n, \qquad \forall \beta_n, \gamma_n \ni \beta_n > \mathcal{S}_{V_0,\Pi}(\gamma_n) \text{ and } W \in \{CS, \Pi, V_0, TW_0\}. \tag{34}$$

Thus, quite remarkably, the only strong result to follow from the analysis of Section 4 is a sufficient condition in which the pooling of perfectly complementary patents actually unambiguously lowers consumer surplus, profit and total welfare. A stark contrast to the conventional wisdom concerning the benefits of pooling complementary patents.

Underlying the result is that when there are large degrees of spillovers in development then for relatively homogenous products there is a lot of free-riding off one's rival's effort in the development process. As a result, increases in spillovers and increases in product homogeneity due to pooling actually reduce overall development efforts to the detriment of all involved.

Several important remarks concerning Theorem 2 are in order. First, the Theorem gives sufficient conditions for pool formation to be welfare reducing for all involved. The conditions are not necessary and indeed there are many other constellations concerning inherent spillovers and degrees of differentiation and how these are affected by pooling that yield the same implication. Second, in all of these cases, because patent holders also are better off without a pool, an inefficient pool would not emerge on its own.

In contrast to the theorem, however, there are also many constellations in which—in congruence with the conventional wisdom—all parties involved strictly benefit from the formation of a pool. The following example depicted in Figure 5 illustrates some of these points.[16]

[16]The examples were calculated and the figures were generated using Mathematica®. The associated files are available from the authors upon request.

Example 1 *Let $\beta_n = 0.7$ and $\gamma_n = 0.2$, that is, products are strongly differentiated and there are strong spillovers in development. Now consider spillover and differentiation effects such that $\beta_p \in [0.7, 1]$ and $\gamma_p \in (0.2, 0.9]$, then there exist functions $\mathcal{F}_{CS}$ and $\mathcal{F}_{V_0,\Pi}$, with $\mathcal{F}_{CS} > \mathcal{F}_{V_0,\Pi}$, such that*

$$CS_p < CS_n, \qquad \forall \beta_p, \gamma_p \ni {}^{\beta_n}\!/\!_{\beta_p} > \mathcal{F}_{CS}\left({}^{\gamma_n}\!/\!_{\gamma_p}\right), \tag{35}$$

$$W_p < W_n, \qquad \forall \beta_p, \gamma_p \ni {}^{\beta_n}\!/\!_{\beta_p} > \mathcal{F}_{V_0,\Pi}\left({}^{\gamma_n}\!/\!_{\gamma_p}\right) \text{ and } W \in \{V_0, \Pi\}. \tag{36}$$

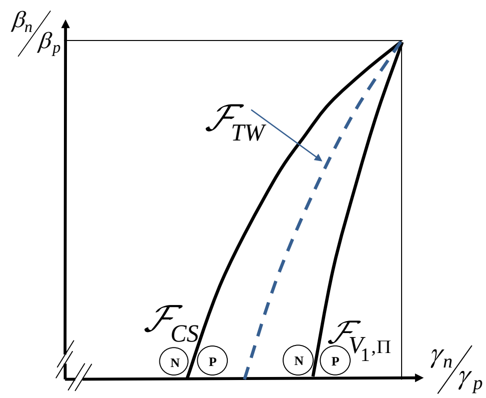

Figure 5: Pooling and Non-Pooling with Fees

Thus, Example 1 shows how pooling can be undesirable, even for initially very differentiated goods, provided that spillover effects are small (*i.e.*, ${}^{\beta_n}\!/\!_{\beta_p}$ large) and differentiation effects are large (*i.e.*, ${}^{\gamma_n}\!/\!_{\gamma_p}$ small). In contrast, if differentiation effects are small, then all parties prefer the pooling outcome.

Moreover, as Figure 5 illustrates, as the differentiation effect becomes smaller (*i.e.*, ${}^{\gamma_n}\!/\!_{\gamma_p}$ increases) or the spillover effect becomes larger (*i.e.*, ${}^{\beta_n}\!/\!_{\beta_p}$ decreases) it is first consumers and only later the fee-charging patent holders who prefer the pooling structure. For this example, this implies two things. First, a sufficient condition for pooling to be overall beneficial is that patent holders prefer to pool. And second, there are constellations for which consumers would prefer the pooling structure, while patent holders do not; and overall welfare would be higher without pooling. Indeed, $\mathcal{F}_{TW}$ in Figure 5 shows the threshold for which pooling becomes beneficial from a total welfare standpoint.

The tradeoffs described in Example 1 and illustrated in Figure 5 are somewhat typical for large areas of the parameter space. In particular, it can be shown that the incentives to pool are much stronger for consumers than for patent holders in most cases. However, a universal policy recommendation to the effect that pool-formation initiated by patent holders would necessarily benefit consumers is unfortunately not possible. This is illustrated in the following example.

Example 2 *Let $\beta_n = 0.2$ and $\gamma_n = 0.8$. Thus, products are fairly homogenous and spillovers in development are moderate when there is no pool. Now suppose that $\beta_p = 0.8$ so that there are large spillover effects from pooling. Consumers will surely not want a pool to form in this case. However, for a small differentiation effect,* i.e., *$\gamma_p \lesssim 0.846$, patent holders wish to pool; which is only overall desirable (from a total welfare perspective) if differentiation effects are truly minimal,* i.e., *if $\gamma_p \lesssim 0.816$.*

Figure 6 depicts the parameter space in which $\beta_n = 0.2, \gamma_n = 0.8, \beta_p \in (0.2, 1]$ and $\gamma_p \in (.8, 0.9]$. The dotted horizontal line gives the case where $\beta_p = 0.8$ and the intersections of that line imply the values of γ_p that are given as cut-offs in Example 2.

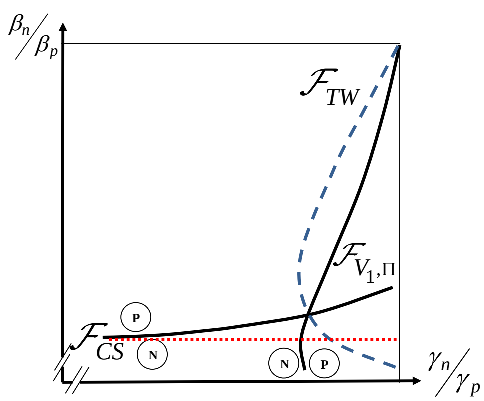

Figure 6: A Case of Profit-Maximizing Pooling that Reduces Total Welfare

Note that to the degree that this type of example is not deemed pathological, in terms of the inherent spillovers and the levels of product differentiation and how they are affected by pooling, then this is cause for some policy concern: Despite patents being perfect complements, this reveals constellations in which patent pools would be expected to form, yet pool formation is against the consumers' interests and also lowers total welfare.

5.2 Royalties

The preceding analysis stems directly from the applications of the spillover and differentiation effects tied to pooling. However, one of the central arguments in the discussion about pool formation is the avoidance of the distortions associated with double-marginalization and royalty-stacking that occur under independently set royalties. We now see if this outweighs other concerns tied to the effect of spillover and differentiation effects on product development. An immediate implication of double-marginalization with royalties is the following theorem.

Theorem 3 *A necessary condition for pooling to be welfare reducing under royalties, is that pooling is welfare reducing under a fee structure.*

Of course an immediate corollary to Theorem 3 is that a sufficient condition for pool-formation to be welfare reducing under fees, is that pools are welfare reducing under royalties;

$$(W_p < W_n | \mathbb{I} = 1) \Longrightarrow (W_p < W_n | \mathbb{I} = 0) \,, \quad W \in \{CS, \Pi, V\}. \tag{37}$$

We now consider when pooling might be of concern. An implication of Theorem 3 is that for wide areas of the parameter space pooling is the preferred structure of all market participants—from which, of course, it readily follows that total welfare is generally also greatest under a pooling structure. Indeed, it turns out that because of the strong distortions that independently-set royalty rates have on output, consumers unambiguously prefer the pool formation. This is so, independent of the degree or product differentiation and the amount of spillovers in development; and independent of the magnitude of spillover and differentiation effects. That is, consumer surplus is always strictly greater under a patent pool when licensing arrangements contain per-unit-of-output royalty rates. This can be viewed as a very robust extension of the Cournot-Shapiro argument concerning patent pools of perfectly complementary patents. Formally:

Theorem 4 *Given per-unit-of-output royalties, the pooling of perfectly complementary patents always generate an increase in consumer surplus,* i.e.,

$$CS_p > CS_n, \quad \forall \beta_n, \beta_p, \gamma_n, \gamma_p. \tag{38}$$

While there is an unambiguous finding for consumers, the picture is more nuanced for firms and, more importantly, in terms of the patent holders' interests as well. As was shown in the previous section, the differentiation effect makes pooling less attractive for firms (Proposition 4), and if spillovers are large then the spillover effect may also make pooling less profitable (Proposition 1). Analogous considerations exist for royalty-charging patent holders as well (see Propositions 5 and 2). Thus, it is typically the case that for either firms or patent holders to want to refrain from pooling, differentiation effects must be very strong. When this is the case, the aversion to pooling can then even be independent of spillover effects; as the following typical example illustrates.

Example 3 *Let* $\beta_n = 0.5$ *and* $\gamma_n = 0.5$*, that is, products are moderately differentiated and there are moderate spillovers in development. Now consider spillover and differentiation effects such that* $\beta_p \in [0.5, 1]$ *and* $\gamma_p \in (0.5, 0.9]$*, then there exist functions* $\mathcal{R}_\Pi$ *and* $\mathcal{R}_{V_1}$*, with* $\mathcal{R}_\Pi > \mathcal{R}_{V_1}$*, such that*

$$\Pi_p < \Pi_n, \qquad \forall \beta_p, \gamma_p \ni {}^{\beta_n}/_{\beta_p} < \mathcal{R}_\Pi \left({}^{\gamma_n}/_{\gamma_p}\right), \tag{39}$$

$$V_p < V_n, \qquad \forall \beta_p, \gamma_p \ni {}^{\beta_n}/_{\beta_p} < \mathcal{R}_V \left({}^{\gamma_n}/_{\gamma_p}\right). \tag{40}$$

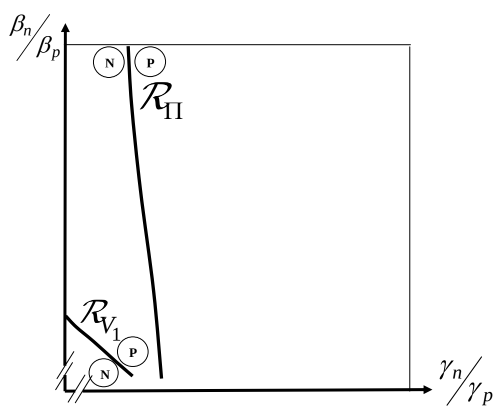

Figure 7: Unprofitable Pooling with Royalty Contracts

Despite the fact that patent holders, and even more so firms, may eschew a pool formation, total welfare is commonly larger with a pool, due to the increase in consumer surplus under a pool. However, the adverse effects of pooling on profits and royalty revenues may be large enough to overcome the advantages of pooling for consumers. This is only the case when the products are intrinsically highly differentiated, but there are very strong differentiation effects that result in goods becoming close substitutes for one another, as is illustrated in the following example.

Example 4 *Let* $\beta_n = 0.8$ *and* $\gamma_n = 0.1$*, that is, products are highly differentiated and there are large spillovers in development. Now consider spillover and differentiation effects such that* $\beta_p \in [0.8, 1]$ *and* $\gamma_p \in (0.1, 1]$*, then there exist* $\mathcal{R}_{TW}$*, such that*

$$TW_p < TW_n, \qquad \forall \beta_p, \gamma_p \ni {}^{\beta_n}/_{\beta_p} < \mathcal{R}_{TW}\left({}^{\gamma_n}/_{\gamma_p}\right). \tag{41}$$

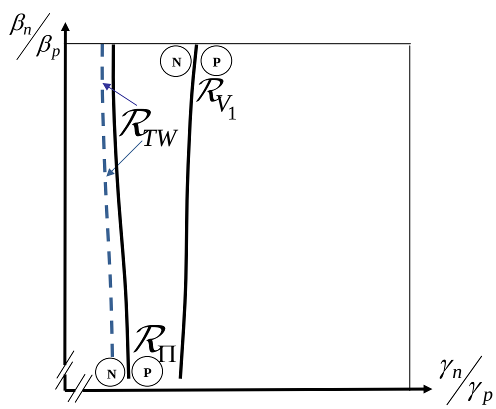

Figure 8: Reduction of Total Welfare due to Pooling with Royalties

The function $\mathcal{R}_{TW}$ from Example 4 is depicted as the blue dashed line in Figure 8. Note that the thresholds for desiring pooling from the firms' and the patent holders' perspectives are also depicted there, using analogous notation. It is worth noting that in contrast to Example 3 and Figure 7 it is now patent holders who more readily reject the pool formation compared to the firms.

An immediate and very important corollary to Theorems 4 and the examples is that the industry's desire to prefer pooling is a sufficient condition to guarantee that overall welfare is increased if a pool is formed. In these instances, then, a good policy guide would be to facilitate the industry's desire. This is in contrast to the findings of Example 2, further demonstrating that general implications concerning the welfare effects of pooling perfectly complementary patents are hard to establish.

6 Conclusion

In the contemporary debate about overcoming the so-called 'patent-thicket,' patent pooling is strongly advocated as a solution, provided that patents placed in the pool are complementary. We contend that this conventional wisdom—present in the academic literature, in policy circles, and antitrust practice—overlooks potential implications of pool formation for downstream product development and commercialization. In particular, largely missing from the debate on intellectual property rights reform is the impact of the transfer of embodied knowledge through either individual patents or the pooling of patents on the subsequent development and commercialization process.

We considered a model in which the pooling of perfectly complementary patents has three potential effects. First, it reduces distortions associated with the double-marginalization caused by royalty stacking. Second, because the pool may also serve as an information-sharing device in product development, the formation of a pool may increase spillovers in subsequent product development. And third, related to this, it may decrease the degree of product differentiation in the final product market.

The first point is generally viewed as the rationale for not only allowing, but actively encouraging patent pools to form for perfectly complementary patents; and the second aspect has also been cited as a strong reason to favor patent pools—in particular in biotechnology. However, we demonstrate that once the development incentives of the downstream firms are accounted for, patent pools—even for perfectly complementary patents—may, in fact, be welfare decreasing.

Nevertheless, there are also many constellations for which patent pools are beneficial. In particular, if consumer surplus is viewed as the relevant criterion for antitrust sanctioning of pools and royalties are paid on a per-unit-of-output basis, the pooling structure is always preferred to the non-pooling structure, regardless of the degree of spillovers and product differentiation and how pooling affects these.

However, when IP is licensed on an up-front fee basis, consumer surplus may be reduced under pooling. This happens, for instance, if products are relatively close substitutes and there are large spillovers in development, because free-riding in the development process lowers development efforts. In these cases firm profit and patent holders' revenues are also diminished under pooling, calling into question the unqualified advocacy for pooling—even when patents are perfectly complementary. Similarly, when using total welfare considerations, pooling is also detrimental when products are not close substitutes, but there are large differentiation effects, regardless of whether spillovers in development are affected by pooling.

A corollary of sorts to this observation has also emerged from our analysis. Thus, another encouraging finding is that, in many instances, a sufficient condition for total welfare to increase under pool formation is that patent holders prefer the pooling structure and therefore would seek it of their own volition. However, we have also been able to find important exceptions to this guide. Specifically, when products are close substitutes and spillovers are initially small, but become large due to pooling, then firms may benefit from reduced costs of effort at the development stage to the detriment of consumers.

In sum, we have found constellations in which even though industry desires to pool, consumer surplus (and even total welfare) is lower under a pool. Also, for the case of royalties, total welfare may decrease under pooling even without any spillover effects, provided that spillovers are already large, products are relatively close substitutes and there are differentiation effects from pooling. Finally, for the case of up-front fees, even minuscule spillover effects alone can decrease welfare when products are relatively similar and spillovers are large.

The model demonstrates that the welfare implications of pooling complementary patents is sensitive to industry specifics, and general policy recommendations based solely on the complementarity of patents ought to be avoided. Although the conventional wisdom may prevail in industries such as consumer electronics where spillovers and product differentiation are not affected by pooling; it may fail in industries such as biotech, where knowledge transfer creates spillover and differentiation effects tied to pooling.

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Appendix A: Derivations

Market Profit

The Bertrand-Nash equilibrium of this game yields:

$$P_i^* = \frac{\frac{(2-\gamma^2)A_i - \gamma A_j}{2+\gamma} + \mathbb{I}R}{2-\gamma}, \qquad i,j = 1,2;\ i \neq j. \tag{42}$$

Hence, since

$$A_i - P_i^* = \frac{\frac{2A_i - \gamma A_j}{2+\gamma} - \mathbb{I}R}{2-\gamma}, \tag{43}$$

substituting (43) into (3), one obtains

$$Q_i^* = \frac{(A_i - P_i^*) - \gamma(A_j - P_j^*)}{(1-\gamma)(1+\gamma)} = \frac{\frac{(2-\gamma^2)A_i - \gamma A_j}{2-\gamma^2-\gamma} - \mathbb{I}R}{(2-\gamma)(1+\gamma)}. \tag{44}$$

Note also that

$$P_i^* - \mathbb{I}R = \frac{(1-\gamma)\left(\frac{(2-\gamma^2)A_i - \gamma A_j}{2-\gamma^2-\gamma} - \mathbb{I}R\right)}{2-\gamma} \ \left(= \left(1-\gamma^2\right) Q_i^*\right); \tag{45}$$

So, from (44) and (45) one obtains profit of

$$\begin{aligned} \pi_i^*(A_i, A_j) &= (P_i^* - \mathbb{I}R)Q_i^* - (1-\mathbb{I})F = (1-\gamma^2)(Q_i^*)^2 - (1-\mathbb{I})F \\ &= \frac{(1-\gamma)\left(\frac{(2-\gamma^2)A_i - \gamma A_j}{2-\gamma^2-\gamma} - \mathbb{I}R\right)^2}{(2-\gamma)^2(1+\gamma)} - (1-\mathbb{I})F, \end{aligned} \tag{46}$$

which is (6).

Effort Equilibrium

Equation (7) has first-order condition

$$e_i^* = \frac{(1-\gamma)\left(a - \mathbb{I}R + \frac{\left(2-\gamma^2-\gamma\beta\right)e_i^* + \left(2\beta-\gamma^2\beta-\gamma\right)e_j}{2-\gamma^2-\gamma}\right)}{(2-\gamma)^2(1+\gamma)} \frac{2-\gamma^2-\gamma\beta}{2-\gamma^2-\gamma}; \tag{47}$$

or

$$e_i^* = \frac{a - \mathbb{I}R + \frac{\left(2-\gamma^2-\gamma\beta\right)e_i^* + \left(2\beta-\gamma^2\beta-\gamma\right)e_j}{2-\gamma^2-\gamma}}{(2-\gamma)^2(1+\gamma)} \frac{2-\gamma^2-\gamma\beta}{2+\gamma}. \tag{48}$$

This yields a best response function of

$$e_i^*(2-\gamma)^2(1+\gamma)(2+\gamma) = \left(a - \mathbb{I}R + \frac{(2-\gamma^2-\gamma\beta)\, e_i^* + (2\beta-\gamma^2\beta-\gamma)\, e_j}{2-\gamma^2-\gamma}\right)(2-\gamma^2-\gamma\beta); \tag{49}$$

or

$$e_i^*\frac{(2-\gamma)^2(1-\gamma^2)(2+\gamma)^2-(2-\gamma^2-\gamma\beta)^2}{2-\gamma^2-\gamma} = \left(a - \mathbb{I}R + \frac{(2\beta-\gamma^2\beta-\gamma)\, e_j}{2-\gamma^2-\gamma}\right)(2-\gamma^2-\gamma\beta); \tag{50}$$

or

$$e_i^*(e_j) = \left(a - \mathbb{I}R + \frac{(2\beta-\gamma^2\beta-\gamma)\, e_j}{2-\gamma^2-\gamma}\right)\frac{(2-\gamma^2-\gamma\beta)(2-\gamma^2-\gamma)}{(2-\gamma)^2(1-\gamma^2)(2+\gamma)^2-(2-\gamma^2-\gamma\beta)^2}. \tag{51}$$

Equilibrium Consumer and Producer Surplus

Substituting the equilibrium effort level (10) into the firm's payoff (7) yields

$$\Pi_i^* = (a-\mathbb{I}R)^2\frac{(2-\gamma)^2(1-\gamma^2)(2+\gamma)^2-(2-\gamma^2-\gamma\beta)^2}{[(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)]^2} - (1-\mathbb{I})F. \tag{52}$$

To derive consumer surplus in the market, we use the representative consumer's preferences that underlie the demand structure $U(Q_i, Q_j) = A_iQ_i + A_jQ_j - (Q_i^2 + 2\gamma Q_iQ_j + Q_j^2)/2$ (see Singh and Vives, 1984). For the symmetric equilibrium this reduces to

$$U^* = 2A^*Q^* + (1+\gamma)Q^{*2}. \tag{53}$$

Substituting (10) into (2) gives

$$A^*(e^*) = \frac{a(2-\gamma)^2(1+\gamma)(2+\gamma) - \mathbb{I}R(1+\beta)(2-\gamma^2-\gamma\beta)}{(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)}. \tag{54}$$

Further substitution into (42) yields

$$P^* = 1 - (a-\mathbb{I}R)\frac{(1+\gamma)(4-\gamma^2)}{(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)}, \tag{55}$$

and substituting (54) and (55) into (3), results in

$$Q^* = (a-\mathbb{I}R)\frac{4-\gamma^2}{(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)}. \tag{56}$$

Consumer surplus is gross utility minus expenditures, *i.e.*, $CS^* := U^* - P^*2Q^*$, so, using (53)–(56),

$$CS^* = (a-\mathbb{I}R)^2\frac{(2-\gamma)^2(1+\gamma)(2+\gamma)^2}{[(2-\gamma)^2(1+\gamma)(2+\gamma)-(1+\beta)(2-\gamma^2-\gamma\beta)]^2}. \tag{57}$$

Appendix B: Proofs

Proofs that are straightforward, or are implied by the discussion in the main text have been omitted.

Proof of Lemma 1 Equilibrium effort is given by (10). After taking the derivative, dropping the denominator and consolidating it follows that $\frac{de^*}{d\beta}$ carries the same sign as

$$-\gamma(2-\gamma)^2(1+\gamma)(2+\gamma)+(2-\gamma^2-\gamma\beta)^2. \tag{58}$$

Setting this equal to zero and solving for β yields

$$\mathcal{S}_e=\frac{2-\gamma^2-\sqrt{\gamma(2-\gamma)^2(1+\gamma)(2+\gamma)}}{\gamma}. \tag{59}$$

$\square$

Proof of Lemma 2 Beginning with (54), the proof follows *mutatis mutandis* that of the previous Lemma with

$$\mathcal{S}_A=\frac{2-\gamma-\gamma^2}{2\gamma}. \tag{60}$$

$\square$

Proof of Proposition 1 By assumption $V_{\mathbb{I}=0}(\cdot)$ is perfectly aligned with equilibrium market profit and hence $\frac{dV^*_{\mathbb{I}=0}}{d\beta}$ carries the same sign as $\frac{d\Pi^*}{d\beta}$. Equilibrium profit Π^* is given by (19). Applying the quotient rule in taking the derivative and dropping the denominator, it follows after some simplification that $\frac{d\Pi^*}{d\beta}$ carries the same sign as

$$\begin{aligned}\left[6+4\gamma+\beta^2\gamma-5\gamma^2-\gamma^3+\gamma^4-\beta\left(2-\gamma-\gamma^2\right)\right]\times\\ \Big[24-44\gamma^2+4\gamma^3+\beta^3\gamma^3+30\gamma^4-\gamma^5-9\gamma^6+\gamma^8-3\beta^2\gamma^2\left(2-\gamma^2\right)-\\ \beta\gamma\left(20+8\gamma-32\gamma^2-6\gamma^3+14\gamma^4+\gamma^5-2\gamma^6\right)\Big].\end{aligned} \tag{61}$$

The first factor can be written as

$$(2-2\beta)+\left(4-4\gamma^2\right)+\left(\gamma-\gamma^2\right)+\left(3\gamma-\gamma^3\right)+\gamma^4+\beta\left(\gamma^2+\gamma\right)+\beta^2\gamma, \tag{62}$$

which is clearly positive. Setting the second factor equal to zero and solving for β yields $\mathbb{S}_{V_0,\Pi}$ and the derivative properties follow. $\square$

Proof of Proposition 2 Taking the derivatives of (21) and (20) with respect to β when $\mathbb{I} = 1$ reveals that the sign is determined by the sign of $-\left(2-\gamma^2-\gamma\beta\right)-(1-\beta)\gamma$, hence $\mathcal{S}_{V_1,CS} = \mathcal{S}_A$. □

Proof of Lemma 3 Equilibrium effort is given by (10). After taking the derivative, dropping the denominator and consolidating it follows that $\frac{de^*}{d\gamma}$ carries the same sign as

$$-(2-\gamma)\left[2\left(2-\gamma-\gamma^2\right)+3\gamma^3+2\gamma^4+\beta\left(4+2\gamma+4\gamma^2+3\gamma^3\right)\right]. \tag{63}$$

Both factors are obviously positive so that the negative of their product is negative; which is also sufficient to prove the second statement. □

Proof of Proposition 4 As remarked in the proof to Proposition 1, $\frac{dV^*_{\mathbb{I}=0}}{d\gamma}$ carries the same sign as $\frac{d\Pi^*}{d\gamma}$. Applying the quotient rule in taking the derivative of (19) with respect to γ, it follows after some simplification that $\frac{d\Pi^*}{d\gamma}$ carries the same sign as

$$\begin{aligned}&\left[-2(\gamma-2)^2(1+\gamma)\right]\left[6+4\gamma+\beta^2\gamma-5\gamma^2-\gamma^3+\gamma^4-\beta\left(2-\gamma-\gamma^2\right)\right]\times\\&\left[12+10\gamma^3+2\gamma^4-3\gamma^5-\gamma^6+\beta\gamma(\gamma-2)(1+\gamma)^2+\beta^2\left(4+2\gamma+3\gamma^2+2\gamma^3\right)\right]\end{aligned} \tag{64}$$

Of the three factors it is straightforward to show that the first is negative and the third is positive. The middle factor is shown to be positive in the proof to Proposition 1, from which it follows that $\frac{d\Pi^*}{d\gamma} < 0$. □

Proof of Proposition 5 We undertake the same steps as in the proof to Proposition 2, but now take derivatives with respect to γ. From this it follows that $\frac{dV_{\mathbb{I}=\mathbb{1}}}{d\gamma}$ has the same sign as

$$-\left[16-28\gamma-8\gamma^2+16\gamma^3+\gamma^4-2\gamma^5+\beta(2+\gamma)^2+\beta^2(4+\gamma^2)\right]. \tag{65}$$

Setting this equal to 0 and solving for β yields

$$\mathcal{D}_{V_1} = \frac{-(2+\gamma)^2+(2-\gamma)\sqrt{-60(1-\gamma)+97\gamma^2+48\gamma^3+28\gamma^4+8\gamma^5}}{4+\gamma^2}. \tag{66}$$

Similarly one derives that $\frac{dCS}{d\gamma}$ has the same sign as

$$\begin{aligned}&\left[-\left(4-\gamma^2\right)\right]\left[6+4\gamma+\beta^2\gamma-5\gamma^2-\gamma^3+\gamma^4-\beta\left(2-\gamma-\gamma^2\right)\right]\times\\&\left[8-40\gamma-46\gamma^2+24\gamma^3+25\gamma^4-3\gamma^5-3\gamma^6+\right.\\&\left.\beta(2+\gamma)^2\left(4-\gamma+\gamma^2\right)+\beta^2\left(8+4\gamma+2\gamma^2+3\gamma^3\right)\right].\end{aligned} \tag{67}$$

Of the three factors it is straightforward to show that the first is negative and in the proof to Proposition 1 it is shown that the second is positive. Setting the third factor equal to zero and solving for β yields

$$\mathcal{D}_{CS} = \frac{(2-\gamma)\sqrt{\gamma\left(384+964\left(\gamma+\gamma^2\right)+669\gamma^3+454\gamma^4+205\gamma^5+36\gamma^6\right)}}{2\left(8+4\gamma+2\gamma^2+3\gamma^3\right)} + (2+\gamma)^2(4-\gamma-\gamma^2). \quad (68)$$

□

Proof of Theorem 2 It can be shown that $\mathcal{S}_{V_0,\Pi} > \mathcal{D}_{CS}$, whereupon the assertion follows immediately as a corollary to Propositions 1, 2, 4 and 5. □

Proof of Theorem 4 Upon setting $\gamma_p = 0.85$ and $\beta_p = 1$ Mathematica's FindInstance[$\{CS_p < CS_n, 0 < \gamma_n < 0.85, 0 < \beta_n < 1\}, \{\gamma_n, \beta_n\}$], shows that no such instance exists on the given domain. Since consumer surplus is concave, it then follows that the theorem holds for the entire domain. □

Proof of Proposition 3:

$\frac{d\mathrm{TW_0}^*}{d\beta}$ carries the same sign as

$(80-8(9\beta-2)\gamma-4(34+12\beta+3\beta^2)\gamma^2+2(\beta^3+40\beta-8)\gamma^3+(\beta^2+28\beta+78)\gamma^4+(7-30\beta)\gamma^5-4(5+\beta)\gamma^6+(4\beta-1)\gamma^7+2\gamma^8))$ = M

Set M = 0, $\beta = S_{TW_0}$

$$S_{TW_0} = \frac{1}{72\gamma^3}(-72\gamma^2(-2+\gamma^2)+(246^{2/3}(1-i\sqrt{3})(-2+\gamma)^2\gamma^4(-6-9\gamma-\gamma^2+3\gamma^3+\gamma^4))/D^{1/3}-66^{1/3}(1+i\sqrt{3})D^{1/3}$$

Where $D = 864\gamma^6+720\gamma^7-1296\gamma^8-792\gamma^9+774\gamma^{10}+261\gamma^{11}-198\gamma^{12}-27\gamma^{13}+18\gamma^{14}+\sqrt{3}\sqrt{((-2+\gamma)^6\gamma^{12}(2+3\gamma+\gamma^2)^3(-2970+81\gamma+2943\gamma^2-1044\gamma^4+128\gamma^6))}$

If $\beta \lesseqqgtr S_{TW_0}$, $\frac{d\mathrm{TW_0}^*}{d\beta} \gtreqqless 0$

$\frac{d\mathrm{TW_1}^*}{d\beta}$ carries the same sign as

$176+32\gamma-320\gamma^2-28\gamma^3+182\gamma^4+9\gamma^5-44\gamma^6-\gamma^7+4\gamma^8+2\beta^3\gamma^2(2\gamma^2+\gamma-8)+6\beta^2\gamma(8-6\gamma-6\gamma^2+2\gamma^3+4\gamma^4)+2\beta(-16-68\gamma-40\gamma^2+80\gamma^3+23\gamma^4-31\gamma^5-3\gamma^6+4\gamma^7)$ = N

Set N = 0, $\beta = S_{TW_1}$

$$S_{TW_1} = -\frac{8-6\gamma-6\gamma^2+2\gamma^3+\gamma^4}{\gamma(-8+\gamma+2\gamma^2)}-(-768\gamma^2+9792\gamma^3+4800\gamma^4-13536\gamma^5-3216\gamma^6+6468\gamma^7+756\gamma^8-1308\gamma^9-60\gamma^{10}+96\gamma^{11})/(32^{2/3}\gamma^2(-8+\gamma+2\gamma^2)E)^{\frac{1}{3}}+E^{1/3}$$

Where $E = 27648\gamma^4-262656\gamma^5-286848\gamma^6+454464\gamma^7+342144\gamma^8-331776\gamma^9-150120\gamma^{10}+117828\gamma^{11}+28296\gamma^{12}-19980\gamma^{13}-1944\gamma^{14}+1296\gamma^{15}+\sqrt{(4(-768\gamma^2+9792\gamma^3+4800\gamma^4-13536\gamma^5-3216\gamma^6+6468\gamma^7+756\gamma^8-1308\gamma^9-60\gamma^{10}+96\gamma^{11})^3+(27648\gamma^4-262656\gamma^5-286848\gamma^6+454464\gamma^7+342144\gamma^8-331776\gamma^9-150120\gamma^{10}+117828\gamma^{11}+28296\gamma^{12}-19980\gamma^{13}-1944\gamma^{14}+1296\gamma^{15})^2)}$

If $\beta \lesseqqgtr S_{TW_1}$, $\frac{d\mathrm{TW_1}^*}{d\beta} \gtreqqless 0$

Proof of Proposition 6:

$$\frac{d\mathrm{TW_0}^*}{d\gamma} = ((-2+\gamma)(112-24\gamma-180\gamma^2+82\gamma^3+130\gamma^4-37\gamma^5-37\gamma^6+5\gamma^7+4\gamma^8$$
$$+\beta^2(48+48\gamma+24\gamma^2+28\gamma^3-\gamma^4-8\gamma^5)+\beta(32+24\gamma-12\gamma^2+6\gamma^3$$
$$+25\gamma^4+5\gamma^5-4\gamma^6)))/(6+4\gamma+\beta^2\gamma-5\gamma^2-\gamma^3+\gamma^4+\beta(-2+\gamma+\gamma^2))^3$$

It carries the same sign as $-(112-24\gamma-180\gamma^2+82\gamma^3+56\gamma^4)$

Because $112-24\gamma-180\gamma^2+82\gamma^3+56\gamma^4>0$, , we can conclude that $\frac{d\mathrm{TW_0}^*}{d\gamma}<0,\ \forall\beta$.

$\frac{d\mathrm{TW_1}^*}{d\gamma}$ =$-(608-576\gamma-1296\gamma^2+1096\gamma^3+794\gamma^4-636\gamma^5-185\gamma^6+147\gamma^7+15\gamma^8-$
$12\gamma^9+4\beta^4\gamma(4+\gamma^2)+4\beta^3(-8+8\gamma+6\gamma^2+2\gamma^3+\gamma^4)+\beta^2(160+160\gamma-144\gamma^2+$
$20\gamma^3+34\gamma^4-15\gamma^5+4\gamma^6)+\beta(32+464\gamma-24\gamma^2-328\gamma^3+48\gamma^4+81\gamma^5-13\gamma^6-$
$4\gamma^7))/(9(6+4\gamma+\beta^2\gamma-5\gamma^2-\gamma^3+\gamma^4+\beta(-2+\gamma+\gamma^2))^3)$

= F/$(9(6+4\gamma+\beta^2\gamma-5\gamma^2-\gamma^3+\gamma^4+\beta(-2+\gamma+\gamma^2))^3)$=0

Set F = 0, $\beta = D_{TW_1}$=$\frac{1}{12}(-\frac{3B}{4\gamma+\gamma^3}-\sqrt{\frac{3}{2}}\sqrt{}(-\frac{1}{\gamma^2(4+\gamma^2)^2}(-6B^2+4C\gamma(4+\gamma^2)+(2\gamma(4+\gamma^2)$

$(C^2+12(-BD+4A\gamma(4+\gamma^2))))/(-216AB^2-C^3+18BCD+576AC\gamma-216D^2\gamma+$

$(-216AB^2-C^3+18BCD+576AC\gamma-216D^2\gamma+144AC\gamma^3-54D^2\gamma^3+\frac{1}{2}\sqrt{}(-4(C^2-$
$12BD+48A\gamma(4+\gamma^2))^3+4(C^3-18BCD+54D^2\gamma(4+\gamma^2)+72A(3B^2-2C\gamma(4+$
$\gamma^2)))^2))^{1/3}+2^{2/3}\gamma(4+\gamma^2)\,(-432AB^2-2C^3+36BCD+288AC\gamma(4+\gamma^2)-108D^2\gamma(4+$
$\gamma^2)+\sqrt{}(-4(C^2-12BD+48A\gamma(4+\gamma^2))^3+4(C^3-18BCD+54D^2\gamma(4+\gamma^2)+$
$72A(3B^2-2C\gamma(4+\gamma^2)))^2))^{1/3}))+6\sqrt{}(\frac{B^2}{2\gamma^2(4+\gamma^2)^2}-\frac{C}{12\gamma+3\gamma^3}+$
$(C^2+12(-BD+4A\gamma(4+\gamma^2)))/(62^{2/3}\gamma(4+\gamma^2)\,(-432AB^2-2C^3+36BCD+$
$288AC\gamma(4+\gamma^2)-108D^2\gamma(4+\gamma^2)+\sqrt{}(-4(C^2-12BD+48A\gamma(4+\gamma^2))^3+4(C^3-$
$18BCD+54D^2\gamma(4+\gamma^2)+72A(3B^2-2C\gamma(4+\gamma^2)))^2))^{1/3})+(-432AB^2-2C^3+36BCD$

$$+288AC\gamma(4+\gamma^2)-108D^2\gamma(4+\gamma^2)+\sqrt{}(-4(C^2-12BD+48A\gamma(4+\gamma^2))^3+4(C^3$$
$$-18BCD+54D^2\gamma(4+\gamma^2)+72A(3B^2-2C\gamma\,(4+\gamma^2)))^2))^{1/3}/(122^{1/3}\gamma(4+\gamma^2))$$
$$-(\sqrt{\frac{3}{2}}(-B^3+BC\gamma(4+\gamma^2)-2D\gamma^2(4+\gamma^2)^2))\Big/(\gamma^3(4+\gamma^2)^3\sqrt{}(-\frac{1}{\gamma^2(4+\gamma^2)^2}(-6B^2+4C\gamma(4+\gamma^2)+$$

$$(2\gamma(4+\gamma^2)(C^2+12(-BD+4A\gamma(4+\gamma^2))))/(-216AB^2-C^3+18BCD+576AC\gamma-216D^2\gamma$$

$$+144AC\gamma^3-54D^2\gamma^3+\frac{1}{2}\surd(-4(C^2-12BD+48A\gamma(4+\gamma^2))^3+4(C^3-18BCD+54D^2\gamma(4+\gamma^2)$$

$$+72A(3B^2-2C\gamma(4+\gamma^2)))^2))^{1/3}+2^{2/3}\gamma(4+\gamma^2)(-432AB^2-2C^3+36BCD+288AC\gamma(4+\gamma^2)$$

$$-108D^2\gamma(4+\gamma^2)+\surd(-4\left(C^2-12BD+48A\gamma(4+\gamma^2)\right)^3$$

$$+4(C^3-18BCD+54D^2\gamma(4+\gamma^2)+72A(3B^2-2C\gamma(4+\gamma^2)))^2))^{1/3})))))$$

Where

$A = 608 - 576\gamma - 1296\gamma^2 + 1096\gamma^3 + 794\gamma^4 - 636\gamma^5 - 185\gamma^6 + 147\gamma^7 + 15\gamma^8 - 12\gamma^9;$

$B = (-8 + 8\gamma + 6\gamma^2 + 2\gamma^3 + \gamma^4);$

$C = (160 + 160\gamma - 144\gamma^2 + 20\gamma^3 + 34\gamma^4 - 15\gamma^5 + 4\gamma^6);$

$D = (32 + 464\gamma - 24\gamma^2 - 328\gamma^3 + 48\gamma^4 + 81\gamma^5 - 13\gamma^6 - 4\gamma^7).$

And we have if $\beta \lesseqgtr D_{TW_1}, \frac{d\mathrm{TW_1}^*}{d\gamma} \gtreqless 0$