The First-Order Approach to Merger Analysis*

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Abstract

Using only information local to the pre-merger equilibrium, we derive approximations of the expected changes in prices and welfare generated by a merger. We extend the pricing pressure approach of recent work to allow for non-Bertrand conduct, adjusting the diversion ratio and incorporating the change in anticipated accommodation. To convert pricing pressures into quantitative estimates of price changes, we multiply them by the *merger pass-through matrix*, which is close under conditions we specify to the pre-merger rate at which cost increases are passed through to prices. Weighting the price changes by quantities gives the change in consumer surplus.

How should we predict the unilateral impact of a merger on prices and welfare? The United States and United Kingdom horizontal merger guidelines released last year incorporate an approach based on the work of Werden (1996), Farrell and Shapiro (2010a) and others that uses information local to pre-merger prices to indicate the directional impacts of the mergers. This "first-order" approach to merger analysis (FOAM) is admirable for adopting both the simplicity and transparency of approaches based on market definition (MD) and the firm grounding in formal economics of market simulations (MS). This paper takes this strategy a step further, attempting to incorporate the remaining strengths of FOAM's rivals: the quantitative precision of MS and the agnosticism about market conduct and cost structures embodied in MD. We show that FOAM, thus modified, provides a simple and general framework for predicting the impact on prices, as well as consumer and social surplus, of a merger based on information local to the pre-merger equilibrium.

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The logic of FOAM is intuitive: when companies A and B merge, company A (and similarly, B) has an additional opportunity cost of selling its products: it now internalizes the loss of profitable sales by company B that occurs when company A lowers its price. The per-unit magnitude of this opportunity cost is the value of the sales diverted from B for each (marginal) sale by A: the fraction of sales gained by A that are cannibalized from B (typically called the *diversion ratio*), multiplied by the profit-value of those sales (*firm B's mark-up*). This quantity, typically called "Upward Pricing Pressure" (UPP), is discussed explicitly in the new guidelines as being critical to determining merger effects; Werden (1996) and Farrell and Shapiro (2010a) advocate using thresholds for UPP to determine merger approval.[1]

However, some significant objections have been raised against the use of FOAM, in its current form, for evaluating mergers:

1. Coate and Simons (2009) object to its near-universal assumption of Nash-in-prices (Bertrand) competition and its reliance, in some settings, on constant marginal costs.
2. Schmalensee (2009) and Hausman et al. (2010) are skeptical of its assumption of default efficiencies and argue that providing only a directional indication of price effects is insufficient.
3. Carlton (2010) emphasizes the difficulty of applying the FOAM approach to mergers between multi-product firms.

While many of these critiques apply to one or all available alternative approaches, there is clearly room for improvement; this paper attempts to address these issues. We consider the most general oligopoly model we are aware of, in which firms have a single strategic variable per product, encompassing Bertrand, Cournot and most supply function equilibrium or conjectural variations models. From this we derive a generalized version of FOAM that predicts the impact of a merger on prices, and thus also on consumer surplus, profits and social surplus, based on information local to the pre-merger equilibrium. In particular, we prove that as long as the merger's effects on prices are small and the supply and demand system is sufficiently smooth (the cases in which quantitative analysis is most useful) its approximate impact on consumer surplus takes the form

$$\Delta CS \approx - \underbrace{\mathbf{g}^{\mathrm{T}}}_{\text{Generalized pricing pressure (GePP) vector}} \cdot \underbrace{\boldsymbol{\rho}^{\mathrm{T}}}_{\text{Merger pass-through matrix}} \cdot \underbrace{\mathbf{Q}}_{\text{Quantity vector}}. \tag{1}$$

The first term, $\mathbf{g}$, is the *Generalized Pricing Pressure* (GePP), which we develop in Section II. GePP, which generalizes UPP to allow for non-Bertrand conduct and general cost systems, is a vector that has zeros for all non-merging firms and, in the case when single

[1] As the US guidelines put it, "[T]he Agencies may seek to quantify the extent of direct competition between a product sold by one merging firm and a second product sold by the other merging firm by estimating the diversion ratio from the first product to the second product" (United States Department of Justice and Federal Trade Commission, 2010). Werden actually advocates the use of a modified version of UPP, the "compensating marginal cost reductions" that we discuss in Subsection VI.A below.

product firms 1 and 2 merge, a first entry of the form

$$g_1 = \underbrace{\overbrace{\tilde{D}_{12}}^{\text{Conjectured diversion ratio}} \cdot \overbrace{m_2}^{\text{Mark-up}}}_{\text{Generalized UPP}} - Q_1 \underbrace{\left(\overbrace{\frac{1}{\frac{d^M Q_1}{dP_1}}}^{\text{Post-merger (inverse) derivative of demand}} - \overbrace{\frac{1}{\frac{dQ_1}{dP_1}}}^{\text{Pre-merger}} \right)}_{\text{End of (accommodating) reactions}} \tag{2}$$

and an analogous second entry.[2] The first term in equation (2) generalizes the basic Bertrand UPP logic by replacing the Bertrand diversion ratio, D_{12} with the *conjectured diversion ratio* $\tilde{D}_{12}$. This is the diversion ratio from good 1 to good 2 (the fraction of a unit of good 2 that goes unsold when one more unit of good 1 is sold) when the impetus for the change in sales is a reduction in the price of good 1 *holding fixed the price of good 2* but *allowing all other prices to adjust as they are conjectured to* by the merged firm.[3] The price of good 2 is now held fixed because it has become, as a result of the merger, one of the quantities over which the merged firm optimizes. The second term in (2) is the quantity of good 1 multiplied by the change in the inverse of the slope of demand induced by the merger: now that the firms are merged, firm 1 no longer anticipates a reaction from firm 2 and thus expects the elasticity of its own demand to be higher (assuming accommodation pre-merger).

Anticipated accommodating reactions will have two effects. First, they will increase the (conjectured) diversion ratio, as they both reduce the number of sales lost by firm 1 and increase those gained by firm 2, whose price is held fixed. Second, they will increase the end of accommodating reactions (EAR) term as the larger are such reactions the more impact their end has on the elasticity of demand. Which of these effects dominates will depend on whether anticipated accommodation between the merging firms and other firms in the industry (first effect) or accommodation between the merging partners (second effect) is stronger. Thus the size of GePP may not differ as much across alternative conduct assumptions as it might at first appear.[4] Thus GePP under assumptions (such as consistent conjectures) that make identification easier can approximate GePP under other (possibly more realistic) assumptions.

The second term in equation (1), $\boldsymbol{\rho}$, is the *merger pass-through* matrix, the rate at which the changes in opportunity cost, the GePP, created by the merger are passed through to changes in prices. As we show in Section III, this quantity, which is a function of local second-order properties of the (conjectured) demand and cost system, converts GePP into a quantitative approximation of the price effects of the merger. In Section IV we argue that in many relevant cases merger pass-through is close to both pre-merger and post-merger pass-through, reconciling divergent strains in recent literature on the relevant pass-through rate. In some special cases, exact merger pass-through may be identified from pre-merger pass-through.

The calculated price changes for various goods can be put in a common denominator of consumer surplus by multiplying by the quantity vector $\mathbf{Q}$, as we discuss in Section V. A

[2]The general formula for multi-product firms is derived in Section II.

[3]We follow the convention in the literature of treating the diversion ratio for substitutes as a positive number – the negative of the ratio of the changes in quantities in the single-product firm case.

[4]See Section VI.B for two examples.

similar approach may be used to estimate social surplus impacts. Furthermore, this broad approach allows for the incorporation of impacts of mergers on consumer welfare not directly mediated by prices, such as changes in network size or product quality.

Section VI discusses an extension of our formula to allow marginal costs efficiencies and thus the calculation of "compensating marginal cost reductions" (Werden, 1996), as well exploring some salient special cases. Section VII discusses the practical implications of our work, including various assumptions that greatly simplify the calculations our formula requires, the comparison of our approach to MS and the stage of merger analysis at which we see our tools applying. At a theoretical level, our approach shows how changes discontinuous in one space (viz. market structure) but local in another (viz. pricing incentives) can be estimated by standard comparative statics techniques, as we emphasize in Subsection VI.D and our conclusion in Section VIII. A companion policy piece (Jaffe and Weyl, 2011) proposes a few potential reforms to the merger guidelines based on our analysis.

I Background

During the 1970's the "Chicago School" of law and economics, culminating in Posner (1976), played a leading role in the growing importance of formal economics in antitrust analysis. The 1982 U.S. Merger Guidelines (United States Department of Justice and Federal Trade Commission, 1982) reflected this growing influence in its move towards more detailed quantitative measures in the delineation of, and measurement of concentration within, antitrust product markets. These standards began with techniques based on market definition (MD) and Herfindahl (1950)-Hirschman (1945) Index (HHI) calculations; they were based on Stigler (1964)'s construction of a model in which the likelihood of collusion is mediated by HHI. However, emphasis during the late 1970's and 1980's on the differentiated nature of most product markets led to increasing concern (Werden, 1982) with the unilateral (non-cooperative) effects of mergers.[5] Farrell and Shapiro (1990) challenged the relationship between MD and the unilateral harms from mergers in the basic undifferentiated Cournot models.[6] Thus, many economists have argued for approaches to merger analysis based more explicitly on differentiated product models.[7]

To help supply this need, Werden and Froeb (1994) proposed a logit demand system, which made merger simulation (MS) techniques practical for policy analysis. During the 1990's merger simulation achieved widespread success in academic circles, exploiting the advances in techniques for demand estimation pioneered by Berry et al. (1995), and culminating in the seminal MS analysis of Nevo (2000). However, Shapiro (1996) and Crooke et al. (1999) argued that the effects of mergers predicted by simulations could differ by an

[5]Formal economic analysis of such unilateral effect began with the pioneering theoretical analysis of Werden (1982), Deneckere and Davidson (1985) and Levy and Reitzes (1992) in the context of differentiated products industries with symmetric firms and Farrell and Shapiro (1990) in homogeneous products industries with heterogeneous firms. Empirical work began with Baker and Bresnahan (1985)'s application of the Bresnahan (1981) consistent conjectures paradigm to the estimation of merger effects. The 1992 Guidelines' primary emphasis on unilateral effects official recognized this shifting focus.

[6]However, Werden (1991) provides an provocative counterpoint.

[7]See Kaplow (2010) for a good summary of these arguments.

order of magnitude or more based on properties of the curvature of demand not typically measured empirically.

To address this concern, Werden (1996) pioneered FOAM by arguing that the "compensating marginal cost reductions" necessary to offset the anticompetitive effects of a merger could be calculated from first-order properties of the demand system.[8] In particular, such efficiencies would have to offset the change in first-order conditions created by the new opportunity cost of a sale due to the diversion from a product of a merger partner. This approach is computationally simple and transparent. Additionally, Shapiro (1996) observed that, regardless of functional form, merger effects appeared to be increasing in this "value of diverted sales" that has come be known as "Upward Pricing Pressure" (UPP). Building on this work, antitrust officials in the United Kingdom, led by Peter Davis and Chris Walters, began to use UPP to evaluate mergers (Walters, 2007).

Froeb et al. (2005) noted that functional forms implying higher pass-through rates of merger efficiencies were closely connected to those generating large anticompetitive merger effects. They proposed an approach, based on Newton's method, for conducting merger simulations in a computationally simpler manner whose first iteration also only required information local to pre-merger prices. Building on the pioneering practical work in the UK and theoretical analysis of Froeb et al., Farrell and Shapiro (2010a,b) translated these ideas into intuitive and widely accessible economic terms; they argued that after subtracting efficiencies from UPP the sign would indicate the direction of merger effects and put forward the measurement of UPP as a practical policy proposal for the evaluation of mergers. Under the leadership of Farrell and Shapiro, UPP was incorporated into the 2010 Guidelines (United States Department of Justice and Federal Trade Commission, 2010) released this past fall. The UK followed close behind with an even more explicit incorporation of FOAM (Commission and of Fair Trading, 2010) and the European Union is considering revising its merger guidelines. The agencies' increased openness (Shapiro, 2010) to a range of simple tools with firm economic grounding has sharpened the focus on the appropriateness of FOAM as a policy proposal and its soundness as a theoretical construct.

II Generalized Pricing Pressure

In this section we develop our oligopoly model and formulae for the changes in incentives firms face post-merger. We first develop the concept in a general space of one-dimension-per-product, static strategies and formulate the Generalize Strategic Pressure. We then transition to treating price as the firms' choice variable, developing the multi-product version of the Generalized Pricing Pressure discussed in the introduction. Extensions of the model – the incorporation of cost efficiencies, an example of the effects of conduct assumptions, and specific examples of formula under Nash-in-prices, Nash-in-quantities, and consistent conjectures – are left to Section VI. Because most of the paper studies the multi-product case in which nearly all objects are multi-dimensional, we henceforth bold neither vectors

[8] As with the first-order approach to principal agent problems pioneered by Mirrlees (1971), FOAM seeks to analyze seemingly global and potentially discrete phenomena using local information. Our paper is similar to Rogerson (1985) in aiming to shore up the foundation for powerful but incompletely formalized techniques of previous work.

nor matrices.

A The general model

Consider a market with N firms denoted $i = (1, \ldots, n)$. Firm i produces m_i goods, and chooses a strategy vector $\sigma_i = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{im_i})$ from a m_i-dimensional strategy space. Thus – following Werden and Froeb (2008) – we allow only one strategic variable per-product.[9] The strategy space could be the space of product prices or product quantities or a different choice parameter, but we require that each combination of strategies $\sigma = (\sigma_1, \ldots, \sigma_n)$ generate a unique demand-equilibrium, defined as a vector of prices $P(\sigma) = (P_{11}, \ldots, P_{1m_1}, \ldots, P_{n1}, \ldots, P_{nm_n})$ and quantities $Q(P(\sigma)) = (Q_{11}, \ldots, Q_{1m_1}, \ldots, Q_{n1}, \ldots, Q_{nm_n})$. Uniqueness places some restriction on the possible space of strategies: if a firm's strategy specifies a twister rather than a shifter (Bresnahan, 1982) of a firm's supply function in the Klemperer and Meyer (1989) model, this will not tie down equilibrium prices and quantities.[10] We additionally require that the map from strategies to quantities be locally invertible and that the demand system be well-behaved and twice-differentiable.

To allow for the possibility of non-Nash equilibria, we permit firms to conjecture changes in other firms' strategies in response to changes in their own (in the spirit of Bowley, 1924).[11] A firm believes that when it changes its strategies, σ_i, its competitors will change their strategies, σ_{-i}, by $\frac{\partial \sigma_{-i}}{\partial \sigma_i}$.[12] Therefore, the total effect of a change in own strategies on a vector of interest is the sum of the direct (partial) effect and the indirect effect working through the effect on others' strategies: $\frac{dA}{d\sigma_i} \equiv \frac{\partial A}{\partial \sigma_i} + \left(\frac{\partial A}{\partial \sigma_{-i}}\right)^{\mathrm{T}} \frac{\partial \sigma_{-i}}{\partial \sigma_i}$. In the case of a Nash equilibrium in σ we have $\frac{dA}{d\sigma_i} = \frac{\partial A}{\partial \sigma_i}$ since $\frac{\partial \sigma_{-i}}{\partial \sigma_i} = 0$.

Pre-merger

Firm i's profit π_i depends on both its strategy vector and its competitors' strategies:

$$\pi_i = P_i(\sigma)^{\mathrm{T}} Q_i(P(\sigma)) - C_i(Q_i(P(\sigma))),$$

where C and Q are the (vector-valued) cost and demand functions and $P(\sigma)$ is the demand-equilibrium price vector generated by σ. For brevity we write P_i for $P_i(\sigma)$ and Q_i for $Q_i(P(\sigma))$ and mc_i for the vector of marginal costs. The firm's first-order conditions pre-merger can be

[9] We thus rule out changes in the non-price determining characteristics of products as considered in the literature on product repositioning (Mazzeo, 2002; Gandhi et al., 2008). See Section V for a discussion of how merger effects on non-price characteristics can be incorporated into our framework.

[10] While it may seem that there are reasonable cases in which strategies do not imply unique prices and quantities, such cases are rarely applied in the industrial organization literature. Multiple equilibria would not create a problem if the firms agreed (and were correct) on which equilibrium would occur.

[11] As we hint at in subsection B and show explicitly in Section VI.C, these conjectured responses can alternatively be used to reformulate a (differentiated) Nash-in-quantities equilibrium as a Nash-in-prices equilibrium.

[12] Throughout the paper we use the notation $\frac{\partial}{\partial}$ to refer to the Jacobian, $\frac{\partial A}{\partial B} \equiv \begin{pmatrix} \frac{\partial A_1}{\partial B_1} & \cdots & \frac{\partial A_1}{\partial B_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial A_n}{\partial B_1} & \cdots & \frac{\partial A_n}{\partial B_n} \end{pmatrix}$.

written as:

$$f_i(\sigma^*) \equiv \underbrace{-\left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}}\right)^{-1}\left(\frac{dP_i}{d\sigma_i}\right)^{\mathrm{T}} Q_i}_{\text{Generalized inverse hazard rate/Cournot distortion}} - \underbrace{(P_i - \mathrm{mc}_i)}_{\text{Mark-up}} = 0,$$

This formula is a natural generalization of the standard, single-product oligopoly first-order condition: the mark-up on each product is equated to the appropriate partial inverse hazard rate or Cournot distortion. Under single-product Nash-in-prices (N-i-p) conduct this is the partial inverse hazard rate of demand, $\frac{Q_i}{\frac{\partial Q_i}{\partial P_i}}$; for multi-product firms it generalizes to the inverse of the Slutsky matrix limited to the firm's products multiplied by that firm's quantities, $\left(\frac{\partial Q_i}{\partial P_i}^{\mathrm{T}}\right)^{-1} Q_i$. Under single-product Nash-in-quantities (N-i-q) conduct it is the partial slope of price multiplied by quantity, $\frac{\partial P_i}{\partial Q_i} Q_i$, which generalizes to that firm's portion of the matrix of derivatives of inverse demand multiplied by its quantities, $\frac{\partial P_i}{\partial Q_i}^{\mathrm{T}} Q_i$. Note that the N-i-p and N-i-q formulae differ only in what actions of *other* firms they hold fixed in the inversion and thus, as we discuss below, a firm playing N-i-q (or N-i-p) can think of itself as choosing either prices or quantities.

For general conduct, where strategies can be arbitrary, both the price and quantity matrices come into play. It is the matrix "ratio" of these that is the generalized version of the inverse quantity slope or direct price slope.

Incentives created by a merger

In studying the impact of a merger on firms' incentives, it is useful to define a generalization of the notion of a diversion ratio as employed in previous work on UPP and in the informal discussion in the introduction. In doing so we use the notation d^M to represent the "post-merger" total derivative, in which any within-merged-firm conjectures are taken to be zero. That is for any function A, after i and j merge, $\frac{d^M A_i}{d\sigma_i} = \frac{\partial A_i}{\partial \sigma_i} + \frac{\partial A_i}{\partial \sigma_{-ij}} \frac{\partial \sigma_{-ij}}{\partial \sigma_i}$. Then we can define the diversion ratio matrix for a pair of merging firms as

$$D^{\sigma}_{ij} \equiv -\left(\frac{d^M Q_i}{d\sigma_i}^{-1}\right)^{\mathrm{T}} \frac{d^M Q_j}{d\sigma_i}^{\mathrm{T}}.$$

That is, rather than simply being the ratio of the quantity gained by the former rival's products to that lost by one's own in response to a change in strategy, it is the ratio of these in the matrix sense. Furthermore, note that it is this matrix ratio *holding fixed the strategy of the merger partner* and *allowing all other strategies to adjust as they are expected to in equilibrium.* We include a superscript σ to indicate the strategy under which the diversion ratio is taken.

Definition 1. *Let a pre-merger equilibrium be defined by $f(\sigma^*) = 0$ and a post-merger equilibrium be defined by $h(\sigma^M) = 0$, where f and h are normalized to be quasi-linear in marginal cost (and price). Then we define $g \equiv h(\sigma^*) - f(\sigma^*)$ to be the* Generalized Strategic Pressure *(GeSP) on that strategy σ created by the merger.*

Thus GeSP is the change in the first-order condition at the pre-merger strategies. It *holds fixed the firms' strategy space and conjectures about other firms' reactions*, thus capturing only the unilateral effects of a merger. The value of GeSP is given in the following proposition.

Proposition 1. *The GeSP on firm i's strategy generated by a merger between firms i and j is*

$$g_i(\sigma) = D^{\sigma}_{ij}(P_j - mc_j) - \left(\frac{d^M Q_i}{d\sigma_i}^{-1}\right)^T \frac{d^M P_j}{d\sigma_i}^T Q_j - \Delta\left(\left(\frac{dQ_i}{d\sigma_i}^{-1}\right)^T \frac{dP_i}{d\sigma_i}^T\right) Q_i. \quad (3)$$

Here $\Delta(\cdot)$ denotes the change from pre- to post-merger value of its argument; the change is due to the merger partner's strategy no longer reacting.

Proof. See Appendix A.

The first and second terms of equation (3) are the changes in firm j's profits induced by a sale by firm i (caused by changing firm i's strategy). Post-merger firm i takes into account the effect of a change in it's strategies on the quantities (first term) and the prices (second term) of its merging partner's products. The last term is the change in firm i's marginal profit due to the end of accommodating reactions: once the firms have merged, the firm no longer anticipates an accommodating reaction from its merger partner.

B Prices as Strategies

In the previous two subsections, we have taken the firms' strategies and conjectures as given exogenously. However, if firms are using a strategy other than prices, then we can still think of the two merging firms as setting prices as long as the merging firms' strategies generate unique prices – no two strategy combinations generate the same set of prices. This, of course, requires that the map from strategies to prices be invertible.[13] Assuming this is true, we can always re-conceptualize the firm's problem as a choice of prices. A firm's conjectures as well as other firms' non-price choosing behavior can be viewed as jointly forming a conjecture on how other firms will adjust price. For example, if firms are actually choosing quantities, we can think of them as choosing prices and expecting the other firms to adjust their prices so as to keep their quantities fixed.[14] The advantage of this approach is that it has a clearer concordance with UPP and the quantitative changes in price that impact welfare. In this subsection we pursue this dual strategy.

If strategies are prices then the second term on the right hand side of equation (3) vanishes because firm i's prices do not change firm j's prices. GeSP simplifies to *Generalized Pricing*

[13]A standard condition to guarantee this is that $\sigma \in \mathbb{R}^{\sum_i m_i}$ and $\frac{\partial P}{\partial \sigma}$ is either globally a P-matrix (a matrix will all positive principal minors, see Hicks (1939)) or globally the negative of a P-matrix. While this may seem a strong condition, it is trivially satisfied in many contexts; for example, if the equilibrium is Cournot (Nash-in-quantities) and consumers have quasi-linear utility then this follows directly from the fact that the Slutsky conditions imply that the Slutsky matrix $\frac{\partial Q}{\partial P}$ and thus its inverse $\frac{\partial P}{\partial Q}$ is negative definite globally, as all negative definite matrices are the negative of P-matrices. Any other sufficient condition for invertibility would be equally suitable.

[14]See the Nash-in-quantities section of VI.C for a fleshing out of this example.

Pressure (GePP):

$$g_i(P) = \tilde{D}_{ij}(P_j - \mathrm{mc}_j) - \Delta\left(\left(\frac{dQ_i}{dP_i}^{-1}\right)^{\mathrm{T}}\right) Q_i.^{15}$$

Here, $\tilde{D}_{ij} \equiv D_{ij}^P$ is the diversion matrix *holding fixed the price of the merger partner* and allowing *all other firms' prices to adjust as they are expected to in equilibrium.* This diversion ratio is the general conjectures and matrix equivalent of the commonly used ratio of the derivatives of demand.

III Price Changes

GePP measures how much firm incentives shift when the firms merge. However, policy makers are typically interested in such shifts in incentives only insofar as they predict changes in prices. We extend the work of Chetty (2009) to show how a comparative static approach without a fully-estimated structural model can be used even to analyze structural changes such as mergers. If the change in incentives is small, the effect of a merger can be approximated the same way the effect of a tax would be, despite the fact that unlike with a tax we cannot imagine a merger "going to zero" to make our formula exact.

Our approach is simply to apply the appropriate envelope theorem, viewing the change in incentives created by the merger, g, as a vector of local changes in the equilibrium conditions; we then apply Taylor's Theorem for inverse functions to approximate the post-merger conditions around the pre-merger equilibrium. This, as with any comparative statics exercise, allows us both to derive an approximation to the effect of the merger based purely on local properties and get a bound on the error of the approximation based on the curvature and the size of the intervention. Theorem 1 gives our main result.

Theorem 1. *If f is the vector of first order conditions and g is the vector of GePPs so $\frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}$ is the Jacobian of the post-merger first-order condition and $(f+g)$ is invertible, then, to a first-order approximation,*

$$\Delta P = -\left(\frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}\right)^{-1}\Bigg|_{P^0} g(P^0).$$

Proof. Let $h(P) = f(P) + g(P)$, if P^0 is the pre-merger price, then we know $h(P^0) = g(P^0) \equiv r$. We want to find P^M (the post-merger price), such that $h(P^M) = 0$. If h is invertible, then

$$\begin{aligned} P^M - P^0 = h^{-1}(0) - h^{-1}(r) &= \frac{\partial h^{-1}}{\partial h}\Bigg|_r (0 - r) + O(\|r\|^2) \qquad (4) \\ &\approx -\left(\frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}\right)^{-1}\Bigg|_{P^0} g(P^0), \end{aligned}$$

which completes the proof.

[15]Note that in the single-product firm case this is exactly equation (2) from the introduction.

As we show in Appendix B, the ith entry of the error vector in equation (4) takes the form

$$E_i = -\frac{1}{2}\sum_j \left[\left(\frac{\partial h}{\partial P}\right)^{-1}\right]_{ij} g^{\mathrm{T}}(P_0)\left(\frac{\partial h^{\mathrm{T}}}{\partial P}\right)^{-1}\left(D_P^2 h_j\right)\left(\frac{\partial h}{\partial P}\right)^{-1} g(P_0),$$

where $[A]_{ij}$ indicates the ij element of matrix A, $D_P^2 h_j$ indicates the Hessian of h_j and the derivatives and Hessian are evaluated at some $P \in [P^0, P^M]$. This error is small whenever g is small and the first-order conditions are not highly curved in the relevant range.[16] Our approximation is equivalent, in the case of constant cost Bertrand conduct, to the first step of the Newton's method approach to merger simulation proposed by Froeb et al. (2005), though the justification is different.[17]

We think these conditions for accuracy are not too restrictive for two reasons: First, mergers leading to large changes in incentives will typically, though not always, lead to large changes in a common direction. If these are great enough to indicate via a local analysis that a large, likely detrimental effect will occur, then that is a strong basis for skepticism about the social desirability of a merger. In such circumstances, precise estimates of price changes are less important, and any quantitative approach relying on pre-merger data will struggle equally. Second, essentially all demand systems used in MS are very smooth (have sharply bounded curvature of equilibrium conditions) and thus our approximation is highly precise for a fairly large range of merger impacts if these demand systems are correct. Thus our approximation is likely to be robust in most cases over the class of common MS models.[18]

IV Pass-Through

Over the past decade an increasing informal consensus among economists interested in mergers has suggested that pass-through rates play an important role in determining the magnitude of merger effects. This section discusses the validity of these conjectures in light of our analysis above, as well as the practical implications for identification of these connections.

Shapiro (1996) and Crooke et al. (1999) showed that MS demand forms with differing curvature, but the same local matrix of cross-price elasticities, might lead to predictions of merger effects differing by an order of magnitude or more. Froeb et al. (2005) argued that the same assumptions about demand that tend to predict large pass-through of efficiencies also predict large anticompetitive merger effects, but did not emphasize whether it was the demand curvature or the pass-through itself that was crucial. They emphasized that post-merger pass-through rates, which they argued were relevant to the pass-through of efficiencies, might in principle differ greatly from pre-merger pass-through rates, though they do not provide examples of such divergences. Weyl and Fabinger (2009) and Farrell

[16]Since the curvature part of the error term is evaluated as some $\tilde{P} \in [P^0, P^M]$, the curvature of the post-merger first-order condition must be bounded over the range, not just at the pre- and post-merger equilibria.

[17]For example, the second step of their approach *does not* correspond to the second-order term that would be derived from our expansion, as theirs relies on non-local but first-order information while ours uses local, higher-order derivatives.

[18]The formula also helps clarify the recent debate over the relationship between MS and FOAM (Epstein and Rubinfeld, 2010; Farrell and Shapiro, 2010c).

and Shapiro (2010a) argued informally that because UPP is essentially the opportunity cost of sales created by the merger, multiplying it by the *pre-merger* pass-through rates should approximate merger effects. Farrell and Shapiro (2010b) and Kominers and Shapiro (2010) prove in the symmetric case that bounds on pre-merger pass-through, in conjunction with those on UPP, over the range between pre- and post-merger prices can be used to establish bounds on merger effects. However, it is not clear whether pass-through or demand curvature is the crucial quantity since they use a constant marginal cost framework under which the two are equivalent. In the following section we reconcile this apparent conflict between pre- and post-merger pass-through rates as the crucial quantities, and resolve the ambiguity between pass-through and demand curvature.

A Pre-merger, post-merger and merger pass-through

Marginal costs (and thus Marshallian specific taxes) enter quasi-linearly into the expression for f_i for an individual firm i. That is $f_i(P) = \tilde{f}_i(P) + \text{mc}_i(P)$ and thus if we were to impose on the firms a vector of Marshallian specific (quantity) taxes t, the post-tax (but pre-merger) equilibrium would be characterized by

$$f(P) + t = 0,$$

so that by the implicit function theorem

$$\frac{\partial P}{\partial t}\frac{\partial f}{\partial P} = -\mathcal{I}.$$

The *pre-merger pass-through matrix* is

$$\rho_{\leftarrow} \equiv \frac{\partial P}{\partial t} = -\left(\frac{\partial f}{\partial P}\right)^{-1}. \tag{5}$$

After the merger between firm i and firm j takes place, the marginal cost of producing good i enters quasi-linearly, with a coefficient of 1, into h_i, but also enters h_j quasi-linearly with a coefficient of $-\tilde{D}_{ji}$. This follows directly from the fact that following the merger, the GePP also enters h_j and includes the mark-up on good i which depends (negatively) on the specific tax applied to this good. Thus if we let

$$K = \begin{pmatrix} 1 & -\tilde{D}_{ij} \\ -\tilde{D}_{ji} & 1 \end{pmatrix},$$

then the post-merger and post-tax equilibrium is characterized by

$$h(P) = -Kt$$

and thus the *post-merger pass-through matrix* is[19]

$$\rho_{\rightarrow} \equiv \frac{\partial P}{\partial t} = -\left(\frac{\partial h}{\partial P}\right)^{-1} K. \tag{6}$$

[19]The term with $\frac{\partial K}{\partial P}t$ drops out because the tax is zero to begin with.

Our result from the previous section is that $P^M - P^0 \approx -\left(\frac{\partial h(P)}{\partial P}\right)^{-1} g\left(P^0\right)$. Thus, *merger pass-through* $-\left(\frac{\partial h}{\partial P}\right)^{-1}$ is not equal to pre-merger pass-through $-\left(\frac{\partial h}{\partial P} - \frac{\partial g}{\partial P}\right)^{-1} = -\left(\frac{\partial f}{\partial P}\right)^{-1}$ nor post-merger pass-through $-\left(\frac{\partial h}{\partial P}\right)^{-1} K$; rather it relies on the curvature of the latter and the cost structure of the former. This is intuitive since the post-merger first-order conditions are relevant, but the opportunity costs are not physical costs so they enter directly, rather than distributed as post-merger marginal costs.[20]

B Calculation and approximation of merger pass-through

Identification

When can we identify the merger pass-through from the pre-merger pass-through? Since $\frac{\partial f(P)}{\partial P}$ is equal to the negative inverse of the pass-through matrix, it is clearly calculable.[21] In the case of two firms merging under N-i-p equilibrium, the pass-through matrix, along with the first derivatives of demand, can be used to calculate $\frac{\partial^2 Q_i}{\partial P_i^2}$, $\frac{\partial^2 Q_i}{\partial P_i \partial P_j}$, $\frac{\partial^2 Q_j}{\partial P_j \partial P_i}$, and $\frac{\partial^2 Q_j}{\partial P_j^2}$. If one assumes Slutzky symmetry $\left(\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i}\right)$, then the other second derivatives are

$$\frac{\partial^2 Q_j}{\partial P_i^2} = \frac{\partial}{\partial P_i}\frac{\partial Q_j}{\partial P_i} = \frac{\partial}{\partial P_i}\frac{\partial Q_i}{\partial P_j} = \frac{\partial^2 Q_i}{\partial P_i \partial P_j}$$

and $\frac{\partial^2 Q_i}{\partial P_j^2} = \frac{\partial^2 Q_j}{\partial P_i \partial P_j}$, which are all that is needed to calculate $\frac{\partial g(P)}{\partial P}$. Since there is little intuition to be gained from the form of $\frac{\partial g(P)}{\partial P}$, we leave it to Appendix C. A similar procedure may be applied under N-i-q competition.

In the case of more than two merging firms, derivatives of the form $\frac{\partial^2 Q_i}{\partial P_j \partial P_k}$ are needed and cannot be calculated from observed pass-through rates and first derivatives unless one places restriction on the form of demand. A slightly more restrictive version of the horizontality assumption of Weyl and Fabinger (2009),

$$Q_i(P) = h\left(p_i + \sum_{j \neq i} f_j(P_j)\right),$$

is sufficient to calculate the necessary second partials. While it is only an approximation, and recent work suggests it may never be exactly consistent with discrete choice demand

[20]This connection between the merger pass-through and price changes is general, allowing for arbitrary conduct and cost functions. Thus, it appears that pass-through, rather than simply demand curvature, is a fundamental determinant of merger effects. In addition to aiding intuition, this is of practical relevance: pass-through rates are potentially observable, in the relevant market or in similar markets in the past. Moreover, Weyl and Fabinger (2009) provide a number of connections between pass-through rates and other potentially observable or intuitively meaningful quantities which may allow intelligent guesses about pass-through rates to be made on the basis of observable properties of industries. Though these connections apply only to pre- and post-merger pass-through, the following section discusses when and how pre-merger pass-through can be used to calculate or approximate merger pass-through.

[21]Note that because a block of an inverted matrix does not equal the inverse of the block, we also need pass-through rates of non-merging firms.

(Jaffe and Kominers, 2011), horizontality seems frequently to be a good approximation to demand in a discrete choice context (Gabaix et al., 2009; Quint, 2010).

In non-Nash equilibrium concepts, calculation becomes even more difficult. Consider the conjectural variation framework. The only way to avoid relying on firms' reports of what they conjecture $\frac{dP_{-i}}{dP_i}$ to be is to assume their conjectures are consistent along the lines of Bresnahan (1981) consistent conjectures. In that case, because there is no guarantee that Slutsky symmetry is satisfied by the relevant residual demand system, calculating the price changes requires a direct observation of the relevant second derivative of demand both when other prices adjust (which requires the derivative of the reaction function) and when they are held fixed. It is possible that a large number of instruments allowing for sufficient variation to identify these higher-order derivatives could be found, but it seems unlikely in practice.

Approximation

However, the difference between pre-merger and merger pass-through (and post-merger pass-through) may in fact be small. For our approximation to be valid, $g(P^0)$ and the curvature of the equilibrium conditions need to be jointly sufficiently "small". If $g(P^0)$ is small, then it seems likely that $\frac{\partial g(P^0)}{\partial P}$ would also be small and thus $\left(\frac{\partial h(P^0)}{\partial P}\right)^{-1}$ would be approximately $\left(\frac{\partial f(P^0)}{\partial P}\right)^{-1}$. If this were not the case, then while $g(P^0)$ is small, if $g(P)$ were evaluated at a relatively close price in the direction of maximal gradient rather than at P^0 it would then no longer be small. To the extent that the smallness of g is "fragile" in this sense, it is unlikely to form a solid basis for using first-order approximations.

Thus, in many cases when the first-order approximation would be valid, the merger pass-through is approximately equal to pre-merger pass-through. Furthermore, if small diversion ratios, rather than other factors, cause $g(P^0)$ to be small, then post-merger pass-through will also be close to merger pass-through as K will be close to the identity matrix. If a merger is likely to have a small impact on prices, then it is likely to have a small impact on pass-through rates and thus both pre- and post-merger pass-through rates will approximate merger pass-through. Of course, using merger pass-through is very likely to be more accurate than using pre- or post-merger pass-through. An extreme example of this effect is the undifferentiated limit of N-i-q competition, where $\frac{\partial g}{\partial P}$ becomes large even though g approaches 0 at the fragile symmetric point.

Nonetheless, the interpretation which views pre-, post- and merger pass-through as close to one another has a number of benefits. First, it is consistent with the apparent coincidence (Froeb et al., 2005) that demand forms that are known to give rise to high pre-merger pass-through rates also have been found to generate high pass-through of merger efficiencies (which are driven by post-merger pass-through) and large anti-competitive effects (which are proportional to merger pass-through). Second, it shows that the Froeb et al. and the Shapiro et al. logic are on some level consistent with one another: to the extent that either is valid as a way to approximate merger effects, they are likely to give similar answers. Finally, it shows that using intuitions about pass-through rates to approximate the rate at which GePP is passed through to prices may not be overly misguided.

V Welfare Changes

The changes in prices calculated in Section III can be converted into estimates of changes in consumer or social surplus. This is useful because we generally care about price changes only in so far as they affect welfare. This normative approach based on consumer or social surplus is concordant with a large body of economic and legal scholarship on the appropriate standards for antitrust policy. While there is still strong disagreement over whether consumer or social surplus is the appropriate standard to apply, there seems to be widespread agreement that *one of these two*, or some mixture of them, should be targeted (Farrell and Katz, 2006). Additionally, focusing on surplus allows for the analysis of mergers that affect multiple products where the changes in price may vary substantially. Also, to the extent that there is substantial uncertainty in the estimates of the relevant parameters, looking at welfare combines the confidence intervals (by plugging in different estimates to the forumulas) in the appropriate way to get the corresponding bounds on the metric that we ultimately care about.[22]

Consumer Surplus

First, consider consumer surplus in the evaluated market (ignoring externalities and potential cross-market effects of the price changes). To a first-order, the change in consumer surplus is, by the classic Jevons formula, just the sum across goods of the change in price times the quantity: $\Delta CS \approx -\Delta P^{\mathrm{T}} Q$.[23] It becomes unit-free, as with any other price index, if it is normalized by the initial value of the price index $P^{\mathrm{T}} Q$ yielding $\Delta I_{CS} \approx -\frac{\Delta P^{\mathrm{T}} Q}{P^{\mathrm{T}} Q}$.

Social Surplus

Estimating the change in social surplus requires an estimate for the expected change in quantity. Multiplying the Slutzky matrix $\frac{\partial Q}{\partial P}$ by the estimated price changes gives a first-order approximation for the change in quantity, $\widehat{\Delta Q} \approx \frac{\partial Q}{\partial P} \widehat{\Delta P}$.[24] Again ignoring externalities and out-of-market effects, the additional deadweight loss from the price increase is the sum of the change in quantities multiplied by the absolute mark-ups:

$$\Delta DWL \approx \Delta Q^{\mathrm{T}} (P - \mathrm{mc}) \approx \left(\frac{\partial Q}{\partial P} \widehat{\Delta P} \right)^{\mathrm{T}} (P - \mathrm{mc}).^{25}$$

The mark-ups can be pre-merger, post-merger or some combination of the two; various approaches, such as normalizing by the value of the market, construct unit-free indices. It would also be natural to include (as an additional term) an expected change in fixed (or more generally infra-marginal) costs due to the merger as in Williamson (1968).[26]

[22] We are grateful to Louis Kaplow for this point.

[23] Since we have calculated the first and second derivatives of Q, we could add higher order terms to this approximation, but since ΔP itself is an approximation that would be adding some second order terms and not others. Still, the formula may be evaluated at pre-merger (in the spirit of Laspeyres) or post-merger (Paasche) quantities or an arithmetic (Marshall-Edgeworth) or geometric (Fisher) average of the two.

[24] In many cases, such as consistent conjectures, the full Slutsky matrix is not necessary.

[25] Using the tax inclusive price includes tax revenue in social surplus in the spirit of Kaplow (2004).

[26] See Section VI.A below for a discussion of changes in marginal cost.

Profits

Our approach gives a simple approximation for the expected change in profits post-merger. While such are not typically an object of regulatory concern, an assumption that these must be positive by the firms' revealed preference for merging may provide some information.[27] If ΔF_i is the (presumably negative) change in firm i's fixed costs and $\Delta \mathrm{mc}_i$ is the (uniform) change inframarginal costs then

$$\Delta\pi_i \approx (\Delta P_i - \Delta \mathrm{mc}_i)^{\mathrm{T}} Q_i + (P - \mathrm{mc}_i)^{\mathrm{T}} \left(\frac{\partial Q}{\partial P} \widehat{\Delta P} \right) - \Delta F_i.$$

The incentive for firms i and j to merge is just $\Delta\pi_i + \Delta\pi_j$.

Advantages of Normative Analysis

Estimating a unified, normatively significant quantity, such as the impact on consumer welfare, may offer several benefits over simply estimating a group of price effects. First, while in some cases it is possible to find remedies addressing particular areas of concern without impacting others, often a package of impacts are inherently tied to one another and must be evaluated as a whole. Such issues are particularly severe in the (quite common) case of mergers between firms with a large number of products that while not identical are broadly thought to compete in the same market. It may frequently be the case that some of these products' prices are predicted to rise (or rise by a large amount) and others to fall (or rise only slightly) after a merger. When making a decision in such a case it is necessary to aggregate all relevant information. Such an aggregation requires some implicit or explicit normative standard; welfare criteria are the natural choice, intuitively putting the greatest weight on the products with the largest market.

Additionally, many of the potential benefits and harms of a merger may arise through channels different from or only indirectly related to a change in price. One example is consumption externalities (viz. network effects): in an industry with advertising-funded media, a primary harm from elevated prices to readers may be the reduction in the readership accessible to advertisers. Accounting for such harms requires a means of making them commensurate with typical price harms. Welfare-based standard makes this straightforward, as illustrated by White and Weyl (2011), who provide a simple extension of our formula to allow for network externality-based benefits and harms in a general framework. The comparative advantages of such a normative framework seem likely to only become greater when more complex effects – such as innovation, the dynamic price paths in an industry, and so forth – are taken into account. Such effects are typically considered entirely separately from simple price effects; in our framework, by contrast, it would be natural to simply extend the formula to include such effects and then to make various assumptions about them to simplify the analysis to the extent required by time constraints.

[27] We do not further explore this avenue here; another natural direction for future research to take such a formula is generalizing Deneckere and Davidson (1985)'s analysis of the incentives for a merger.

VI Extensions and Examples

A Marginal cost efficiencies

The GePP formula derived above assumes no cost effeciencies generated by the merger and as such can be seen as the baseline case. However, if estimates of expected efficiencies are available, they can easily be incorporated. If post-merger firm i's marginal costs are expected to be $\widetilde{\mathrm{mc}}_i$, then the GePP for firm i after a merger of firms i and j is

$$\tilde{g}_i(P) = \tilde{D}_{ij}(P_j - \widetilde{\mathrm{mc}}_j) - \Delta\left(\left(\frac{dQ_i}{dP_i}^{-1}\right)^{\mathrm{T}}\right) Q_i - (\mathrm{mc}_i - \widetilde{\mathrm{mc}}_i).$$

This adjusted GePP can be used in the calculation of consumer or social surplus effects or to calculate the generalized version of Werden (1996)'s "compensating marginal cost reductions." For the marginal cost reductions to counterbalance the other incentive effects and lead to no price change, it must be that

$$\begin{pmatrix} \tilde{g}_i(\sigma) \\ \tilde{g}_j(\sigma) \end{pmatrix} = 0,$$

which yields that compensating cost reductions of

$$\begin{pmatrix} e_i^\star \\ e_j^\star \end{pmatrix} \equiv \begin{pmatrix} \mathrm{mc}_i \\ \mathrm{mc}_j \end{pmatrix} - \begin{pmatrix} \widetilde{\mathrm{mc}}_i \\ \widetilde{\mathrm{mc}}_j \end{pmatrix} = \begin{pmatrix} \mathcal{I} & -\tilde{D}_{ij} \\ -\tilde{D}_{ji} & \mathcal{I} \end{pmatrix}^{-1} \begin{pmatrix} g_i(\sigma) \\ g_j(\sigma) \end{pmatrix}.$$

which simplifies to Werden's formula in the case of single-product Bertrand. Alternatively, if one wishes to apply the natural generalization of Farrell and Shapiro (2010a)'s more permissive standard, the off diagonal terms are ignored and the GePP itself is contrasted to (assumed default) efficiencies.

B How much does conduct matter?

A natural concern here, is that, especially in differentiated product industries, it may be difficult to determine empirically (Nevo, 1998) or even grasp intuitively what conduct is appropriate. While for many questions this is a serious worry, it may not be as severe a problem for merger analysis since, as we discuss in the introduction, changes in the conduct (or solution concept) may have offsetting effects in the two terms of GePP. With more accommodating behavior the increased diversion ratio pushes GePP in the opposite direction as the increased change from the end of the merging partner's accommodating reactions.

To illustrate this, we consider the role of conduct in two simple examples. First, a symmetric industry with n single-product firms playing a symmetric equilibrium, earning mark-up m, selling quantity q each, with an aggregate (Bertrand) diversion ratio D to the $n-1$ other firms in the industry. Each firm anticipates an increase in λ by all other firms in response to a one unit local increase in their own price.

Proposition 2. *In the symmetric example, the GePP from a merger of any two firms is*

$$D \cdot m \frac{1 + \tilde{\lambda}(n-3) - D(n-1)\frac{\tilde{\lambda}^2}{1-\tilde{\lambda}}}{\left(1 - D\tilde{\lambda}\right)(n-1) + D\tilde{\lambda}} \approx D \cdot m \frac{1 + \tilde{\lambda}(n-3)}{\left(1 - D\tilde{\lambda}\right)(n-1) + D\tilde{\lambda}}, \tag{7}$$

where $\tilde{\lambda} = \frac{\lambda}{1+\lambda}$ is the post-merger accommodation by the un-merged firms and the approximation is valid for small λ.

Proof. See Apprendix D.

In analyzing (7), we begin by focusing on the approximate formula. Note that $\tilde{\lambda}$ is strictly increasing in λ. When $n = 2$, we are considering a merger to monopoly, equation (7) is proportional to $1 - \tilde{\lambda}$, which is clearly decreasing in λ. That is, as discussed above, if accommodation by the merger partner is the only issue, GePP declines with the degree of accommodation as Farrell and Shapiro (2010a) conjecture. However, when $n = 3$ equation (7) is proportional to $\frac{1}{2-D\tilde{\lambda}}$ which is clearly increasing in λ. This effect gets stronger as $n \to \infty$; in the limit the expression is proportional to $\frac{\tilde{\lambda}}{1-D\tilde{\lambda}}$ which increases even more quickly in λ. Thus, in this basic example, "somewhere between" a merger to a monopoly and a merger by two firms within a triopoly the effect of accommodation on GePP switches from negative to positive. Using the precise rather than the approximate formula weights things further towards GePP decreasing with λ as it subtracts a term strictly increasing in λ.

Often, the two merging firms are closer competitors (and potential accommodators) with each other than with other firms in the industry. Therefore, we now consider a three firm model, with the two merging firms being symmetric but the third-firm being asymmetric, representing a reduced form for the rest of the industry. To keep things simple, though, we assume that the quantity of all firms (q) and all firms' (Bertrand) demand slopes are the same, but now we have two diversion ratios: d, the (Bertrand) diversion to and from the third firm from and to each of the two merger partners and δ, the diversion from each merger partner to the other. The mark-ups of the two merger partners are m. We assume that conjectures are in proportion to diversion: each merger partner anticipates an accommodating reaction of $\lambda\delta$ from its partner and λd from the third firm, while the third firm excepts λd from each of the merger partners.

Proposition 3. *In the three-firm example, GePP from a merger of the two close firms is*

$$m \frac{\delta + \tilde{\lambda}\left(d^2 - \delta^2\right) - \left(d^2 + \delta^2\right)\frac{\tilde{\lambda}^2}{1-\delta\tilde{\lambda}}}{1 - d^2\tilde{\lambda}} \approx m \frac{\delta + \tilde{\lambda}\left(d^2 - \delta^2\right)}{1 - d^2\tilde{\lambda}}, \tag{8}$$

where $\tilde{\lambda} \equiv \frac{\lambda}{1+\delta\lambda}$ and again the approximation is valid for small λ. Approximate GePP is thus increasing (decreasing) in λ if and only if d is greater (less) than $\frac{\delta}{\sqrt{1+\delta}}$. Approximate GePP is constant in λ if and only if $d = \frac{\delta}{\sqrt{1+\delta}}$. Precise GePP decreases in strictly more cases than does approximate GePP.

Proof. See Appendix D.

If the strength of the within-merger interaction is small compared to that outside the merger, GePP increases with anticipated accommodation. Conversely, if the strength of within-merger interaction is sufficiently greater than the total outside interaction then accommodation decreases GePP. Some relevant cases may be close to the point where the degree of accommodation anticipated has little effect. To the extent that the effect of conduct on GePP is not too large, our general formulation becomes particularly useful because solution concepts such as consistent conjectures are more identifiable than are those standardly applied, as discussed in the next subsection. Furthermore, it is reassuring for the theory of oligopoly that, even if the levels of prices may be quite sensitive to conduct, their comparative statics under interventions of interest may be less so.

C Specific equilibrium concepts

As further illustration, we now explore the model under a few common equilibrium concepts. The formulae for Nash-in-prices, Nash-in-quantities and consistent conjectures are below; we give an additional supply function example in Appendix E.

Nash-in-Prices

In the case of Nash-in-prices, the expected accommodating reactions are zero, so GePP trivially simplifies to the standard (multi-product) UPP formula:

$$g_i(P) = -\left(\frac{\partial Q_i}{\partial P_i}^{\mathrm{T}}\right)^{-1}\left(\frac{\partial Q_j}{\partial P_i}^{\mathrm{T}}\right)(P_j - \mathrm{mc}_j)$$

or with single product firms

$$g_i(P) = -\frac{\frac{\partial Q_j}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i}}(P_j - \mathrm{mc}_j).$$

Nash-in-Quantities

In a (differentiated products) Nash-in-quantities equilibrium each firm takes competitors' quantities as fixed. Instead of thinking of each firm as setting quantity, we can think of it as setting price with the expectation that other firms will adjust their prices so as to keep their quantities fixed. Using single-product firms for simplicity, pre-merger we have the first-order condition

$$Q_i + \frac{\partial Q_i}{\partial P}\frac{dP}{dP_i}(P_i - \mathrm{mc}_i) = 0.$$

We have $\frac{dP_i}{dP_i} = 1$ and can pin down $\frac{\partial P_{-i}}{\partial P_i}$ because

$$\frac{\partial Q_{-i}}{\partial P_i} + \frac{\partial Q_{-i}}{\partial P_{-i}}\frac{dP_{-i}}{dP_i} = 0,$$

which implies

$$\frac{dP_{-i}}{dP_i} = -\frac{\partial Q_{-i}}{\partial P_{-i}}^{-1}\frac{\partial Q_{-i}}{\partial P_i}.$$

This gives us a pre-merger condition of

$$f_i(P) = -(P_i - \mathrm{mc}_i) - \frac{Q_i}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-i}} \frac{\partial Q_{-i}}{\partial P_{-i}}^{-1} \frac{\partial Q_{-i}}{\partial P_i}} = 0.$$

After the firms merger, firm i starts taking firm j's price as given, so, following the same logic as above, the GePP is

$$g_i(P) = \underbrace{-\frac{\frac{\partial Q_j}{\partial P_i} - \frac{\partial Q_j}{\partial P_{-ij}} \frac{\partial Q_{-ij}}{\partial P_{-ij}}^{-1} \frac{\partial Q_{-ij}}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-ij}} \frac{\partial Q_{-ij}}{\partial P_{-ij}}^{-1} \frac{\partial Q_{-ij}}{\partial P_i}}}_{\text{Diversion Ratio}} (P_j - \mathrm{mc}_j)$$
$$\underbrace{- Q_i \left(\frac{1}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-ij}} \frac{\partial Q_{-ij}}{\partial P_{-ij}}^{-1} \frac{\partial Q_{-ij}}{\partial P_i}} - \frac{1}{\frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{-i}} \frac{\partial Q_{-i}}{\partial P_{-i}}^{-1} \frac{\partial Q_{-i}}{\partial P_i}} \right)}_{\text{End of Accomodating Reactions}}.$$

The limit as one approaches undifferentiated N-i-q competition demonstrates the importance of the assumption of the invertibility of the map from strategies to prices. In the undifferentiated case, the GePP is 0, but this is meaningless, because it is impossible to change the price of one firm holding fixed the other firm's price. Under no differentiation, one would need to apply the GeSP formula above when strategies are quantities and use the merger quantity pass-through (Weyl and Fabinger, 2009), but we do not pursue this further here.[28]

Consistent Conjectures

Bresnahan (1981) proposed a method for empirically tying down firms' beliefs about other firms' reaction to changes in their strategy (for example prices). He argued that firms' beliefs should be *consistent* with what actually occurs when they are induced, say by a cost shock, to change their price. More formally, if we consider the case of prices as strategies, firms' conjectures are said to be consistent if $\frac{dP^k}{dP^i} = \frac{dP^k}{dt^i}\left(\frac{dP^i}{dt^i}\right)^{-1}$ where $k \neq i$ is any other firm and t^i is a vector of specific quantity taxes on the m_i goods of firm i or, equivalently, any other shifter of only firm i's marginal cost vector.

There has been long debate about the attractiveness of consistent conjectures as a theoretical concept, both over how compelling it is as an idea (Dockner, 1992) and how predictive it is as a theoretical construct starting from primitives of supply and demand (Makowski, 1987). What is clear, however, both by example of its application (Baker and Bresnahan, 1985, 1988) and from theory (Weyl, 2009), is its pragmatic empirical benefits: it offers simple procedures for empirically tying down the relevant elasticities with fewer instruments than those needed under Bertrand competition. The following proposition provides, as far as we are aware, the first general formalization of this folk intuition in the merger context.

Proposition 4. *Suppose an exogenous vector of variables x of dimension $m_1 + m_2$ (the total number of products of the two merging firms) has the property that $\frac{\partial f_k}{\partial x} = 0$ for all $k \neq 1, 2$*

[28]See Farrell and Shapiro (1990) and Moresi (2010) for a related analyses.

while the matrix formed by $\begin{pmatrix} \frac{df_1}{dx} \\ \frac{df_2}{dx} \end{pmatrix}$ *is non-singular. Then observing* $\frac{dP_1}{dx}$, $\frac{dP_2}{dx}$, $\frac{dQ_1}{dx}$ *and* $\frac{dQ_2}{dx}$ *identifies* $\frac{dQ_1}{dP_1}$, $\frac{dQ_1}{dP_2}$, $\frac{dQ_2}{dP_1}$ *and* $\frac{dQ_2}{dP_2}$ *and thus* $\tilde{D}_{12}$, $\frac{d^M Q_1}{dP_1}$, $\frac{d^M Q_2}{dP_2}$ *and finally the Generalized Pricing Pressures,* g_1, g_2, *under consistent conjectures. The partial derivatives (i.e.* $\frac{\partial Q_1}{\partial P_2}$*) are not identified, so further variation is required to identify these parameters under N-i-p.*

Proof. See Appendix F.

This shows in a more abstract context the result that Baker and Bresnahan (1988) implicitly rely on in the case of linear demand: if conjectures are consistent then the relevant elasticity of demand for a single firm is that which would be observed in the data based on a cost shock to that firm alone. Under the N-i-p concept, in order to predict the behavior of even a single firm, enough instruments must be available to hold fixed all other firms' prices (as they do not in equilibrium stay fixed in response to a single-firm cost shock), leading to the classic curse of dimensionality (Ackerberg et al., 2007) in empirical industrial organization. Under consistent conjectures only shocks to the firms whose incentives one wishes to identify are necessary.

D Other applications

While our focus has been almost exclusively on merger analysis, some of our results and approach may apply to problems beyond this narrow context. Our formulation of oligopoly theory in terms of arbitrary conduct, conjectural variations and choice variables, each independent of one another, may have been known to some of the leading practitioners in industrial economics. Nevertheless, it is much more general than any formula we have actually seen applied formally in past work. Such a general formulation may be a useful starting point for other general work on oligopoly, as it may help clarify exactly which assumptions are needed for which conclusions.

Additionally, our approach to first-order approximation illustrates how local approximations may be used even in analyzing interventions that may at first blush seem discrete or discontinuous. Of course, this is valid only when the intervention is in some relevant sense small. However, there are many cases of interest, at least in industrial economics, when an intervention (such as the introduction of a new product or the entry of a new firm) may have only a small impact on consumer welfare and the prices of other products even though it may seem to constitute a discrete change. In these cases, our technique allows the sufficient statistics or first-order identification approach advocated by Chetty (2009) and Weyl (2009) to be applied more broadly than was originally envisioned.

VII Practical implications

In this section we discuss implications of our results for applied merger analysis. While we have highlighted ways in which our approach combines the benefits FOAM, MS and MD, this balancing clearly come at some cost: direct use of the formulae we derive, while conceptually simple, would require many more inputs than the simple calculation of UPP appears to. In this section, we illustrate how one might go about applying our formulae in practice.

A Simplifying the formula

While it seems that UPP is, in some sense, a simpler calculation than those we suggest, this is simply because a UPP-based calculation imposes simplifying assumptions. For example, if we were to assume all firms produced a single product, that conduct were Bertrand, that all cross-product pass-through rates were zero, then our formula would simplify to $\sum_i Q_i \rho_i UPP_i$, where ρ_i is the own-pass-through rate of each product.

Of course this is a very extreme example, but the general point is that beginning with our formula there are numerous simplifying assumptions one might make to reduce the complexity of the analysis. A few categories of assumptions one might consider are:

1. Pass-through: one could assume all cross pass-through rates (across firms and/or within particular products of a given firm) are zero so that we can ignore the impact of change in one merging firm's (opportunity) cost on the price of the other's product. One could impose symmetry on own- and cross- pass-through rates or, through an assumption akin to the horizontality assumption discussed in Subsection IV.B, assume some general relationship between pass-through rates and elasticities. Any of the assumptions discussed in Section IV above would aid in the identification of pass-through rates.

2. Heterogeneity: imposing some form of symmetry, either between the two merging firms, among all non-merging firms, between the merging and non-merging firms or all of the above would simpfly the equations. Or one could summarize all non-merging firms into a single firm, as in Subsection VI.B above. Any of these would greatly reduce the number of parameters to be estimated. Just the imposition of Slutsky symmetry across products would somewhat reduce the number of parameters.

3. Conduct: an assumption such as Bertrand, Cournot, consistent conjectures (which also aids in identification as shown in Subsection VI.C), would simplify implimentation.

Given time and judicial constraints, some potentially unattractive assumptions will inevitably be imposed. Certainly, the full force of our general formula is only likely to be used in exceptional cases. However, our general formulation allows easy selection and application of any combination of assumptions – it does not force all industries into one mold. Furthermore, it is easy to conduct GePP analysis under several combinations of assumptions, facilitating the comparison of the resulting conclusions and thus clarifying the exact role each of these assumptions plays. This makes much clearer the robustness (or weakness) of claims made in any special case.

B Comparison to merger simulation

It is instructive to compare the practical merits of a formula like ours to those of an explicit structural MS. While the greatest potential advantage of UPP over our approach is simplicity, the greatest potential advantage of MS over our approach is precision. We only *approximate* the effects of mergers, while MS provides a precise answer, up to any estimation error.

However, this increased precision is a direct result of the additional assumptions that MS requires. In our approach, if one were to assume a functional form for demand, that would generate all the higher order terms for the Taylor expansion and yield the same precise result as MS. In practice, these assumptions typically go further than tying down higher-order effects and actually restrict quantities, such as pass-through rates and elasticities (Crooke et al., 1999; Weyl and Fabinger, 2009).[29]

Thus MS is accurate in cases when the local information available prior to the merger is determinative of the predicted effect *and* the misspecification of functional forms does not overly restrict the implications of this local information. Our approximation is likely to be precise whenever the first of these conditions is satisfied. Furthermore, in any case where the local information *is* determinative (the effects are small), our results guarantee that our formula will closely agree with the predictions of MS, so long as the functional form assumptions used in MS are not misspecified. Furthermore, the robustness of conclusions derived from MS to differing functional form, cost-side, conduct and other assumptions can easily be examined without building a whole new computational model by simply changing some of the numbers that enter the relevant matrices. This is the sense in which our approach incorporates, and generalizes, the strengths of both MS and the traditional approaches to FOAM.

Of course, many well-worn approaches to estimation of the demand and cost parameters required by our formula invoke parametric demand structures, such as Hausman (1997)'s application of Deaton and Muellbauer (1980)'s Almost Ideal demand system and the influential characteristics-based models of Berry et al. (1995, 2004) and Berry and Pakes (2007). Our formula is entirely consistent and in fact highly complementary with such an approach to demand estimation, as the relevant elasticities and pass-through rates may simply be extracted from such a model and used in our formula without requiring re-computation of the equilibrium. However, our formula is also consistent with any other approach to obtaining these numbers, hence the estimates from many approaches, or ranges of confidence about such estimates derived from several approaches, may be easily combined together to generate a picture of the range of plausible outcomes.

C At which stage should our tools apply?

Merger review typically proceeds in stages, beginning with an initial screen, proceeding through a more thorough investigation if the screen indicates danger and, if no settlement can be reached, proceeding to a full court case. As Werden and Froeb (2011) emphasize, FOAM is typically touted as appropriate as an initial screen, with some value during an investigation, but inadequate for a thorough investigation or in-court proceedings where a detailed merger simulation will typically be more compelling.

An advantage of our approach is that it avoids a sharp distinction between these different phases. A version of the formula with many assumptions like those proposed in Subsection A above may be imposed initially to accommodate limited time and data. As more time and data become available these assumptions can gradually be relaxed and replaced with estimates from data or detailed intuitions. If network effects, product repositioning or other factors are thought to be important they may be incorporated into the analysis from the

[29] We understand that pass-through can be very difficult to measure, but we believe that using any information available or being explicit about what it is assumed to be is preferable to indirectly constraining it via functional form assumptions.

first stages (using extensions of our formula as described in V), initially in a highly restricted way and then, again, these restrictions may gradually be relaxed as the analysis progresses. Thus our approach aims to incorporate all of the standard stages of an analysis continuously into a unified framework.

VIII Conclusion

Our work provides a general modeling framework for the quantitative analysis of oligopoly behavior. It shows how such a framework may be analyzed using inversion techniques to study general conduct and illustrates how first-order approximations may be applied to apparently discontinuous events such as mergers. Perhaps most substantively, it provides the appropriate generalization of the notion of "Upward Pricing Pressure" that applies for general conduct and the appropriate pass-through rates that convert this "Generalized Pricing Pressure" into a quantitative approximation of merger price effects. Our primary aim is that this general formula will be directly useful in the formulation of future guidelines for merger evaluation and in the interpretation of those already in place.

We also hope to stimulate further work in this direction. Perhaps the simplest and most natural extension of our analysis would be to conduct a more broad ranging quantitative analysis of the accuracy of the first-order approximation for various demand and cost systems. Another step would be to consider an actual second-order approximation to the merger effect, with a focus on what variation would be needed to identify such an approximation and its intuitive interpretation.

On the more ambitious theoretical side, it will be important going forward to allow for dynamics: both to generalize our formula to allow for dynamic time paths of adjusting prices and to incorporate effects like entry and and product repositioning typically studied in dynamic contexts. Such an analysis might either proceed through an explicit dynamic model, which might be amenable to first-order analysis only with substantially new techniques, or through the application of some form of Marshallian long-run analysis, which might be more directly connected to our analysis here. In a similar spirit, we only consider the unilateral effects of a merger: the change in incentives *holding fixed the strategy space and conjectures.* It would be natural to add coordinated effects, changes in the strategy space and conjectures, using a more explicit model of dynamic coordination. The incorporation into our model of non-Jevons effects on consumer welfare, such as those arising when firms choose quality or prices affect network size, is an active area of research being pursued by White and Weyl (2011) and Gaudin and White (2011).

Empirical work oriented towards measuring pass-through rates and how they vary across markets will be crucial in helping to calibrate policymakers' intuitions about these important, but often difficult-to-measure parameters. Similarly, work on understanding the empirical relationship between pre-merger, post-merger and merger pass-through rates will be important. Such work will help policy makers determine reasonable simplifications that can safely be made to the general formulae without sacrificing too much accuracy. The formulation of such simplifications is central to making the work here directly relevant to the often severely time-constrained analysis of particular mergers.

Appendix

A Deriving GeSP

Proof of Proposition 1. Writing P_i for $P_i(\sigma)$ and Q_i for $Q_i(P(\sigma))$ for conciseness, the firm's first order conditions are

$$\left(\frac{\partial P_i}{\partial \sigma_i} + \frac{\partial P_i}{\partial \sigma_{-i}}^{\mathrm{T}} \frac{\partial \sigma_{-i}}{\partial \sigma_{\mathbf{i}}}\right)^{\mathrm{T}} Q_i + \left(\frac{\partial Q_i}{\partial \sigma_i} + \frac{\partial Q_i}{\partial \sigma_{-i}}^{\mathrm{T}} \frac{\partial \sigma_{-i}}{\partial \sigma_i}\right)(P_i - \mathrm{mc}_i(Q_i)) = 0.$$

Remembering that $\frac{dA}{dB_i} = \frac{\partial A}{\partial B_i} + \left(\frac{\partial A}{\partial B_{-i}}\right)^{\mathrm{T}} \frac{\partial B_{-i}}{\partial B_i}$, the matrix of full derivatives including the effects of other firms adjusting their strategies as expected, and then multiplying by $-\left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}}\right)^{-1}$ the firm's first-order conditions can be rewritten as:

$$f_i(\sigma) \equiv -\left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}}\right)^{-1} \frac{dP_i}{d\sigma_i}^{\mathrm{T}} Q_i - (P_i - \mathrm{mc}_i(Q_i)) = 0.$$

After a merger of firms i and j, the newly formed firm takes into account the effect of σ_i on π_j and no longer expects σ_j to react to σ_i since the two are chosen jointly. The merged firm's first-order derivatives with respect to σ_i can be written:

$$\begin{aligned}
&-(P_i - \mathrm{mc}_i(Q_i)) - \left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}} - \frac{\partial Q_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{-1} \left(\frac{dP_i}{d\sigma_i} - \frac{\partial P_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{\mathrm{T}} Q_i \\
&-\left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}} - \frac{\partial Q_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{-1} \left[\left(\frac{dP_j}{d\sigma_i} - \frac{\partial P_j}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{\mathrm{T}} Q_j + \left(\frac{dQ_j}{d\sigma_i} - \frac{\partial Q_j}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i})\right)^{\mathrm{T}} (P_j - \mathrm{mc}_j(Q_j))\right]
\end{aligned}$$

Subtracting $f_i(\sigma)$ from these first-order conditions gives the Generalized Pricing Pressure, $g(\sigma)$, so that post merger $f(\sigma_{ij}) + g(\sigma_{ij}) = 0$. This is given by:

$$\begin{aligned}
g_i(\sigma) = &-\left(\left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}} - \frac{\partial Q_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{-1} \left(\frac{dP_i}{d\sigma_i} - \frac{\partial P_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{\mathrm{T}} - \left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}}\right)^{-1} \frac{dP_i}{d\sigma_i}^{\mathrm{T}}\right) Q_i + \\
&-\left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}} - \frac{\partial Q_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{-1} \left(\left(\frac{dP_j}{d\sigma_i} - \frac{\partial P_j}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{\mathrm{T}} Q_j + \left(\frac{dQ_j}{d\sigma_i} - \frac{\partial Q_j}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)^{\mathrm{T}} (P_j - \mathrm{mc}_j(Q_j))\right).
\end{aligned}$$

Using the convention $\frac{d^M Q_i}{d\sigma_i} = \left(\frac{dQ_i}{d\sigma_i}^{\mathrm{T}} - \frac{\partial Q_i}{\partial \sigma_j}\frac{\partial \sigma_j}{\partial \sigma_i}\right)$ and similarly for price, we get the formulation in Proposition 1.

B Taylor Series Error Term

For notational convenience let $x = h^{-1}$. The error term is

$$\frac{1}{2}\begin{pmatrix} \sum_i \sum_j \frac{\partial^2 x_1}{\partial h_i \partial h_j} g_i(P^0) g_j(P^0) \\ \vdots \\ \sum_i \sum_j \frac{\partial^2 x_n}{\partial h_i \partial h_j} g_i(P^0) g_j(P^0) \end{pmatrix} = \frac{1}{2}\left[\sum_i \begin{pmatrix} \frac{\partial^2 x_1}{\partial h_i \partial h_1} & \cdots & \frac{\partial^2 x_1}{\partial h_i \partial h_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 x_n}{\partial h_i \partial h_1} & \cdots & \frac{\partial^2 x_n}{\partial h_i \partial h_n} \end{pmatrix} g_i(P^0)\right] g(P^0)$$

$$\equiv \frac{1}{2}\left[\sum_i \left(\frac{\partial^2 x}{\partial h_i \partial h} g_i(P^0)\right)\right] g(P^0). \tag{9}$$

We know $\frac{\partial x}{\partial h}\frac{\partial h}{\partial x} = \mathcal{I}$. Differentiating with respect to h_i gives:

$$\frac{\partial^2 x}{\partial h_i \partial h}\frac{\partial h}{\partial x} + \frac{\partial x}{\partial h}\begin{pmatrix} \sum_k \frac{\partial^2 h_1}{\partial x_i \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_1}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \\ \vdots & \ddots & \vdots \\ \sum_k \frac{\partial^2 h_n}{\partial x_i \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_n}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \end{pmatrix} = 0.$$

Solving for $\frac{\partial^2 x}{\partial h_i \partial h}$, using $\frac{\partial x}{\partial h} = \left(\frac{\partial h}{\partial x}\right)^{-1}$ and substituting into (9) gives

$$E = -\frac{1}{2}\sum_i \frac{\partial x}{\partial h}\begin{pmatrix} \sum_k \frac{\partial^2 h_1}{\partial x_1 \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_1}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \\ \vdots & \ddots & \vdots \\ \sum_k \frac{\partial^2 h_n}{\partial x_1 \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_n}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \end{pmatrix}\frac{\partial x}{\partial h} g_i\left(P_0\right) g\left(P_0\right).$$

If we look at just the ath entry of the vector, we have

$$\begin{aligned}
E &= -\frac{1}{2}\sum_i \sum_j \frac{\partial x_a}{\partial h_j}\begin{pmatrix} \sum_k \frac{\partial^2 h_j}{\partial x_1 \partial x_k}\frac{\partial x_k}{\partial h_i} & \cdots & \sum_k \frac{\partial^2 h_j}{\partial x_n \partial x_k}\frac{\partial x_k}{\partial h_i} \end{pmatrix}\frac{\partial x}{\partial h} g_i\left(P_0\right) g\left(P_0\right) \\
&= -\frac{1}{2}\sum_i \sum_j \frac{\partial x_a}{\partial h_j}\sum_k \sum_l \frac{\partial^2 h_j}{\partial x_l \partial x_k}\frac{\partial x_k}{\partial h_i}\frac{\partial x_l}{\partial h} g_i\left(P_0\right) g\left(P_0\right) \\
&= -\frac{1}{2}\sum_i \sum_j \sum_m \sum_k \sum_l \frac{\partial x_a}{\partial h_j}\left(\frac{\partial x_k}{\partial h_i}\frac{\partial^2 h_j}{\partial x_l \partial x_k}\frac{\partial x_l}{\partial h_m}\right) g_i\left(P_0\right) g_m\left(P_0\right) \\
&= -\frac{1}{2}\sum_i \sum_j \sum_m \frac{\partial x_a}{\partial h_j}\left(\frac{\partial x^T}{\partial h_i} D_x^2 h_j \frac{\partial x}{\partial h_m}\right) g_i\left(P_0\right) g_m\left(P_0\right) \\
&= -\frac{1}{2}\sum_j \sum_m \frac{\partial x_a}{\partial h_j}\left(\sum_i \frac{\partial x^T}{\partial h_i} g_i\left(P_0\right)\right) D_x^2 h_j \frac{\partial x}{\partial h_m} g_m\left(P_0\right) \\
&= -\frac{1}{2}\sum_j \sum_m \frac{\partial x_a}{\partial h_j} g^T\left(P_0\right)\left(\frac{\partial x}{\partial h}\right)^T D_x^2 h_j \frac{\partial x}{\partial h_m} g_m\left(P_0\right) \\
&= -\frac{1}{2}\sum_j \frac{\partial x_a}{\partial h_j} g^T\left(P_0\right)\left(\frac{\partial x}{\partial h}\right)^T D_x^2 h_j \frac{\partial x}{\partial h} g\left(P_0\right).
\end{aligned}$$

Where $D_x^2 h_j$ denotes the Hessian. Letting $[A]_{ij}$ indicate the ij element of matrix A,

$$E_a = -\frac{1}{2}\sum_j \left[\left(\frac{\partial h}{\partial x}\right)^{-1}\right]_{aj} g^T\left(P_0\right)\left(\left(\frac{\partial h}{\partial x}\right)^{\mathrm{T}}\right)^{-1}\left(D_x^2 h_j\right)\left(\frac{\partial h}{\partial x}\right)^{-1} g\left(P_0\right).$$

Where the Hessian and derivatives are evaluated at some price in $[P^0, P^M]$.

C Calculating $\frac{\partial g}{\partial P}$

In the case of single product firms in a Bertrand equilibrium, we know that

$$-\frac{\partial f(P)}{\partial P} = -\begin{pmatrix} 2-\frac{Q_i\frac{\partial^2 Q_i}{\partial P_i^2}}{(\frac{\partial Q_i}{\partial P_i})^2} & \frac{\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_i}-Q_i\frac{\partial^2 Q_i}{\partial P_i\partial P_j}}{(\frac{\partial Q_i}{\partial P_i})^2} \\ \frac{\frac{\partial Q_j}{\partial P_i}\frac{\partial Q_j}{\partial P_j}-Q_j\frac{\partial^2 Q_j}{\partial P_j\partial P_i}}{(\frac{\partial Q_j}{\partial P_j})^2} & 2-\frac{Q_j\frac{\partial^2 Q_j}{\partial P_j^2}}{(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix} = \rho^{-1} \equiv -\begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix}.$$

Also,

$$\frac{\partial g(P)}{\partial P} = \begin{pmatrix} -(P_j-C_j)\frac{\frac{\partial^2 Q_j}{\partial P_i^2}\frac{\partial Q_i}{\partial P_i}-\frac{\partial^2 Q_i}{\partial P_i^2}\frac{\partial Q_j}{\partial P_i}}{(\frac{\partial Q_i}{\partial P_i})^2} & -\frac{\frac{\partial Q_j}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i}}-(P_j-C_j)\frac{\frac{\partial^2 Q_j}{\partial P_i\partial P_j}\frac{\partial Q_i}{\partial P_i}-\frac{\partial^2 Q_i}{\partial P_i\partial P_j}\frac{\partial Q_j}{\partial P_i}}{(\frac{\partial Q_i}{\partial P_i})^2} \\ -\frac{\frac{\partial Q_i}{\partial P_j}}{\frac{\partial Q_j}{\partial P_j}}-(P_i-C_i)\frac{\frac{\partial^2 Q_i}{\partial P_j\partial P_i}\frac{\partial Q_j}{\partial P_j}-\frac{\partial^2 Q_j}{\partial P_j\partial P_i}\frac{\partial Q_i}{\partial P_j}}{(\frac{\partial Q_j}{\partial P_j})^2} & -(P_i-C_i)\frac{\frac{\partial^2 Q_i}{\partial P_j^2}\frac{\partial Q_j}{\partial P_j}-\frac{\partial^2 Q_j}{\partial P_j^2}\frac{\partial Q_i}{\partial P_j}}{(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix},$$

so, using Slutzky symmetry of $\frac{\partial^2 Q_j}{\partial P_i^2} = \frac{\partial^2 Q_i}{\partial P_i \partial P_j}$ and $\frac{\partial^2 Q_i}{\partial P_j^2} = \frac{\partial^2 Q_j}{\partial P_i \partial P_j}$, we have

$$\frac{\partial g(P)}{\partial P} = \begin{pmatrix} v_i & v_j \end{pmatrix},$$

where

$$v_i = \begin{pmatrix} -\frac{(P_j-C_j)(\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_i}-m_2(\frac{\partial Q_i}{\partial P_i})^2)\frac{\partial Q_i}{\partial P_i}-(\frac{\partial Q_i}{\partial P_i})^2(2-m_1)\frac{\partial Q_j}{\partial P_i}}{Q_i(\frac{\partial Q_i}{\partial P_i})^2} \\ -\frac{\frac{\partial Q_i}{\partial P_j}}{\frac{\partial Q_j}{\partial P_j}}-\frac{(P_i-C_i)((\frac{\partial Q_i}{\partial P_i}-m_2(\frac{\partial Q_i}{\partial P_i})^2)\frac{1}{Q_i}\frac{\partial Q_j}{\partial P_j}-(\frac{\partial Q_j}{\partial P_i}\frac{\partial Q_j}{\partial P_j}-m_3(\frac{\partial Q_j}{\partial P_j})^2)\frac{1}{Q_j}\frac{\partial Q_i}{\partial P_j}}{(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix},$$

$$v_j = \begin{pmatrix} -\frac{\frac{\partial Q_j}{\partial P_i}}{\frac{\partial Q_i}{\partial P_i}}-\frac{(P_j-C_j)((\frac{\partial Q_j}{\partial P_j}-m_3(\frac{\partial Q_j}{\partial P_j})^2)\frac{1}{Q_j}\frac{\partial Q_i}{\partial P_i}-(\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_i}-m_2(\frac{\partial Q_i}{\partial P_i})^2)\frac{1}{Q_i}\frac{\partial Q_j}{\partial P_i}}{(\frac{\partial Q_i}{\partial P_i})^2} \\ -\frac{(P_i-C_i)(\frac{\partial Q_j}{\partial P_i}\frac{\partial Q_j}{\partial P_j}-m_3(\frac{\partial Q_j}{\partial P_j})^2)\frac{\partial Q_j}{\partial P_j}-(\frac{\partial Q_j}{\partial P_j})^2(2-m_4)\frac{\partial Q_i}{\partial P_j}}{Q_j(\frac{\partial Q_j}{\partial P_j})^2} \end{pmatrix}.$$

D Conjectural variations examples

Proof of Proposition 2. The first-order condition for a single firm requires that

$$m = -\frac{q}{\frac{dQ^i}{dP^i}}.$$

Prior to the merger, by the variables we have set up and symmetry $\frac{dQ^i}{dP^i} = \frac{\partial Q^i}{\partial P^i} + (n-1)\lambda\frac{\partial Q^i}{\partial P^j}$, where partials represent Bertrand derivatives (elements of the Slutsky matrix). But the definition of aggregate diversion we gave and symmetry imply that $\frac{\partial Q^i}{\partial P^j} = -\frac{\partial Q^i}{\partial P^i}\frac{D}{n-1}$. Solving out we obtain

$$\frac{\partial Q^i}{\partial P^i} = -\frac{q}{m\,(1 - D\lambda)}.$$

Post-merger the price of the merger partner is held fixed rather than increasing by λ in response to an increase in the firm's price. By symmetry, therefore, the post-merger symmetric increase in the $n-2$ remaining firms' prices in response to an increase in one of the partners' prices, $\tilde{\lambda}$, must satisfy

$$\lambda = \tilde{\lambda} + \lambda\tilde{\lambda},$$

by the chain rule. Thus $\tilde{\lambda} = \frac{\lambda}{1+\lambda}$. With these quantities in hand, we can calculate the relevant post-merger derivatives.

First consider $\frac{d^M Q^1}{dP^1}$. This is composed of the direct Slutsky effect and the indirect effect from the change in the $n-2$ non-merging firm prices:

$$\frac{d^M Q^1}{dP^1} = \frac{\partial Q^i}{\partial P^i}\left(1 - \tilde{\lambda}\frac{n-2}{n-1}D\right),$$

while for the merger partner, firm 2, the sales gained are the direct diversion plus the indirect diversion from the $n-2$ non-merging firms' accommodation:

$$\frac{d^M Q^2}{dP^1} = -\frac{\partial Q^i}{\partial P^i}\frac{D}{n-1}\left(1 + \tilde{\lambda}\,[n-2]\right)$$

Thus,

$$\tilde{D}_{12}m = \frac{\frac{D}{n-1}\left(1 + \tilde{\lambda}\,[n-2]\right)}{1 - \tilde{\lambda}\frac{n-2}{n-1}D}m = \frac{D\left[1 + \tilde{\lambda}(n-2)\right]}{(n-1)\left(1 - \tilde{\lambda}D\right) + \tilde{\lambda}D}m,$$

while

$$Q_1\left(\frac{1}{-\frac{dQ^1}{dP^1}} - \frac{1}{-\frac{d^M Q^1}{dP^1}}\right) = q\left(\frac{1}{\frac{q}{m}} - \frac{1 - D\lambda}{\frac{q}{m}\left(1 - \tilde{\lambda}\frac{n-2}{n-1}D\right)}\right) = Dm\frac{\left(\lambda - \tilde{\lambda}\right)(n-1) + \tilde{\lambda}}{(n-1)\left(1 - \tilde{\lambda}D\right) + \tilde{\lambda}D};$$

where the last step follows by some tedious and thus omitted algebra. Subtracting these two terms yields the expression in the text given that $\lambda - \tilde{\lambda} = \frac{\tilde{\lambda}}{1-\tilde{\lambda}} - \tilde{\lambda} = \frac{\tilde{\lambda}^2}{1-\tilde{\lambda}}$.

It is easily verified that whenever the approximate formula is increasing in λ (and λ is positive) so is the exact expression, but that sometimes (when λ is large enough) the exact expression is decreasing in λ despite the approximate expression in the text being increasing. Thus the conclusion that GePP is may decrease in λ so long as the number of firms is small enough is stronger in the exact case than in the approximate case.

Proof of Proposition 3. Our proof here is almost entirely analogous to that of Proposition 2. The first-order condition now requires that for the merging firms

$$m = -\frac{q}{\frac{\partial Q^1}{\partial P^1}\left(1 - \left[d^2 + \delta^2\right]\lambda\right)},$$

so

$$\frac{\partial Q^1}{\partial P^1} = -\frac{q}{m\left(1 - \left[d^2 + \delta^2\right]\lambda\right)}.$$

On the other hand by the logic of conjectures discussed in the proof of Proposition 2, if l represents the pre-merger merging-firm-to-non-merging-firm conjecture, L represents the same between the merging firms and $\tilde{l}$ represents the post-merger version of l then

$$l = \tilde{l}\,(1 + L) \iff \tilde{l} = \frac{l}{1 + L}.$$

Plugging in our definitions of $l = d\lambda$ and $L = \delta\lambda$ we obtain

$$\tilde{l} = \frac{d\lambda}{1 + \delta\lambda}.$$

Now we can compute

$$\frac{d^M Q^1}{dP^1} = \frac{\partial Q^1}{\partial P^1}\left(1 - \tilde{l}d\right) = -\frac{q\left(1 - d^2\tilde{\lambda}\right)}{m\left(1 - \left[d^2 + \delta^2\right]\lambda\right)}$$

and

$$\frac{d^M Q^2}{dP^1} = -\frac{\partial Q^1}{\partial P^1}\left(\delta + \tilde{l}d\right) = \frac{q\left(\delta + d^2\tilde{\lambda}\right)}{m\left(1 - \left[d^2 + \delta^2\right]\lambda\right)},$$

so that

$$\tilde{D}_{12} = \frac{\delta + d^2\tilde{\lambda}}{1 - d^2\tilde{\lambda}},$$

while

$$Q_1\left(\frac{1}{-\frac{dQ^1}{dP^1}} - \frac{1}{-\frac{d^M Q^1}{dP^1}}\right) = m\left(1 - \frac{1 - \left[d^2 + \delta^2\right]\lambda}{1 - d^2\tilde{\lambda}}\right) = m\frac{d^2\tilde{\lambda} - \left(d^2 + \delta^2\right)\lambda}{1 - d^2\tilde{\lambda}}.$$

And thus, with a little algebra, subtraction yields the formula in the text given that $\tilde{\lambda} - \lambda = \frac{\tilde{\lambda}^2}{1 - \delta\tilde{\lambda}}$. Again, it can easily be verified, as before, that the more sophisticated formula is decreasing in λ whenever the simpler version is, but also decreases in some cases (for larger λ) when the simpler version does not.

Returning to the simpler formula and taking the derivative with respect to λ yields and expression proportional to

$$\left(d^2 - \delta^2\right)\left(1 - d^2\lambda\right) + d^2\left(\delta + \lambda\left[d^2 - \delta^2\right]\right) = d^2\left(1 + \delta\right) - \delta^2,$$

which is clearly positive or negative depending on the sign of the inequality in the proposition.

E GePP with Supply Functions

To show that the analysis is not limited to prices or quantities as strategies, we briefly outline the formulation when a firm's strategy is a supply function. Take a case of single product firms, each of whom chooses a linear supply function $P_i = \sigma_i Q_i$. Rotating the supply curve traces out the residual demand curve so we have $\frac{\partial P_i}{\partial \sigma_i} = \frac{\partial P_i}{\partial Q_i}\frac{\partial Q_i}{\partial \sigma_i}$ which gives us:

$$\begin{aligned} f(\sigma_i) &= -\frac{\partial Q_i(P(\sigma))}{\partial \sigma_i}^{-1}\frac{\partial P_i}{\partial \sigma_i}Q_i(P(\sigma)) - (P_i(\sigma) - \mathrm{mc}_i(Q_i(P(\sigma)))) \\ &= -\frac{\partial P_i}{\partial Q_i}Q_i(P(\sigma)) - (P_i(\sigma) - \mathrm{mc}_i(Q_i(P(\sigma)))). \end{aligned}$$

The pricing pressure is:

$$-\frac{\partial Q_j}{\partial P_i}\frac{\partial P_i}{\partial Q_i}\left(\frac{\partial P_j}{\partial Q_j}Q_j(\sigma) + (P_j(\sigma) - \mathrm{mc}_j(\sigma))\right).$$

F Consistent conjectures

Proof of Proposition 4. Equilibrium is given by $f(P, x) = 0$ so by the implicit function theorem

$$\frac{dP}{dx} = -\left(\frac{\partial f}{\partial P}\right)^{-1}\begin{pmatrix} \frac{df_1}{dx} \\ \frac{df_2}{dx} \\ 0 \end{pmatrix}$$

and thus by the chain rule

$$\frac{dQ}{dx} = -\frac{\partial Q}{\partial P}\left(\frac{\partial f}{\partial P}\right)^{-1}\frac{\partial f}{\partial x}$$

where $\frac{\partial f}{\partial x} \equiv \begin{pmatrix} \frac{df_1}{dx} \\ \frac{df_2}{dx} \\ 0 \end{pmatrix}$.

However note that, as discussed in Section IV of the text, if t is a vector of specific taxes on each of the goods

$$\frac{dP}{dt} = -\left(\frac{\partial f}{\partial P}\right)^{-1} \quad \text{and} \quad \frac{\partial Q}{\partial t} = -\frac{\partial Q}{\partial P}\left(\frac{\partial f}{\partial P}\right)^{-1}.$$

Thus, using the above formulae and letting the subscript on a vector 12 denote the sub-vector corresponding to the first two firms' entries and for a matrix consist of the principal submatrix formed by those to firms' row-column pairs,

$$\left(\frac{dP_{12}}{dt_{12}}\right)^{-1}\frac{dP}{dt_{12}} = \left(\frac{dP_{12}}{dx}\right)^{-1}\frac{dP}{dx},$$

where invertibility follows from the non-singularity of $\begin{pmatrix} \frac{df_1}{dx} \\ \frac{df_2}{dx} \end{pmatrix}$ and $\frac{\partial f}{\partial P}$. Thus by the definition of consistent conjectures we have that

$$\frac{dP}{dP_{12}} = \frac{dP}{dx}\left(\frac{dP_{12}}{dx}\right)^{-1}.$$

We wish to solve for

$$\frac{dQ}{dP_{12}} = \left(\frac{\partial Q}{\partial P}\frac{dP}{dP_{12}}\right)_{12} = \left(\frac{\partial Q}{\partial P}\frac{dP}{dx}\left(\frac{dP_{12}}{dx}\right)^{-1}\right)_{12},$$

but from the chain rule we have that $\frac{\partial Q}{\partial P}\frac{dP}{dx} = \frac{dQ}{dx}$ and thus

$$\frac{dQ}{dP_{12}} = \left(\frac{dQ}{dx}\left(\frac{dP_{12}}{dx}\right)^{-1}\right)_{12}.$$

Breaking this up by row blocks yields $\frac{dQ}{dP_1}$ and similarly $\frac{dQ}{dP_2}$; breaking these resultant matrices down by columns yields the individual effects on Q_1 and Q_2. From this the desired pre-merger quantities are extracted and the post-merger quantities calculated as in the text. For example:

$$\frac{d^M Q_1}{dP_1} = \frac{dQ_1}{dP_1} - \frac{dQ_1}{dP_2}\frac{dP_2}{dP_1}.$$

To establish that the result fails under N-i-p consider the simple linear demand system with N firms given by

$$Q = \alpha - \beta P$$

where α is an N-dimensional vector, β is an N-dimensional matrix with $N + \frac{(N-1)N}{2} = \frac{N(N+1)}{2}$ independent dimensions assuming Slutsky symmetry. Assume a constant marginal cost system c. Let D_β be the diagonal matrix with the same diagonal entries as β. Then with single-product firms

$$f = P - c - D_\beta^{-1}Q = P - c - D_\beta^{-1}\alpha + D_\beta^{-1}\beta P = \left[I + D_\beta^{-1}\beta\right]P - c - D_\beta^{-1}\alpha.$$

Taking c_1 and c_2 as the exogenous variables for simplicity we can solve for equilibrium prices

$$P = \left[I + D_\beta^{-1}\beta\right]^{-1}c + \left[I + D_\beta^{-1}\beta\right]^{-1}D_\beta^{-1}\alpha = \left[I + D_\beta^{-1}\beta\right]^{-1}c + \left[D_\beta + \beta\right]^{-1}\alpha$$

and quantities

$$Q = \left[I - \beta\left[D_\beta + \beta\right]^{-1}\right]\alpha - \beta\left[I + D_\beta^{-1}\beta\right]^{-1}c.$$

Thus observing $\frac{dQ}{dc_{12}}$ and $\frac{dP}{dc_{12}}$ reveals the first two columns of $[I + D_\beta\beta]^{-1}$ and $\beta\left[I + D_\beta^{-1}\beta\right]^{-1} = \left[\beta^{-1} + D_\beta^{-1}\right]^{-1}$. This places $4N$ linear restrictions on β, but does not directly reveal the first two columns of β, which we wish to obtain, since under N-i-p $\frac{dQ}{dP} = -\beta$. For $N \geq 8$, $4N < \frac{N(N+1)}{2}$ and thus the rank conditions for identification fail. Thus, in general under N-i-p identification does not hold. Note that typically, 8 firms are not necessary to ensure failure of N-i-p identification: it is only in this special linear case. More broadly, flexibility of second-order effects can cause failure even with only three firms.

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