The First-Order Approach to Merger Analysis

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Recently there has been a lot of work in what we refer to as “the first-order approach to merger analysis” (FOAM)

- Werden (1996): uses only information local to the pre-merger equilibrium to calculate the hypothetical efficiency gains necessary to offset the pressure to increase prices
- Farrell and Shapiro (2010): develop *Upward Pricing Pressure* (UPP) that uses pre-merger information and efficiency credits to predict whether a merger will increase prices
- Implemented by Davis (in UK) and Farrell and Shapiro (in US)
Our Contribution

1. Generalize UPP to “GePP”
   - allow for non-pricing conduct and non-Nash equilibrium
   - generalize the diversion ratio
   - add an “end of accommodating reaction” term

Jaffe and Weyl (2011)

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   - an intuitive combination of pre- and post-merger pass-through
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3. Combine price changes into an aggregate metric of the changing consumer surplus
   - weight by quantities

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The General Model

An industry with multi-product firms and generic strategies

- Each firm \( i = 1, \ldots, n \) has \( m_i \) products
- It chooses strategies \( \sigma_i = (\sigma_{i1}, \ldots, \sigma_{im_i}) \)
- It has quantity and price vectors: \( Q_i \) and \( P_i \)
- Its profit function is:

\[
\pi_i = P_i(\sigma)^T Q_i(P(\sigma)) - C_i(Q_i(P(\sigma))),
\]

- Firms can conjecture other firms’ reactions to changes in their strategies
  - We denote the total derivative \( \frac{dA}{d\sigma_i} \equiv \frac{\partial A}{\partial \sigma_i} + \left( \frac{\partial A}{\partial \sigma_{-i}} \right)^T \frac{\partial \sigma_{-i}}{\partial \sigma_i} \)
Prices as Strategies

Comparison to UPP and estimation of price changes are easier if we think of firms as setting prices

- As long as the map from strategies to prices is invertible, this is without loss of generality
- Other firms’ non-price setting behavior is incorporated into the conjectures concerning their reactions
  - Cournot can be reformulated as a firm setting prices and conjecturing that other firms will adjust their prices to keep their quantities fixed
  - Again, the total derivative is $\frac{dQ}{dP_i} \equiv \frac{\partial Q}{\partial P_i} + \left( \frac{\partial Q}{\partial P_{-i}} \right)^T \frac{\partial P_{-i}}{\partial P_i}$

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  \[
  \frac{dQ_i}{dP_i} = \frac{\partial Q_i}{\partial P_i} + \left( \frac{\partial Q_i}{\partial P_{-i}} \right)^T \frac{\partial P_{-i}}{\partial P_i}
  \]

The pre-merger first-order condition simplifies to:

\[
f_i^\sigma (\sigma) \equiv - \left( \frac{dQ_i}{dP_i} \right)^T Q_i - (P_i - mc_i) = 0
\]
Post-merger, the merging partner $j$ does not react:

- $d^M$ holds fixed partner’s strategy $\frac{d^MA_i}{dP_i} = \frac{\partial A_i}{\partial P_i} + \frac{\partial A_i}{\partial P_{-ij}} \frac{\partial P_{-ij}}{\partial P_i}$

- The diversion ratio matrix is $D^P_{ij} \equiv - \left( \frac{d^M Q_i}{dP_i} -1 \right)^T \frac{d^M Q_j}{dP_i}^T$
Generalized Pricing Pressure (GePP)

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The Generalized Pricing Pressure (GePP) is

$$g_i \equiv \tilde{D}_{ij} (P_j - MC_j)$$

Note: The two changes (from UPP) go in opposite directions

Examples

Nash-in-Prices

- In single-product case, exactly UPP
- In multi-product case, diversion by matrices

Price Response

Diversion Ratio

\( \frac{(P_j - mc_j)}{Q_i \left( \frac{\partial Q_i}{\partial P_i} - \frac{\partial Q_i}{\partial P_{ij}}\right)} \)
Examples

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Nash-in-Quantities

- Only works for differentiated products

\[ g_i(P) = - \left( \frac{\partial Q_j}{\partial P_i} - \frac{\partial Q_j}{\partial P_{ij}} \right) \left( \frac{-1}{\partial Q_{-ij}} \right) \frac{\partial Q_{-ij}}{\partial P_i} \left( P_j - mc_j \right) \]

Price Response

Diversion Ratio

\[ \left( \frac{1}{\partial Q_i} - \frac{1}{\partial P_{ij}} \right) \left( \frac{-1}{\partial Q_{-ij}} \right) \frac{\partial Q_{-ij}}{\partial P_i} - \left( \frac{1}{\partial Q_i} - \frac{1}{\partial P_{-ij}} \right) \left( \frac{-1}{\partial Q_{-i}} \right) \frac{\partial Q_{-i}}{\partial P_i} \]

End of Accommodating Reactions
GePP, like UPP, only gives pricing pressure, not price changes

- **Pass-through** is the rate at which changes in marginal cost are passed through to prices
- It’s intuitive to think that pass-through rates should be used to convert pricing pressure into price changes
- Disagreement in the literature over which pass-through rate choice
  - Froeb, Tschantza, Werden (2005) claim post-merger
  - Farrell and Shapiro (2010) claim pre-merger
We use a Taylor expansion to find the relevant pass-through rate

- Even though mergers are a discrete change in industry structure, we can use standard comparative static approaches as long as the changes in incentives are small
- Requires that the first-order conditions be invertible

**Theorem**

If $f$ is the vector first-order conditions and $g$ is the vector of GePPs and $(f + g)$ and $\left(\frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}\right)$ are invertible, then,

$$
\Delta P \approx -\left(\frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}\right)^{-1}\bigg|_{P^0} \underbrace{g(P^0)}_{pricing\ pressure}.
$$

merger pass-through
Pre-, post- and merger pass-through

- **Pre-merger:** \( \rho^- = - \left( \frac{\partial f}{\partial P} \right)^{-1} \)
- **Post-merger:** \( \rho^+ = - \left( \frac{\partial f}{\partial P} + \frac{\partial g}{\partial P} \right)^{-1} \tilde{D} \)
  - where \( \tilde{D} \equiv \begin{bmatrix} I & -\tilde{D}_{12} & 0 \\ -\tilde{D}_{21} & I & 0 \\ 0 & 0 & I \end{bmatrix} \)
- **Merger:** \( \rho = - \left( \frac{\partial f}{\partial P} + \frac{\partial g}{\partial P} \right)^{-1} \)

**Pre-merger cost impacts**
- Opportunity, not physical costs,

**Post-merger curvature**
- Post-merger incentives
Identifying merger pass-through

We can’t directly observe merger pass-through...two approaches:

1. **Exact identification**
   - Two merging firms
     - Use Slutsky symmetry to identify w/ Bertrand or Cournot
     - With more firms, need stronger assumptions
       - Weyl-Fabinger horizontality or independent discrete choice

2. **Approximating merger pass-through with pre-merger pass-through**
   - If $g$ is small
     - Then likely that $\frac{\partial g}{\partial P}$ also small $\implies \rho \approx \rho_{\leftarrow}$
     - Otherwise “smallness” not very robust
     - If $g$ is small because $D$ is small $\implies \rho \approx \rho_{\rightarrow}$
   - If merger small, effect on pass-through are likely to also be small

Jaffe and Weyl (2011)
The error term $\frac{1}{2} \frac{\partial^2 h^{-1}}{\partial x^2} r^2$, is small whenever:

- $g$ is small
- The first-order conditions are not highly curved in the relevant range.
Approximation Error and Validity

The error term \( \frac{1}{2} \frac{\partial^2 h^{-1}}{\partial x^2} r^2 \), is small whenever:

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Validity

1. Smoothness
   - Standard demand systems in MS have very smooth focs
   - Easier to estimate...and based on pre-merger information
     \( \Rightarrow \) To the extent not smooth, non-parametric unreliable anyway

2. Smallness
   - If merger has large effects, likely to be easier to judge
     \( \Rightarrow \) Detailed quantitative analysis most useful when small

Jaffe and Weyl (2011)
Welfare as a common currency

In the end, we care about Welfare

- **Consumer Surplus**
  - $\Delta P^T Q$
    - Laspeyres, Paasche, Marshall-Edgeworth or Fisher
    - Normalize by value of trade for unit-free measure
      \[ \frac{\Delta P^T Q}{P^T Q} \]

- **Social Surplus**
  - $\Delta DWL \approx (P - mc)^T \Delta Q \approx (P - mc)^T \left( \frac{\partial Q}{\partial P} \Delta P \right)$.
  - Ignores externalities and any other out-of-market affects
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  $$\Delta P^T Q / P^T Q$$

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In addition to being what we care about, these

- Aggregate multiple price changes (possibly with mixed signs)
- Allow comparison to other effects
Comparison to merger simulation

“Approximateness”

- With MS’s assumptions, we would get the same results
  - Functional forms tie down the higher order Taylor terms
    - But they also tie down pass-through rates
    - We think it’s better to try to measure these empirically
  - Our approach makes it clear what the assumptions are
    - And allows for easy testing of different sets of assumptions

Computational ease

- No “simulation” or computation: explicit formula
- Easier to explain

Complementarity

- Might use elasticities from structural estimation
Simplifying the formula for applications

The more general, robust formula requires more inputs

- Many possible simplifying assumptions
  1. Pass-through
  2. Firm heterogeneity
  3. Conduct: Bertrand, Cournot, consistent

- Bertrand conduct, zero cross-pass-through, and unit own pass-through give \( \text{UPP}^T \cdot Q \)

- Our value added diminishes, but useful for robustness checks
This paper:

1. Generalizes UPP
2. Converts GePP to price changes and welfare effects
3. Extends comparative statics approaches to seemingly discrete changes

Future Directions:

1. How accurate is the first-order approximation and when?
2. Add more richness: dynamics, products, quality choice
3. Best ways to simplify the formula in salient cases
4. Empirical work on pass-through

Jaffe and Weyl (2011)
Conclusion

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