Sequential Pricing: Theory and Experiments

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Firms are currently using item level RFID technology to track items in a shopping cart.
- Theft Detection
- Restocking

Technology could also be used for
- Dynamic Pricing

A seller can know you have peanut butter in your cart as you approach the jelly.
Setting Sequential Prices

- straightforward in online markets cookies, “shopping carts”, etc.

- more difficult in stores?

From HGTV.com

**What's New: Intelligent Shopping Cart…**

...the store’s directory is right on the screen. When you find your item, scan the barcode under the LED light indicator then add it to your cart. The computer keeps a running tally of your purchases...
What are the Market Implications?

Sellers have information on preferences scanner data correlated values frequent buyer cards complements/substitutes

Traditionally, sellers have set prices in advance. Now sellers can adjust prices in real time and tailor them to specific shopper.

What would a monopolist do in this setting? What would happen in competitive markets?
Simultaneous Pricing
(Pure Components)

Sellers know distribution of buyer types and set optimal prices given this information.
1. Sellers post single price for each product.
2. Buyers observe prices and make decisions.

Sellers can attempt to increase profits by engaging in price discrimination: Quantity Discounts, Coupons (potentially based upon previous behavior), etc.
(Mixed) Bundling

Adams and Yellen (1976) show that bundling can improve profitability on two unrelated items by raising prices on single items and offer a bundle at a discount.

Venkatesh and Kamakura (2003) consider the case where the bundle has an additive value: $V_{AB} = (1+\theta)(V_A + V_B)$

- $\theta > 0 \Rightarrow A$ and $B$ are complements
- $\theta < 0 \Rightarrow A$ and $B$ are substitutes

Again, prices are set before customers make purchases.
Sequential Pricing

Seller can set prices incrementally during the shopping trip.

*Example:* Suppose we have two types of people, $X$ and $Y$, with $V_A^X = V_B^X = 100$ and $V_A^Y = V_B^Y = 20$.

If each type is equally likely then the expected profit is

- $100 under pure components ($P_A = 100$ & $P_B = 100$)
- $100 with mixed bundling ($P_{AB} = 200$) &
- $110 under sequential pricing
  ($P_A = 100$, $P_{B|A} = 100$, & $P_{B|\neg A} = 20$)
Sequential Pricing – No Discrimination

1. Seller posts a price for Good A
2. Buyer makes purchase decision for Good A
3. Seller sets unconditional price of Good B
4. Buyer makes purchase decision for Good B

The monopolist will first maximize

$$\max_{P_B} (P_B - C_B) \int_{P_B}^{P_A} \int_0^{\infty} f_A(V_A) f_B(V_B) dV_A dV_B + (P_B - C_B) \int_{P_B - \theta V_A}^{\infty} \int_0^{\infty} f_A(V_A) f_B(V_B) dV_A dV_B$$

Given the optimal $P_B^*$ the monopolist will maximize

$$\max_{P_A} (P_A - C_A) \int_{P_A}^{\infty} \int_0^{\infty} f_A(V_A) f_B(V_B) dV_A dV_B + (P_B^* - C_B) \int_{P_B^* - \theta V_A}^{\infty} \int_0^{\infty} f_A(V_A) f_B(V_B) dV_A dV_B + (P_A - C_A + P_B^* - C_B) \int_{P_A}^{\infty} \int_{P_B^* - \theta V_A}^{\infty} f_A(V_A) f_B(V_B) dV_A dV_B$$
Sequential Pricing – No Discrimination

Under the assumption that \( f(V_A) \sim U[0,100] \) & \( f(V_B) \sim U[0,100] \)

\[
P^*_B = \frac{C_B(P_A\theta - 100) - \frac{100^2}{2} (2 + 3\theta) + \frac{\theta P_A^2}{2}}{-2(P_A\theta + 100)}
\]

and the first stage problem becomes

\[
\max_{P_A} \left( \frac{P_A - C_A}{100^2} \right) \left[ 100^2 - P_A P^*_B + \frac{P_A P^*_B - 100 P^*_B}{1 + \theta} + \frac{\theta (100^2 - P_A^2)}{2(1 + \theta)} \right]
\]

which can be solved using Mathematica.

If \( C_A = C_B = \theta = 0 \) then \( P_A^* = P_B^* = 50 \).
Sequential Pricing - Discrimination

Step 3 becomes 3’. Seller sets conditional price of Good B

The monopolist maximizes

$$\max_{P_B \mid q_A = 0} (P_B - C_B) \int_{P_B}^{\infty} f_B(V_B) dV_B$$

and

$$\max_{P_B \mid q_A = 1} (P_B - C_B) \int_{P_A}^{\infty} \frac{f(V_B) f(V_A \mid V_A \geq P_A) dV_A dV_B}{1 + \theta}$$

Using the optimal Good B prices from above, the monopolist then chooses $P_A$ to maximize expected profit
Sequential Pricing - Discrimination

Under the assumption of a uniform distribution of values

\[ P_B^* \mid (q_A = 0) = \frac{100 + C_B}{2} \]

\[ P_B^* \mid (q_A = 1) = \frac{100(1 + \theta) + \frac{\theta(100 - P_A)}{2} + C_B}{2} \]

and therefore

\[ P_A^* = \frac{50 - C_B - \frac{C_B^2}{200} + 100 + C_A - \frac{1}{100} \left[ 100(1 + \theta) + \frac{\theta(100 - P_A)}{2} - C_B \right] \left[ 50 + \frac{\theta(100 - P_A)^2}{2(1 + \theta)} - C_B \right]}{2} \]

Again, if \( C_A = C_B = \theta = 0 \) then \( P_A^* = P_B^*|A = P_B^*|\neg A = 50 \).
## Numerical Comparison

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<thead>
<tr>
<th>$\theta$</th>
<th>Pure Components</th>
<th>Sequential Pricing w/o Discrimination</th>
<th>Mixed Bundling</th>
<th>Sequential Pricing w/ Discrimination</th>
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# Numerical Comparison

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Distributions generated by removing corners of $[0,100] \times [0,100]$
Competitive Markets

Many retailers are not monopolists!
Buyers have the ability to comparison shop.

Following Varian (1980), a fraction $\alpha$ of shoppers are uninformed and visit one of the $n$ sellers selected randomly and $1-\alpha$ are informed.

Informed shoppers know $P_{Ai} \forall i = 1,\ldots,n$ and only visit the lowest price seller if $V_A \geq \min \{P_{A1},\ldots,P_{AN}\}$

Only buyers who actually visit seller $i$ observe $P_{Bi}$. 
Experimental Design

3x2 across subjects design

3 underlying relationships between goods

Baseline: $\theta = 0$, $f(V_A) \sim U[0,100]$, & $f(V_B) \sim U[0,100]$

Complements: like baseline except $\theta = 0.3$

Correlated Goods: $\theta = 0$, $f(V_A, V_B) \sim U\{S\}$ $\Rightarrow \rho = +0.5$
where $S = \text{all } (a,b) \text{ s.t. } 0 \leq a, b \leq 100 \text{ and } |a-b| \leq 50$

2 sequential pricing strategies
with and without conditional pricing
On Screen Tool

Positive Correlation Treatment ($\rho = 0.5 \& \theta = 0$)

Each cell represents a potential buyer.

<table>
<thead>
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<th>Row Heading is Value of A</th>
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Expected profit if only seller visited = 51.

Shading indicates what potential buyers would do if they only visit you.
Experimental Design

Other Parameters: $C_A = C_B = 0, \ n = 4, \ \alpha = 0.8$

Markets lasted 750 periods @ 3 seconds each
Automated buyers
Sellers could adjust prices at any point
Sellers could observe prices and profits of rivals

4 replications of each of the 6 experimental conditions
Sessions lasted 90 minutes including directions and handout
Average payment $\approx$ $18.00 + $7.50 participation payment
Findings: Baseline

- Ability to price discriminate *nominally* increases $P_A$ and $P_B$.
- Sellers do not discriminate based upon purchase of $A$.
- Low price seller of $A$ does not charge more for $B$.
- Ability to price discriminate does not affect welfare.
Findings: Complements

- Ability to price discriminate \textit{nominally} increases $P_A$.
- Purchasing A leads to a higher $P_B$ when the seller can discriminate. Without discrimination all buyers are charged the same price as those who purchased A.
- Low price seller of A does not charge more for B.
- Ability to price discriminate does not affect welfare.
Findings: Correlated Goods

- Ability to price discriminate does not affect $P_A$.
- Buyers who purchased Good A were quoted the same $P_B$ as buyers could not be tracked. Buyers who could be indentified as not purchasing A were quoted a lower $P_B$.
- Low price seller of A does not charge more for B.
- Ability to price discriminate does not affect welfare.
Findings: Comparison with Bundling

Aloysius and Deck (2008) examines bundling using a similar design (parameters, interface, subjects).

- Efficiency is lower with sequential pricing.
- Average within session profits are universally higher with sequential pricing.

![Efficiency by Session](image-url)
Summary

1. Evolving technology will enable sellers to track intended purchases and adjust prices.

2. For a monopolist, sequential pricing is more attractive than bundling for goods that are highly positively correlated or close substitutes.

3. The effects of the ability to discriminate are weakened by competition.

4. There are benefits from this technology (recommendations, etc.) but redefining the problem sequentially has costs and reduces welfare relative to bundling.

5. More research is needed with strategic buyers, different information acquisition, endogenous selection of bundling or sequential pricing.