

An Experimental Approach to Merger Evaluation *

Christopher T. Conlon[†]
Julie Holland Mortimer[‡]

October 29, 2013

Abstract

The 2010 Department of Justice and Federal Trade Commission Horizontal Merger Guidelines lay out a new standard for assessing proposed mergers in markets with differentiated products. This new standard is based on a measure of “upward pricing pressure,” (UPP) and the calculation of a “gross upward pricing pressure index” (GUPPI) in turn relies on a “diversion ratio,” which measures the fraction of consumers of one product that switch to another product when the price of the first product increases. One way to calculate a diversion ratio is to estimate own- and cross-price elasticities. An alternative (and more direct) way to gain insight into diversion is to exogenously remove a product from the market and observe the set of products to which consumers actually switch. In the past, economists have rarely had the ability to experiment in this way, but more recently, the growth of digital and online markets, combined with enhanced IT, has improved our ability to conduct such experiments. In this paper, we analyze the snack food market, in which mergers and acquisitions have been especially active in recent years. We exogenously remove six top-selling products (either singly or in pairs) from vending machines and analyze subsequent changes in consumers’ purchasing patterns, firm profits, diversion ratios, and upward pricing pressure. Using both nonparametric analyses and structural demand estimation, we find significant diversion to remaining products. Both diversion and the implied upward pricing pressure differ significantly across manufacturers, and we identify cases in which the GUPPI would imply increased regulatory scrutiny of a proposed merger.

*We thank Mark Stein, Bill Hannon, and the drivers at Mark Vend Company for implementing the experiments used in this paper, providing data, and generally educating us about the vending industry. Tom Gole, Adam Kapor and Sharon Traiberman provided exceptional research assistance. Financial support for this research was generously provided through NSF grant SES-0617896. Any remaining errors are our own.

[†]Department of Economics, Columbia University, 420 W. 118th St., New York City, NY 10027. email: cconlon@columbia.edu

[‡]Department of Economics, Boston College, 140 Commonwealth Ave., Chestnut Hill, MA 02467, and NBER. email: julie.mortimer.2@bc.edu

1 Introduction

Since 1982, one of the primary tools in evaluating the potential anticompetitive effects of horizontal mergers has been the Herfindahl Index (HHI) which measures both the level of concentration in a particular market, as well as how a proposed merger would change the level of concentration. The Herfindahl Index relates marketshares to markups when firms are engaged in a symmetric Cournot game. Because HHI requires marketshares as inputs, one practical challenge has been defining the relevant market in terms of both geography and competitors. The 2007 merger between *Whole Foods* and *Wild Oats* highlighted the problems with the black-and-white distinction of market definitions. In that case, the FTC argued that the proposed merger was essentially a merger to monopoly in the market for “premium organic groceries,” while Whole Foods argued they faced competition from traditional grocery stores, and thus the merger represented little change in concentration.

In 2010, the Department of Justice (DOJ) and the Federal Trade Commission (FTC) released a major update to the Horizontal Merger Guidelines which shifted the focus away from traditional concentration measures like HHI, and towards methods that better accounted for product differentiation and the closeness of competition, utilizing intuition from the differentiated products Bertrand framework. Rather than use a full structural equilibrium simulation of post-merger prices and quantities such as in Nevo (2001), the guidelines instead hold all post-merger quantities fixed, as well as the prices of all goods outside the merger, and consider the unilateral effects of the merger on the prices of the merged entity’s products. This exercise produces the two key measures: Upward Pricing Pressure (UPP) and Generalized Upward Pricing Pressure Index (GUPPI). Both of these depend on the prices and costs of merged products, and are monotonic functions of the “Diversion Ratio” between the products of the two merged firms. In general, one expects prices and profit margins to be directly observed, leaving only the Diversion Ratio to be estimated. In this sense, the Diversion Ratio serves as a sufficient statistic to determine whether or not a proposed merger is likely to increase prices (and be contested by the antitrust authorities).

This sentiment is captured in the Guidelines’ definition of the diversion ratio:

In some cases, the Agencies may seek to quantify the extent of direct competition between a product sold by one merging firm and a second product sold by the other merging firm by estimating the diversion ratio from the first product to the second product. The diversion ratio is the fraction of unit sales lost by the first product due to an increase in its price that would be diverted to the second

product. Diversion ratios between products sold by one merging firm and products sold by the other merging firm can be very informative for assessing unilateral price effects, with higher diversion ratios indicating a greater likelihood of such effects. Diversion ratios between products sold by merging firms and those sold by non-merging firms have at most secondary predictive value.

There is a growing literature that examines the potential advantages and disadvantages of the use of diversion ratios as the primary input into merger evaluation. Many of these advantages are discussed in Farrell and Shapiro (2010), and similar to those found in the literature on sufficient statistics (Chetty 2009). One benefit is that the Diversion Ratio does not require data from all firms in an industry, but rather only those firms engaged in a potential merger. Another benefit is that the diversion ratio need not be measured with reference to a specific demand system, nor does it necessarily assume a particular type of conduct within the industry. Some potential disadvantages include the fact that GUPPI and UPP may not always correctly predict the sign of the price effect of a merger, and that these measures may either overpredict and underpredict pricing effects; in general this will depend on the nature of competition among non-merging firms, and whether prices are strategic substitutes or strategic complements. Cheung (2011) provides empirical evidence comparing UPP with econometric merger simulation and finds evidence of both type I and type II errors. There is a growing literature that examines the theoretical conditions under which the predictions of UPP and GUPPI accurately predict merger effects, including Carlton (2010), Schmalensee (2009), Willig (2011).

A limitation of the UPP/Diversion approach is that it ignores how mergers change the incentives of non-merging firms, as well as the equilibrium quantities. This has led some to consider more complicated (and accurate) alternatives based on pass-through rates (i.e., Jaffe and Weyl (Forthcoming)), with recent empirical work by Miller, Remer, Ryan, and Sheu (2012). This paper remains silent on whether or not Diversion, UPP, and GUPPI accurately capture the price effects of mergers, but instead focuses on how the Diversion Ratio might be measured by empirical researchers and antitrust practitioners. Farrell and Shapiro (2010) suggest that firms themselves track diversion in their normal course of business, Reynolds and Walters (2008) examine the use of consumer surveys in the UK. Alternatively, diversion could be measured as an outcome of a parametric demand estimation exercise, and Hausman (2010) argues this is the only acceptable method of measuring diversion. We explore an experimentally motivated identification argument for diversion, and then design and conduct a series of field experiments.

Diversion measures the fraction of consumers who switch from product 1 to product 2 as product 1 becomes less attractive (often by raising its price). While it would be impossible (or at least a very bad idea) to randomize whether or not mergers take place, we show that an unintended benefit of the *ceteris parabis* approach embedded in the diversion ratio is that it lends itself to experimental sources of identification. It is relatively straightforward to consider an experiment increasing the price of a single good and measuring sales to substitute goods. Another option would be to eliminate product 1 from the choice set and measure substitution directly. By narrowing the focus to the diversion ratio, rather than a full merger simulation, it is now possible to employ an additional set of tools that have become common in the economics literature on field experiments and randomized controlled trials.

In this paper we design a series of experiments in order to measure diversion. However, even though experimental approaches to identification allow treating the diversion ratio as a “treatment effect” of an experiment, we must be careful in considering which treatment effect our experiment measures. We show that the measure of diversion required in UPP analysis represents a marginal treatment effect (MTE), but our experimental design measures an average treatment effect (ATE). We derive an expression for the variance of the diversion ratio, and show that a tradeoff exists. For small price increases, an assumption of constant diversion may be a reasonable assumption, but small changes in the sales of product 1 imply more variable estimates of diversion. At the same time, large changes in p_1 imply a more efficient estimate of diversion, but introduce the possibility for bias if the diversion ratio is not constant. We also derive expressions for diversion under several parametric models of demand, and show how diversion varies over a range of price increases. For example, we show that both the linear demand model and the IIA logit model exhibit constant diversion (treatment effects) for any price increase.

While removing products from consumers’ choice sets (or changing prices) may be difficult to do on a national scale, one might be able to measure diversion accurately using smaller, more targeted experiments. In fact, many large retailers such as Target and Wal-Mart frequently engage in experimentation, and online retailers such as Amazon.com and Ebay have automated platforms in place for “A/B-testing.” As information technology continues to improve in retail markets, and as firms become more comfortable with experimentation, one could imagine antitrust authorities asking the parties in a proposed merger to submit to an experiment executed by an independent third party. One could even imagine both parties entering into an *ex ante* binding agreement that mapped experimental outcomes into a decision on the proposed merger.

Our paper considers several hypothetical mergers within the single serving snack foods industry, and demonstrates how to design and conduct experiments to measure the diversion ratio. We measure diversion by exogenously removing one or two top-selling products from each of three leading manufacturers of snack food products and observing substitution patterns. We use a set of sixty vending machine in secure office sites as our experimental “laboratory” for the product removals. We then compare our estimated experimental diversion ratio to those found using structural econometric models, such as a random-coefficients logit model, and a nested logit model. We document cases where these different approaches agree, and where they disagree, and analyze what the likely sources of disagreement are. In general we show that while all approaches generate qualitatively similar predictions, the parametric models tend to under predict substitution to the very closest substitute, and over-predict substitution to non-substitute products when compared to the experimentally-measured treatment effects.

The paper also contributes to a recent discussion about the role of different methods in empirical work going back to Leamer (1983), and discussed recently by Heckman (2010), Leamer (2010), Keane (2010), Sims (2010), Nevo and Whinston (2010), Stock (2010), and Einav and Levin (2010). A central issue in this debate is what role experimental or quasi-experimental methods should play in empirical economic analyses in contrast to structural methods. Angrist and Pischke (2010) complain about the general lack of experimental or quasi-experimental variation in many IO papers. Nevo and Whinston (2010)’s response highlights that most important IO questions, such as prospective merger analysis and counterfactual welfare calculations are not concerned with measuring treatment effects and do not lend themselves to experimental or quasi-experimental identification strategies. Of course, as screening of potential mergers shifts towards diversion-based measures, this may open the door towards more experimentally-motivated identification strategies.

Several of these recent papers essentially argue that both types of approaches have advantages and drawbacks. Our setting provides the opportunity to examine empirically the trade-offs to which these papers refer. For example, while our experimental estimates are quite informative in many respects, there are cases in which we would not want to infer causality (e.g., when sales of substitute goods decrease in response to a focal good’s removal from the market). The structural demand models use economic theory to rule out such an effect, but cannot fully capture the degree of substitution that occurs from a focal product to other goods. This is especially true when a product’s most important characteristics are less easy to measure (e.g., packaging differences, or possibly unobserved advertising campaigns).

The paper proceeds as follows. Section 2 lays out a theoretical framework, section 3 describes the snack foods industry, our data, and our experimental design. We describe our calculation of the treatment effect under various models in section 4, and present the results of the analyses in section 5. Section 6 concludes.

2 Theoretical Framework

The first part of this section is purely expositional and does not introduce new results beyond those presented in Farrell and Shapiro (2010) and closely follows Cheung (2011).

For simplicity, consider a single market composed of $f = 1, \dots, F$ multi-product firms and $j = 1, \dots, J$ products, where firm f sets the prices of products in set \mathcal{J}_f to maximize profits:

$$\pi_f = \sum_{j \in \mathcal{J}_f} (p_j - c_j) s_j(\mathbf{p}) - C_j$$

Under the assumption of constant marginal costs c_j the FOC for firm f becomes

$$s_j(\mathbf{p}) + \sum_{k \in \mathcal{J}_f} (p_k - c_k) \frac{\partial s_k(\mathbf{p})}{\partial p_j} = 0$$

Let the superscripts (0) and (1) denote pre- and post merger quantities respectively. Consider the pre- and post-merger FOC of a single product firm who owns product j and is acquiring product k :

$$\begin{aligned} s_j(\mathbf{p}^{(0)}) + (p_j^{(0)} - c_j) \frac{\partial s_j(\mathbf{p}^{(0)})}{\partial p_j} &= 0 \\ s_j(\mathbf{p}^{(1)}) + (p_j^{(1)} - (1 - e_j) \cdot c_j) \frac{\partial s_j(\mathbf{p}^{(1)})}{\partial p_j} + (p_k^{(1)} - c_k) \frac{\partial s_k(\mathbf{p}^{(1)})}{\partial p_j} &= 0 \end{aligned}$$

Then UPP_j represents the change in the price of j : $p_j^{(1)} - p_j^{(0)}$ that comes about from the difference in the two FOCs, where all other quantities are held fixed at the pre-merger values (i.e.: $\mathbf{p}^{(1)} = \mathbf{p}^{(0)}$ and $p_k^{(1)} = p_k^{(0)}$) and the merger results in cost savings e_j . That is:

$$UPP_j = (p_k^{(0)} - c_k) \cdot \underbrace{\left(\frac{\partial s_j(\mathbf{p}^{(0)})}{\partial p_j} \right)^{-1} \cdot \frac{\partial s_k(\mathbf{p}^{(0)})}{\partial p_j}}_{D_{jk}(\mathbf{p}^{(0)})} - e_j \cdot c_j \quad (1)$$

Here UPP_j measures how the merger affects the opportunity cost of selling an extra unit of j traded off against the marginal cost efficiency. Notice that now the firm internalizes the effect that some fraction of sales of j would have become sales of k . The key input into merger analysis is the quantity $D_{jk}(\mathbf{p}^{(0)})$ or the Diversion Ratio at the pre-merger prices (and quantities). In words, the diversion ratio measures the fraction of consumers who switch from j to k under pre-merger prices $\mathbf{p}^{(0)}$ when the price of j is increased just enough for a single consumer to cease purchasing j .

The other basic extension we want to consider is when the owner of firm j merges with a firm that controls not only product k , but also product l . In the case where k and l have the same margins $p_k - c_k = p_l - c_l$ then the impact of the three product merger on UPP_j is identical to that in (1) with the exception that it depends on the sum of the diversion ratios $D_{jk} + D_{jl}$.

2.1 Empirically Measuring Diversion

The goal of our paper is to show how the diversion ratio might be estimated experimentally. Part of the rationale given in Farrell and Shapiro (2010) is that the diversion ratio no longer needs to be measured with respect to a particular parametric functional form of demand, and one way to identify D_{jk} might be through conducting an experiment. An obvious experiment would be to exogenously manipulate the price of product j to some random subset of consumers and to measure how the sales of j and k respond. If we let $(\mathbf{p}^{(0)}, \mathbf{p}^{(1)})$ represent the price vectors during the control and treatment, so that $p_l^{(0)} = p_l^{(1)}, \forall l \neq j$ and $p_j^{(1)} = p_j^{(0)} + \Delta p_j$. The diversion ratio can be computed:

$$\widehat{D}_{jk} = \left| \frac{\Delta Q_k}{\Delta Q_j} \right| = \left| \frac{Q_k(\mathbf{p}^{(1)}) - Q_k(\mathbf{p}^{(0)})}{Q_j(\mathbf{p}^{(1)}) - Q_j(\mathbf{p}^{(0)})} \right| = \frac{\int_{p_j^0}^{p_j^1} \frac{\partial q_k(p_j, p_{-j})}{\partial p_j} \partial p_j}{\int_{p_j^0}^{p_j^1} \frac{\partial q_j(p_j, p_{-j})}{\partial p_j} \partial p_j} \quad (2)$$

First we consider the possible bias introduced by considering a larger than infinitesimal increase in price $\Delta p_j \gg 0$. We derive an expression by considering a second order expansion

of demand at $\mathbf{p}^{(0)}$:

$$\begin{aligned}
q_k(\mathbf{p} + \Delta p_j) &\approx q_k(\mathbf{p}) + \frac{\partial q_k}{\partial p_j} \Delta p_j + \frac{\partial^2 q_k}{\partial p_j^2} (\Delta p_j)^2 + O((\Delta p_j)^3) \\
\frac{q_k(\mathbf{p} + \Delta p_j) - q_k(\mathbf{p})}{\Delta p_j} &\approx \frac{\partial q_k}{\partial p_j} + \frac{\partial^2 q_k}{\partial p_j^2} \Delta p_j + O(\Delta p_j)^2
\end{aligned} \tag{3}$$

This allows us to compute an expression for the bias in D_{jk} :

$$Bias(\widehat{D}_{jk}) \approx \frac{D_{jk} \frac{\partial^2 q_j}{\partial p_j^2} - \frac{\partial^2 q_k}{\partial p_j^2}}{\frac{\partial q_j}{\partial p_j} + \frac{\partial^2 q_j}{\partial p_j^2} \Delta p_j} \Delta p_j \tag{4}$$

Our expression in (4) shows that the bias of our diversion estimate depends on two things: one is the magnitude of the price increase Δp_j , the second is the curvature of demand $\frac{\partial^2 q_k}{\partial p_j^2}$. This suggests that bias is minimized by experimental designs that consider small price changes. However, small changes in price may lead to noisy measures of Δq . More formally we assume constant diversion $\Delta q_k \approx D_{jk} \Delta q_j$ and construct a measure for the variance of the diversion ratio:

$$Var(\widehat{D}_{jk}) \approx Var\left(\frac{\Delta q_k}{|\Delta q_j|}\right) \approx \frac{1}{\Delta q_j^2} \left(D_{jk}^2 \sigma_{\Delta q_j}^2 + \sigma_{\Delta q_k}^2 - 2D_{jk} \rho \sigma_{\Delta q_j} \sigma_{\Delta q_k} \right) \tag{5}$$

This implies a bias-variance tradeoff when estimating diversion. If our primary concern is that the curvature of demand is steep, it suggests considering a small price increase. However, if our primary concern is that sales are highly variable, we should consider a larger price increase.

Instead of viewing our experiment as a biased measure of the marginal treatment effect, we can instead ask: “What quantity does our experimental diversion measure estimate?” We can derive an approximate relationship for our experimental diversion measure ED_{jk} :

$$\widehat{ED}_{jk} = \frac{1}{\Delta q_j} \int_{p_j^0}^{p_j^0 + \Delta p_1} \underbrace{\frac{\partial q_k}{\partial q_j}}_{D_{jk}(\mathbf{p})} \left| \frac{\partial q_j}{\partial p_j} \right| dp_j \tag{6}$$

Thus an experiment varying the price p_j measures the weighted average of diversion ratio, where the weights correspond to the lost sales of j at a particular p_j as a fraction of all lost sales of j .

The experiments in our empirical example do not employ actual price changes, instead the focal product is removed from the consumer’s choice set. This is equivalent to increasing the price p_j to the choke price where $q_j(p_j^c, \mathbf{p}_{-j}^{(0)}) = 0$. This has the advantage that it minimizes the variance expression in (5). The other derivations in this section provide insight in to when a product removal provides an accurate measure of diversion: (a) when the curvature of demand is small ($\frac{\partial^2 q_k}{\partial p_j^2} \approx 0$), (b) when the true diversion ratio is constant (or nearly constant) $D_{jk}(\mathbf{p}) = D_{jk}$, or (c) when demand for j is steepest near the market price $\left| \frac{\partial q_j(p_j, \mathbf{p}_{-j}^{(0)})}{\partial p_j} \right| \gg \left| \frac{\partial q_j(p_j + \delta, \mathbf{p}_{-j}^{(0)})}{\partial p_j} \right|$.

In our example, it might seem reasonable that customers who substitute away from a *Snickers* bar after a five cent price increase switch to *Reese’s Peanut Butter Cup* at the same rate as after a 25 cent price increase, where the only difference is the number of overall consumers leaving *Snickers*. However in a different industry, this may no longer seem as reasonable. For example, we might expect buyers of a Toyota Prius to substitute primarily to other cheap, fuel-efficient cars when faced with a small price increase (from the market price of \$25,000 to \$25,500), but we might expect substitution to luxury cars when facing a larger price increase (from \$45,000 to \$45,500). Again, if demand (in units) for the Prius falls rapidly with a small price increase, so that residual demand (and the potential impact of further price increases) are small, then an experiment that considers removing it from the choice set may provide an accurate measure of diversion, even though the diversion ratio itself is not constant.

In the Appendix we derive explicit relationships for the Diversion Ratio for several well known parametric demand functions. The important result is that both linear demand, and the IIA logit demand model exhibit constant diversion ratios that do not vary with prices. We also show that random coefficients logit demand, and CES demands (including log-linear demand) do not generally exhibit constant diversion.

3 Description of Data and Industry

Globally, the snack foods industry is a \$300 Billion a year business, comprised of a number of large well-known firms and some of the most heavily advertised global brands. Mars Incorporated reported over \$50 Billion in Revenue in 2010, and represents the third largest privately held firm in the US; other substantial players include Hershey, Nestle, Kraft, Kellogg, Frito-Lay division of PepsiCo. While the snack-food industry as a whole might not appear highly concentrated, sales within product categories can be very concentrated. For

example, Frito-Lay comprises around 40% of all savory snack sales in the United States, and reported over \$13 Billion in US revenues last year, but it’s sales outside the salty-snack category are minimal, coming mostly through parent PepsiCo’s Quaker Oats brand and the sales of *Quaker Chewy Granola Bars*.¹

Over the last 25 years, the industry has been characterized by a large amount of merger and acquisition activity, both on the level of individual brands and entire firms. For example, the *Famous Amos* cookie brand has been owned by the Kellogg Company since 2001. Between 1985 and 2001, Famous Amos cookies were owned by at least seven firms, including the Keebler Cookie Company (acquired by Kellogg in 2001), Presidential Baking Company (acquired by Keebler in 1998), as well as by other snack food makers and private equity firms. *Zoo Animal Crackers* have a similarly complicated history owned by cracker manufacturer Austin Quality Foods; before they too were acquired by the Keebler Cookie Co. (who in turn was acquired by Kellogg).²

Our study measures diversion through the lens of a single medium-sized retail vending operator in Chicago metropolitan area, MarkVend. Each of MarkVend’s machines internally records price and quantity information. The data track total vends and revenues since the last service visit on an item-level basis (but do not include time-stamps for each sale). Any given machine can carry roughly 35 products at one time, depending on configuration. We observe retail and wholesale prices for each product at each service visit during our 38-month panel. There is relatively little price variation within a site, and almost no price variation within a category (e.g., chocolate candy) at a site. This is helpful from an experimental design perspective, but can pose a challenge to demand estimation. Very few “natural” stock-outs occur at our set of machines.³ Most changes to the set of products available to consumers are a result of product rotations, new product introductions, and product retirements. Over all sites and months, we observe 185 unique products. We consolidate some products with very low levels of sales using similar products within a category produced by the same manufacturer, until we are left with the 73 ‘products’ that form the basis of the

¹Most analysts believe Pepsi’s acquisition of Quaker Oats in 2001 was unrelated to its namesake business but rather for Quaker Oats’ ownership of Gatorade, a close competitor in the soft drink business.

²A landmark case in market definition was brought by *Tastykake* in attempt to block the acquisition of *Drake* (the maker of Ring-Dings) by *Ralston-Purina’s Hostess* brand (the maker of Twinkies). That case established the importance of geographically significant markets, as Drake’s had only 2% marketshare nationwide but a much larger share in the Northeast (including 50% of the New York market). *Tastykake* also successfully argued that the relevant market was single-serving snack cakes rather than a broad category of snack foods involving cookies, candy bars. [*Tasty Baking Co. v. Ralston Purina, Inc.*, 653 F. Supp. 1250 - Dist. Court, ED Pennsylvania 1987]

³Mark Vend commits to a low level of stock-out events in its service contracts.

rest of our exercise.⁴

In addition to the data from Mark Vend, we also collect data on the characteristics of each product online and through industry trade sources.⁵ For each product, we note its manufacturer, as well as the following set of product characteristics: package size, number of servings, and nutritional information.⁶

3.1 Experimental Design

We ran four experimental treatments with the help of the Mark Vend Company. These represent a subset of a larger group of experiments we have used in other projects, such as Conlon and Mortimer (2013). Our experiments followed a subset of Mark Vend’s operation, 60 snack machines located in professional office buildings, for which demand was historically quite stable.⁷ Most of the customers at these sites are ‘white-collar’ employees of law firms and insurance companies. Our goal in selecting the machines was to choose machines that could be analyzed together, in order to be able to run each experiment over a shorter period of time across more machines.⁸ Finally, we selected machines on routes that were staffed by experienced drivers, so that the implementation of the experiments would be successful. The 60 machines used for each experiment were distributed across five of Mark Vend’s clients, which had between 3 and 21 machines each. The largest client had two sets of floors serviced on different days, and we divided this client into two sites. Generally, each site is spread across multiple floors in a single high-rise office building, with machines located on each floor.

For each experiment, we removed a product from all machines at a client site for a period of 2.5 to 3 weeks. We conducted four experiments. In two of the experiments we removed the two best-selling products from chocolate maker Mars Incorporated (Snickers and Peanut M&Ms’), in another we removed the two best-selling products from the salty snack category (PepsiCo’s: Doritos Nacho Cheese and Cheetos Crunchy); and for our third and fourth

⁴For example, we combine Milky Way Midnight with Milky Way, and Ruffles Original with Ruffles Sour Cream and Cheddar.

⁵For consolidated products, we collect data on product characteristics at the disaggregated level. The characteristics of the consolidated product are computed as the weighted average of the characteristics of the component products, using vends to weight. In many cases, the observable characteristics are identical.

⁶Nutritional information includes weight, calories, fat calories, sodium, fiber, sugars, protein, carbohydrates, and cholesterol.

⁷More precisely, demand at these sites is “relatively” stable compared to the population of sites serviced by the vending operator.

⁸Many high-volume machines are located in public areas (e.g., museums or hospitals), and have demand (and populations) that varies enormously from one day to the next, so we did not use machines of this nature. In contrast, the work-force populations at our experimental sites are relatively homogenous.

experiment we removed two products owned by Kellogg's: Famous Amos Chocolate Chip Cookies, and Zoo Animal Crackers. We chose to run these two experiments separately in part because the Animal Crackers are a difficult to categorize product; and close substitutes are less obvious.

Whenever a product was experimentally stocked-out, poster-card announcements were placed at the front of the empty product column. The announcements read "This product is temporarily unavailable. We apologize for any inconvenience." The purpose of the card was two-fold: first, we wanted to avoid dynamic effects on sales as much as possible, and second, the firm wanted to minimize the number of phone calls received in response to the stock-out events. The dates of the interventions range from June 2007 to September 2008, with all removals run during the months of May - October. We collected data for all machines for just over three years, from January of 2006 until February of 2009. During each 2-3 week experimental period, most machines receive service visits about three times. However, the length of service visits varies across machines, with some machines visited more frequently than others.

The cost of the experiment consisted primarily of driver costs. Drivers had to spend extra time removing and reintroducing products to machines, and the driver dispatcher had to spend time instructing the drivers, tracking the dates of each experiment, and reviewing the data as they were collected. Drivers are generally paid a small commission on the sales on their routes, so if sales levels fell dramatically as a result of the experiments, their commissions could be affected. Tracking commissions and extra minutes on each route for each driver would have been prohibitively expensive to do, and so drivers were provided with \$25 gift cards for gasoline during each week in which a product was removed on their route to compensate them for the extra time and the potential for lower commissions. With the exception of an individual site in one treatment, implementation was successful.⁹

Our experiment differs somewhat from an ideal experiment. Ideally, we would be able to randomize the choice set on an individual level, though technologically that is difficult in both vending and traditional brick and mortar contexts. In contrast, online retailers are capable of showing consumers different sets of products and prices simultaneously. This leaves our design susceptible to contamination if for example, Kraft runs a large advertising campaign for Planters Peanuts that corresponds to the timing of one of our experiments. Additionally, because we remove all of the products at an entire client site for a period of 2.5 to 3 weeks,

⁹In the unsuccessful run, the driver at one site forgot to remove the focal product, so no intervention took place.

we lack a contemporaneous group of untreated machines. We chose this design, rather than randomly staggering the product removals, because we (and the participating firms) were afraid consumers might travel from floor to floor searching for stocked out products. This design consideration prevents us from using control machines in the same building, and makes it more difficult to capture weekly variation in sales due to unrelated factors, such as a law firm taking a big case to trial, or accountants during quarterly reporting season. In balance, we thought that people traveling from floor to floor was a larger concern. It also has the additional benefit that we can aggregate over all machines at a client site, and treat the entire site as if it were a single machine.

4 Analyses of the Experimental Outcomes

4.1 Paired Differences

One goal of our experiment is to determine how sales are diverted away from best-selling products. All of these data are recorded at the level of a service visit to a vending machine. Because machines are serviced on different schedules it is sometimes more convenient to organize observations by week, rather than by visit. When we do this, we assume that sales are distributed uniformly among the business days in a service interval, and assign those to weeks. Because different experimental treatments start on different days of the week, we allow our definition of when weeks start and end to depend on the client site and experiment.¹⁰ The next step we take is to aggregate our data to the level of a client site-week rather than a machine-week. We do this for two reasons, the first is that sales at the individual machine level are quite small and can vary quite a bit over time. This would make it hard to measure any kind of effect. The second is that we don't want to worry about consumers going from machine to machine searching for missing products, or additional noise in demand created by a long meeting on a particular floor, etc.

Let's begin by defining some basic quantities. We let q_{jt} denote the sales of product j in site-week t , and we use a superscript 1 to denote sales when a focal product(s) is removed, and a superscript 0 to denote sales when a focal product(s) is available. We denote the set of available products as A , and F as the set of products we remove for our experiment. Then $Q_t^1 = \sum_{j \in A \setminus F} q_{jt}^1$ and $Q_s^0 = \sum_{j \in A} q_{js}^0$ are the overall sales during a treatment week, and control week respectively. It is also convenient to write the sales of the removed products $q_{fs}^0 = \sum_{j \in F} q_{js}^0$. Our goal is to compute $\Delta q_{kt} = q_{kt}^1 - E[q_{kt}^0]$, the treatment effect on the sales

¹⁰At some site-experiment pairs, weeks run Tuesday to Monday, while others run Thursday to Wednesday.

of product k of the experiment, and the diversion ratio $D_{fs,k} = \frac{\Delta q_{kt}}{q_{fs}^0}$. In the context of our experiment, the diversion ratio has a simple interpretation: the change in sales for product k during our experiment as a fraction of sales of the focal product(s) during the control period.

In principle, this calculation is straightforward. In practice, however, there are two challenges in implementing the experiments and interpreting the data generated by them. The first challenge is that there is a large amount of variation in overall sales at the weekly level independent of our experiments. This weekly variation in overall sales is common in retail environments. It is not uncommon for week over week sales to vary by over 20%, while no single product enjoys more than 4.5% market share. This can be seen in Figure 1 which plots the overall sales of all machines from one of the sites in our sample on a weekly basis. For example, a law firm may have a large case going to trial in a given month, and vend levels will increase at the firm during that period. In our particular setting, many of the experiments were run during the summer of 2007, which was a high-point in demand at these sites, most likely due to macroeconomic conditions. In this case, using a simple measure like previous weeks' sales, or overall average sales for $E[q_{jt}^0]$ could result in unreasonable treatment effects, such as sales increasing due to stock-out events, or sales decreasing by more than the sales of the focal products.

We deal with this challenge by imposing a simple restriction born out of consumer theory. After we have aggregated our data across machines to the level of a client-week observation, we restrict the set of possible control weeks, so that aggregate sales do not increase as the result of our experiment. Likewise, we also impose that aggregate sales do not decline by more than the sales of the product we removed. Mathematically the set of control weeks s corresponding to treatment week t is defined by:

$$\{s : s \neq t, Q_t^0 - Q_s^1 \in [0, q_{fs}^0]\} \quad (7)$$

Notice that this is the same as placing a restriction on the sum of the diversion ratios:

$$\sum_{j \in A \setminus F} D_{fs,j} \in [0, 100\%] \quad (8)$$

While this has the nice property that it imposes the restriction on our selection of control weeks that all products are weak-substitutes, it has the disadvantage that it introduces the potential for selection bias. The bias results from the fact that weeks with unusually high sales of the focal product q_{fs}^0 are now more likely to be included in our control. Because it

would make the denominator too large, the bias would likely understate the diversion ratio.

We propose a slight modification of (7) which removes the bias. That is, we can replace $q_{f_s}^0$ with $\widehat{q}_{f_s}^0 = E[q_{f_s}^0|Q_s^0]$. An easy way to obtain the expectation is to run an OLS regression of $q_{f_s}^0$ on Q_s^0 (for the subsample where focal products are available), at the machine level and use the predicted value. This has the nice property that the error is orthogonal to Q_s^0 , which ensures that our choice of weeks is now unbiased. We run one regression for each client-site and report the results for one of the client-sites in Table 1.

We use our definition of control weeks s to compute the expected control sales that correspond to treatment week t as:

$$S_t = \{s : s \neq t, Q_t^0 - Q_s^1 \in [0, \hat{b}_0 + \hat{b}_1 Q_s^0]\} \tag{9}$$

The second challenge is that, although the experimental design is relatively clean, the product mix presented in a machine is not necessarily fixed across machines, or within a machine over long periods of time because we rely on observational data for the control weeks. For example, manufacturers may change their product lines, or Mark Vend may change its stocking decisions over time). Thus while our experiments intend to isolate the treatment effect of removing *Snickers* and *M&M Peanut*, we might instead compute the treatment effect of removing the Mars products jointly with changing pretzel suppliers. There are two possible options to mitigate this problem. The first would be to further restrict the set of potential control weeks so that control weeks had similar availability to our experimental weeks (at least among some subset of products). If we were considering one specific merger, it would make sense to focus our attention so that the set of available products was the same during treatment and control for the two merging firms. For example, if *3 Musketeers* was unavailable during our Mars experiment, we would restrict our control weeks only to weeks where *3 Musketeers* was also unavailable. This removes some contamination where substitution from *3 Musketeers* to *Reese's Peanut Butter Cups* is also attributed to diversion from *Snickers* and *M&M Peanut*. The disadvantage of this approach is that as we want to control more carefully for the set of available products, we reduce the number of potential control weeks.

An alternative approach is to adjust the product assortment from the control period to correspond to the product assortment in the treatment period. We do this by computing the percentage of consumers who would have seen the product available when making their purchase for the treatment and control period. For each machine, and each week we determine which products were available and then weight each week by it's corresponding sales at

the machine-level to form a consumer-weighted average level of availability at the site-week level. Thus busier machines and busier weeks are given more weight than less busy ones. This should roughly correspond to the probability that a randomly sampled consumer sees a product available for purchase. We construct this measure which we call “% Availability” for both treatment and control periods.¹¹ We then use the ratio $\frac{\%Avail_1}{\%Avail_0}$ to rescale the control vends to correspond to the availability during the treatment period.

The basic idea of the approach is as follows. If during the treatment period *Reese’s Peanut Butter Cups* was available during 75% of sales-weighted machine-weeks, and during the control it was available during 25% of sales-weighted machine weeks, then even without any change in the availability of the focal products, we might expect a threefold increase in the sales of *Reese’s Peanut Butter Cups*; therefore we multiply our observed control vends by $\frac{\%Avail_1}{\%Avail_0} = 3$ and use that to compute the change in vends. Likewise, a product that had 25% availability during the treatment period, but was available for 50% of the time during the control period would be adjusted so that $Vends_0$ was half as large. This avoids the problem of computing negative diversion just because products were less likely to be available during the treatment than during the control. This adjustment only addresses changes in “own product” availability. It does not account for the fact that sales of *Reese’s Peanut Butter Cups* might increase because availability of a close competitor such as *3 Musketeers* was reduced during the control period, and still attributes all of those sales to the effect of the experiment. The results of these adjustments are reported in Table 3 for the subset of products with positive diversion for the Mars experiment.¹² While this adjustment addresses changes in availability of substitute products, it no longer gets the aggregate diversion patterns correct. Even with our restriction on control weeks in (9), after the adjustments we can get diversion ratios that are less than zero or greater than 100% in aggregate. Therefore we rescale the adjusted change in vends for products with positive diversion by a constant (experiment-specific) factor, γ , for all products so that the aggregate diversion ratio is the same before and after our availability adjustment. This is demonstrated in Table 2.

When we review Table 3 we see that the adjustments are generally quite small, as availability is roughly the same during the treatment and control weeks. Though in a few cases the adjustment is crucial. For example, *Reese’s Peanut Butter Cups* is a close competitor to the two Mars products *Snickers* and *Peanut M&M’s* and sees the largest change in unadjusted sales. After the adjustment, the change in sales is considerably smaller (33.4 units

¹¹When we compute % Availability for control periods, we use the sales at the corresponding treatment weeks as determined by (9), rather than the sales at the control weeks themselves.

¹²The full set of adjustments for all product and all experiments are available in an online appendix.

instead of 118.2 units) but still quite substantial. Similarly, *Salty Other* shows a negative change in sales as a result of the Mars experiment implying that it is a complement rather than a substitute, but after adjusting for declining availability the sign changes.

And for each treatment week t we can compute the treatment effect and diversion ratio as:

$$\Delta q_{kt} = q_{kt}^1 - \frac{AV_k^{(1)}}{AV_k^{(0)}} \cdot \frac{\gamma}{\#S_t} \sum_{s \in S_t} q_{ks}^0 \quad (10)$$

$$D_{jk} = \frac{\Delta q_{kt}}{q_{jt}^{(0)}} \quad (11)$$

Once we have constructed our restricted set of treatment weeks and the set of control weeks that corresponds to each, inference is fairly straightforward. We use (10) to construct a set of pseudo-observations for the difference, and employ a t-test for differences.

4.2 Regression Based Approach

An alternative to the paired differences specification presented above, is to consider a regression based identification strategy for measuring diversion. Begin with data on the sales of each product j in week t and machine m :

$$q_{jmt} = \alpha_{jm} + \beta_j \times treatment_{mt} + \tilde{\epsilon}_{jmt} \quad (12)$$

In this specification α_{jm} is a product-machine specific intercept, and β_j is the effect the experiment has on the sales of j . Just like in the paired-differences setup we face two major challenges when identifying β_j . The first is that the level of overall sales varies quite a bit within machines from week to week. The second is that the availability of other (non-experimental) products may be correlated with our treatment. To deal with the second problem, we can include indicator variables for the availability of other products as controls, we label these coefficients γ_{jk} . To control for the overall sales level, we might like to include week specific fixed effects, however those would be collinear with our treatment. Instead we could consider interacting all of our right hand side variables with the a control for the size of the market at machine m in week t , M_{mt} . We can just rescale q_{jmt} by M_{mt} and work with the marketshare s_{jmt} instead of the quantity:

$$s_{jmt} \equiv \frac{q_{jmt}}{M_{mt}} = \alpha_{jm} + \beta_j \times treatment_{mt} + \sum_k \gamma_{jk} \times avail_{kmt} + \epsilon_{jmt} \quad (13)$$

Under this specification we construct the expected change in sales of k , and the corresponding diversion ratio implied by the experiment:

$$E[\Delta q_k] = \beta_k \times \mathbb{I}[\beta_k \geq 0] \times \sum_{(m,t):k \in A_{m,t}} M_{mt} \quad (14)$$

$$D_{jk} = \frac{E[\Delta q_k]}{E[\Delta q_j]}$$

In words, after computing the treatment effect β_k in marketshare terms, we compute diversion by predicting the change in quantity for each machine-week for which k is available, and divide that by the overall sales of the focal product across our dataset. We use the historical maximum weekly sales at the machine level as the market size. We can construct our diversion measure over just the treatment weeks (which corresponds to the paired differences result of the previous section), or all of the weeks in our observational data (which corresponds to the parametric models of the next section). One advantage that this regression based approach may have over the paired differences is that we are able to incorporate controls for β_k while still utilizing the entire dataset.

4.3 Parametric Specifications

In addition to computing treatment effects, we also specify two parametric models of demand: nested logit and random-coefficients logit, which are estimated from the full dataset (including weeks of observational data that do not meet any of our control criteria).

We consider a model of utility where consumer i receives utility from choosing product j in market t of:

$$u_{ijt} = \delta_{jt} + \mu_{ijt} + \varepsilon_{ijt} \quad (15)$$

The parameter δ_{jt} is a product-specific intercept that captures the mean utility of product j in market t , and μ_{ijt} captures individual-specific correlation in tastes for products.

In the case where $(\mu_{ijt} + \varepsilon_{ijt})$ is distributed generalized extreme value, the error terms allow for correlation among products within a pre-specified group, but otherwise assume no correlation. This produces the well-known nested-logit model of McFadden (1978) and Train (2003). In this model consumers first choose a product category l composed of products g_l , and then choose a specific product j within that group. The resulting choice probability for

product j in market t is given by the closed-form expression:

$$p_{jt}(\delta, \lambda, a_t) = \frac{e^{\delta_{jt}/\lambda_l} (\sum_{k \in g_l \cap a_t} e^{\delta_{kt}/\lambda_l})^{\lambda_l - 1}}{\sum_{\forall l} (\sum_{k \in g_l \cap a_t} e^{\delta_{kt}/\lambda_l})^{\lambda_l}} \quad (16)$$

where the parameter λ_l governs within-group correlation, and a_t is the set of available products in market t .¹³

The random-coefficients logit allows for correlation in tastes across observed product characteristics. This correlation in tastes is captured by allowing the term μ_{ijt} to be distributed according to $f(\mu_{ijt}|\theta)$. A common specification is to allow consumers to have independent normally distributed tastes for product characteristics, so that $\mu_{ijt} = \sum_l \sigma_l \nu_{ilt} x_{jl}$ where $\nu_{ilt} \sim N(0, 1)$ and σ_l represents the standard deviation of the heterogeneous taste for product characteristic x_{jl} . The resulting choice probabilities are a mixture over the logit choice probabilities for many different values of μ_{ijt} , shown here:

$$p_{jt}(\delta, \theta, a_t) = \int \frac{e^{\delta_{jt} + \sum_l \sigma_l \nu_{ilt} x_{jl}}}{1 + \sum_{k \in a_t} e^{\delta_{kt} + \sum_l \sigma_l \nu_{ilt} x_{kl}}} f(\nu) d\nu \quad (17)$$

Estimation then proceeds by full information maximum likelihood (FIML) in the case of nested logit or maximum simulated likelihood (MSL) in the case of the random coefficients. The log-likelihood is:

$$\hat{\theta}_{MLE} = \arg \max_{\theta} \sum_{j,t} q_{j,t} \log(p_{jt}(\delta, \theta; a_t))$$

In both the nested-logit and random-coefficient models we let $\delta_{jt} = d_j + \xi_t$, that is we separate mean utility into a product intercept and a market specific demand shifter. We include an additional ξ_t for each of our 15,256 machine-visits, or 2710 unique choice sets. For the nested-logit model, we allow for heterogeneous tastes across five major product categories or nests: chocolate candy, non-chocolate candy, cookie, salty snack, and other.¹⁴ For the random-coefficients specification, we allow for three random coefficients, corresponding to

¹³Note that this is not the IV regression/‘within-group share’ presentation of the nested-logit model in Berry (1994), in which σ provides a measure of the correlation of choices within a nest. Roughly speaking, in the notation used here, $\lambda = 1$ corresponds to the plain logit, and $(1 - \lambda)$ provides a measure of the ‘correlation’ of choices within a nest (as in McFadden (1978)). The parameter λ is sometimes referred to as the ‘dissimilarity parameter.’

¹⁴The vending operator defines categories in the same way. ‘Other’ includes products such as peanuts, fruit snacks, crackers, and granola bars.

consumer tastes for salt, sugar, and nut content.¹⁵

We provide information on how to calculate diversion ratios under these parametric models in the Appendix. An important decision is how to determine the choice set used in the counterfactual prediction exercise. We use a single choice set based on the most commonly available products during our treatment period. An alternative would be to consider a representative choice set based on some other period, or to predict the change in vends for each market observed in the data and then aggregate to obtain a choice-set weighted diversion prediction (similar to how we construct the OLS prediction). We choose the simpler representative choice set, because we believe it better approximates the approach likely to be taken by empirical researchers.

4.4 Identification and Parameter Estimates

The treatment effects approach and the parametric model rely on two different sources of identification. Formal nonparametric identification results for random utility models such as Berry and Haile (2010) or Fox and Gandhi (2012) often rely on variation across markets in continuous characteristics such as price. This is unavailable in the vending setting, since there is little to no price variation. Instead, the parametric models are identified through discrete changes in the choice set, primarily through product rotations. The intuition is that *Snickers* and *Milky Way* may look similar in terms of observable characteristics (a *Snickers* is essentially a *Milky Way* with peanuts), but have different marketshares. In fact, *Snickers* often outsells *Milky Way* 3:1 or better, this leads us to conclude that Snickers offers higher mean utility to consumers. At the same time, sales may respond differently to the availability of a third product. For example, if *Planters Peanuts* is introduced to the choice set, and it reduces sales of *Snickers* relatively more than *Milky Way* we might conclude there are heterogeneous preferences for peanuts. One challenge of this approach, is that while we might observe many differences in relative substitution patterns across products, we must in essence project them onto a lower dimensional basis of random coefficients. Thus if our model did not include a random parameter for peanuts, we would have to explain those

¹⁵We do not allow for a random coefficient on price because of the relative lack of price variation in the vending machines. We also do not include random coefficients on any discrete variables (such as whether or not a product contains chocolate). As we discuss in Conlon and Mortimer (Forthcoming), the lack of variation in a continuous variable (e.g., price) implies that random coefficients on categorical variables may not be identified when product dummies are included in estimation. We did estimate a number of alternative specifications in which we include random coefficients on other continuous variables, such as carbohydrates, fat, or calories. In general, the additional parameters were not significantly different from zero, and they had no appreciable effect on the results of any prediction exercises.

tastes with something else, like “salt”.

In the absence of experimental variation, many of the best-selling products are essentially always stocked by the retailer. Therefore we learn about how closely popular products compete primarily through how marketshares respond to the availability of a third often much less popular product. The identifying variation comes through the fact that we observe 2,710 different choice sets in 15,256 service visits. If for example, all machines stocked exactly the same set of products every week, we would only have a single choice set, and would struggle to identify nonlinear parameters.¹⁶

While the product rotations are crucial to the identification of the parametric models, they are somewhat of a nuisance to the identification of the treatment effects model. Product rotations introduce additional heterogeneity that must either be specifically controlled for, or risks introducing bias into the estimated treatment effect. The ideal identification setting for the treatment effect would be if there were no non-experimental variation in either prices or the set of available products. Thus the treatment effect estimator should perform best precisely when we worry the parametric demand model may not be identified and vice versa. This creates an inherent problem in any setting where the two approaches are pitted head to head in a “horse race” scenario.

Both the treatment effects approach and the discrete choice models benefit from experimental variation in the choice set. Because we consider diversion from popular products, there are very few cases “in the wild” where these items are not in stock. Thus, in the absence of experimental variation, the parametric model forecasts the effect of removing *Snickers* from how its sales are reduced when *Reese’s Peanut Butter Cups* (or some other substitute) were present. Obviously, with experimental variation this becomes an “in-sample” prediction exercise for the parametric model.

In Tables 4 and 5, we report the parameter estimates for our preferred specification of the random coefficients and nested logit models. We considered a large number of alternative models, but presented the results selected by the Bayesian Information Criteria (BIC). All of our estimates included 73 product-specific intercepts. In the case of the nested logit model, this included 5 nests (Chocolate, Non-Chocolate Candy, Cookie/Pastry, SaltySnack, and Other). For the random coefficients model, this included only three normally distributed random coefficients (Salt, Sugar, and Nut Content), though we considered additional specifications including coefficients on Fat, Calories, Chocolate, Protein, Cheese, and other product characteristics. Additionally we estimated both sets of models with both choice set fixed

¹⁶Typically this problem could be alleviated by seeing variation in prices within a product over time.

demand shifters ξ_t and machine-visit level demand shifters. While the extra demand shifters improved the fit, BIC preferred the smaller set of fixed effects.

To further explore the identification issues, we estimated each specification excluding the data from different experiments one at a time, and another where excluded all treatment periods. The results are clearest in the nested logit model. In that model all of the non-linear parameters λ increase in magnitude. As λ represents the dissimilarity parameter of McFadden (1978), where 0 is perfect correlation within the nest and $\lambda \rightarrow 1$ implies IIA logit substitution; this implies that without experimental variation there is less within nest correlation and the nested logit behaves more logit-like. This means that without experimental variation, the nested logit model would predict diversion patterns that would be explained mostly by ex-ante marketshares and less by the grouping of products. The results for the random coefficient model are less obvious, as withholding experimental variation leads to more heterogenous tastes for *nuts* but no statistically significant heterogeneous tastes for *sugar*.

5 Results

Our primary goal has been both to show how to measure diversion ratios experimentally, but also to understand how those experimentally measured diversion ratios compare to diversion ratios obtained from common parametric models of demand.

Table 6 shows the diversion computed for the top 5 substitutes under the treatment effects approach and the diversion computed under random coefficients, and nested logit models for each experiment. There is a general pattern that emerges. The logit-type models and the treatment effects approach predict a similar order for substitutes (i.e.: the best substitute, the second best substitute, and so on). Also, they tend to predict similar diversion ratios for the third, fourth and fifth best substitute. For example in the Snickers and M&M Peanut experiment, the treatment effects approach and the random coefficients logit approach predict around 4% diversion to *Reese's Peanut Butter Cups* and *M&M Milk Chocolate*. However, there tends to be a big discrepancy across all experiments for the diversion to the best-substitute product. The treatment effect predicts nearly 14.2% of consumers substituting to Twix, while the random coefficients model predicts only 6.4%. This effect is large enough that it could lead to incorrect conclusions about upward pricing pressure or a potential merger.

It is important to point out that our results should not be interpreted as showing that the treatment effects approach is always preferred to the parametric demand estimation

approach. In the *Famous Amos Cookie* experiment, the treatment effect shows that 20.7% of consumers switch from *Famous Amos Chocolate Chip Cookies* to *Sun Chips*, while the random coefficient model predicts diversion $< 1\%$ (the products quite dissimilar). While this effect appears large in the raw data, it represents an effect that most industry experts would be unlikely to believe. Moreover, the effect is not precisely estimated as indicated by table 7. In part this may be because we have imperfectly adjusted for increased availability of Sun Chips, or because we have failed to adjust for some close competitor of Sun Chips, or that overall sales levels of Sun Chips are highly variable.¹⁷ While the parametric demand models observe the reduced availability of *Smartfood Popcorn* and use that to improve its estimates of parameters, the treatment effects model is now faced with confounding variation.

Table 7 shows diversion ratios aggregated to the level of a manufacturer. The results exhibit the same pattern as before where the treatment effects predict much more substitution to the closest substitutes than the random coefficients and nested logit models do. In some sense, for merger analysis, correctly predicting diversion to one or two closest substitutes might be the most important feature of a model or identification strategy, since those are the potential mergers we should be most concerned about. The other potential anomaly is that after adjusting for product availability, the aggregate diversion measures from the paired differences approach can be unrealistically high ($> 240\%$). In the columns labeled “Adj. Differences” and “Adj. OLS” we re-scale all of the diversion estimates, so that the aggregate diversion (among manufacturers with positive diversion) matches the original aggregate diversion in the paired (but unadjusted for availability) data. Much of the differences between the parametric models and the treatment effects approach can be attributed to substitution to the outside good. The parametric models predict that consumers will be diverted to some other product at rate of approximately 30-45%, while the paired and OLS approaches predict that consumers will be diverted at a rate of 65-75%. All of the models broadly predict the same relative magnitudes, but the parametric models predict smaller aggregate magnitudes. A different parametrization of the outside share, might lead to more similar estimates.

In table 8 we compute the gross upward pricing pressure index (GUPPI). Guppi is $GUPPI_j = D_{jk} \frac{p_k - c_k}{p_j}$. Here we exploit the fact that we observe the wholesale contracts

¹⁷Imagine that our treatment weeks at our largest location are correlated with reduced availability of *Smartfood Popcorn* for reasons unrelated to our experiment, such as supply problems at the warehouse (It is!). In that case we could think about the actual experiment as removing *Famous Amos Cookies* and *Smartfood Popcorn* simultaneously, but the one we measure as attributing the entire effect to the *Famous Amos Cookies*.

between the retailer and the manufacturer, and let the wholesale price serve as p_j, p_k in the above expression. We assume that all products have a manufacturing cost of \$0.15, and exploit the fact that within manufacturer-category there is no wholesale price variations: (*Snickers* and *3 Musketeers* have the same wholesale price even though one is much more popular than the other). Under a symmetry assumption (not true in our example) the critical value for GUPPI is generally 10%, because it corresponds to SSNIP of 5% under the hypothetical monopolist test. If we apply the same 10% threshold to our results we find that a Mars-Hershey merger would likely attract further scrutiny, but not a Mars-Nestle merger (at least not for the prices of Snickers and M&M Peanut). Meanwhile the acquisition of the Mars or Pepsi product portfolio might place upward pricing pressure on either the Animal Crackers, as they exhibit large point estimates for GUPPI under the treatment effects approach, though diversion is not statistically significant from zero, and the effects are imprecise. Similarly the Famous Amos cookies show a large but imprecise point estimate for the acquisition of the Pepsi portfolio (driven by an inexplicable rise in the sales of Sun Chips).

6 Conclusion

Under the revised 2010 Horizontal Merger Guidelines, the focus on the diversion ratio and the “single product merger simulation” approach in UPP and GUPPI where all other prices and quantities are held fixed, imply that the diversion ratio serves as a single sufficient statistic for at least the initial screening stage of merger evaluation. We show that the diversion ratio has the attractive empirical property that it can be estimated via experimental or quasi-experimental techniques in a relatively straightforward manner. Our hope is that this makes a well-developed set of tools available both to IO researchers, and also to antitrust practitioners.

At the same time, we are quick to point out that while the diversion ratio can be obtained experimentally, it is not trivial, and researchers should think carefully about which treatment effect their experiment (or quasi-experiment) is actually identifying; as well as what identifying assumptions required for estimating the diversion ratio implicitly assume about the structure of demand. An important characteristic of many retail settings is that even category level sales are much more variable than most product level market shares. We partially alleviate this problem by employing full removal of the product, rather than marginal price increases, and derive conditions under which this may provide a reasonable approximation to the marginal diversion ratio. Even under our design, the overall variabil-

ity in the sales levels can make diversion ratios difficult to measure. If the agencies were to require that firms submit to similar experiments as part of the merger review process, they would likely incorporate prior information and focus experiments on a narrower set of potential products than we considered in our setting.

Perhaps someday soon, potential mergers could even be evaluated by agreeing to an experiment in lieu of lengthy and costly court proceedings.

References

- ANGRIST, J., AND J.-S. PISCHKE (2010): “The Credibility Revolution in Empirical Economics: How Better Research Design is Taking the Con out of Econometrics,” *The Journal of Economic Perspectives*, 24(2), 3–30.
- BERRY, S. (1994): “Estimating discrete-choice models of product differentiation,” *RAND Journal of Economics*, 25(2), 242–261.
- BERRY, S., AND P. HAILE (2010): “Nonparametric Identification of Multinomial Choice Demand Models with Heterogeneous Consumers,” Working Paper.
- CARLTON, D. W. (2010): “Revising the Horizontal Merger Guidelines,” *Journal of Competition Law and Economics*, 6(3), 619–652.
- CHETTY, R. (2009): “Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods,” *Annual Review of Economics*, 1(1), 451–488.
- CHEUNG, L. (2011): “The Upward Pricing Pressure Test for Merger Analysis: An Empirical Examination,” Working Paper.
- CONLON, C., AND J. H. MORTIMER (2013): “All Units Discount: Experimental Evidence from the Vending Industry,” Working Paper.
- (Forthcoming): “Demand Estimation Under Incomplete Product Availability,” *American Economic Journal: Microeconomics*.
- EINAV, L., AND J. LEVIN (2010): “Empirical Industrial Organization: A Progress Report,” *The Journal of Economic Perspectives*, 24(2), 145–162.
- FARRELL, J., AND C. SHAPIRO (2010): “Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition,” *The B.E. Journal of Theoretical Economics*, 10(1), 1–41.
- FOX, J., AND A. GANDHI (2012): “Nonparametric Identification and Estimation of Random Coefficients in Nonlinear Economic Models,” Working Paper.
- FOX, J. T., K.-I. KIM, S. P. RYAN, AND P. BAJARI (2011): “A Simple Estimator for the Distribution of Random Coefficients,” *Quantitative Economics*, 2(3), 381–418.
- HAUSMAN, J. A. (2010): “2010 Merger Guidelines: Empirical Analysis,” Working Paper.
- HECKMAN, J. J. (2010): “Building Bridges Between Structural and Program Evaluation Approaches to Evaluating Policy,” *Journal of Economic Literature*, 48(2), 356–398.
- JAFFE, S., AND E. G. WEYL (Forthcoming): “The First Order Approach to Merger Analysis,” *American Economic Journal: Microeconomics*.

- KEANE, M. (2010): “A Structural Perspective on the Experimentalist School,” *The Journal of Economic Perspectives*, 24(2), 47–58.
- LEAMER, E. (1983): “Let’s Take the Con Out of Econometrics,” *American Economic Review*, 75(3), 308–313.
- (2010): “Tantalus on the Road to Asymptopia,” *The Journal of Economic Perspectives*, 24(2), 31–46.
- MCFADDEN, D. (1978): “Modelling the Choice of Residential Location,” in *Spatial Interaction Theory and Planning Models*, ed. by A. Karlqvist, L. Lundsvist, F. Snickars, and J. Weibull. North-Holland.
- MILLER, N. H., M. REMER, C. RYAN, AND G. SHEU (2012): “Approximating the Price Effects of Mergers: Numerical Evidence and an Empirical Application,” Working Paper.
- NEVO, A. (2001): “Measuring Market Power in the Ready-to-Eat Cereal Industry,” *Econometrica*, 69, 307–342.
- NEVO, A., AND M. WHINSTON (2010): “Taking the Dogma out of Econometrics: Structural Modeling and Credible Inference,” *The Journal of Economic Perspectives*, 24(2), 69–82.
- REYNOLDS, G., AND C. WALTERS (2008): “The use of customer surveys for market definition and the competitive assessment of horizontal mergers,” *Journal of Competition Law and Economics*, 4(2), 411–431.
- SCHMALENSSEE, R. (2009): “Should New Merger Guidelines Give UPP Market Definition?,” *Antitrust Chronicle*, 12, 1.
- SIMS, C. (2010): “But Economics Is Not an Experimental Science,” *The Journal of Economic Perspectives*, 24(2), 59–68.
- STOCK, J. (2010): “The Other Transformation in Econometric Practice: Robust Tools for Inference,” *The Journal of Economic Perspectives*, 24(2), 83–94.
- TRAIN, K. (2003): *Discrete Choice Methods with Simulation*. Cambridge University Press.
- WILLIG, R. (2011): “Unilateral Competitive Effects of Mergers: Upward Pricing Pressure, Product Quality, and Other Extensions,” *Review of Industrial Organization*, 39(1-2), 19–38.

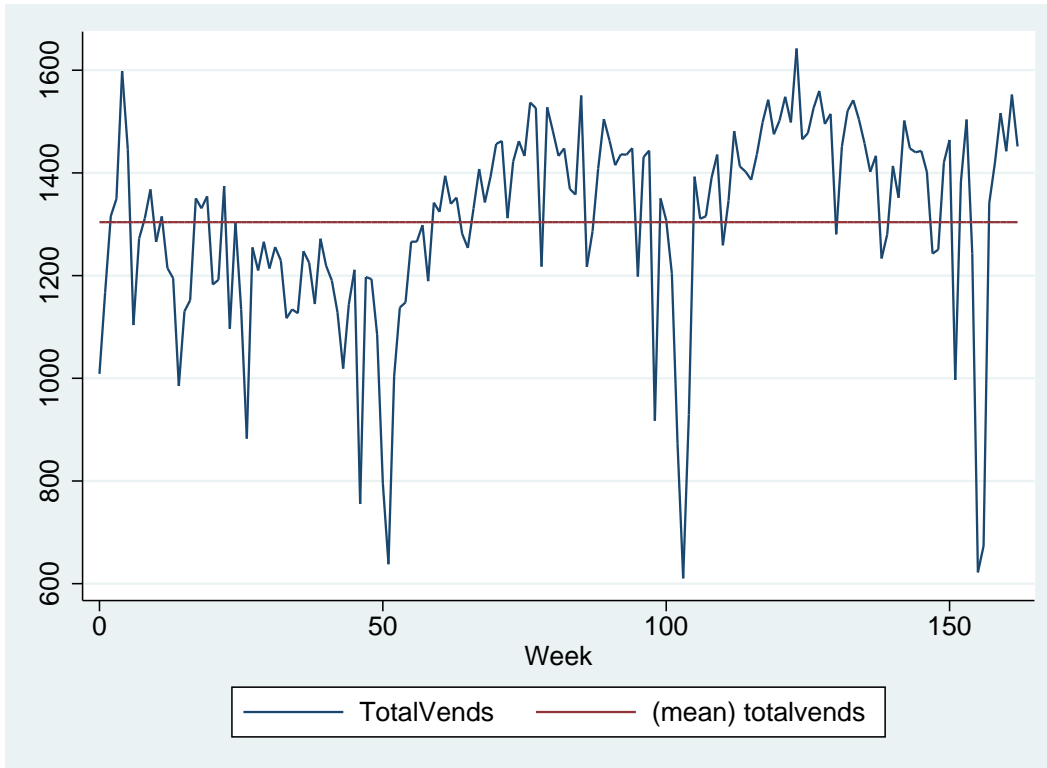


Figure 1: Total Sales by Week, Site 93

Focal Sales	β_0	$\beta_{totalsales}$	R^2
Zoo Animal Cracker	6.424	0.039	0.503
Famous Amos Cookie	6.937	0.025	0.395
Doritos and Cheetos	-16.859	0.078	0.628
Snickers and Peanut M&Ms	-1.381	0.127	0.704

Table 1: Selection of Control Weeks: Regression for Focal Sales at Site 93

Experiment	Unadjusted	Adjusted	Adjusted (> 0)	Rescaling Factor
Zoo Animal Cracker	77.35	84.58	246.11	0.31
Famous Amos Cookie	65.30	-178.83	181.98	0.36
Cheetos and Doritos	66.22	-7.57	101.62	0.65
Snickers and Peanut M&M	66.47	17.69	79.00	0.84

Table 2: Selection of Control Weeks: Regression for Focal Sales at Site 93

Manufacturer	Product	$Vends_0$	$Vends_1$	(% Avail ₀)	(% Avail ₁)	Adjusted Control
Hershey	Payday	1.2	6.6	1.3	2.5	2.3
Hershey	Reeses Peanut Butter Cups	56.5	174.7	29.0	72.5	141.3
Hershey	Twizzlers	24.3	57.3	17.7	37.0	50.7
Hershey	Choc Herhsey (Con)	37.2	65.9	29.9	35.5	44.2
Kellogg	Brown Sug Pop-Tarts	5.9	5.9	4.6	3.7	4.7
Kellogg	Strwbry Pop-Tarts	127.6	139.6	77.7	78.7	129.2
Kellogg	Cheez-It Original SS	190.0	206.2	84.6	82.3	184.8
Kellogg	Choc Chip Famous Amos	189.7	209.5	99.9	100.0	189.9
Kellogg	Rice Krispies Treats	26.1	82.7	30.7	60.8	51.7
Kraft	Ritz Bits Chs Vend	21.9	27.1	37.0	45.3	26.8
Kraft	100 Cal Oreo Thin Crisps	32.4	46.9	25.3	33.5	42.8
Mars	M&M Milk Chocolate	113.2	144.6	61.0	57.6	106.7
Mars	Milky Way	17.0	64.3	17.9	26.9	25.5
Mars	Twix Caramel	182.7	301.4	78.6	80.0	186.0
Mars	Nonchoc Mars (Con)	33.3	44.5	26.1	25.2	32.2
Nestle	Butterfinger	41.3	61.8	31.3	31.8	41.9
Nestle	Raisinets	149.2	199.3	80.8	83.1	153.5
Pepsi	Frito LSS	160.7	187.1	75.6	78.9	167.8
Pepsi	Grandmas Choc Chip	78.9	83.7	55.3	52.0	74.2
Pepsi	Lays Potato Chips 1oz SS	117.8	181.2	53.5	78.8	173.5
Pepsi	Baked Chips (Con)	208.1	228.0	91.7	97.5	221.3
Snyders	Snyders (Con)	367.6	418.6	82.6	87.2	387.9
Sherwood	Ruger Wafer (Con)	116.6	151.9	63.7	80.9	148.1
Kar's Nuts	Kar Sweet&Salty Mix 2oz	111.2	134.9	61.5	66.9	121.1
Misc	Rasbry Knotts	63.7	72.4	82.5	77.9	60.1
Misc	Farleys Mixed Fruit Snacks	66.6	78.8	57.3	67.4	78.3
Misc	Other Pastry (Con)	0.5	10.5	1.6	5.0	1.5
Misc	Salty Other (Con)	35.7	29.1	20.4	15.4	27.1

Table 3: Adjustments of Control Weeks: Mars (Snickers, Peanut M&M's) Experiment

Table 4: Random Coefficients Model Estimates

	Base		Holdout Analyses			
	Visit FE	Choice FE	All	Choc	Cookie	Salty
σ_{Salt}	0.506 [.006]	0.458 [.010]	0.567 [.006]	0.532 [.011]	0.511 [.012]	0.448 [.009]
σ_{Sugar}	0.673 [.005]	0.645 [.012]	0.001 [.001]	0.277 [.024]	0.585 [.011]	0.620 [.011]
σ_{Peanut}	1.263 [.037]	1.640 [.028]	1.896 [.033]	1.530 [.016]	1.780 [.009]	1.671 [.035]
# Nonlinear Params	3	3	3	3	3	3
Product FE	73	73	73	73	73	73
# Fixed Effects ξ_t	15256	2710	1985	2507	2517	2563
Total Parameters	15332	2786	2061	2583	2593	2639
LL	-4372750	-4411184	-3598162	-4136723	-4242030	-4196647
Total Sales	2960315	2960315	2431633	2778944	2851096	2820548
BIC	8973960	8863881				
AIC	8776165	8827939				

Table 5: Nested Logit Model Estimates

	Base		Holdout Analyses			
	Visit FE	Choice FE	All	Choc	Cookie	Salty
$\lambda_{Chocolate}$	0.828 [.003]	0.810 [.005]	0.917 [.006]	0.873 [.005]	0.813 [.005]	0.814 [.005]
$\lambda_{CandyNon-Choc}$	0.908 [.007]	0.909 [.009]	0.947 [.010]	0.916 [.009]	0.918 [.009]	0.910 [.009]
$\lambda_{Cookie/Pastry}$	0.845 [.004]	0.866 [.006]	0.933 [.007]	0.901 [.007]	0.869 [.006]	0.874 [.006]
λ_{Other}	0.883 [.005]	0.894 [.006]	0.937 [.007]	0.926 [.007]	0.895 [.006]	0.897 [.006]
$\lambda_{SaltySnack}$	0.720 [.003]	0.696 [.004]	0.767 [.004]	0.729 [.004]	0.702 [.004]	0.704 [.004]
# Nonlinear Params	5	5	5	5	5	5
Product FE	73	73	73	73	73	73
# Fixed Effects ξ_t	15256	2710	1985	2507	2517	2563
Total Parameters	15334	2788	2063	2585	2595	2641
LL	-4372147	-4410649	-3597958	-4136342	-4241561	-4196159
Total Sales	2960315	2960315	2431633	2778944	2851096	2820548
BIC	8972783	8862840				
AIC	8774962	8826873				

Manufacturer	Product	Adj. Diversion	Estimate(RC)	Estimate(NL)	Blank
Zoo Animal Cracker Experiment					
Mars	M&M Peanut	11.9	1.9	1.5	
Mars	M&M Milk Chocolate	4.2	1.0	0.8	
Mars	Snickers	7.6	1.8	1.4	
Pepsi	Sun Chip LSS	4.2	0.9	0.9	
Pepsi	Rold Gold (Con)	10.0	2.2	1.4	
Famous Amos Experiment					
Hershey	Choc Herhsey (Con)	6.0	8.5	0.8	
Pepsi	Sun Chip LSS	20.7	0.7	1.2	
Pepsi	Rold Gold (Con)	9.1	1.3	1.8	
Planters	Planters (Con)	10.5	0.6	1.8	
Misc	Rasbry Knotts	3.9	0.3	1.3	
Doritos and Cheetos Experiment					
Pepsi	Frito LSS	15.4	1.2	5.0	
Pepsi	Baked Chips (Con)	6.3	1.1	4.8	
Pepsi	FritoLay (Con)	11.1	1.4	3.7	
Pepsi	Ruffles (Con)	7.3	1.3	6.0	
General Mills	Nature Valley Swt&Salty Alm	3.9	0.6	0.5	
Snickers and Peanut M&Ms Experiment					
Hershey	Reeses Peanut Butter Cups	4.1	4.1	3.1	
Mars	M&M Milk Chocolate	4.7	4.0	3.6	
Mars	Milky Way	4.8	2.4	2.5	
Mars	Twix Caramel	14.2	6.4	4.3	
Nestle	Raisinets	5.6	3.5	2.9	

Table 6: Top 5 Substitutes by Experiment

Manufacturer	Differences	Adj. Differences	Adj. OLS	Nested Logit	RC Logit
Animal Crackers Experiment					
Misc	23.6	8.3	11.3	3.8	2.5
General Mills			.		
Kar's Nuts	7.0	2.4	1.7	1.3	1.0
Sherwood	28.7	10.0	4.5	3.5	0.6
Snyders			.		
Pepsi	82.6	28.9	23.3	8.2	8.9
Nestle	0.2	0.1	4.4	1.1	1.5
Mars	6.1	2.2	12.7	5.5	6.8
Kraft	35.6	12.5	4.2	6.1	3.4
Kellogg	1.0	0.4	2.6	5.9	1.7
Hershey			.	1.5	1.9
Total	184.8	64.7	64.7	37.0	28.3
Famous Amos Cookie Experiment					
Misc	23.3	6.3	20.0	8.3	6.4
General Mills			.	1.0	0.8
Kar's Nuts			.		
Sherwood	19.2	5.2	.		
Snyders			.		
Pepsi	150.0	40.3	35.3	8.5	8.0
Nestle			.		
Mars			.	0.9	4.4
Kraft	34.8	9.4	5.6	4.1	5.2
Kellogg			.	5.9	4.4
Hershey	14.7	4.0	4.1	2.3	9.5
Total	241.9	65.1	65.1	30.9	38.8
Cheetos and Doritos Nacho Experiment					
Misc	8.2	4.1	13.0	6.0	3.6
General Mills	0.3	0.2	8.6	0.5	0.6
Kar's Nuts			.		
Sherwood	9.3	4.7	.		
Snyders	7.0	3.6	2.8		
Pepsi	94.4	47.7	37.8*	30.2	7.9
Nestle			.		
Mars			.	2.7	3.7
Kraft	8.2	4.1	1.9	1.3	1.8
Kellogg			.	0.7	0.9
Hershey	23.8	12.0	12.2	2.7	4.8
Total	151.2	76.4	76.4	44.1	23.4
Snickers and M&M Peanut Experiment					
Misc	3.6	3.1	1.7	1.8	1.1
General Mills			.		
Kar's Nuts	4.0	3.4	2.6	0.7	0.9
Sherwood	5.0	4.3	.	0.7	0.6
Snyders	0.6	0.6	2.3	2.0	1.5
Pepsi	1.4	1.2	8.9	3.7	4.2
Nestle	8.5	7.3	7.7**	4.9	5.9
Mars	25.2	21.7	9.7	10.8	13.1
Kraft	3.2	2.8	6.8*	1.4	3.3
Kellogg	1.9	1.7	10.5*	3.2	5.3
Hershey	20.6	17.8	13.8***	6.8	8.7
Total	74.1	63.9	63.9	36.1	44.6

Table 7: Diversion: Manufacturer Level Results

Manufacturer	Adj. Differences	Adj. OLS	Nested Logit	RC Logit
Animal Crackers Experiment				
Pepsi	18.6	18.3	5.3	5.7
Mars	2.3	16.6	5.9	7.3
Sherwood	6.3	2.2	2.2	0.4
Kraft	5.9	2.3	2.9	1.6
Nestle	0.1	4.0	1.0	1.3
Famous Amos Cookie Experiment				
Pepsi	23.1	28.4	4.9	4.6
Hershey	4.2	3.1	2.4	10.1
Mars			0.9	4.2
Kraft	3.9	3.2	1.7	2.2
Cheetos and Doritos Nacho Experiment				
Pepsi	23.8	24.9	15.1	4.0
Hershey	11.1	7.7	2.5	4.5
General Mills	0.1	3.2	0.2	0.2
Mars			2.3	3.1
Snickers and M&M Peanut Experiment				
Mars	13.6	7.0	6.8	8.2
Hershey	12.3	5.8	4.7	6.0
Pepsi	0.4	3.9	1.4	1.6
Nestle	3.7	3.8	2.5	3.0
Kellogg	0.5	3.5	0.9	1.5

Table 8: Gross Upward Pricing Pressure Estimates

A Appendix:

A.1 Diversion Ratio under Parametric Demands

This section derives explicit formulas for the diversion ratio under common parametric forms for demand. The focus is whether or not a demand model implies that the diversion ratio is constant with respect to the magnitude of the price increase. It turns out that the IIA Logit and the Linear demand model exhibit this property, while the log-linear model, and mixed logit model do not necessarily exhibit this property.

We go through several derivations below:

Linear Demand

The diversion ratio under linear demand has the property that it does not depend on the magnitude of the price increase. To see this consider that the linear demand is given by:

$$Q_k = \alpha_k + \sum_j \beta_{kj} p_j$$

Which implies a diversion ratio corresponding to a change in price p_j of Δp_j :

$$D_{jk} = \frac{\Delta Q_k}{\Delta Q_j} = \frac{\beta_{kj} \Delta p_j}{\beta_{jj} \Delta p_j} = \frac{\beta_{kj}}{\beta_{jj}} \quad (18)$$

This means that for any change in p_j from an infinitesimal price increase, up to the choke price of j ; the diversion ratio, D_{jk} is constant. This also implies that under linear demands, divergence is a global property, under any initial set of prices, quantities, or any magnitude of price increase will result in the same diversion.

Log-Linear Demand

The log-linear demand model does not exhibit constant diversion with respect to the magnitude of the price increase. The log-linear model is specified as:

$$\ln(Q_k) = \alpha_k + \sum_j \epsilon_{kj} \ln(p_j)$$

If we consider a small price increase Δp_j the diversion ratio becomes:

$$\begin{aligned} \frac{\Delta \log(Q_k)}{\Delta \log(Q_j)} &\approx \underbrace{\frac{\Delta Q_k}{\Delta Q_j}}_{D_{jk}} \cdot \frac{Q_j(\mathbf{p})}{Q_k(\mathbf{p})} = \frac{\epsilon_{kj} \Delta \log(p_j)}{\epsilon_{jj} \Delta \log(p_j)} = \frac{\epsilon_{kj}}{\epsilon_{jj}} \\ D_{jk} &\approx \frac{Q_k(\mathbf{p})}{Q_j(\mathbf{p})} \cdot \frac{\epsilon_{kj}}{\epsilon_{jj}} \end{aligned} \quad (19)$$

This holds for small changes in p_j . However for larger changes in p_j we can no longer use the simplification that $\Delta \log(Q_j) \approx \frac{\Delta Q_j}{Q_j}$. So for a large price increase (such as to the choke price $p_j \rightarrow \infty$, log-linear demand can exhibit diversion that depends on the magnitude of the price increase.

IIA Logit Demand

The plain logit model exhibits IIA and proportional substitution. This implies that the diversion ratio does not depend on the magnitude of the price increase. Here we consider two price increases, an infinitesimal one and an increase to the choke price $p_j \rightarrow \infty$.

Consider the derivation of the diversion ratio D_{jk} under simple IIA logit demands. We have utilities and choice probabilities given by the well known equations, where a_t denotes the set of products available

in market t :

$$u_{ijt} = \underbrace{x_{jt}\beta - \alpha p_{jt}}_{\tilde{v}_{jt}} + \varepsilon_{ijt}$$

$$P_{jt} = \frac{\exp[\tilde{v}_{jt}]}{1 + \sum_{k \in a_t} \exp[\tilde{v}_{kt}]} = \frac{V_{jt}}{IV(a_t)}$$

Under logit demands, an infinitesimal price change in p_1 exhibits identical diversion to setting $p_1 \rightarrow \infty$ (the choke price):

$$\widehat{D}_{jk} = \frac{\frac{\partial S_k}{\partial p_j}}{\left| \frac{\partial S_j}{\partial p_j} \right|} = \frac{\alpha S_k S_j}{\alpha S_j (1 - S_j)} = \frac{S_k}{(1 - S_j)}$$

$$\overline{D}_{jk} = \frac{\frac{e^{V_k}}{1 + \sum_{l \in a \setminus j} e^{V_l}} - \frac{e^{V_k}}{1 + \sum_{l' \in a} e^{V_{l'}}}}{0 - \frac{e^{V_j}}{1 + \sum_{l \in a} e^{V_l}}} = \frac{S_k}{(1 - S_j)}$$

As an aside $\frac{S_k}{1 - S_k} = \frac{Q_k}{M - Q_k}$, so we either need market shares or market size (back to market definition!). In both cases diversion is merely the ratio of the marketshare of the substitute good divided by the share not buying the focal good (under the initial set of prices and product availability). It does not depend on any of the econometric parameters (α, β) .

Random Coefficients Logit Demand

Random Coefficients Logit demand relaxes the IIA property of the plain Logit model, which can be undesirable empirically, but it also means that the diversion ratio varies with original prices and quantities, as well as with the magnitude of the price increase. Intuitively a small price increase might see diversion from the most price sensitive consumers, while a larger price increase might see substitution from a larger set of consumers. If price sensitivity is correlated with other tastes, then the diversion ratio could differ with the magnitude of the price increase.

We can repeat the same exercise for the logit model with random coefficients, by discretizing a mixture density over $i = 1, \dots, I$ representative consumers, with population weight w_i :

$$u_{ijt} = \underbrace{x_{jt}\beta_i - \alpha_i p_{jt}}_{\tilde{v}_{ijt}} + \varepsilon_{ijt}$$

Even when consumers have a common price parameter $\frac{\partial V_{ik}}{\partial p_j} = \alpha$,

$$\widehat{D}_{jk} = \frac{\frac{\partial S_k}{\partial p_j}}{\left| \frac{\partial S_j}{\partial p_j} \right|} = \frac{\int s_{ij} s_{ik} \frac{\partial V_{ik}}{\partial p_j}}{\int s_{ij} (1 - s_{ij}) \frac{\partial V_{ij}}{\partial p_j}} \rightarrow \frac{\int s_{ij} s_{ik}}{\int s_{ij} (1 - s_{ij})}$$

$$\overline{D}_{jk} = \frac{\int \frac{e^{V_{ik}}}{1 + \sum_{l \in a \setminus j} e^{V_{il}}} - \frac{e^{V_{ik}}}{1 + \sum_{l' \in a} e^{V_{il'}}}}{\int - \frac{e^{V_{ij}}}{1 + \sum_{l \in a} e^{V_{il}}}} = \frac{1}{s_j} \int \frac{s_{ij} s_{ik}}{(1 - s_{ij})}$$

Now, each individual exhibits constant diversion, but weights on individuals vary with p , so that diversion is only constant if $s_{ij} = s_j$. Otherwise observations with larger s_{ij} are given more weight in correlation of $s_{ij} s_{ik}$. The more correlated (s_{ij}, s_{ik}) are (and especially as they are correlated with α_i) the greater the discrepancy between marginal and average diversion.