Search, Design, and Market Structure[1]

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Abstract

The Internet has made consumer search easier, with consequences for prices, industry structure and the kinds of products offered. We explore these consequences in a rich but tractable model that allows for strategic design choices. A polarized market structure results, where some firms choose designs aiming for broad-based audiences, while others target narrow niches. Such an industry structure can arise even when all firms and consumers are ex-ante identical. We analyze the effect of reduced search costs and find results consistent with the reported prevalence of niche goods and the long-tail and superstar phenomena.

JEL: D83, L11, L86, M31

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The Internet has dramatically changed the nature of demand and competition. A familiar example is the book-publishing industry. Easier access to information on

available titles has increased the number of specialized books that cater to ever more specific audiences. In recent years, while the overall size of the market has remained roughly constant, the composition of book sales has changed. From 2002 to 2007, the number of new titles and editions grew at the astonishing rate of approximately ten percent each year; indeed, the number of new titles in 2007 alone surpassed the total published throughout the 1970s. More broadly, observers have highlighted that new production and search technologies have changed the pattern of sales and market structure to the benefit of fringe goods in the "long tail" in many industries.[2] However, another common view, informed by search models, is that cheaper access to information leads to very competitive markets with low price dispersion and a few, high-quality, superstar products, such as the *Harry Potter* series or *The Da Vinci Code*.[3]

While the long-tail and superstar phenomena are often portrayed as opposed to each other, reality has proven more subtle, as the publishing-industry suggests. In another example, Jeff Bewkes, the Chairman and C.E.O. of Time Warner, points out that in media industries, the two effects arise simultaneously:

> Audiences are at once fragmenting into niches and consolidating around blockbusters. Of course, media consumption has not risen much over the years, so something must be losing out. That something is the almost but not quite popular content that occupies the middle ground between blockbusters and niches. The stuff that people used to watch or listen

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[2]The term "long tail" refers to the well-documented and dramatic increase in the market share for goods in the tail of the sales distribution (that is, with relatively low sales). The phrase was coined in an article in *Wired* (Chris Anderson, 2004) and was later expanded and developed in Anderson (2006 and 2009) who focussed on books, music, and film sales and rentals but argued that the phenomena is much broader. See Erik Brynjolfsson, Jeffrey Yu Hu and Michael D. Smith (2006) for a discussion and references to academic work and emprical examples, and, in particular, Brynjolfsson, Hu and Duncan Simester (2007) who examine data from a physical and online women's clothing catalog.

[3]The facts in this paragraph are collected from a variety of sources. That the overall size of the book market is relatively unchanged (revenues fell by 5%) can be found in the U.S. Census, which also shows that the number of book publishers (NAICS-51113) fell by 10%. It also shows that the number of smaller publishers (revenue under $50m) has increased (by 27% in the year 2009!) totalling an unprecedented (in the data available) 68% of the revenue of the whole market. The data on new book titles from 2002-2007 is available from Bowker (the exclusive U.S. ISBN and SAN Agency). The comparison with the 1970s brings this information together with data in Albert N. Greco (2005). The startling success of the *Harry Potter* series anf the *Da Vinci Code* can be verified by examining sales figures across several issues of *Publishers Weekly*.

> to largely because there was little else on is increasingly being ignored.
> (*The Economist*, 2009)

In this paper, we allow for a richer choice of firm strategies than the search literature has typically considered. Specifically, firms choose the "design" of their products in addition to price. Our notion of design is broad and can accommodate not only physical design, but also marketing and information disclosure. In the book publishing example, it is broad enough to accommodate a publisher's decisions both about commissioning particular topics or accepting particular kinds of manuscripts and such marketing decisions as making sample chapters available on line. Our approach allows us to address how designs adapt as search costs fall and to consider the equilibrium effects on market structure, prices and consumer surplus. In particular, our analysis leads naturally to long-tail and superstar effects arising simultaneously, and to prices and industry profits that are non-monotonic in search costs.[4]

Formally, our notion of design choice builds on a recent and growing literature, notably Justin P. Johnson and David P. Myatt (2006), with an important antecedent in Tracy R. Lewis and David E. M. Sappington (1994).[5] While this literature has focused on design choices by monopolists, this paper extends this analysis to a competitive environment. To do so, we introduce product design, along the lines of Johnson and Myatt (2006), into a search model (Asher Wolinsky, 1986; Yannis Bakos, 1997; or Anderson and Renault, 1999). In particular, firms choose designs ranging from broad market designs that are inoffensive to all consumers to more niche or quirky designs that consumers either love or loathe. Meanwhile, each consumer searches

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[4]There is a small related literature that considers firms that vary design in response to falling search costs. Nathan Larson (2008) studies horizontal differentiation in a model of sequential search with a particular emphasis on welfare considerations in what can be viewed as a special case of our model. Dimitri Kuksov (2004) presents a duopoly model where consumers know the varieties available (but not their location) prior to search, and different designs come with different costs associated; Simon Anderson and Régis Renault (forthcoming) also consider duopoly and, in a result similar to one in this paper, show that it is the low-quality firm that has the greater incentives to release information on horizontal characteristics; Gérard Cachon, Christian Terwiesch and Ye Xu (2008) and Randall Watson (2007) focus specifically on multi-product firms' choices of product range. Our model allows for a wide range of designs and a much more general demand specification. It, also, has a different focus and results from these papers, which, for example, do not consider sales distributions explicitly and so do not address long-tail and superstar effects.

[5]More recently, Heski Bar-Isaac, Guillermo Caruana and Vicente Cuñat (2008, 2010) put more emphasis on consumers' information-gathering decisions and highlight that these are co-determined with the firm's pricing, design and marketing strategies in equilibrium.

among firms, paying a small cost to obtain a price quote from an additional firm and to learn about the extent to which that firm's product suits his tastes.

The model allows us to address the impact of search engines, the Internet, communication technologies and information technologies in general, by considering these as a fall in search costs. We show, first, that firms choose extremal strategies—that is, either a most-broad or a most-niche design. Second, more-advantaged firms choose most-broad designs, while disadvantaged firms prefer most-niche designs. Our central results are on the equilibrium market outcomes. Allowing for an endogenous choice of product design reveals that lower search costs have an indirect effect on prices and profits through changes to the offered designs. Lower search costs induce more firms to choose niche designs, effectively softening competition. Consequently, prices and profits can be non-monotonic in search costs. In particular, we show that profits increase as search costs fall only when ex-ante differences between firms are relatively small. Instead, if firms are sufficiently vertically differentiated, then reducing search costs intensifies price competition and leads to lower industry profits.

Reduced search costs and endogenous designs also have interesting effects on sales distributions. Lower search costs allow consumers to search longer and find "better" firms. This leads to a superstar effect where better firms are even more successful.[6] Furthermore, consumers are also more likely to buy better-suited products. In our model, the overall size of the market stays constant. However, and consistent with Jeff Bewkes's comments quoted above, the stars and the tail can both increase market share at the expense of middling firms.[7] These firms, facing a more competitive environment, switch to niche designs with lower sales and higher markups, thereby releasing additional buyers. Some of these buyers will end up purchasing from niche firms, and, so, boost their sales and allow for a long-tail effect. Overall, our model shows how a reduction on search costs can simultaneously explain both superstar and long-tail effects. Imposing functional forms, as we do in Section 6 provides a

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[6]Maris Goldmanis, Ali Hortaçsu, Emre Onsel and Chad Syverson (2010) consider better firms to be low-cost (rather than high-quality) and find such a superstar effect both theoretically and empirically in their study of bookstores, travel agencies and new auto dealers.

[7]Empirical evidence on simultaneous long-tail and superstar appears in Anita Elberse and Felix Oberholzer-Gee (2006) on video film rentals, Gal Oestreicher-Singer and Arun Sundararajan (2008) on books and Catherine Tucker and Juanjuan Zhang (2007) on wedding service vendors. Andrés Hervás-Drane (2009) provides further references and a model that contrasts two different channels (sequential search and ex-ante recommendations) through which the Internet might generate superstar and long-tail effects.

simple condition under which the profits of the worst firms in the industry increase as search costs fall. This condition has an intuitive economic interpretation: If there is considerable vertical heterogeneity in the industry and the effect of firms changing designs is relatively limited then the profits of the worst firms necessarily fall. This is consistent with the evidence of Goldmanis et al. (2010) on bookstores, new auto dealers and travel agencies where different firms are often selling identical goods. Instead, if moving from a broad to a niche design has a large effect and there is more limited vertical heterogeneity, profits of the worst firms increase as search costs fall, as is consistent with the book-publishing industry, where it is easy to imagine that there is more scope for different book titles to appeal to niches and, as Brynjolfsson et al. (2003) document, product variety has increased dramatically (reflecting that small sellers are more profitable).

1   Model

There is a continuum of risk-neutral firms and consumers of measure 1 and m, respectively. Each firm i produces a single product. Each consumer l has tastes described by a conditional utility function (net of any search costs) of the form

$$u_{li}(p_i) = v_i + \varepsilon_{li} - p_i \tag{1}$$

if she buys product i at price p_i. The term v_i captures a natural advantage of firm i. A higher v_i can be thought of as a lower marginal production cost, but it also can be interpreted as better innate vertical quality.[8] Meanwhile, $\varepsilon_{li} \sim F_i$ is a match value between consumer l and product i. It captures idiosyncratic consumer preferences for certain products over others. We assume that realizations of ε_{li} are independent across firms and individuals.[9]

A consumer incurs a search cost c to learn the price p_i and the match value ε_{li}

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[8]Note that our results do not impose that $v_i > 0$, taking $v_i < 0$ and interpretting it as a marginal cost would lead the derived p_i to be interpretted as an absolute mark-up above the marginal cost but otherwise derivations would not change at all, and our results are consistent with either interpretation.

[9]Taking these realizations to be independent, while consistent with previous literature on search (Wolinsky, 1986; and Anderson and Renault, 1999), is not without loss of generality. It does not allow firms to target specific niches. That is, there is no spatial notion of differentiation or product positioning. However, given that we assume a continuum of firms and no ability for consumers to determine location in advance, this assumption may be more reasonable.

for the product offered by any particular firm i. Consumers search sequentially. The utility of a consumer l is given by

$$u_{lk}(p_k) - kc \tag{2}$$

if she buys product k at price p_k at the kth firm she visits. From now on, and for simplicity, we will omit the firm and consumer subscripts, unless there is ambiguity.

Firms cannot affect v, the exogenous quality of the good, which is distributed according to some continuously differentiable distribution $H(v)$ with support $[\underline{v}, \overline{v}]$. In Section 5, we analyze the case where the distribution is degenerate so that, ex-ante, all firms are identical.

We introduce strategic design choice by assuming that the firm can affect the distribution of the match-specific component of consumer tastes F_s by picking a design $s \in S = [B, N]$. That is, designs range from a most-broad (B) to a most-niche (N) design. A design s leads to ε_{li} distributed according to $F_s(\cdot)$ with support on some bounded interval $(\underline{\theta}_s, \overline{\theta}_s)$, and with a logconcave density $f_s(\cdot)$ which is positive everywhere.[10] Regardless of design and intrinsic quality, the firm produces goods at a marginal cost of 0.[11]

The strategy for each firm, therefore, consists of a choice of price p and a product design $s \in S$. We suppose that there are no costs associated with choosing different designs s.[12]

We follow Johnson and Myatt (2006) in assuming that different product designs induce demand rotations. Formally, there is a family of rotation points $\theta_s^\dagger$ such that $\frac{\partial F_s(\theta)}{\partial s} < 0$ for $\theta > \theta_s^\dagger$ and $\frac{\partial F_s(\theta)}{\partial s} > 0$ for $\theta < \theta_s^\dagger$; further $\theta_s^\dagger$ is increasing in s. The

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[10]See Mark Bagnoli and Ted Bergstrom (2005) for a broad discussion of logconcavity and functions that do and do not satisfy this condition. The assumption of logconcavity ensures that the failure rate $f_s(\theta)/(1 - F_s(\theta))$ is monotonic, and, so, guarantees existence of a profit-maximizing monopolist price which is continuous and increasing in constant marginal costs.

[11]Assuming constant marginal costs and no fixed costs simplifies the analysis considerably, though it can be relaxed in a similar fashion to Section IIB of Johnson and Myatt (2006). Within a framework of constant marginal costs, setting them to zero is without loss of generality. As already mentioned, differences in marginal costs play an identical role to differences in v.

[12]Relaxing this assumption would affect the results. However, it is not obvious how costs should vary with design. In particular, when interpreted as information provision, assuming that different designs come at the same cost is reasonable. More generally, targeting a specific audience may be costly, but so is producing a fully compatible wide-reaching product. Thus, we remain agnostic and focus fully on the demand-induced effect.

concept of a demand rotation is a formal approach to the notion that some designs lead to a wider spread in consumer valuations than others. In particular, a higher value of s should be interpreted as "quirkier" product that appeals more to certain consumers and less to others; the bounds on s correspond to the most broad (B) and the most niche (N) designs. Alternatively, in the marketing interpretation, a higher value of s corresponds to more information that shifts priors (up or down) and leads to more dispersed valuations. The formalization of design choices is general enough to accommodate a wide range of concepts of product design. One of the contributions of Johnson and Myatt (2006) is to show that natural models of physical product design and information-release provide micro-foundations for such demand rotations and to argue that it is natural to focus on instances where information or physical designs lead to more or less dispersed valuations.

As is standard in the search literature, we assume that consumers keep the same (passive) expectations about the distribution of future prices and design, independent of today's observed realization. This implies that a consumer's search and purchase behavior can be described by a threshold rule U: She buys the current product, obtaining $u_{li}(p_i)$, if this is more than or equal to U, and continues searching otherwise. This allows us to use Nash as our equilibrium concept. Consumers choose a threshold U and each v firm a pair (p, s).[13] One advantage to this notation is that U also represents the consumer surplus from participating in the market. Note that there always exist equilibria where consumers do not search and firms choose prohibitively high prices. We do not consider such equilibria if others exist.

2 Equilibrium

2.1 Consumer behavior

Suppose that a consumer expects each firm of type v to choose strategy (p_v, s_v).[14] Consider a consumer who can stop searching and obtain utility u. If the consumer, instead, samples an additional firm of type v, she will prefer the new product if $v + \varepsilon - p_v > u$. In this case, the additional utility obtained is $v + \varepsilon - (u + p_v)$, and

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[13]More broadly, we can allow firms to mix, so that each firm chooses an element $\sigma_v \in \Delta(\mathbb{R} \times [B, N])$.

[14]With a continuum of firm types and no atoms in the distribution, it is without loss of generality to assume that each type of firm chooses a pure strategy in design and price.

so the expected incremental utility from searching at one more firm that is expected to have design s_v and price p_v and to be of quality v is

$$E_\varepsilon(\max\{v + \varepsilon - p_v - u, 0\}) = \int_{u+p-v}^{\infty} (v + \varepsilon - p_v - u)f_{s_v}(\varepsilon)d\varepsilon. \tag{3}$$

It is worth visiting one more firm if and only if the expected value of the visit is worth more than the cost, where the final expectation is taken over v (with an implicit firm strategy for both price and design); that is, as long as $E_v[E_\varepsilon(\max\{v+\varepsilon-p_v-u, 0\})] \geq c$, or, equivalently, if $u < U$ where U is implicitly defined by:

$$\int_{-\infty}^{\infty} \left(\int_{U+p_v-v}^{\infty} (v + \varepsilon - p_v - U)f_{s_v}(\varepsilon)d\varepsilon \right) h(v)dv = c. \tag{4}$$

There is, at most, one solution to (4) since the left-hand side is strictly decreasing in U (as the integrand is decreasing in U and the lower limit of the inner integral is increasing in U). For large enough c, there is no feasible positive U that satisfies (4): No consumer would ever continue searching, and firms would have full monopoly power (as in Peter Diamond, 1971). In other words, the consumer initiates search if and only if $U \geq 0$.

2.2 Firm profit maximization

Consumers who visit a firm of type v buy as long as they receive a match ε such that $v + \varepsilon - p > U$ and, thus, purchase with probability $1 - F_s(p + U - v)$.

We define ρ as the expected probability that a consumer who visits a random firm buys from that firm; this is exogenous from the perspective of firm v. This definition allows us to calculate the demand for a given firm v. The expected number of consumers who visit it in the first round is m. A further $m(1 - \rho)$ visit v in the second round after an unsuccessful visit to some other firm, a further $m(1 - \rho)^2$ visit in the third round, and so on. We can, therefore, write demand for firm v that chooses a design s and price p as

$$\frac{m}{\rho}(1 - F_s(p + U - v)), \tag{5}$$

and its profits as

$$\Pi = \frac{m}{\rho} p(1 - F_s(p + U - v)). \tag{6}$$

It is useful to define $p_{vs}(U)$ as firm v's profit-maximizing price when the consumer's threshold is U and the design strategy is s. This price is implicitly determined by

$$p_{vs}(U) = \frac{1 - F_s(p_{vs}(U) + U - v)}{f_s(p_{vs}(U) + U - v)}. \tag{7}$$

Our first result, a consequence of the logconcavity assumption, ensures that p_{vs} is well-defined and behaves in a way that is intuitive: Higher-quality firms charge higher prices, and firms charge lower prices if they face pickier consumers—that is, if customers have higher values of U and, so, will only purchase if they obtain higher net surplus. Note that all proofs in the paper are relegated to the appendices.

Lemma 1 *The profit-maximizing price, $p_{vs}(U)$, associated with a design s is uniquely defined. It is continuously decreasing in the consumers' reservation threshold U, and continuously increasing in the firm's quality v. Further, $p_{vs}(U) + U$ is continuously increasing in U, and $p_{vs}(U) - v$ is continuously decreasing in v.*

Substituting for $p_{vs}(U)$, in (6), the firm's problem is to maximize the resulting expression with respect to its remaining strategic variable s. Note that neither the optimal price nor the optimal design choice depends on m or ρ, as these are just constant factors in profits.[15]

Johnson and Myatt (2006) show that, when designs are rotation-ordered and all designs cost the same, monopoly profits are quasi-convex in design. Thus, a monopolist would always choose an extremal design. In our competitive environment, since every firm has a local monopoly power, and the resulting residual demand is a truncation of the original one, it is still rotation-ordered, and, as a consequence, the same result applies.

Proposition 1 *Firms choose extremal designs. That is, every firm chooses either the most-niche $(s = N)$ or the most-broad $(s = B)$ design.*

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[15]This highlights that search costs play a qualitatively different role from that of scale effects. This is a central point of Wolinsky (1986) and is discussed by Anderson and Renault (1999).

To gain some intuition for this result, first consider the case when the optimal price at a given design s is below the point of rotation, so that the profit-maximizing quantity is greater than the quantity at the point of rotation $1 - F_s(\theta_s^\dagger)$. Then, decreasing s (and so "flattening" out demand) will lead to a greater quantity being sold even if the price is kept fixed. Therefore, decreasing s must lead to higher profits. A similar argument applies when the optimal price is above the point of rotation.[16]

Proposition 1 allows us to restrict attention to equilibrium strategies in which firm v chooses either a broad design (p_{vB}, B) or a niche one (p_{vN}, N), where p_{vB} and p_{vN} are defined by (7) for $s = B, N$, respectively.

Next, define $V(U)$ as the solution to

$$p_{VB}(U)(1 - F_B(p_{VB}(U) + U - V)) = p_{VN}(U)(1 - F_N(p_{VN}(U) + U - V)). \quad (8)$$

If $V(U)$ lies in the feasible range $[\underline{v}, \overline{v}]$, then $V(U)$ captures the firm that is indifferent between choosing the broad or the niche strategy. If $V(U)$ falls outside this range, with some abuse of notation, we redefine it to be the appropriate extreme of the range.[17] This definition allows us to characterize firm behavior.[18]

Proposition 2 *Given a consumer search rule, U, there is a threshold type of firm $V(U)$, as characterized by (8), such that all firms with lower quality than this threshold, $v < V(U)$, choose a niche design, and all firms with $v > V(U)$ choose a broad one. Moreover, $V(U)$ is continuously increasing in U; that is, as consumers search more intensively, more firms choose niche designs.*

Proposition 2 shows that the more severe the competition a firm faces (either because consumers are pickier and require more utility in order to purchase, or because the firm faces a disadvantage as compared to other firms), the more likely it is to choose a niche strategy. Loosely, the intuition here is that a firm in a disadvantageous position needs the consumer to "love" the good in order to buy it. The

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[16]For an alternative intuition, consider in the price-quantity space all feasible demand curves that arise from different designs. The rotation-ordering condition ensures that the upper envelope of these demands is traced out by the most-niche and most-broad designs.

[17]Mathematically, we redefine V to be $\max\{\underline{v}, \min\{\overline{v}, \cdot\}\}$ of the solution to (8).

[18]Note that firms that make no sales are indifferent about the design they choose. However, it is convenient for the statement of results (while having no effect on equilibrium transactions) to assume that such firms respect the design choices implied by Proposition 2.

chance that this happens increases with a design that leads to dispersed valuations—
a niche design. Instead, a high-value firm can appeal to many consumers by adopting
the broad strategy and, thereby, minimize the chance that a well-disposed consumer
observes that a product is such a bad match that she would prefer not to purchase.

This result is economically rich and appealing. First, when interpreting v as re-
lating to marginal costs, it states that low-cost firms try to attract a broad market,
while high-cost firms, who must charge higher prices to be profitable, target niches.
Second, as an example of the quality interpretation, consider five-star hotels com-
peting in a city. Although they are in the same category, they differ in an important
dimension: location. Our model predicts that hotels that are well located (center
of the city, close to the airport or other facilities) are more likely to deliver stan-
dard services. Meanwhile, those with less-desirable locations are more likely to be
specialized—for example, boutique hotels with distinctive styling or those catering
to specific groups, such as customers with pets.

2.3 Equilibrium Summary

Given the analysis above, we can express an equilibrium as a pair (U, V), where U
summarizes the search and purchase behavior of consumers, and V determines which
firms choose the broad or the niche strategy. These two values have to satisfy the
following conditions. First, rearranging (4), consumers optimize their behavior when

$$c = \int_{-\infty}^{V} \left(\int_{p_{vN}(U)+U-v}^{\infty} (v + \varepsilon - p_{vN}(U) - U) f_N(\varepsilon) d\varepsilon \right) h(v) dv \qquad (9)$$
$$+ \int_{V}^{\infty} \left(\int_{p_{vB}(U)+U-v}^{\infty} (v + \varepsilon - p_{vB}(U) - U) f_B(\varepsilon) d\varepsilon \right) h(v) dv.$$

Second, as explained above, firms' maximizing behavior is summarized by the
indifference of V, as in (8). Third, associated with broad and niche designs are
profit-maximizing prices $p_{vB}(U)$ and $p_{vN}(U)$, as determined in (7). Finally, it must
be worthwhile for a consumer to initiate search; that is, $U \geq 0$.

Finally, we can compute the expected probability that a consumer buys when she

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visits a random firm. This is given by

$$\rho(U,V) \equiv \int_{-\infty}^{V} (1-F_N(p_{vN}(U)+U-v))h(v)dv + \int_{V}^{\infty} (1 - F_B(p_{vB}(U) + U - v)) h(v)dv.$$
(10)

Note there always exist equilibria where consumers prefer not to search (and firms charge sufficiently high prices that this is optimal behavior for consumers). When search costs are sufficiently high, that is for $c > c_0$; this is the unique equilibrium. For lower search costs, equilibria involve some firms choosing niche designs and others broad designs, or all firms choosing niche or broad designs. The latter case is, relatively easy to characterize and we summarize results in the Proposition below. The other, more interesting case—where different firms choose different kind of designs—is the focus of our analysis is more involved and analyzed below.

Proposition 3 *Let U_B denote the consumer stopping rule that makes the lowest quality firm indifferent between choosing a niche and broad design, and U_N, the consumer stopping rule that makes the highest quality firm indifferent between choosing a niche and broad design.*

There exist thresholds on search costs c_N and c_B, such that there exists an equilibrium where all firms choose the broad design if and only if $U_B > 0$ and $c \in [c_B, c_0)$ and there exists an equilibrium where all firms choose the niche design if and only if $c < \underline{c}$, where $\underline{c} = \begin{cases} c_N & \text{if } U_N > 0 \\ c_0 & \text{if } U_N \leq 0 \end{cases}$.

3 Comparative Statics: the effect of lower search costs

The goal of this paper is to understand how changes in online information, recommender systems and other search technologies, summarized in the model as a fall in search costs affect equilibrium market structure and product design. Without imposing further structure on the distributions of quality $h(\cdot)$ and matches $f_s(\cdot)$, we cannot perform analytic comparative statics exercises on equilibria characterized in Section 2.3. Indeed, while Section 2.3 characterizes equilibria, we have not established uniqueness and so a comparative statics exercise can be difficult to interpret. Even, for parameters such that all firms choose the same designs, characterized in Proposition 3, there can be multiplicity of equilibria; for example, this is necessarily

the case if $c_N > c_B$.[19] More generally, for a given search cost, there may be equilibria where the consumers' search threshold is relatively low and, consistent with Proposition 2, many firms choose the broad design. Alternatively, firm and consumer expectations may be aligned so that many firms choose a niche strategy, and the consumer threshold is relatively high.

We analyze particular distributions $h(\cdot)$ and $f_S(\cdot)$ in Sections 5 and 6. This allows us to perform meaningful comparative statics exercises, for instance, by showing that there is a unique equilibrium (other than the trivial equilibrium with no activity). Through these cases, we demonstrate, for example, the possibility of simultaneous long tail and superstar effects.

Here, we argue that some equilibria are better behaved than others in terms of their stability properties. We can consider how the demand, through changes to U, and supply sides, through changes to V, respond to perturbations from an equilibrium. Given the continuum of firms and consumers, the game has an infinite-dimensional space of strategy profiles, and so in Appendix B, we propose a simplified dynamic system that captures the interplay between V and U. The steady states of this system are the Nash equilibria of our model. Thus, we refer to our Nash equilibria as stable if they are asymptotically stable within that dynamic system and, from now on, focus our attention on stable equilibria. In the online appendix B, we show that an equilibrium is stable if and only if $U(\cdot)$ has a slope less than 1 at the equilibrium. This observation allows us to return to our central concern, which is to understand how the industry changes in response to a reduction in the search costs:

Proposition 4 *At any stable equilibrium, decreasing the search cost c raises both consumer surplus (higher U) and the fraction of niche firms (higher V).*

As highlighted at the start of the paper, there has been much recent discussion of the long tail of the Internet. Proposition 4 provides a theoretical result that speaks to the issue by demonstrating that, for stable equilibria, lower search costs bring more niche firms.

This does not necessarily imply that niche products sell more or are more profitable. There are competing effects that arise from consumers being more picky (that

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[19]We demonstrate that this case can arise in Footnote 22.

is, having higher U). First, there is a direct effect that leads firms to drop prices and sell less per consumer-visit. But, second, there is a countervailing effect: More consumers will visit any given firm (i.e., ρ is lower), not only because consumers are more picky but also because more intermediate firms opt for a low-sales, high-mark-up strategy. The overall effect on sales and profits is, therefore, ambiguous. Indeed, in Sections 5 and 6, we show that either effect can dominate. First, we formally define our notions of long tail and superstar effects.

4 Long tails and superstars

Definition 1 *We say that a* superstar effect *is present if the firm with the highest sales captures an increasing market share as search costs fall.*

Definition 2 *We say that a* long-tail effect *is present if the firm with the lowest sales captures an increasing market share as search costs fall.*

Our definitions of long-tail and superstar effects may seem somewhat extreme in focusing only on one firm. But in this model, because of continuity, if the extreme firm behaves in a certain way, so do adjacent ones. Thus, our definitions imply a mass of firms at the head or tail of the sales distribution gaining market share.

Below, we study distributional changes when different designs coexist in equilibrium. However, we start by stressing the importance of design heterogeneity for the emergence of long-tail effects. In Appendix B, we show that over a class of distribution functions then if all firms choose the same design, there are always superstar effects but never long-tail effects. In particular, a sufficient condition is that the density function, $f_s(.)$ is increasing, though Proposition 7 shows a somewhat more general result.

This suggests that the documented long-tail effect may not be solely a consequence of a fall in the cost of search. If firms continued delivering the same type of products, we expect to see low-quality firms losing market share. It is through a change towards more-niche designs that the long tail arises.[20] Note, also, that

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[20] An alternative and plausible explanation is that the nature of the search technology has changed, and, in particular, the quality of "targeted" search has improved. We consider this channel complementary to the one we study. They would reinforce each other and exacerbate the shift towards niche designs and the phenomena studied in this paper.

holding design constant firm profits decrease as search costs decrease, which appears contrary to the rise of new firms on the Internet.[21]

While it is plausible that the Internet has reduced fixed costs of entry, we demonstrate that when firms' designs are strategic choices, the long-tail effect arises naturally, and that as search costs fall firm profits can increase, potentially leading to new-firm entry. We show these effects clearly by adding some further structure to the model. In Section 5, we assume ex-ante symmetry of all firms, and in Section 6, we allow for heterogeneous types and consider uniform distributions.

5   Homogeneous Firms

Here, we simplify our model by assuming that all firms are ex-ante identical. The purpose is to show that the key ingredient to obtain our results is the endogenous design choice, and not firm heterogeneity *per se*, as in most papers in the long-tail literature.

Without loss of generality, we assume that $v = 0$ for all firms. To simplify notation, we drop the v subscripts throughout this section. Given that all firms are now alike, we need to consider the possibility of mixed-strategy equilibria. In particular, we denote λ as the proportion of firms that choose a niche rather than a broad design. Analogous to the characterization of Section 2.3, equilibria can be summarized by (U, λ), and conditions (8)-(10) can be adapted as:

$$c = \lambda \int_{p_N(U)+U}^{\infty} (\varepsilon - p_N(U) - U) f_N(\varepsilon) d\varepsilon + (1 - \lambda) \int_{p_B(U)+U}^{\infty} (\varepsilon - p_B(U) - U) f_B(\varepsilon) d\varepsilon. \quad (11)$$

$$\lambda \in \arg\max\{\lambda p_B(U)(1 - F_B(p_B(U) + U)) + (1 - \lambda)p_N(U)(1 - F_N(p_N(U) + U))\}. \quad (12)$$

$$\rho(U) = \lambda(1 - F_N(p_N(U) + U)) + (1 - \lambda)(1 - F_B(p_B(U) + U)). \quad (13)$$

Note that the characterization of prices, given by (7), and the consumer's participation constraint ($U \geq 0$) still apply.

Given that all firms are identical, U_B and U_N, coincide. Recall these are the consumer search rules that induce all firms to choose broad or niche designs; they are characterized in Equations (21) and (23) for the general case. Here, we write $\overline{U} = U_B = U_N$. For $U > \overline{U}$, therefore, all firms prefer a niche design, whereas, for

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[21]This follows from Lemma 1 and Proposition 4 and does not require any restriction on $f_s(.)$.

$U < \overline{U}$, all firms prefer a broad design. It is only at $U = \overline{U}$ that firms might mix. However, a mixed-strategy equilibrium can exist over a wide range of search costs. This is immediate, by noting that at $U = \overline{U}$, expression (11) can be rewritten as

$$c = \lambda c_N + (1 - \lambda)c_B, \tag{14}$$

where c_B and c_N are the search cost thresholds introduced in Proposition 3 and are formally characterized in Equations (22) and (24) in the Appendix. Note that c_B and c_N have interpretations as the expected consumer surplus from visiting a broad or a niche firm, respectively, when the reservation utility $\overline{U}$ is such that a firm makes identical profits whether choosing a broad or a niche design.

If $c_N < c_B$, then the mixed-strategy equilibrium exactly fills the gap between the regions where all-broad and all-niche equilibria exist, and λ is linear and decreasing in c. If $c_N > c_B$, then in this region, there are, in principle, three equilibria: one all-broad, one mixed and one all-niche. However, note that the mixed equilibrium in this case is unstable. Thus, for $c \in (c_B, c_N)$, only two pure equilibria remain. Finally, if $c_N = c_B$, the mixed-strategy equilibrium has no mass. There are examples of each of these three cases.[22]

When search costs are sufficiently high (low) so that all firms choose the broad (niche) design, then comparative statics are standard and familiar: As c falls, consumer surplus rises, profits decrease and the sales of each firm stay constant at m.[23] Instead, when search costs are intermediate, results are more interesting.

Proposition 5 *When all firms are ex-ante identical, then as search costs fall within the region, $c \in (c_N, c_B)$, where both designs are offered: (i) consumer surplus is constant at $\overline{U}$; (ii) there are more niche firms (λ increases); (iii) consumers search more (ρ decreases); (iv) every firm's profits increase; and (v) both long-tail and superstar effects arise.*

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[22]When demands are linear, the ratio of consumer surplus to firm profits for a monopolist is constant at $\frac{1}{2}$. Therefore, two firms facing linear demands (regardless of their slopes) who earn the same profits must generate the same consumer surplus and, so, $c_N = c_B$. Instead, if demand is concave, the ratio of consumer surplus to profits is always lower than it would be in the linear case. Consequently, if F_N is linear and F_B is concave, then $c_N > c_B$ and multiplicity arises, whereas in the opposite case, with F_N concave and F_B linear, a unique equilibrium exists.

[23]See Proposition 8 in Appendix B for a formal statement and proof.

First, note that although a fall in search costs represents a direct benefit to consumers, this gain is exactly offset by the negative impact from searching more (ρ decreases) and from the increased preponderance of niche firms that provide less surplus in expectation ($c_B > c_N$). Next, since the consumer threshold is constant throughout the region, a firm's expected profit per consumer visit does not change. However, given that there are more consumer visits (ρ decreases), profits increase.[24]

Finally, we turn to market structure. Consistent with "long-tail" stories, we observe that as search costs fall, each niche firm sells more. In addition, there are more niche firms and, since the total volume of sales is constant, it follows that the niche firms account for a greater proportion of overall sales. Note, also, that superstar effects are present. The "top" firm chooses a broad design and sells more as c goes down. The tail is niche throughout and also sells more as c goes down. The middle region, where the mix of broad and niche is changing, is the one that loses sales to both the head and the tail of the sales distribution. This is illustrated in Figure 1 below.
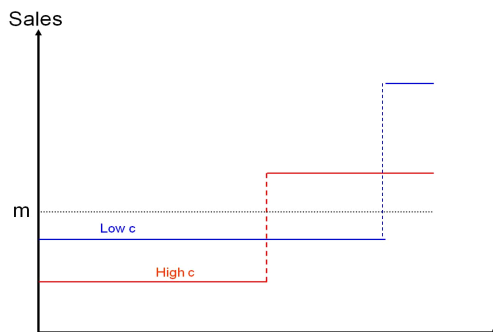

Fig 1: Distribution of sales at
different search costs.

When search costs are low enough or high enough, all firms choose the same design and all of them sell m. Thus, as Figure 1 shows, sales are non-monotonic. Profits are also non-monotonic: They decrease in search costs when these are low or

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[24]Although the probability of making a sale for any given visit stays constant for any given type of firm, this is consistent with more consumers visiting since the composition of firms changes. There are more niche firms as c falls, and niche firms sell less than broad firms.

high, but increase in search costs in the intermediate region (as shown in Propositions 5 and 8).

6 Uniformly distributed quality and linear demands

We once again consider heterogeneous firms, but impose further structure that allows us to derive additional analytic results. These highlight that the results of Section 5 extend naturally to more-general settings. We analyze the case where the distribution of firm quality is uniform $v \sim U[L, H]$, and the distributions $F_s(\cdot)$ are uniform, leading to linear demand functions. In particular, the niche and broad product designs are, respectively, $\varepsilon \sim U[\underline{\theta}_N, \overline{\theta}_N]$ and $\varepsilon \sim U[\underline{\theta}_B, \overline{\theta}_B]$. We impose that $\underline{\theta}_N < \underline{\theta}_B$ and $\overline{\theta}_N > \overline{\theta}_B$. This ensures that these distributions represent demand rotations.

In this environment, we derive the following results.

Proposition 6 *When the distribution of firm's vertical quality is uniform with support $[L, H]$ and the distribution of matches for niche and broad designs are uniform on $[\underline{\theta}_N, \overline{\theta}_N]$ and $[\underline{\theta}_N, \overline{\theta}_N]$ respectively. When all firms are active, then: (i) there is a unique equilibrium (U, V) for each search cost c. When different firms choose different design strategies, then as the search cost decreases: (ii) consumer surplus U increases; (iii) there are more niche firms (V increases); (iv) profits of the highest- and lowest-quality firms increase if and only if $\overline{\theta}_N - \overline{\theta}_B > H - L$; (v) the superstar effect arises; and (vi) the long-tail effect can, but need not, arise; a sufficient condition for it to arise is $\overline{\theta}_N - \overline{\theta}_B > H - L$.*

It is worth highlighting that as the extent of vertical differentiation among firms diminishes (that is, as $H - L \to 0$), we recover the results of Proposition 5. This shows that the case of homogeneous firms is not knife-edged. In other words, one can regard the vertical component as a device to purify the mixed strategies that arise in the homogeneous case.

But, more importantly, Proposition 6 shows that while superstar effects are robust, the long-tail effect and the comparative statics of profitability depend on how $\overline{\theta}_N - \overline{\theta}_B$ compares to $H - L$. While $H - L$ captures the extent of vertical differentiation, $\overline{\theta}_N - \overline{\theta}_B$ captures horizontal differentiation: It measures the importance of changing from broad to niche designs in terms of the dispersion of match values. Thus, when designs have a relatively greater impact on horizontal differentiation, the

competition-softening effect of firms switching to niche designs more than compensates for the intensified vertical competition that arises as search costs fall.

Note that if firms' types are very dispersed then a low quality firm must be forced out of the market when search costs are sufficiently low; following our definition, trivially, in such circumstances, long tail effects cannot arise. Proposition 6, therefore, focuses on parameter ranges where all firms remain active even for low values of c.

We illustrate some results of Proposition 6 in the case that $\overline{\theta}_N - \overline{\theta}_B > H - L$. Specifically, consider $f_N(x) = \frac{1}{16}$ on $[-12, 4]$, $f_B(x) = \frac{1}{6}$ on $[-3, 3]$ and $h(x) = 1$ on $[0, 0.75]$. We use this example to demonstrate the non-monotonicity of prices and profits, and the superstar and long-tail effects.

Figure 2 illustrates how prices vary with search costs for a particular firm (at $v = 0.5$). As one would anticipate, in general, prices increase with search costs. However, when the firm changes design from niche to broad, prices drop substantially, leading to prices that are non-monotonic in search costs. The price pattern for other values of v is qualitatively the same.

Next, consider how firm profits for the worst firm, the best firm, and the industry's average profits, vary with search costs, as illustrated in Figure 3.[25] Note the two points where the derivative is discontinuous. These are the search-cost thresholds at which the equilibrium changes from all-niche or all-broad to one in which there is a mix of designs: Below $c_N = .038$, all firms are niche, but as search costs increase, the high-quality firms gradually start switching to a broad design. At $c_B = .08$ and beyond, all firms choose a broad design. Figure 3 illustrates that profits may be non-monotonic. The intuition is the, by now, familiar one that as search costs fall in the intermediate region, more firms choose a niche design and, thereby, soften competition.

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[25]Since there is a mass 1 of firms, the graph of average profits also represents total industry profits.
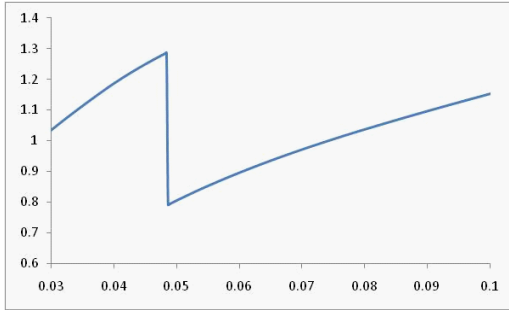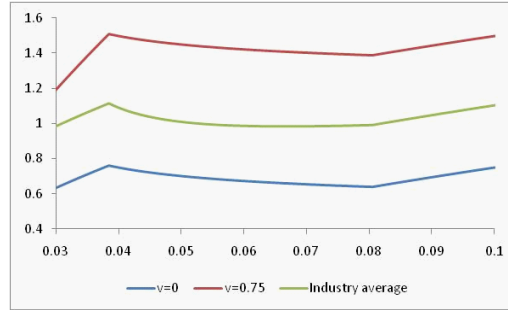
Fig 2: Price against search cost.

Fig 3: Profits against search costs.

Finally, we consider sales distributions. Figure 4 is the analogue of Figure 1 and plots the distribution of sales for two different search costs. Naturally, higher-quality firms sell more than low-quality firms, regardless of the search costs. Comparing sales at different search costs, both the highest- and lowest-quality firms sell more at the lower level of search costs, illustrating that superstar and long-tail effects arise simultaneously. These are also illustrated at intermediate levels of search costs (where there is dispersion in designs offered) in Figure 5, which plots sales against search costs for the best and worst firms.
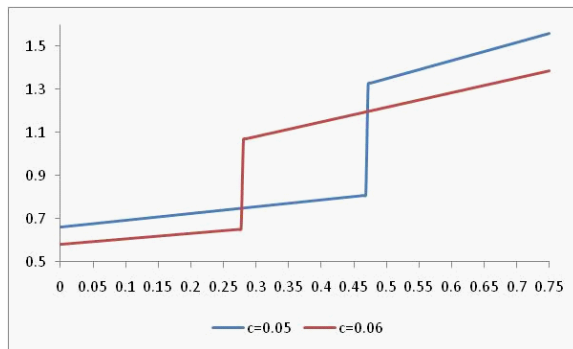

Fig 4: Sales against quality (v) at
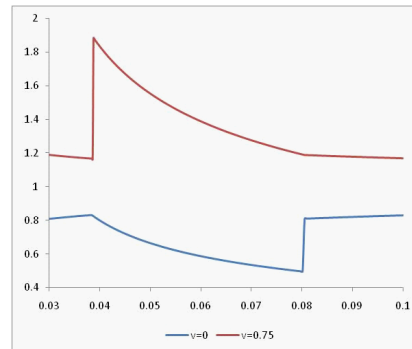$c = 0.05$ and $c = 0.06$.

Fig 5: Sales against search
cost for best and worst firms.

7   Conclusions

There has been considerable attention on the influence of the Internet on the kind of products offered and the distribution of their sales. In particular, academic and

popular commentators have highlighted both long-tail and superstar effects for various industries (including publishing, media, and travel destinations, among others). This paper presents a simple and tractable model integrating consumer search and firms' strategic product-design choices that is useful to analyze these phenomena.

We show that, in equilibrium, different product designs coexist. More-advantaged firms prefer broad-market strategies, seeking a very broad design and choosing a relatively low price, while less-advantaged firms take a niche strategy with quirky products priced high to take advantage of the (relatively few) consumers who are well-matched to the product. Such design diversity arises even when all firms are homogeneous.

Prices and profits can be non-monotonic in consumer search costs. There is an intuitive rationale for this: As search costs fall, and as long as the product designs remain unchanged, prices fall. However, at ever lower prices, the broad-market strategy becomes less appealing to firms, some of whom adopt a niche strategy, charging a high price to the (few) consumers who are well-matched for the product. Moreover, the firms' decision to adopt a niche strategy acts as a form of differentiation that softens price competition, and effectively create a positive externality on other firms. Indeed, this observation suggests a rationale for industry coordination: since profits can be non-monotonic in search costs, as search costs fall exogenously, industries might benefit from reducing them further (for example, through industry-sponsored comparison sites).

Finally, our comparative statics analysis provides a demand-side explanation of the long-tail effect. As search costs fall, a greater proportion of firms choose the niche strategy. Consumers search to a much greater extent and, consequently, niche firms may account for a larger proportion of the industry's sales. In addition, lower search costs can simultaneously account for a superstar effect, with sales to both the head and tail of the sales distribution coming from middling firms whose designs change from broad to niche.

An aspect that our model did not explicitly address is the entry of new firms into the market. Empirically, entry is an important phenomenon, as highlighted by the increase of book titles, mentioned in the Introduction. We abstract from entry for clarity of presentation. However, firm entry can be endogenized simply by assuming a fixed entry cost. Qualitatively, the results and intuitions of the paper

would remain unchanged. Further, the Internet has had broader impacts that go beyond search costs, and long-tail and superstar phenomena may reflect changes to production costs.[26] In this paper we have focused on changes to the demand-side to isolate their effects, as we believe they are economically significant.

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[26]It is worth highlighting that the 10% figure on new titles mentioned in the Introduction included on-demand and short-run titles that are likely to be affected by changes in publishing costs. After excluding these, Bowker still reports a 6% annualized growth in new titles from 2002 to 2007.

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A Proofs

Proof of Lemma 1 First, note that since $f_s(x)$ is logconcave, $\frac{1-F_s(x)}{f_s(x)}$ is strictly decreasing in x (See, for example, Corollary 2 of Bagnoli and Bergstrom, 2005). Suppose (for contradiction) that at some value of U, $p_{vs}(U)$ is increasing in U; then, $p_{vs}(U) + U$ is also increasing in U, and so $\frac{1-F_s(p_{vs}(U)+U-v)}{f_s(p_{vs}(U)+U-v)} = p_{vs}(U)$ is decreasing in U, which provides the requisite contradiction. A similar argument ensures that $p_{vs}(U) + U$ is increasing in U, that $p_{vs}(U)$ is increasing in v, and that $p_{vs}(U) - v$ is decreasing in v. ∎

Proof of Proposition 1 The optimal design is chosen to maximize $p_{vs}(U)(1-F_s(p_{vs}(U)+U-v))$. Now, given that $p_{vs} + U - v$ is an affine transformation of p_s, it follows that $D_v(p_{vs}, s)$, as in (5), are rotation-ordered. The proof then follows immediately from Proposition 1 in Johnson and Myatt (2006), p. 761. ∎

Proof of Proposition 2 For a fixed value of U, in principle, there may be more than one V solving equation (8). We show later that this is not the case. Consider one such solution and notice that

$$p_{VB}(U)(1 - F_B(p_{VB}(U) + U - V)) = p_{VN}(U)(1 - F_N(p_{VN}(U) + U - V)) \geq \qquad (15)$$

$$\geq p_{VB}(U))(1 - F_N(p_{VB}(U) + U - V)). \qquad (16)$$

It follows that

$$1 - F_B(p_{VB}(U) + U - V) \geq 1 - F_N(p_{VB}(U) + U - V). \qquad (17)$$

Similarly,

$$1 - F_N(p_{VN}(U) + U - V) \geq 1 - F_B(p_{VN}(U) + U - V). \qquad (18)$$

We use these facts to show that $p_{VN}(U) > p_{VB}(U)$. Suppose (for contradiction) that

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$p_{VN}(U) < p_{VB}(U)$. Note that since N and B are drawn from a family of demand rotations, it follows that there is some $\widetilde{x}$ such that $1 - F_N(x) > 1 - F_B(x)$ if and only if $x > \widetilde{x}$. If $p_{VB}(U) + U - V > \widetilde{x}$, then, $1 - F_N(p_{VB}(U) + U - V) > 1 - F_B(p_{VB}(U) + U - V)$ in contradiction to (17). If, instead, $\widetilde{x} \geq p_{VB}(U) + U - V > p_{VN}(U) + U - V$, then (18) is contradicted. Thus, $p_{VN}(U) > p_{VB}(U)$ and from (8), trivially,

$$1 - F_B(p_{VB}(U) + U - V) > 1 - F_N(p_{VN}(U) + U - V). \tag{19}$$

Define $\pi_{vs} := p_{vs}(1 - F_s(p_{vs} + U - v))$ with $s = B, N$. Since the price is chosen to maximize profits, by the envelope theorem, we have that $\frac{d\pi_{vs}}{dv} = p_{vs}f_s(p_{vs} + U - v) = 1 - F_s(p_{vs}(U) + U - v)$ where the second equality follows from (7). Now, given (19), it follows that $\frac{d\pi_{VN}}{dv} < \frac{d\pi_{VB}}{dv}$. This ensures that $\pi_{vB} - \pi_{vN}$ always crosses zero from above and the uniqueness of $V(U)$ follows trivially.

Finally, by definition, $V(U)$ satisfies

$$p_{V(U)B}(U)(1 - F_B(p_{V(U)B}(U) + U - V(U))) = p_{V(U)N}(U)(1 - F_N(p_{V(U)N}(U) + U - V(U))).$$

Taking the derivative of both sides with respect to U, applying the envelope theorem, and using (7), we obtain

$$-(1 - F_N(p_{vN} + U - V(U)))(1 - \frac{dV(U)}{dU}) = -(1 - F_B(p_{vB} + U - V(U)))(1 - \frac{dV(U)}{dU}),$$

which, given (19) implies that $\frac{dV(U)}{dU} = 1$. ∎

Proof of Proposition 2 Consider, first, a situation in which consumers use a $U = 0$ search rule. Firms would react using a $V(0)$ strategy. Now, using (9), one can compute the search cost c_0 that delivers $(0, V(0))$ as an equilibrium:

$$c_0 = \int_{-\infty}^{V(0)} \left(\int_{p_{vN}(0)-v}^{\infty} (\varepsilon - p_{vN}(0) + v)f_N(\varepsilon)d\varepsilon \right) h(v)dv + \int_{V(0)}^{\infty} \left(\int_{p_{vB}(0)-v}^{\infty} (\varepsilon - p_{vB}(0) + v)f_B(\varepsilon)d\varepsilon \right) h(v)dv. \tag{20}$$

Clearly, for all $c > c_0$, consumers would not search.

Next, consider the consumer stopping rule U_B that makes all firms prefer the broad strategy and the lowest-quality firm indifferent. This is the highest level of consumer search compatible with all firms offering a broad product and is characterized by:

$$p_{\underline{v}B}(U_B)(1 - F_B(p_{\underline{v}B}(U_B) + U_B - \underline{v})) = p_{\underline{v}N}(U_B)(1 - F_N(p_{\underline{v}N}(U_B) + U_B - \underline{v})). \tag{21}$$

Using (9), we can compute the search cost c_B that results in an equilibrium with firm and consumer behavior of $(U_B, \underline{v})$:

$$c_B := \int_{-\infty}^{\infty} \left(\int_{p_{vB}(U)+U_B-v}^{\bar{\theta}_B} (v + \varepsilon - p_{vB}(U_B) - U_B)f_B(\varepsilon)d\varepsilon \right) h(v)dv. \tag{22}$$

If $U_B \leq 0$, clearly there is no all-broad equilibrium. If $U_B > 0$, by the definition of

c_0, there is no positive search equilibrium with $c > c_0$. Take, now, $c \in [c_B, c_0)$. Using the definitions of c_B and c_0 and by looking at condition (9), one can easily see that there exists a $U \in (0, U_B]$ such that $(U, \underline{v})$ constitutes an equilibrium. Finally, for $c < c_B$, there cannot be an all-broad equilibrium. By looking at (22), note that the induced U had to be bigger than U_B, but this would imply that the type $\underline{v}$ firm prefers a niche strategy, providing a contradiction.

Analogous to the all-broad case, we can consider all firms choosing the niche design, so that $V = \overline{v}$, together with the consumer stopping rule that makes the highest-quality firm indifferent in its design choice, U_N, and the associated search cost, c_N. These are defined by the following conditions:

$$p_{\overline{v}B}(U_N)(1 - F_B(p_{\overline{v}B}(U_N) + U_N - \overline{v})) = p_{\overline{v}N}(U_N)(1 - F_N(p_{\overline{v}N}(U_N) + U_N - \overline{v})), \quad (23)$$

$$c_N = \int_{-\infty}^{\infty} \left(\int_{p_{vN}(U) + U_N + - v}^{\overline{\theta}_N} (v + \varepsilon - p_{vN}(U_N) - U_N) f_N(\varepsilon) d\varepsilon \right) h(v) dv. \quad (24)$$

Again, it is straightforward that for sufficiently low search costs, it is an equilibrium for all firms to choose the niche design. From the definitions of c_N and c_0 that $\underline{c}$ is the highest search cost value that supports an all-niche equilibrium. We argue now that an all-niche equilibrium exists for any $c' < \underline{c}$.

Consider (9) then following Lemma 1, the lower limits of the integrals are increasing and the integrands are decreasing in U. As a result, the left-hand side above is decreasing in U. This is sufficient to show that $U(V, c)$ is decreasing in c and, in particular, $U' = U(\overline{v}; c') > U$. Now, Proposition 2 shows that in response to c' and U' firms would like to increase V, but this is already at its highest value $\overline{v}$. Thus, $(U', \overline{v})$ is an all-niche equilibrium at c'. ∎

Proof of Proposition 4 Consider a stable equilibrium $(U, V(U))$. According to our definition, this means that $\frac{\partial U}{\partial V}(V(U)) < 1$, where $U(V)$ is implicitly defined by expression (9). Denote its left-hand side as $H(U, V)$. Note that

$$\frac{\partial U}{\partial V}(V(U)) = \frac{\frac{\partial H}{\partial V}(U, V(U))}{-\frac{\partial H}{\partial U}(U, V(U))} < 1 \Leftrightarrow \frac{\partial H}{\partial U}(V(U)) + \frac{\partial H}{\partial V}(V(U)) < 0.$$

Now, since $\frac{\partial V}{\partial U}(U) = 1$, it follows that stability is satisfied if and only if

$$\frac{\partial H}{\partial U}(V(U)) + \frac{\partial H}{\partial V}(V(U))\frac{\partial V}{\partial U}(U) < 0. \quad (25)$$

Now, at an equilibrium, $\overline{H}(U) \doteq H(U, V(U)) = c$. Moreover, following (25), we see that $\frac{\partial \overline{H}}{\partial U}(U) < 0$. Finally, if c falls, $\overline{H}(U)$ needs to decrease as well, which implies that U needs to increase to restore equilibrium. Finally, using Proposition 2, we know that $V(U)$

increases as well.   ∎

Proof of Proposition 5   Following the argument in the text, part (i) is immediate and consumer surplus is constant at $\overline{U}$ throughout this region. Next, part (ii) follows immediately from (14) since $c_B > c_N$. From the proof of Proposition 2, $1 - F_B(p_B(\overline{U}) + \overline{U}) > 1 - F_N(p_N(\overline{U}) + \overline{U})$; then, part (iii) follows immediately.

Firm profits for niche and broad firms are identical and given by $\frac{m}{\rho(U)} p_N(\overline{U})(1 - F_N(p_N(\overline{U}) + \overline{U})) = \frac{m}{\rho(U)} p_B(\overline{U})(1 - F_B(p_B(\overline{U}) + \overline{U}))$. Following part (iii) of the proposition, part (iv) follows immediately.

Finally, sales for a broad and a niche firm are $\frac{m}{\rho(U)}(1 - F_B(\overline{U} + p_B(\overline{U})))$ and $\frac{m}{\rho(U)}(1 - F_N(\overline{U} + p_N(\overline{U})))$. Again, following part (iii) of the proposition, part (v) follows immediately.

Proof of Proposition 6   We use the functional forms for $F_N(\cdot)$, $F_B(\cdot)$ and $h(\cdot)$ to rewrite the equations in Section 2.3 that characterize equilibrium assuming that all firms are active (that is, they make positive sales).

First, consider prices. Condition (7) delivers

$$p_{vB}(U) = \frac{\overline{\theta}_B + v - U}{2}, \text{ and } p_{vN}(U) = \frac{\overline{\theta}_N + v - U}{2}. \tag{26}$$

Next, we focus on the firms' decision V. We rewrite condition (8) as:

$$\frac{(\overline{\theta}_B + V - U)^2}{b^2} = \frac{(\overline{\theta}_N + V - U)^2}{n^2},$$

where we introduce the notation $b^2 = \overline{\theta}_B - \underline{\theta}_B$ and $n^2 = \overline{\theta}_N - \underline{\theta}_N$ for convenience. Note that $n > b$.

Recalling footnote 17 and rearranging the previous expression, we obtain

$$V = \min\{H, \max\{U + K, L\}\}, \tag{27}$$

where $K = \frac{\overline{\theta}_N b - \overline{\theta}_B n}{n - b}$ is a constant that depends on exogenous parameters.

Finally, we rewrite the consumer condition (9) as:

$$c = \int_L^V \left(\int_{\frac{\overline{\theta}_N - v + U}{2}}^{\overline{\theta}_N} (\varepsilon - \frac{\overline{\theta}_N - v + U}{2}) \frac{d\varepsilon}{n^2} \right) \frac{dv}{H - L} + \int_V^H \left(\int_{\frac{\overline{\theta}_B - v + U}{2}}^{\overline{\theta}_B} (\varepsilon - \frac{\overline{\theta}_B - v + U}{2}) \frac{d\varepsilon}{b^2} \right) \frac{dv}{H - L}.$$

Suppose that there are some firms choosing both a niche and a broad design. Then, we can write $V = U + K \in (L, H)$ and simplify the previous expression to

$$c = \frac{1}{24} \Big(\frac{V - L}{H - L} \frac{(V - L)^2 + 3(K + \overline{\theta}_N)(K + L + \overline{\theta}_N - V)}{n^2} + (H - V) \frac{(H - V)^2 + 3(K + \overline{\theta}_B)(K + H + \overline{\theta}_B - V)}{b^2 (H - L)} \Big).$$

Note that the right-hand side is a polynomial in V. Denote it by $A(V)$.

Since $A(V)$ is a cubic, it has, at most, three roots. Note that $n > b$ so as $V \to -\infty$

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that $A \to \infty$ and as $V \to \infty$ then $A \to -\infty$. Consider

$$\frac{dA}{dV} = \frac{1}{8} \frac{\left(Lb + \bar{\theta}_B n - Vb - Ln - \bar{\theta}_N n + Vn\right)^2}{n^2 (H - L)(n - b)^2} - \frac{1}{8} \frac{\left(\bar{\theta}_B b + Hb - \bar{\theta}_N b - Hn - Vb + Vn\right)^2}{b^2 (H - L)(n - b)^2} \text{ and}$$

$$\frac{d^2 A}{dV^2} = \frac{1}{4} \frac{Hn^2 - Lb^2 + bn(\bar{\theta}_N - \bar{\theta}_B)}{b^2 n^2 (H - L)} - \frac{1}{4} \frac{n^2 - b^2}{b^2 n^2 (H - L)} V.$$

Now $V \in (\min\{K, L\}, H)$. Note that $\frac{d^2 A}{dV^2}\big|_{V=H} = \frac{1}{4} \frac{(H-L)b + n(\bar{\theta}_H - \bar{\theta}_B)}{bn^2(H-L)} > 0$, and since $\frac{d^3 A}{dV^3} < 0$, this means that $\frac{d^2 A}{dV^2} > 0$ throughout the relevant region. Consider $\frac{dA}{dV}\big|_H = -\frac{1}{8} \frac{2n(\bar{\theta}_N - \bar{\theta}_B) - (H-L)(n-b)}{n^2(n-b)}$. If $\frac{dA}{dV}\big|_H = -\frac{1}{8} \frac{2n(\bar{\theta}_N - \bar{\theta}_B) - (H-L)(n-b)}{n^2(n-b)} < 0$, then, since $\frac{d^2 A}{dV^2} > 0$ through the region, $\frac{dA}{dV} < 0$ and there can be, at most, one solution to $A = 0$. This is the case if and only if

$$2n \frac{\bar{\theta}_N - \bar{\theta}_B}{n - b} > H - L. \tag{28}$$

Note that, throughout, we assumed that all firms are active. Consider, now, the limiting case where all firms choose niche designs and the marginal firm is indifferent, so that $V = H$ (which we know must arise when c is sufficiently small, following Proposition 3). Then, the lowest-quality firm makes positive sales as long as $p_{LN}(H - K) > 0$. Note that

$$p_{LN}(H - K) = \frac{\bar{\theta}_N + L - H + K}{2} = \frac{1}{2} \frac{n\left(\bar{\theta}_N - \bar{\theta}_B\right) - (H - L)(n - b)}{n - b}.$$

So, $p_{LN}(H - K) > 0$ if and only if

$$n \frac{\bar{\theta}_N - \bar{\theta}_B}{n - b} > H - L,$$

which, trivially, implies (28).

This shows that $\frac{dA}{dV}\big|_H < 0$, and so also that $\frac{dA}{dV} < 0$ for all $V \in (\min\{K, L\}, H)$; thus, there is a unique solution to $A = 0$ and, moreover, V is decreasing in c. This proves (i) and (iii) of the Proposition. Part (ii) follows trivially from (27).

Turning to part (iv), we can write the profits of the H firm and the L firm, respectively:

$$\pi_{HB}(U) = m(H - L)(\bar{\theta}_N - \underline{\theta}_N) \frac{(\bar{\theta}_B + H - U)^2}{(\bar{\theta}_B + H - U)^2(\bar{\theta}_N - \underline{\theta}_N) - (\bar{\theta}_N + L - U)^2(\bar{\theta}_B - \underline{\theta}_B)}, \text{ and}$$

$$\pi_{LN}(U) = m(H - L)(\bar{\theta}_B - \underline{\theta}_B) \frac{(\bar{\theta}_N + L - U)^2}{(\bar{\theta}_B + H - U)^2(\bar{\theta}_N - \underline{\theta}_N) - (\bar{\theta}_N + L - U)^2(\bar{\theta}_B - \underline{\theta}_B)}.$$

Taking the derivative of each with respect to U, we obtain

$$\frac{d\pi_{HB}(U)}{dU} = 2m(H-L)(\overline{\theta}_N - \underline{\theta}_N)\left(\overline{\theta}_B - \underline{\theta}_B\right)\frac{\left(\overline{\theta}_B + H - U\right)\left(\overline{\theta}_N + L - U\right)\left(\overline{\theta}_N - \overline{\theta}_B - (H-L)\right)}{((\overline{\theta}_B + H - U)^2(\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_N + L - U)^2(\overline{\theta}_B - \underline{\theta}_B))^2}, \text{ and}$$

$$\frac{d\pi_{LN}(U)}{dU} = 2m(H-L)(\overline{\theta}_B - \underline{\theta}_B)(\overline{\theta}_N - \underline{\theta}_N)\frac{\left(\overline{\theta}_B + H - U\right)\left(\overline{\theta}_N + L - U\right)\left(\overline{\theta}_N - \overline{\theta}_B - (H-L)\right)}{((\overline{\theta}_B + H - U)^2(\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_N + L - U)^2(\overline{\theta}_B - \underline{\theta}_B))^2}.$$

Note that since all firms are active, $p_{HB}(U)$ and $p_{LN}(U)$ must be positive. Following that prices must be non-negative and using (26), it follows that $\frac{d\pi_{HB}(U)}{dU}$ and $\frac{d\pi_{LN}(U)}{dU}$ have the same sign as $(\overline{\theta}_N - \overline{\theta}_B) - (H - L)$.

Finally, turning to sales, we can write the sales of the highest-quality and lowest-quality firms as

$$S_{HB}(U) = m\frac{(H-L)(\overline{\theta}_N - \underline{\theta}_N)}{2}\frac{(\overline{\theta}_B + H - U)}{(\overline{\theta}_B + H - U)^2(\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_N + L - U)^2(\overline{\theta}_B - \underline{\theta}_B)}, \text{ and}$$

$$S_{LN}(U) = m\frac{(H-L)(\overline{\theta}_B - \underline{\theta}_B)}{2}\frac{(\overline{\theta}_N + L - U)}{(\overline{\theta}_B + H - U)^2(\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_N + L - U)^2(\overline{\theta}_B - \underline{\theta}_B)}.$$

The superstar effect arises immediately; as c falls, U increases and

$$\frac{dS_{HB}(U)}{dU} = 2m\left(H - L\right)\left(\overline{\theta}_N - \underline{\theta}_N\right)\frac{\left(\overline{\theta}_B - \underline{\theta}_B\right)\left(\overline{\theta}_B + H - L - \overline{\theta}_N\right)^2 + \left(\overline{\theta}_B + H - U\right)^2\left(\overline{\theta}_N - \overline{\theta}_B + \underline{\theta}_B - \underline{\theta}_N\right)}{((\overline{\theta}_B + H - U)^2(\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_N + L - U)^2(\overline{\theta}_B - \underline{\theta}_B))^2} > 0.$$
$$(29)$$

Analyzing the long-tail effect is slightly more involved.

$$\frac{dS_{LN}(U)}{dU} = 2m\left(H - L\right)\left(\overline{\theta}_B - \underline{\theta}_B\right)\frac{(\overline{\theta}_N - \underline{\theta}_N)\left(\left(L - U + \overline{\theta}_N\right)^2 - (\overline{\theta}_N - \overline{\theta}_B - H + L)^2\right) - (\overline{\theta}_B - \underline{\theta}_B)\left(L - U + \overline{\theta}_N\right)^2}{((\overline{\theta}_B + H - U)^2(\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_N + L - U)^2(\overline{\theta}_B - \underline{\theta}_B))^2}.$$

First, note that the sign of $\frac{dS_{LN}(U)}{dU}$ is the same as the sign of the numerator of the fraction; that is:

$$(\overline{\theta}_N - \underline{\theta}_N)\left(\left(L - U + \overline{\theta}_N\right)^2 - (\overline{\theta}_N - \overline{\theta}_B - H + L)^2\right) - (\overline{\theta}_B - \underline{\theta}_B)\left(L - U + \overline{\theta}_N\right)^2.$$

This is a quadratic in H, which takes its maximum at $H = \overline{\theta}_N - \overline{\theta}_B + L$, at which point it takes the value $((\overline{\theta}_N - \underline{\theta}_N) - (\overline{\theta}_B - \underline{\theta}_B))\left(L - U + \overline{\theta}_N\right)^2 > 0$. It is monotonically increasing in H for $H < \overline{\theta}_N - \overline{\theta}_B + L$, and recall that $H \geq L$. At the minimum, $H = L$, we can apply the results of Proposition 5 to obtain that $\frac{dS_{LN}(U)}{dU} > 0$, and so it follows that $\frac{dS_{LN}(U)}{dU} > 0$ for all $H \leq \overline{\theta}_N - \overline{\theta}_B + L$. Finally, there exist parameter values where the long-tail effect does not arise; in particular, this is the case at $\overline{\theta}_N = 4$, $\underline{\theta}_N = -8$, $\overline{\theta}_B = 3$, $\underline{\theta}_B = -3$, $H = 5$ and $L = 0$ and for all values of c. ∎

B   Omitted results

B.1   Results related to Section 3

Existence of Equilibria: Consider $V(\cdot)$ and $U(\cdot)$ which are, respectively detemined as the solution for V to Equation (8) as a function of U and the solution for U to Equation (9) as a function of V. These are well-behaved continuous functions. The composition $V(U(\cdot))$ is, therefore, a continuous function of $[\underline{v}, \overline{v}]$ into itself. Given that $[\underline{v}, \overline{v}]$ is compact, $V(U(\cdot))$ has a fixed point V^*. It is immediate that $(U(V^*), V^*)$ constitutes a Nash equilibrium of the game.

Concept of Stability: We define the following differential dynamic system

$$\dot{V} = V(U) - V$$
$$\dot{U} = U(V) - U.$$

One can immediately see that the Nash equilibrium of our game coincides with the steady states of this system. Now, a steady state (V^*, U^*) of this system is asymptotically stable if and only if the eigenvalues of the Jacobian of the dynamic system evaluated at the steady state have strictly negative real parts (see Angel De la Fuente, 2000, p. 488 for more details). In this case the Jacobian is

$$\begin{pmatrix} -1 & \frac{\partial V}{\partial U}(U^*) \\ \frac{\partial U}{\partial V}(V^*) & -1 \end{pmatrix}$$

and the eigenvalues λ defined by

$$\begin{vmatrix} -1 - \lambda & \frac{\partial V}{\partial U}(U^*) \\ \frac{\partial U}{\partial V}(V^*) & -1 - \lambda \end{vmatrix} = (-1 - \lambda)^2 - \frac{\partial V}{\partial U}(U^*)\frac{\partial U}{\partial V}(V^*) = 0 \Leftrightarrow$$

$$\lambda = -1 \pm \sqrt{\frac{\partial V}{\partial U}(U^*)\frac{\partial U}{\partial V}(V^*)}.$$

Clearly, λ has a strictly negative real part iff $\frac{\partial V}{\partial U}(U^*)\frac{\partial U}{\partial V}(V^*) < 1$. Since Proposition 2 shows that $\frac{\partial V}{\partial U}(\cdot) = 1$, stability is equivalent to $\frac{\partial U}{\partial V}(V^*) < 1$.

Proposition 7 *Suppose that all firms choose the same design s. A sufficient condition for the superstar, but not the long-tail effect, to arise is that the distribution of consumer valuations satisfies the following condition:*

$$f'(P) > -\frac{(f'(P)^2 - f(P)f''(P))(1 - F(P))}{f'(P)^2\frac{(1-F)^2}{f^2} + 5(f'(P)(1 - F(P)) + f^2(P))}. \tag{30}$$

Proof. As shown in Lemma 1, $p_v(U) + U$ is increasing in U. Now, since design is fixed, by considering (9), we can conclude that a fall in c implies an increase in U. Given that the only effect of a change of c is through U, we can study changes in U directly.

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The superstar effects arise if and only if

$$\frac{\partial\left(\frac{m(1-F(p_{\overline{v}}(U))+U-\overline{v}))}{\rho(U)}\right)}{\partial U} = m\frac{\partial}{\partial U}\left(\frac{[1-F(p_{\overline{v}}(U)+U-\overline{v})]}{\int_{\underline{v}}^{\overline{v}}[1-F(p_v(U)+U-v)]\,h(v)dv}\right) > 0.$$

A sufficient condition, therefore, is that

$$\frac{\partial}{\partial U}\left(\frac{1-F(p_{\overline{v}}(U)+U-\overline{v})}{1-F(p_v(U)+U-v)}\right) > 0 \text{ for all } v < \overline{v}. \tag{31}$$

Similarly, a sufficient condition to ensure that no long-tail effect arises is

$$\frac{\partial}{\partial U}\left(\frac{1-F(p_{\underline{v}}(U)+U-\underline{v})}{1-F(p_v(U)+U-v)}\right) < 0 \text{ for all } v > \underline{v}. \tag{32}$$

Writing $W = U - v$ (and the corresponding $\overline{W}$ and $\underline{W}$), we can write $1 - F(p_v(U) + U - v) = q(W)$. Then, (31) is equivalent to $\frac{d}{dW}(\frac{q(\overline{W})}{q(W)}) > 0$ and (32) to $\frac{d}{dW}(\frac{q(\underline{W})}{q(W)}) < 0$. Note that Lemma 1 shows that $q(\overline{W}) > q(W)$ and that $\frac{d}{dW}q(W) < 0$. But neither of these conditions is enough to guarantee (31) and (32). A sufficient condition, though, is

$$\frac{d^2}{dW^2}q(W) < 0 \text{ for } W \in (\underline{W}, \overline{W}). \tag{33}$$

It remains to verify this condition. Consider the firm's maximization problem $p\,[1 - F_s(p + U - v)]$; this is equivalent to maximizing $(P - W)(1 - F(P))$ and $q(W) = 1 - F(P)$. It follows that we can write:

$$\frac{d^2 q}{dW^2} = -f\frac{d^2 P}{dW^2} - f'(\frac{dP}{dW})^2. \tag{34}$$

By differentiating the firm's first-order condition with respect to W, and differentiating again, and rearranging both expressions, we obtain $\frac{dp}{dW} = \frac{1}{2+\frac{1-F(P)}{f(P)}\frac{f'(P)}{f(P)}}$ and $\frac{d^2 p}{dW^2} = \frac{1+2\frac{1-F(P)}{f(P)}\frac{f'(P)}{f(P)}-\frac{1-F(P)}{f(P)}\frac{f''(P)}{f'(P)})}{(2+\frac{1-F(P)}{f(P)}\frac{f'(P)}{f(P)})^3}\frac{f'(P)}{f}$. Then, we can substitute these expressions into (34) and rearrange to obtain:

$$\frac{d^2 q}{dW^2} = -f(P)^4\frac{(f'(P)^2 - f(P)f''(P))(1 - F(P)) + f'(P)\left(f'(P)^2\frac{(1-F(P))^2}{f(P)^2} + 5(f'(P)(1 - F(P)) + f^2)\right)}{(f'(P)(1 - F(P)) + 2f(P)^2)^3}.$$

Logconcavity of $f(\cdot)$ implies that $f'(P)^2 - f(P)f''(P) > 0$, and that $1 - F(\cdot)$ is logconcave. This, in turn, implies that $f'(P)(1 - F(P)) + f(P)^2 > 0$, and so also $f'(P)(1 - F(P)) + 2f(P)^2 > 0$. It follows that (33) is satisfied as long as

$$f'(P) > -\frac{(f'(P)^2 - f(P)f''(P))(1 - F(P))}{f'(P)^2\frac{(1-F)^2}{f^2} + 5(f'(P)(1 - F(P)) + f^2(P))}.$$

This is necessarily the case when $f'(\cdot) > 0$ or, more generally, when $F(\cdot)$ is not too concave. ■

B.2 Results related to Section 5

Proposition 8 *In the homogeneous firms model of Section 5, if $c \leq c_N$ or $c > c_B$, then as c falls: (i) consumer surplus U is increasing; (ii) consumers search more (ρ decreases); (iii) every firm's profits decrease; and (iv) every firm's sales stays constant.*

Proof. Consider the case $c \leq c_N$ (the other case is analogous). Then, (11) and (13) can be written simply as

$$c = \int_{p_N(U)+U}^{\infty} (\varepsilon - p_N(U) - U)f_N(\varepsilon)d\varepsilon,$$

$$\rho(U) = 1 - F_N(p_N(U) + U).$$

By Lemma 1, $U + p_N(U)$ is increasing in U; parts (i) and (ii) follow immediately.

Profits as in (6) are given by $\frac{m}{\rho(U)}p_N(U)(1 - F_N(p_N(U) + U)) = mp_N(U)$, which is decreasing in U by Lemma 1. Finally, the sales of any firm are $\frac{m}{\rho}(1 - F_N(p_N(U)+U)) = m$, and thus constant. ■