

HMG Revision Project B Comment, Project No. P092900

Comment submitted by Michael Baumann of Economists Incorporated and Paul Godek of Compass Lexecon regarding the revision of the Horizontal Merger Guidelines; May 4, 2010.

At §4.1.3, the draft Horizontal Merger Guidelines describe “critical loss analysis.” That description ignores relevant economic literature and accepted economic principles, and reflects a substantive misunderstanding of the issue. In particular, the guidelines state:

Critical loss analysis asks whether imposing at least a SSNIP on one or more products in a candidate market would raise or lower the hypothetical monopolist’s profits. ... A price increase raises profits on sales made at the higher price, but this will be offset to the extent customers substitute away from products in the candidate market. Critical loss analysis compares the magnitude of these two offsetting effects resulting from the price increase. The “critical loss” is defined as the number of lost unit sales that would leave profits unchanged. The “predicted loss” is defined as the number of lost unit sales that the hypothetical monopolist is predicted to lose due to the price increase. ... [H]igh pre-merger margins normally indicate that each firm’s product individually faces demand that is not highly sensitive to price. Higher pre-merger margins thus indicate a smaller predicted loss and make it more likely that the predicted loss is less than the critical loss and that the candidate market satisfies the hypothetical monopolist test. (§4.1.3, ¶¶3-4.)

As we have pointed out in two recent publications, the inference that high margins are informative about actual demand elasticities is plausible only if the calculated margin is based on the true marginal cost of the firm, as economic theory defines marginal cost. (See “Reconciling the Opposing Views of Critical Elasticity,” *Global Competition Policy*, September 2009; “A New Look at Critical Elasticity,” *The Antitrust Bulletin*, Summer 2006.) The inevitable sources of the margin data used in critical loss analysis, however, are the accounting costs found in the financial statements of the firms in question. Such accounting costs, at best, can be used to estimate short-run variable cost, not marginal cost.

The inappropriateness of inferring market power from accounting costs has been long established and generally accepted, primarily due to the observations of Professor Franklin Fisher. (See Franklin Fisher, “On the Misuse of the Profits-Sales Ratio to Infer Monopoly Power,” *Rand Journal of Economics*, Fall 1987.) Professor Timothy Bresnahan describes the issue as follows: “Firms’ price-cost margins are not taken to be observables; economic marginal cost cannot be directly or straightforwardly observed.” Professor Bresnahan further describes the refutation of the idea that “economic price-cost margins ... could be directly observed in the accounting data.” (See Timothy Bresnahan, “Empirical Studies of Industries with Market Power,” in *Handbook of Industrial Organization*, Richard Schmalensee and Robert Willig, Editors, 1989.) The issue is summarized in the comprehensive industrial organization textbook, *Modern Industrial Organization*, which states that the use of margins based on average variable costs to infer market power can “lead to serious biases.” (See Dennis Carlton and Jeffrey Perloff, *Modern Industrial Organization*, 4th Edition, 2005, Chapter 8.)

The guidelines offer another mistaken observation regarding critical loss:

While this “breakeven” analysis differs from the profit-maximizing analysis called for by the hypothetical monopolist test ... merging parties sometimes present this type of analysis to the Agencies. (§4.1.3, ¶13.)

While we cannot speak to the presentations made by others, a refinement of the original critical loss analysis – consistent with the hypothetical monopolist test – has been available for about 15 years. (See our “Could and Would Understood: Critical Elasticities and the Merger Guidelines,” *The Antitrust Bulletin*, Winter 1995.)

The concept of critical elasticity is operational because merger policy relates to industries with substantial sunk costs, and because in such industries prices tend to be well in excess of short-run variable costs. Without sunk costs, antitrust concerns are mitigated by ease of entry. Without prices above short-run variable costs, industries with sunk costs would not exist.

In sum, it is well recognized that margin calculations based on accounting costs do not allow for the inference of market power and are not informative about actual elasticities of demand. To assume the contrary renders the guidelines inconsistent with existing economic literature and basic economic principles.



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Reconciling the Opposing Views of Critical Elasticity

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Reconciling the Opposing Views of Critical Elasticity

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I. INTRODUCTION

Market definition is the core of antitrust analysis, and the concepts of “critical elasticity” and “critical loss” have often been applied to the task of defining relevant antitrust markets, in both differentiated-product and homogenous-product scenarios. The critical elasticity is that elasticity of demand that is just high enough to prevent a hypothetical monopolist from profitably increasing price by a threshold “small but significant” amount; critical loss is the fraction of sales lost by the hypothetical monopolist, as implied by the critical elasticity. Evidence that the actual demand elasticity exceeds the critical elasticity indicates that the product in question is not a relevant market.

In recent articles, several researchers have offered an alternative approach to the issue of critical elasticity.² Their view is that the traditional application of the concept is flawed and that critical elasticity calculations contain information about actual elasticities of demand. We believe that alternative view to be half wrong. While it offers a misleading interpretation of critical elasticity analysis, the alternative view does reveal a flaw in the conventional approach—a flaw that proponents of the conventional approach have failed to recognize. Here

¹ Michael Baumann is with Economists Incorporated, and Paul Godek is with Compass Lexecon, both in Washington, DC. This article reflects and expands our previous paper on the subject: *A New Look at Critical Elasticity*, ANTITRUST BULL., (Summer 2006.) The mathematical appendix to that article was printed with several errors – a corrected version of which appears here.

² The list of relevant articles in this debate includes:

- Malcolm Coate & Jeffrey Fischer, *Critical Loss: Implementing the Hypothetical Monopolist Test*, GLOBAL COMPETITION POL’Y, (April 2008);
- Kevin Murphy & Robert Topel, *Critical Loss Analysis in the Whole Foods Case*, GLOBAL COMPETITION POL’Y, (March 2008);
- Gregory Werden, *Beyond Critical Loss: Properly Applying the Hypothetical Monopolist Test*, GLOBAL COMPETITION POL’Y, (February 2008);
- Joseph Farrell & Carl Shapiro, *Improving Critical Loss Analysis*, ANTITRUST SOURCE, (February 2008);
- Daniel O’Brien & Abraham Wickelgren, *The State of Critical Loss Analysis: Reply to Scheffman and Simons*, ANTITRUST SOURCE, (March 2004);
- Michael Katz & Carl Shapiro, *Further Thoughts on Critical Loss*, ANTITRUST SOURCE, (March 2004);
- David Scheffman & Joseph Simons, *The State of Critical Loss Analysis: Let’s Make Sure We Understand the Whole Story*, ANTITRUST SOURCE, (November 2003);
- Daniel O’Brien & Abraham Wickelgren, *A Critical Analysis of Critical Loss Analysis*, ANTITRUST L.J., (2003); and
- Michael Katz & Carl Shapiro, *Critical Loss: Let’s Tell the Whole Story*, ANTITRUST, (Spring 2003).

we present a revised and more general critical elasticity model, one that might reconcile the competing arguments. The new model also implies substantially higher critical elasticities than previous models would indicate.

II. CRITICAL ELASTICITY REVIEWED

The concepts of critical elasticity and critical loss have been used to determine the appropriate relevant market for antitrust purposes, in a way that is intended to be consistent with the approach taken by the U.S. federal antitrust authorities.³ As noted, the critical elasticity is the elasticity of demand that is just high enough to prevent a hypothetical monopolist from profitably increasing price—by whatever percentage represents the threshold “small but significant non-transitory” amount. Evidence that the hypothetical monopolist faces a demand elasticity greater than the critical elasticity indicates that the product in question is not a relevant market. Critical loss is the fraction of sales lost by the hypothetical monopolist, as implied by the critical elasticity.

The basic result of critical elasticity analysis is straightforward. For any given price increase, the critical elasticity decreases as the initial price-cost margin increases. A monopolist more readily gives up low margin sales to achieve a higher price on remaining sales, whereas high margin sales are more costly to relinquish. Thus, when the initial margin is low, a higher elasticity is necessary to prevent a monopolist from raising price by a given percentage. In previous work, we derived the following critical elasticity formula: the critical elasticity equals

$$1 / (2t + m)$$

where m is the initial margin over short-run variable cost and t is the percentage price increase of interest.⁴

The concept of critical elasticity is operational because merger policy relates to firms with substantial sunk costs, and because firms with substantial sunk costs charge prices that tend to be well in excess of short-run variable costs. Without sunk costs, antitrust concerns are mitigated by ease of entry. Without prices above short-run variable costs, firms with sunk costs would not exist.

³ United States Department of Justice and Federal Trade Commission, *Horizontal Merger Guidelines* (Revised April 1997). Critical elasticity analysis played an important role in the recent case *FTC v. Whole Foods Market, Inc.*, 502 F. Supp. 2d 1 (D.D.C. 2007). In the *Whole Foods* litigation, the reports filed by Defendant’s expert (David Scheffman) and Plaintiff’s expert (Kevin Murphy) reprise some of the same issues discussed in this paper. For an overview of that case, see Deborah Feinstein & Michael Bernstein, *A Perspective on the Whole Foods Decision: Would the Most Important Evidence Please Stand Up?*, COMPETITION POL’Y INT’L, (Spring 2008). See also the discussion of critical elasticity in Ken Heyer & Nicholas Hill, *The Year in Review: Economics at the Antitrust Division, 2007-2008*, REV. INDUS. ORG., (November 2008).

⁴ Our derivation of this “would” elasticity differs from previous treatments in that it is based on profit maximization. The appropriate way to compute the critical elasticity follows from what the monopolist would do, given profit maximization, rather than from what the monopolist could do, given that profits are no less after the price increase than before. See Michael Baumann & Paul Godek, *Could and Would Understood: Critical Elasticities and the Merger Guidelines*, ANTITRUST BULL., (Winter 1995). Farrell & Shapiro (2008) discuss the same issue, but they seem to have been unaware of our derivation.

III. AN ALTERNATIVE VIEW

An alternative view proposes to use a calculated critical elasticity to infer information about the actual demand elasticity:

When gross margins are large, defense claims that the elasticity of demand is high should be treated with a healthy dose of skepticism. More specifically, we advocate an approach under which there is a presumption that high gross margins go along with a low elasticity of demand faced by the hypothetical monopolist.⁵

and

Here, one can make inferences about demand sensitivity, as gauged by a real firm based on its premerger choice of price. In particular, if (before the merger) a firm chooses a high margin on its product, the firm evidently thinks that demand for its product is not very sensitive to price.⁶

The gross margin is the difference between price and marginal cost, relative to the price, which in economic theory is equal to the inverse of the elasticity of demand facing the firm and is known as the Lerner index. In other words, this approach proposes to infer information about an actual elasticity from the critical elasticity calculation.

Those statements and the ensuing analyses are plausible, however, only if the critical elasticity calculation is based on the true marginal cost of the firm, as economic theory defines marginal cost. But economic marginal cost is not easily ascertained. If it were easy to know marginal cost, either short-run or long-run, then determining the elasticity of demand would be easy, and it isn't.

The inappropriateness of inferring market power from accounting costs, we thought, had been long ago established and generally accepted, primarily due to the observations of Frank Fisher. Professor Fisher analyzed and condemned the use of accounting data to infer the Lerner Index:

The profits-sales ratio is an unreliable estimate of the Lerner Index. Simulated examples show that the errors involved in using it may be large in practice, ... even the direction of error cannot be easily determined, nor is there a simple way to recast profits/sales so as to recover the Lerner index from accounting data.⁷

Timothy Bresnahan's observations about this issue are also directly relevant. Professor Bresnahan described one of the central ideas of what he labels the "New Empirical Industrial Organization" ("NEIO") as follows:

Firms' price-cost margins are not taken to be observables; economic marginal cost ("MC") cannot be directly or straightforwardly observed. The analyst infers MC from firm behavior, uses differences between closely related markets to trace

⁵ Katz & Shapiro, *Critical Loss*, *supra* note 2.

⁶ Farrell & Shapiro, *Improving Critical Loss*, *supra* note 2. These two papers also include the concept of a "diversion ratio" in the analysis. That concept is an extension that requires the resolution of the critical loss issue and is not necessary to the discussion here.

⁷ Franklin Fisher, *On the Misuse of the Profits-Sales Ratio to Infer Monopoly Power*, RAND, (Fall 1987); reprinted in INDUSTRIAL ORGANIZATION, ECONOMICS, AND THE LAW, Ch. 6, (1991).

the effects of changes in MC, or comes to a quantification of market power without measuring cost at all.⁸

Professor Bresnahan further noted that the NEIO is motivated by “dissatisfactions” over maintained hypotheses in the structure conduct performance paradigm, one of which is that “economic price-cost margins (performance) could be directly observed in the accounting data.”⁹

We observe that the critical elasticity calculation is based on something better understood and more readily observed in the business world than marginal cost. In practice, actual critical elasticity analysis is based on the margin over short-run variable cost.¹⁰ We will use the term variable-cost margin (or “VCM”) to refer to *the excess of price over short-run average variable cost*. The margin over short-run variable cost, also known as quasi-rent, determines whether a firm is earning enough to justify its investment in fixed assets. A brief description of quasi-rents is worth recalling here:

A quasi-rent is the return to a durable and specialized productive instrument. ... In the long run—in a period long enough to build new instruments or wear out old ones—the return to the instrument must equal the current rate of return on capital (with appropriate allowance for risk). If the machine’s quasi-rents are less than interest plus depreciation, it will not be replaced; if the quasi-rents exceed interest plus depreciation, more will be built until equilibrium is restored. The long-run net return on capital goods must yield the appropriate interest rate; their short-run gross return is a quasi-rent.¹¹

Whether a firm’s VCM is sufficient to justify the investment made to acquire and maintain capital goods helps to determine whether such investments were worthwhile and whether similar investments will be made in the future. And capital goods are more appropriately considered to include all sunk costs, such as advertising, research and development, and the failures necessary to achieve success (known as “dry holes” in not only the oil industry).

Critical elasticity analysis, based as it is on variable cost, does not allow for the inference of market power or the calculation of a Lerner index. In other words, the existence of sunk costs does not imply the existence of market power. And a calculated critical elasticity does not reveal information about an actual elasticity.

⁸ Timothy F. Bresnahan, *Empirical Methods of Industries with Market Power*, in HANDBOOK OF INDUSTRIAL ORGANIZATION, 1012, (Schmalensee & Willig, eds.), (1989).

⁹ *Id.*, pp. 1012-1013. Some of the discussion here is reiterated in JEFFREY M. PERLOFF, LARRY S. KARP, & AMOS GOLAN, ESTIMATING MARKET POWER AND STRATEGIES, Ch. 2 (2007).

¹⁰ We do not suggest that the calculation of the appropriate variable cost margin is trivial. The determination of which costs to include can be problematic. For a related discussion, see William Baumol, *Predation and the Logic of the Average Variable Cost Test*, J.L. & ECON., (April 1996).

¹¹ GEORGE STIGLER, THE THEORY OF PRICE, 4th Edition, 263, (1987).

IV. CALCULATING THE NEW CRITICAL ELASTICITY

The alternative view, however, has revealed a flaw in the previous analysis of the subject. The critical elasticity formulae in the literature depend on the assumption that marginal costs and average variable costs are constant—and therefore equal. Under that assumption, knowing average variable cost would be equivalent to knowing marginal cost.¹²

A more realistic paradigm would acknowledge that marginal cost is unknown—except that it is not equal to average variable cost and is, therefore, upward sloping at the firm's profit-maximizing level of output: in other words, the standard textbook description of cost curves.¹³ When thinking about incremental costs, it should be remembered that production does not sell itself. Short-run marginal cost is the cost of producing *and selling* incremental output. It seems likely that even if production costs are fairly stable, selling costs—the costs of obtaining another sale—begin to increase at some point. That paradigm would seem to be a fair characterization of how competing firms with relatively constant costs of production arrive at an equilibrium distribution of sales.

It is possible to derive a revised critical elasticity formula based on that economic paradigm of rising marginal cost. Assuming linearity of demand and marginal cost the critical elasticity formula becomes:

$$\frac{-1 + \sqrt{1 + (2m/t)}}{2m}$$

where, as before, m is the initial margin over average variable costs, the VCM, and t is the percentage price increase of interest. (See the Appendix for the derivation.) The derivation of this formula assumes that price is equal to marginal cost at the initial equilibrium. That assumption is employed not to imply the equality between marginal cost and price, but rather because that assumption produces an upper bound on the critical elasticity. That is, if the initial price is actually above marginal cost, the formula overstates the critical elasticity. (Proof is available from the authors.)

Note that the new formula derives uniformly and substantially higher critical elasticities than those derived from previous models. The following table shows various critical elasticities for price increase thresholds of 5 percent, 10 percent, and 15 percent, for both the increasing-marginal-cost model derived here and the conventional constant-cost model. (Again, both formulae are “would” elasticities; they are based on the assumption of profit maximization.) For the values shown in the table, the new critical elasticity is 1.1 to 2.8 times greater than the

¹² Some of the defenders of the original approach to critical elasticity, in desperation it would seem, have invoked the theory of the “kinked demand curve”—a theory that was debunked more than 60 years ago. See George Stigler, *The Kinky Oligopoly Demand Curve and Rigid Prices*, J. POL. ECON., (October 1947). But wait, there's more. Thirty years later George Stigler again singled out this theory for its unjustified longevity in economics textbooks. See George Stigler, *The Literature of Economics: The Case of the Kinked Oligopoly Demand Curve*, ECON. INQUIRY, (April 1978). It seems that bad theories never die—ever.

¹³ See, for example, JACK HIRSHLEIFER, AMIHAI GLAZER, & DAVID HIRSHLEIFER, *PRICE THEORY AND APPLICATIONS: DECISIONS, MARKETS, AND INFORMATION*, 7th Edition, Ch. 7 (2005) or any other price theory textbook.

conventional approach would indicate. This result should not be surprising. The previous critical elasticity model has the hypothetical monopolist losing the same profit “margin” on all sales, while the new model has the hypothetical monopolist losing very little profit margin on its initial lost sales.

Critical Elasticities										
Assumption	Price Increase	Variable Cost Margin								
		10%	20%	30%	40%	50%	60%	70%	80%	90%
Constant Marginal Cost	5%	5.0	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0
	10%	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9
	15%	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9	0.8
Increasing Marginal Cost	5%	6.2	5.0	4.3	3.9	3.6	3.3	3.1	3.0	2.8
	10%	3.7	3.1	2.7	2.5	2.3	2.2	2.1	2.0	1.9
	15%	2.6	2.3	2.1	1.9	1.8	1.7	1.6	1.5	1.4

It follows that, since higher critical elasticities imply that more sales must be lost to deter a given price increase, this new approach will tend to result in narrower market definitions. In sum, the recent critique of critical elasticity, while misguided, has revealed a need for a new formulation of critical elasticity.

V. CONCLUSION

Based on our analysis, parties seeking to assist the antitrust agencies or the courts in defining relevant markets should continue to feel comfortable making the usual critical elasticity argument, which is that higher variable cost margins indicate that fewer lost sales are needed to deter a price increase. (And it is easier to lose fewer sales than more sales.) It should be understood, however, that the conventional critical elasticity formula is not appropriate. It should also be understood that calculated critical elasticities are not informative about actual elasticities.

VI. APPENDIX: A NEW CRITICAL ELASTICITY

Determining whether a hypothetical monopolist would raise its price a certain threshold percentage implies a comparison of the profit-maximizing monopoly price, P_M , and the initial price, P_0 . In percentage terms, the price increase is less than the price increase threshold, t , if $(P_M - P_0) / P_0 < t$. The critical elasticity is the elasticity of demand at P_0 that is just high enough to prevent a monopolist from increasing price by t . Here, the critical elasticity is derived under the assumption that the marginal cost and demand functions are linear.

A. *Marginal Cost*

Assuming linearity, marginal cost (MC) can be written

$$MC = \alpha + \beta Q, \quad (1)$$

where α and β are positive constants and Q is quantity. This assumption implies that total cost (TC) is a quadratic function:

$$TC = \gamma + \alpha Q + \frac{1}{2} \beta Q^2. \quad (2)$$

And variable cost (VC) equals:

$$VC = \alpha Q + \frac{1}{2} \beta Q^2. \quad (3)$$

Average variable cost (AVC) is equal to VC divided by Q :

$$AVC = \frac{VC}{Q} = \alpha + \frac{1}{2} \beta Q. \quad (4)$$

Taking the difference between marginal cost and average variable cost derives:

$$\beta = 2 \left(\frac{MC - AVC}{Q} \right). \quad (5)$$

Initial quantity, marginal cost, and average variable cost are denoted as Q_0 , MC_0 , and AVC_0 , respectively; equations (1) and (5) imply that marginal cost at any quantity is given by the following:

$$MC(Q) = MC_0 + 2 \left(\frac{MC_0 - AVC_0}{Q_0} \right) (Q - Q_0). \quad (6)$$

B. Demand

Assuming linear demand, the inverse demand function can be written

$$P = A - BQ \quad (7)$$

where P is price, Q is quantity, and A and B are positive constants. Let ε denote the elasticity of demand at the initial equilibrium stated as a positive value:

$$\varepsilon = - \frac{\partial Q}{\partial P} \frac{P_0}{Q_0} = - \frac{P_0}{-BQ_0} = \frac{P_0}{BQ_0}, \quad (8)$$

where Q_0 and P_0 are the initial quantity and price. Together, (7) and (8) imply that

$$B = \frac{P_0}{\varepsilon Q_0} \text{ and} \quad (9)$$

$$A = P_0 + BQ_0 = P_0 \left(1 + \frac{1}{\varepsilon} \right).$$

C. Profit maximization

Total revenue is price times quantity and price is given by equation (7), so marginal revenue can be written as

$$MR = \frac{\partial(PQ)}{\partial Q} = \frac{\partial((A - BQ)Q)}{\partial Q} = A - 2BQ.$$

Substituting for A and B using (9) yields

$$MR = P_0 \left(1 + \frac{1}{\varepsilon} \right) - 2 \frac{P_0}{\varepsilon Q_0} Q. \quad (10)$$

Applying equation (6) and assuming that the market is initially competitive, $MC_0 = P_0$, yields the following expression for marginal cost:

$$MC = P_0 + 2 \left(\frac{P_0 - AVC_0}{Q_0} \right) (Q - Q_0)$$

or

$$MC = P_0 + 2 \frac{P_0}{Q_0} mQ - 2P_0 m \quad (11)$$

where

$$m = \left(\frac{P_0 - AVC_0}{P_0} \right)$$

is the average variable cost margin (*VC**M*) at the initial equilibrium.

Setting $MR = MC$ and applying equations (10) and (11) yields the following first-order condition for profit maximization:

$$P_0 \left(1 + \frac{1}{\varepsilon} \right) - 2 \frac{P_0}{\varepsilon Q_0} Q = P_0 + 2 \frac{P_0}{Q_0} mQ - 2P_0 m.$$

Grouping terms, dividing through by P_0 , and multiplying through by ε gives

$$\frac{2}{Q_0} (1 + m\varepsilon) Q = 1 + 2m\varepsilon,$$

which implies that the profit-maximizing quantity, Q_M , is given by

$$Q_M = \left(\frac{Q_0}{2} \right) \left(\frac{1 + 2m\varepsilon}{1 + m\varepsilon} \right). \quad (12)$$

The profit-maximizing price, P_M , is given by $P_M = A - BQ_M$. By substitution using equations (9) and (12):

$$P_M = P_0 \left(1 + \frac{1}{\varepsilon} \right) - \frac{P_0}{\varepsilon Q_0} \left(\left(\frac{Q_0}{2} \right) \left(\frac{1 + 2m\varepsilon}{1 + m\varepsilon} \right) \right) = P_0 \left(1 + \frac{1}{\varepsilon} - \frac{1 + 2m\varepsilon}{2\varepsilon(1 + m\varepsilon)} \right)$$

or

$$P_M = P_0 \left(1 + \frac{1}{2\varepsilon(1 + m\varepsilon)} \right). \quad (13)$$

D. Critical elasticity

The critical elasticity is the lowest elasticity of demand at P_0 such that a hypothetical monopolist would not increase price by t . Using (13), the percentage increase in price from the initial price, P_0 , to the profit-maximizing price, P_M , is

$$\frac{P_M - P_0}{P_0} = \frac{P_0 \left(1 + \frac{1}{2\varepsilon(1+m\varepsilon)} \right) - P_0}{P_0} = \frac{1}{2\varepsilon(1+m\varepsilon)}. \quad (14)$$

For the percentage increase in price to be less than the critical value, t , the following condition must hold:

$$\frac{1}{2\varepsilon(1+m\varepsilon)} < t$$

or

$$m\varepsilon^2 + \varepsilon - \frac{1}{2t} > 0. \quad (15)$$

Solving the quadratic equation (15) for ε and taking the positive root gives the following value for the critical would elasticity:

$$\varepsilon > \frac{-1 + \sqrt{1 + \frac{2m}{t}}}{2m}. \quad (16)$$

Could and would understood: critical elasticities and the merger guidelines

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I. Introduction

Market definition is the core of antitrust analysis. The market definition paradigm employed by the federal antitrust agencies is well-known. The Justice Department's 1984 Merger Guidelines state it in the following manner:

Formally, a market is defined as a product or group of products and a geographic area in which it is sold such that a hypothetical, profit-maximizing firm, not subject to price regulation, that was the only present and future seller of those products in that area would impose a "small but significant and nontransitory" increase in price above prevailing or likely future levels.¹

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AUTHORS' NOTE: *We thank Barry Harris, Kent Mikkelsen, John Morris, Jeffrey Smith, and Greg Werden for helpful comments.*

¹ United States Department of Justice, Merger Guidelines (June 14, 1984), § 2.0, *reprinted in* 4 Trade Reg. Rep. (CCH) ¶ 13,103 [hereinafter 1984 Guidelines].

The concept of "critical elasticity" can often be helpful in determining the extent of the relevant market. The critical elasticity is that elasticity of demand that is just high enough to prevent the hypothetical monopolist from profitably increasing price by whatever percentage represents the threshold "small but significant" amount. Evidence that the hypothetical monopolist faces a demand elasticity greater than the critical elasticity indicates that the product in question is not a relevant market.

As with many concepts in antitrust, there exists some confusion about the critical elasticity. It has been defined as the lowest elasticity of demand such that a hypothetical monopolist of a putative relevant market *would not* raise price by a given percentage over the initial price. Alternatively, the critical elasticity has also been defined as the lowest elasticity of demand such that the hypothetical monopolist *could not* profitably raise price by a threshold percentage over the initial price. As we shall see, the distinction is an important one. The "would" approach has a more appropriate theoretical basis and it leads to lower critical elasticities than the "could" approach. To say that one or the other approach is correct, however, is to assign an undue amount of precision to a particular price increase threshold. Here we will derive, explain, and compare the two approaches.²

II. The could-elasticity

The distinction and the confusion between the could-elasticity and the would-elasticity seem to have arisen from a poor choice of language in the 1984 Guidelines. The excerpt cited above asks whether the monopolist "would impose" a price increase. However, the 1984 Guidelines also state the following:

² We will be discussing and extending the analyses of this issue presented by Harris and Simons, *Focusing Market Definition: How Much Substitution Is Necessary*, 11 RES. L. & ECON. 207 (1989); Johnson, *Market Definition Under the Merger Guidelines: Critical Demand Elasticities*, 11 RES. L. & ECON. 227 (1989); Johnson, *Two Approaches to Market Definition Under the Merger Guidelines*, 11 RES. L. & ECON. 235 (1989); and Werden, *Four Suggestions on Market Delineation*, 37 ANTITRUST BULL. 107 (1992).

In general, the Department will include in the product market a group of products such that a hypothetical firm that was the only present and future seller of those products (a "monopolist") *could* profitably impose a "small but significant and nontransitory" increase in price. (emphasis added)³

In all, when referring to potential price increases, the 1984 Guidelines use the phrase "could profitably" six times and "would profitably" once.

It is not immediately clear what the phrase "could profitably" was supposed to mean. It could mean that charging the higher price would result in profits that are at least as great as profits at the old price. The monopolist could charge that price without decreasing its profits. That is the interpretation used by Harris and Simons to derive the concept of critical elasticity. On the other hand, if the phrase was meant to imply profit maximization, then it means "would profitably." Is the profit-maximizing price of the monopolist significantly above the current price; that is, would a profit-maximizing monopolist raise price by the threshold percentage? The 1992 Horizontal Merger Guidelines consistently use the phrase "would profitably."⁴

To derive a could-elasticity, Harris and Simons set the difference in profits at the two prices equal to zero. (The new price is some given percentage greater than the initial price, the percentage representing "the small but significant and non-transitory" price increase envisioned by the Guidelines.) They then express that equation in terms of the elasticity of demand, the price increase, and the initial margin of price over average variable costs. We will use the following notation throughout the article:

P_0 = initial price,

t = critical percentage price increase,

$m = (P_0 - C) / P_0$ = the margin of initial price over cost.

³ 1984 Guidelines § 2.11.

⁴ United States Department of Justice and Federal Trade Commission. Horizontal Merger Guidelines (April 2, 1992), § 1, *reprinted in* 4 Trade Reg. Rep. (CCH) ¶ 13,104 [hereinafter 1992 Guidelines].

The margin can be thought of as the contribution margin (price over average variable costs), as Harris and Simons assume. Alternatively, it can be thought of as the margin of price over marginal cost, as Werden assumes.⁵

The insight of Harris and Simons is that the initial margin is an important element in determining the critical elasticity. For any given price increase and form of the demand function, the critical elasticity increases as the initial margin decreases. That result is intuitively clear. A monopolist readily gives up low-margin sales to achieve a higher price, whereas high-margin sales are more costly to relinquish. Thus, when the initial margin is low, a higher elasticity is necessary to prevent a monopolist from raising price by a given percentage.

Using the simplifying assumptions of linear demand and constant average variable costs, Harris and Simons derive the following expression for the critical could-elasticity:

$$1 / (t + m).$$

It is also possible to derive the could-elasticity assuming a constant-elasticity demand curve:

$$\ln(1 + t/m) / \ln(1 + t),$$

where $\ln$ represents the natural logarithm. All of the critical elasticities discussed here are derived in the appendix.

III. The would-elasticity

The would-elasticity is the elasticity of demand that generates a given percentage price increase, where the price is determined by profit maximization. Johnson recognizes and solves the would-elasticity problem. There seem to be two shortcomings to his approach, however. First, his model does not employ a constant

⁵ We need to assume that the margin is over marginal cost to derive the would-elasticities, as that is the relevant margin for profit maximization. See the appendix. Werden, *supra* note 2, recognizes that the contribution margin is likely to provide the most appropriate estimate available for marginal cost.

marginal cost. His analysis, therefore, depends on estimating or speculating on the value of another parameter, namely, the elasticity of supply. Second, and more importantly, his model does not incorporate the effect of any pre-price-increase margin. Thus, his approach appears to be more complicated, with little benefit from the increased complexity, and less general than it needs to be.

Werden presents more general and useful results, but he does not show how they are derived. Also, his results are expressed in terms of the markup, $(P_0 - C)/C$, instead of the more conventional margin, $(P_0 - C)/P_0$. Again, the would-elasticity is the elasticity that generates a given percentage price increase by a profit-maximizing monopolist. Based on the initial margin and the threshold price increase, the linear-demand would-elasticity is the following:

$$1 / (2t + m).$$

The constant-elasticity-of-demand would-elasticity is the following:

$$(1 + t) / (m + t).$$

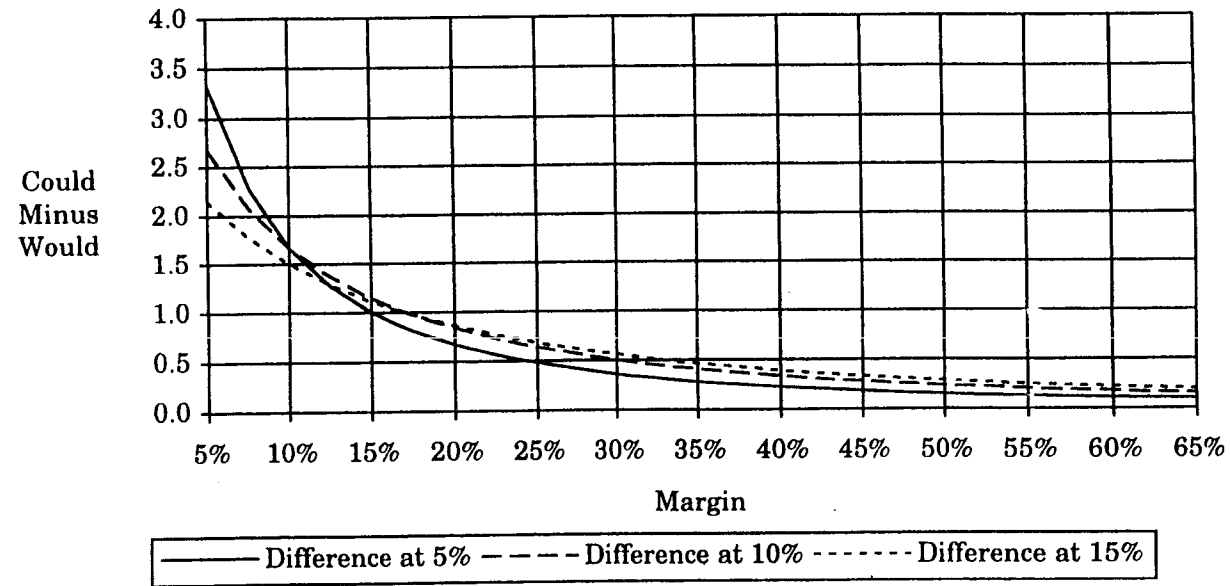
The table shows the critical elasticities, under the two approaches and the two forms of the demand function, at various margins and price increases.

Note that, as discussed above, the higher the margin the lower the critical elasticity. Also, for any given price increase, the would-elasticity is always lower than the could-elasticity for the same margin and the same form of the demand function. By the Harris and Simons definition of "could," the amount that a monopolist could raise price will always exceed the amount that it would raise price. If it could raise price 10% then it would raise price that much only if demand were less elastic. The difference between the two values, substantial at low margins, decreases as the margin increases. The differences between the two elasticity measures, at varying margins and selected price increase thresholds, are depicted in the figure.

Table
Critical Elasticities Table

<i>Approach</i>	<i>Demand</i>	<i>Price increase</i>	<i>Price-cost margin</i>									
			<i>10%</i>	<i>20%</i>	<i>30%</i>	<i>40%</i>	<i>50%</i>	<i>60%</i>	<i>70%</i>	<i>80%</i>	<i>90%</i>	<i>100%</i>
Would	Linear	5%	5.0	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9
		10%	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9	0.8
		15%	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9	0.8	0.8
Would	Constant elasticity	5%	7.0	4.2	3.0	2.3	1.9	1.6	1.4	1.2	1.1	1.0
		10%	5.5	3.7	2.8	2.2	1.8	1.6	1.4	1.2	1.1	1.0
		15%	4.6	3.3	2.6	2.1	1.8	1.5	1.4	1.2	1.1	1.0
Could	Linear	5%	6.7	4.0	2.9	2.2	1.8	1.5	1.3	1.2	1.1	1.0
		10%	5.0	3.3	2.5	2.0	1.7	1.4	1.3	1.1	1.0	0.9
		15%	4.0	2.9	2.2	1.8	1.5	1.3	1.2	1.1	1.0	0.9
Could	Constant elasticity	5%	8.3	4.6	3.2	2.4	2.0	1.6	1.4	1.2	1.1	1.0
		10%	7.3	4.3	3.0	2.3	1.9	1.6	1.4	1.2	1.1	1.0
		15%	6.6	4.0	2.9	2.3	1.9	1.6	1.4	1.2	1.1	1.0

Figure
 Margins and Critical Elasticity Differences: Could Minus Would at Price Thresholds 5%, 10%, and 15%



IV. Which approach is correct?

Johnson and Werden both criticize Harris and Simons for applying the could-elasticity approach. The criticism has a point. The would-elasticity generates a given percentage price increase, where price is determined by profit maximization. For the sake of theoretical integrity, it would seem better to know what the hypothetical monopolist would do, not what it could do without reducing profits below the initial level. That the hypothetical monopolist could raise price 10% and still be as profitable as it was at the old price is irrelevant if, given the elasticity of demand, it would raise price only 4%. As we shall see, however, the two elasticities are closely linked and to call one or the other correct is problematic.

Note that there is a one-to-one correspondence between the two formulas. The linear could-elasticity equals $1/(t + m)$ and the linear would-elasticity equals $1/(2t + m)$. One is a simple transformation of the other. If we let E_c represent the linear could-elasticity and E_w represent the linear would-elasticity, it is easy to demonstrate that

$$E_c = 2E_w / (1 + E_w m).$$

Thus, for any given price increase and initial margin (t and m) there is a unique could-elasticity and a unique would-elasticity. In the absence of a theoretically justifiable critical value of the price increase (t), saying that the would-elasticity is more correct than the could-elasticity is like saying that centigrade is more correct than Fahrenheit.

The 1984 and the 1992 Guidelines both state that the threshold price increase used in practice will be, in general, 5%, although the 1992 Guidelines are evasive on this point:

In attempting to determine objectively the effect of a "small but significant and nontransitory" increase in price, the Agency, in most contexts, will use a price increase of five percent lasting for the foreseeable future. However, what constitutes a "small but significant and nontransitory" increase in price will depend on the nature of the

industry, and the Agency at times may use a price increase that is larger or smaller than five percent.⁶

In sum, we agree with Werden that, for this purpose, average variable cost is a reasonable substitute for marginal cost. We also agree that, when significance is being attached to a specific price-increase threshold, the linear would-elasticity is appropriate. The would-elasticity is more correct, however, only to the extent that the price increase threshold has a valid theoretical or empirical basis for purposes of market definition. We are unaware of any such justification for any specific level of the critical price-increase threshold.⁷

V. Conclusion

The would-elasticity is the appropriate critical elasticity to use in the discussion of market definition. By consistently describing the problem as whether or not a monopolist "would profitably impose a price increase," the 1992 Guidelines remove one element of confusion concerning the problem of defining relevant markets. To say that the could-elasticity is incorrect, however, attributes an unwarranted precision to a particular price-increase threshold.

⁶ 1992 Guidelines § 1.11.

⁷ Even if the theoretically correct threshold were known, it is not necessarily the one that should be used in merger investigations. The correct threshold is the one that induces the lawyers and economists conducting the investigation to arrive at the correct conclusion about the relevant market. For example, even if 10% is a reasonable standard, it may still be that the agencies should pose the question to private parties—if it should be posed at all—as 20%. It depends on how people think about hypothetical price increases. See G. J. STIGLER, *THE INTELLECTUAL AND THE MARKETPLACE* ch. 11 (1984) for an amusing and insightful discussion of how people tend to respond to such questions.

APPENDIX

Calculation of the Critical Demand Elasticities

Here we derive the various critical demand elasticities. The calculations are done for two forms of the demand function, constant elasticity and linear, and under two approaches, "would-raise-price" and "could-raise-price." The "would" approach is the elasticity that generates a given price increase by a profit-maximizing monopolist. The "could" approach compares the profits after a certain percentage price increase (whether profit maximizing or not) to the profits at the initial price.

We assume that marginal cost is constant. This is equivalent to assuming that the average variable cost curve is flat over the relevant range.

The following four definitions are used throughout:

- (i) C = marginal cost, which is assumed to be constant;
- (ii) P_0 = initial price;
- (iii) $m = (P_0 - C) / P_0$ = the price-cost margin, or the contribution margin, at the initial price;
- (iv) t = the critical price increase threshold.

The would-raise-price approach

Under this approach, the question being asked is whether or not a hypothetical profit-maximizing monopolist *would* raise price a certain percentage above the initial price. This requires, first, the calculation of the profit-maximizing price, and second, a comparison of that price to the initial price. In examining the would approach, an additional definition is employed:

- (v) P_M = monopoly price, the profit-maximizing price that would be charged by the hypothetical monopolist.

The monopoly price is less than the critical price increase threshold if

$$(P_M - P_0) / P_0 < t.$$

Constant elasticity of demand

A constant-elasticity demand function can be written as $Q = AP^{-\varepsilon}$, where Q is the quantity demanded, ε is the elasticity of demand, and A is a constant. The monopolist's profit function is

$$\pi = P Q - C Q = AP^{1-\varepsilon} - C AP^{-\varepsilon}.$$

Maximizing this with respect to P yields the profit-maximizing condition

$$P_M = C \left(\frac{\varepsilon}{\varepsilon-1} \right).$$

From (iii), the initial price can be expressed as $P_0 = C / (1 - m)$. Therefore, for the profit-maximizing price increase to be less than the critical threshold, the following condition must hold:

$$\frac{P_M - P_0}{P_0} = \frac{C \left(\frac{\varepsilon}{\varepsilon-1} \right) - C \left(\frac{1}{1-m} \right)}{C \left(\frac{1}{1-m} \right)} < t.$$

This implies that

$$\frac{\varepsilon}{\varepsilon-1} > \frac{1+t}{1-m}$$

and solving for ε yields

$$\varepsilon > \frac{1+t}{m+t}.$$

Linear demand

A linear demand function can be written as $Q = A - BP$, where Q is the quantity demanded and A and B are constants. At the initial price, P_0 , the quantity sold is $Q_0 = A - BP_0$. The elasticity of demand at the initial price and quantity, ε , can be calculated using the following definition of elasticity:

$$\varepsilon = - \frac{\Delta Q}{\Delta P} \frac{P_0}{Q_0},$$

where ΔQ is the change in quantity and ΔP is the change in price. Given the linear demand function, $\Delta Q = -B \Delta P$. Therefore, the elasticity at P_0 is

$$(1) \quad \varepsilon = \frac{BP_0}{Q_0} = \frac{BP_0}{A - BP_0} = \frac{P_0}{\lambda - P_0}$$

where $\lambda = A / B$.

The monopolist's profit function is

$$\pi = P Q - C Q = P (A - BP) - C (A - BP).$$

Maximizing π with respect to P yields the profit-maximizing price condition

$$(2) \quad P_M = \frac{\lambda + C}{2}.$$

Solving (1) for λ in terms of P_0 and ε yields

$$\lambda = \frac{P_0(1 + \varepsilon)}{\varepsilon}.$$

Substituting this expression into (2) yields

$$P_M = \frac{P_0(1 + \frac{1}{\varepsilon}) + C}{2}.$$

From (iii), marginal cost, C , can be expressed as $C = P_0 (1 - m)$. Therefore, for the profit-maximizing price increase to be less than the threshold, the following condition must hold:

$$\frac{P_M - P_0}{P_0} = \frac{\frac{P_0(1 + \frac{1}{\varepsilon}) + P_0(1 - m)}{2} - P_0}{P_0} < t$$

or

$$(1 + \frac{1}{\varepsilon}) + (1 - m) < 2(1 + t).$$

Solving for ε yields

$$\varepsilon > \frac{1}{2t + m}.$$

The could-raise-price approach

Under this approach, the question being asked is whether or not a hypothetical monopolist *could* raise price a certain percentage above the initial price and earn more profits than at the initial price. The question is whether or not a price increase of a certain magnitude is profitable relative to initial profits, not whether it is the most profitable. This approach requires a comparison of profits at the increased price to profits at the initial price. Here, an additional definition is employed:

(vi) $P_1 = P_0(1 + t)$, where, as before, t is the critical price increase threshold.

Constant elasticity of demand

A constant-elasticity demand function can be written as $Q = A P^{-\varepsilon}$, where Q is the quantity demanded, ε is the elasticity of demand, and A is a constant. The monopolist's profit function is

$$\pi = P Q - C Q = A P^{1-\varepsilon} - C A P^{-\varepsilon}.$$

If the critical price threshold is t , then under the could approach the relevant question is whether it is more profitable to price at P_0 or at $P_1 = P_0(1 + t)$.

At P_0 , quantity is $Q_0 = A P_0^{-\varepsilon}$. At a price of P_1 , quantity will be

$$Q_1 = A P_1^{-\varepsilon} = A P_0^{-\varepsilon} (1 + t)^{-\varepsilon} = Q_0 (1 + t)^{-\varepsilon}.$$

It will not increase profits to raise price to P_1 if $(P_0 - C) Q_0 > (P_1 - C) Q_1$ or

$$(P_0 - C) Q_0 > (P_0 (1 + t) - C) Q_0 (1 + t)^{-\varepsilon}.$$

Eliminating the Q_0 terms, and, from (iii), substituting $C = P_0(1 - m)$, yields that it will not increase profits to raise price to P_1 if

$$P_0 - P_0(1 - m) > [P_0(1 + t) - P_0(1 - m)] (1 + t)^{-\varepsilon},$$

or

$$m > (m + t) (1+t)^{-\varepsilon}.$$

Solving for ε shows that the price increase is unprofitable if

$$\varepsilon > \frac{\ln(1 + \frac{t}{m})}{\ln(1 + t)}.$$

Linear demand

A linear demand function can be written as $Q = A - BP$, where Q is the quantity demanded and A and B are constants. At the initial price P_0 , the quantity sold is $Q_0 = A - BP_0$. If the critical price threshold is t , then under the could approach the relevant question is whether it is more profitable to price at P_0 or at $P_1 = P_0(1 + t)$.

At a price of P_1 , quantity will be

$$Q_1 = A - BP_1 = A - BP_0(1 + t) = Q_0 - BP_0t.$$

It will not increase profits to raise price to P_1 if $(P_0 - C) Q_0 > (P_1 - C) Q_1$ or

$$(P_0 - C) Q_0 > (P_0(1 + t) - C) (Q_0 - BP_0t).$$

This condition reduces to

$$0 > P_0t (Q_0 - BP_0(1 + t) + CB).$$

Dividing through by BP_0 , the condition can be written as

$$0 > \frac{Q_0}{BP_0} + \frac{C}{P_0} - (1 + t).$$

From (1), $\varepsilon = \frac{BP_0}{Q_0}$ or alternatively, $\frac{Q_0}{BP_0} = \frac{1}{\varepsilon}$.

From (iii), $\frac{C}{P_0} = (1 - m)$. Therefore, the condition becomes

$$0 > \frac{1}{\varepsilon} + (1 - m) - (1 + t).$$

Solving for ε yields

$$\varepsilon > \frac{1}{t + m}.$$