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Output Under
Second-Degree Price Discrimination

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I

Pigovian price discrimination of the second degree is characterized by a single price schedule to all buyers with different prices for successive groups of units. As such, it constitutes an imperfect form of first-degree discrimination, and its analytic properties are usually taken as approximations to those of perfect price discrimination. This approach, however, overlooks important --if subtle--differences between these forms and differences in their effects. 1/ The present note undertakes to show that, contrary to the conventional wisdom, output under second-degree discrimination may be less than single-price monopoly output under perfectly reasonable, non-pathological circumstances. 2/ Some public policy implications of this revisionist view are offered at the conclusion of the analytical discussion.

II

Familiar examples of second-degree price discrimination include electric utility rates within the class of residential customers ("block pricing") and quantity discounts in a variety of commercial and retail transactions. The usual depiction of such pricing is given in Fig. 1 (ignoring D_{II} and D_{III} for the moment). The inframarginal blocks, P_1 up to quantity Q_1 and P_2 up to Q_2, succeed in monetizing additional consumer surplus under D_I, compared with a simple monopolist charging the single price P_2. 3/ And that consumer will purchase the quantity $D_I(P_3)$ rather than $D_I(P_2)$ as long as the area B exceeds area A. If, as shown, P_3 is set to coincide with constant marginal costs C, consumption is identical to that under condition of competitive supply and hence exceeds output under single-pricing monopoly (Q_2). Profits to the seller exceed those secured with any single price, and depending on the arrangement of the blocks, may approach levels under perfect price discrimination.

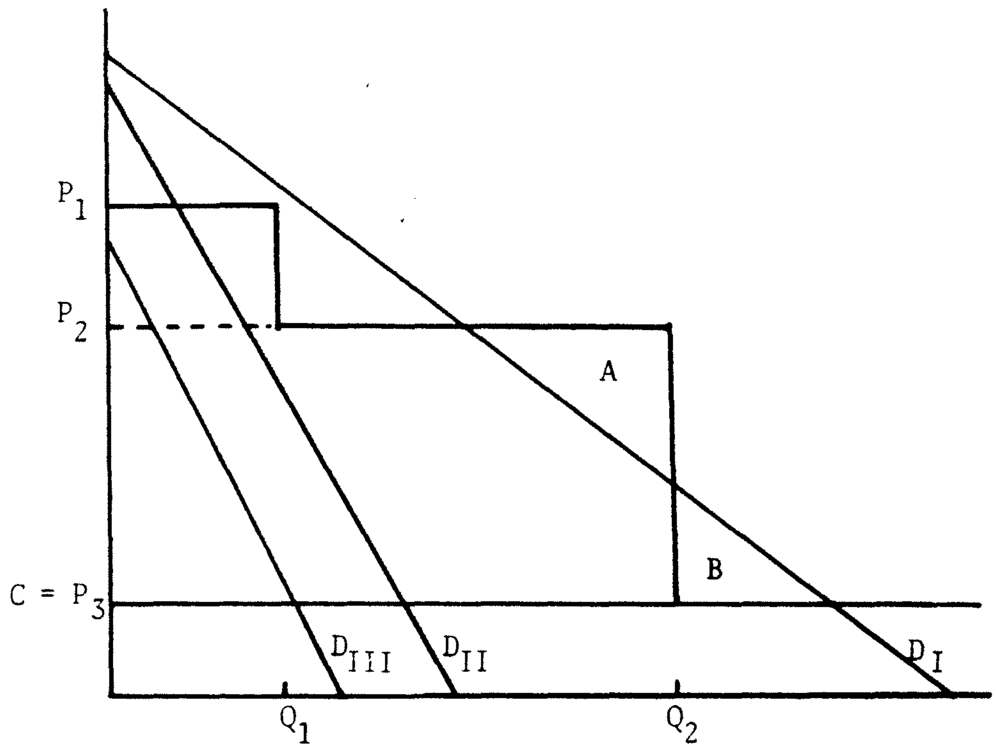

FIG. 1

The basic deficiency of this analysis is that it ignores the question of optimal (i.e., profit-maximizing to the seller) blocks and hence the effects of such blocks on other demanders in the market. For example, in Fig. 1, in addition to D_I, two other kinds of consumers need to be recognized. D_{II} represents an individual whose consumption falls from $D_{II}(P_2)$ under simple monopoly to $D_{II}(P_1)$ under block pricing. D_{III} denotes a consumer of an amount $D_{III}(P_2)$ under a single-pricing monopoly who is altogether squeezed out of the market with block prices. If type II and III consumers are sufficiently numerous, their consumption declines might threaten to outweigh the increase by D_I. 4/

But could that occur? After all, the design of the blocks themselves is within the discretion of the seller, and it might be precisely that diminution of total output which would reduce his profits and instead cause him to define different block prices. Neither this proposition nor its converse is intuitive. In the remainder of this paper, we develop the problem analytically and offer a numerical counter-example to the usual conclusion that output rises under block pricing.

III

We seek to develop a set of conditions on the structure of demand in the market which yield a smaller output under profit-maximizing price discrimination than with single-pricing monopoly. In order to make this problem tractable, but still with some generality, we make the following assumptions:

(1) There are two consumers in the market, with linear (inverse) demand functions:

D_1: $P = a_1 - b_1 Q_1$ (1a)

D_2: $P = a_2 - b_2 Q_2$ (1b)

where a_1, a_2, b_1, b_2 > 0. For concreteness, we also assume $a_1 > a_2$.

(2) Production is costless. This assumption will be relaxed in later discussion, but the present mathematics and intuitive understanding are facilitated by taking both marginal and fixed costs as zero. 5/

(3) The seller is limited to two price blocks. This implies he has three parameters: the two prices P_1 and P_2, and the quantity which delimits the blocks. 6/

Since there are a number of discrete alternatives available to the seller, this problem cannot be solved by global optimization techniques. We begin by identifying and comparing the alternatives for single-pricing monopolist, as shown in Fig. 2. D represents the lateral summation of the two demand curves, coincident with D_1 down to $P=a_2$ where the second consumer enters the market. The parameters of the demand functions determine whether a point like E, profit-maximizing with only one market served, or a point like F, profit-maximizing when both are served, yields the larger total profit. Since no profit-maximizing pricing scheme produces output less than Q_E, the case we seek (where simple monopoly output is larger) must entail that monopoly operating at point F. Hence for this example,

$$\pi_F > \pi_E \qquad (2)$$

where π_F and π_E denote total profits at the two points shown. Recalling that C = 0, we quickly find that 7/

$$\pi_E = \frac{a_1^2}{4b_1} \qquad (3a)$$

and $$\pi_F = \frac{(b_2 a_1 + a_2 b_1)^2}{4b_1 b_2 (b_1+b_2)} \qquad (3b)$$

Condition (2) is therefore equivalent to

$$(b_2 a_1 + a_2 b_1)^2 > a_1^2 b_2 (b_1 + b_2) \qquad (4)$$

Next we develop the optimum block pricing strategy. The monopolist

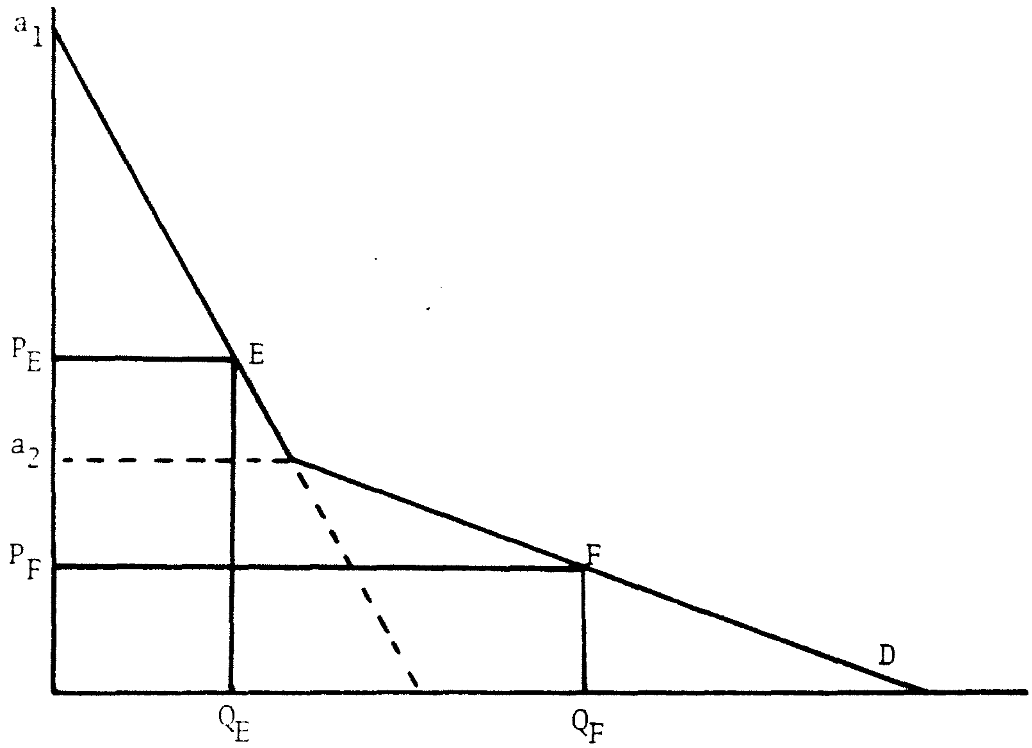

FIG. 2

can secure all the consumer surplus under any single demand curve by designing appropriate blocks. In Fig. 3, the total surplus under D_1 is given by $\frac{a_1^2}{2b_1}$. This surplus can be appropriated by a price schedule which offers the last unit at $P_2 = 0$ (making consumption of $\frac{a_1}{b_1}$ attractive at the margin) and which prices preceding units such that area A equals area B (thereby exhausting inframarginal surplus). One such scheme is shown by $P_1 = P_1^L$, designating the lowest possible initial price. In fact, all points on the rectangular hyperbola marked $(P_1 Q_1)^*$ down to P_1^L yield equivalent profits in the amount $(P_1 Q_1)^* = \frac{a_1^2}{2b_1}$. 8/

Thus the discriminating monopolist can create two blocks which exhaust the larger (since he can exhaust either) of the two surpluses of his customers, or which capture twice the smaller surplus (once from each consumer). In the former case his profit equals that surplus, plus possibly an increment due to some purchases by the other consumer. 9/ With the latter strategy his basic profit of twice the smaller surplus may, under certain circumstances, be enlarged by below-cost pricing on the second block. Although some units sold thereby produce apparent losses, the lower price raises total surplus which is collected from both customers. 10/ Hence total profits may increase, though for most cases, the quantitative effect is minimal.

Though this is not a necessary condition, it will prove easier and clearer to develop conditions based on the former strategy. In that case, the surplus under D_1, denoted CS_1, is pursued, 11/ and that surplus must exceed twice D_2's. Thus, the following condition is sufficient:

$$\frac{a_1^2}{2b_1} > \frac{a_2^2}{b_2} \qquad (5)$$

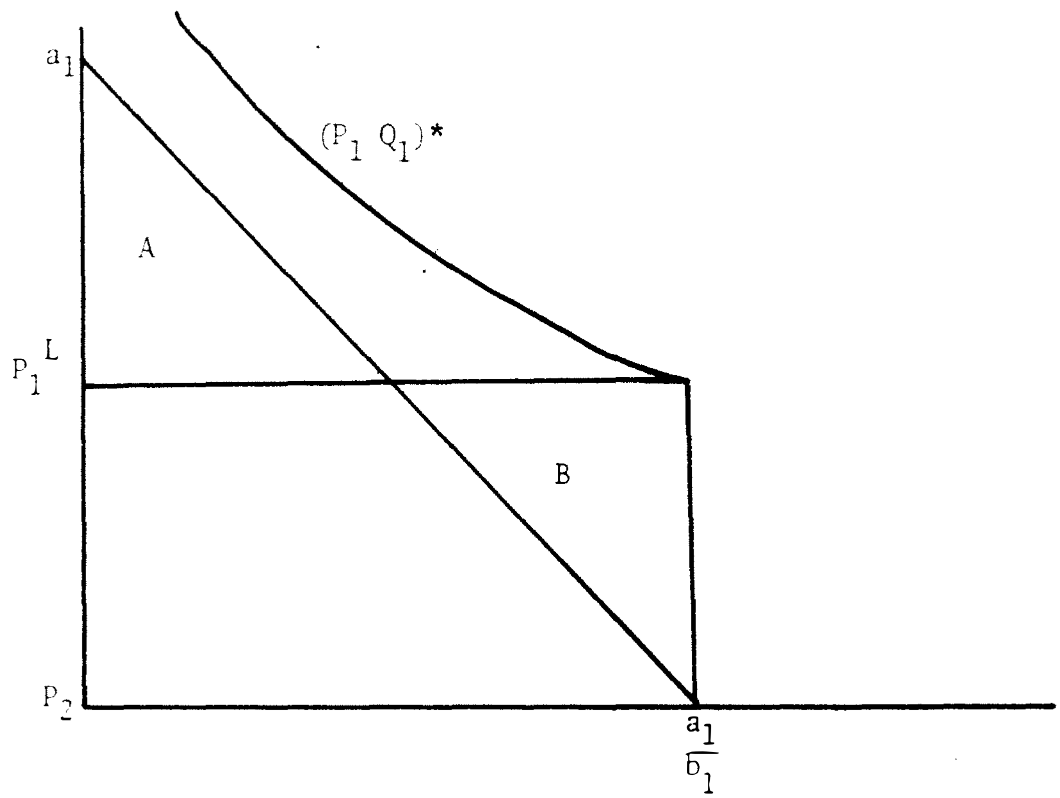

FIG. 3

The last condition concerns quantity itself. For the present counter-example, single-pricing monopoly output (Q_F in Fig. 2) necessarily exceeds output under this block pricing scheme:

$$\frac{b_2 a_1 + a_2 b_1}{2b_1 b_2} > \frac{a_1}{b_1} \qquad (6a)$$

and

$$a_2 b_1 > a_1 b_2 \qquad (6b)$$

Since $a_1 > a_2$, (6b) implies that $b_1 > b_2$ to an even greater degree, i.e., that D_1 is steeply sloped and intersects the horizontal axis at a smaller quantity than does D_2.

The conditions expressed in (5) and (6b) jointly imply that $a_1 > 2a_2$. Since $P_1 \geq \frac{1}{2} a_1$, the first block price never falls to a level where D_2 is induced into the marekt and the profit-maximizing arrangement excludes him entirely.

Conditions (4), (5), and (6b) can be simultaneously met by a considerable class of plausible demand functions of the form shown in Fig. 4. For example,

$$D_1: \quad P = 5 - 8 Q_1 \qquad (7a)$$

$$D_2: \quad P = 2 - 3 Q_2 \qquad (7b)$$

can readily be shown to satisfy these conditions. The relevant feature is that D_2 is flat enough so that its surplus is insufficient to forsake D_1's, but steep enough so that the single-price monopolist gains by serving that consumer. 12/ In Part A of Table I, the prices, profits and total quantity have been calculated for the example of (7a) and (7b) with costless production. As discussed, the single-pricing monopolist serves both markets and produces output equal to .646, while the block-pricing discriminator maximizes his profits by capturing all D_1's surplus of 1.56 and producing a smaller quantity .625. Note that when P_2 is not constrained to zero, profit-maximization

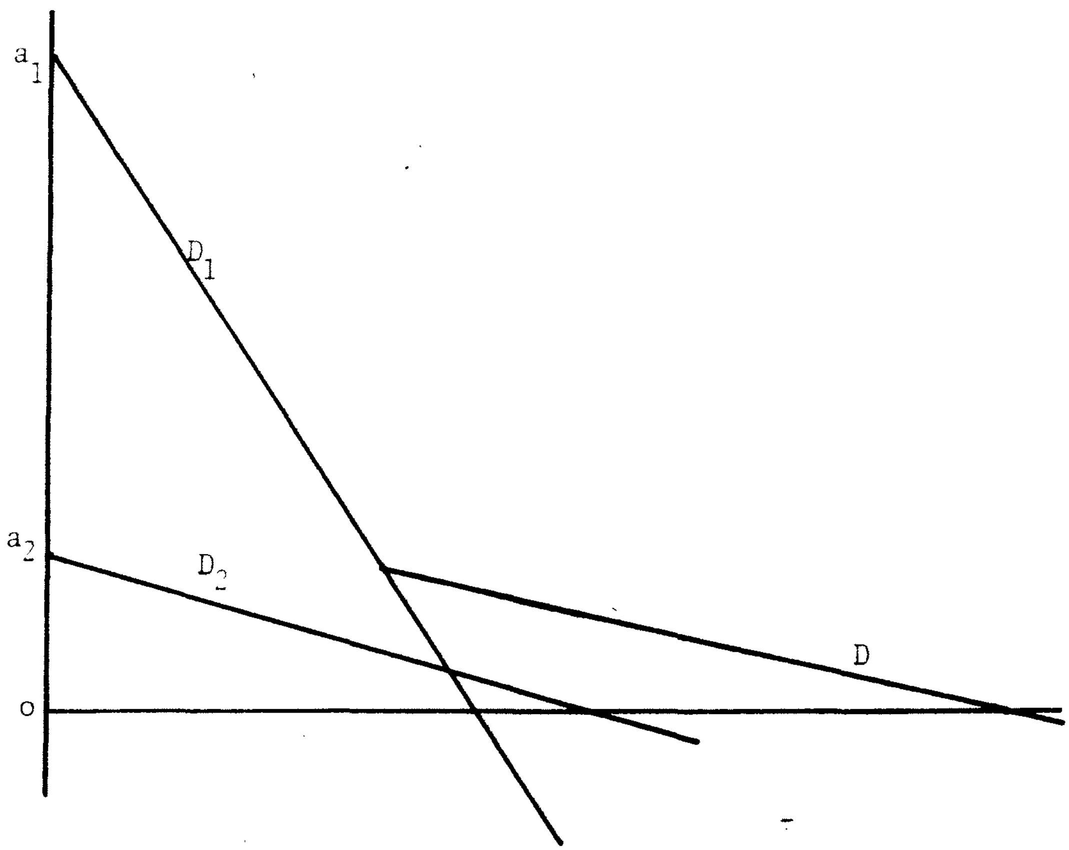

FIG. 4

Table I

Pricing Strategy	Part A				Part B			
	P_1	P_2	π	Q	P_1	P_2	π	Q
Single price to D_1	2.50	-	.78	.312	2.55	-	.75	.306
Single price to $D_1 + D_2$	1.41	-	.91	.646	1.46	-	.85	.623
Block price to D_1	>2.50	0	1.56	.625	>2.45	.096	1.50	.613
Block price to D_2 ($P_2 = C$)	>1.00	0	1.333	1.292	>.95	.098	1.204	1.268
Block price to D_2	>1.08	-.17	1.336	1.370	>1.01	-.016	1.205	1.299

directed at D_2 requires a below-cost price on the final block (see footnote 10 and the Appendix).

One important extension of this example involves non-constant costs. 13/ Since the case for block-pricing would appear strongest in a declining cost industry, it is interesting to demonstrate the possibility of a reduced output effect in that circumstance. To the demand functions of (7a) and (7b) are added the following average and marginal costs curves:

$$AC = .1 - .001\ Q \qquad (8a)$$

$$MC = .1 - .002\ Q \qquad (8b)$$

The prices, quantity, and profits under each of the pricing strategies already discussed are reported in Part B of Table I. The global maximum profit again occurs by block pricing to D_1, which yields a smaller output than under the preferred single price which involves serving both markets. Although steeper cost curves can reverse this phenomenon, clearly declining costs are insufficient to insure larger output with block pricing.

IV

Generalization to include all cases of declining output under second-degree price discrimination would surely be useful, though immensely difficult. Despite the particular assumptions of the present example, it is suggestive of one set of demand characteristics yielding that conclusion. The thrust of this example is that one consumer has relatively large surplus so that the seller maximizes his profit by capturing that surplus and ignoring other consumers. Here the second consumer is effectively excluded from the block-priced market. That is not a necessary outcome, however, as can be seen by adding to Fig. 4 a third demand curve with vertical intercept $a_3 > a_1$ and a very steep slope. His minimal surplus is likewise ignored in block construction, but he remains (at reduced consumption) in the market still designed to exhaust D_1's surplus.

The implications of an ambiguous output effect from second-degree price discrimination are of some importance for public policy. Long a feature of most electric utility rate structures, block pricing has been justified as expanding output in a declining-cost industry. That presumption is now seen to require analytical or empirical evidence of a sort rarely offered. Despite adverse court rulings, quantity discounts are not uncommon in commodity transactions. 14/ Considerable criticism of these rulings and the laws on which they are based has been voiced by economists, arguing that such discrimination raises output and moderates the allocative inefficiency of a single-pricing monopoly. The present result shows this to be an open question.

Footnotes

1. A considerable recent literature has developed in this area, testifying to the recognition of such subtleties. See Oi (1971), Adams and Yellen (1976), and Leland and Meyer (1976).

2. Even some recent work, in the process of clarifying other issues, simply repeats this over-simplification. Yamey (1974), for example, states (pp. 377-78):

    > With perfect price discrimination (or even with Pigou's discrimination of the second degree) the profit-maximizing output is the same as the competitive output.

3. P_2 is used both as the single-price and as one block of a discriminating monopolist only for graphical simplicity. The choice of three blocks is arbitrary. Throughout we assume zero income effects.

4. An additional type of consumer is one whose reservation price never exceeds monopoly price P_2. Since that consumer appears in neither pricing regime, we ignore him here. None of these other cases are discussed in the seminal articles by Buchanen (1953) and Gabor (1955).

5. Taking fixed costs as zero is of course irrelevant to the marginal conditions throughout. Zero marginal costs are assumed for ease of exposition and differ from any constant costs only by rescaling one axis.

6. This problem with a larger number of blocks (or with two blocks but more demand curves, as in Leland and Meyer (1976)) cannot be solved analytically.

7. Somewhat less elliptic derivations for this and later results appear in the Appendix.

8. The large number of equivalent strategies is the result of our particular assumptions which give the seller two blocks and three parameters to operate on one (or later two) demand curves. It is interesting to note that the highest leading block price, P_1^H, is given by $\frac{a_1^2}{2b_1}$ itself, all on the initial unit purchased. Such a pricing scheme is formally equivalent to a "two-part tariff". See Gabor (1955), Oi (1971).

9. That is, if the other consumer's demand curve resembles D_{II} in Fig. 1. As we show below, this is precluded in the present example.

10. If D_2 is the market with smaller total surplus, profit-maximizing $P_2=(a_1b_2-a_2b_1)/2b_2$, which can be of either sign. (See Appendix. Further proof available from author on request) Subsequent assumptions of the present case will make P_2 negative (really below cost, where C=0), since the marginal revenue collected from two consumers exceeds price. For analogous conclusions with respect to two-part tariffs, see Oi (1971).

11. This is due to the fact that when $a_1 > a_2$, CS_2 can only exceed 1/2 CS_1 when total output (quantity demanded by both consumers) is larger than simply D_1's.

12. One arrangement of demand curves which can only yield higher output under price discrimination is parallel curves. Leland and Meyer's analysis (1976) proceeds with this rather restrictive assumption.

13. I wish to thank Knox Lovell for emphasizing this case and suggesting an approach.

14. The ruling opinion is from FTC v. Morton Salt Co. [334 U.S. (1948)]. The court struck down Morton's quantity discount system with the following argument:

> Respondent's basic contention, which it argues this case hinges on, is that its "standard quantity discounts, available to all on equal terms, . . . are not discriminatory within the meaning of the Robinson-Patman Act." Theoretically, these discounts are equally available to all, but functionally they are not. (p.42)

The concept of "functional availability" applies directly to Types II and III demanders in Fig. 1, though the Court's concern is more with distributive than allocative efficiency. See Leland and Meyer (1976) and Crockett (1976).

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Appendix

At the point E, profits are simply $P_E Q_E$, or

$$\pi_E = \frac{a_1}{2}\left(\frac{1}{2}\cdot\frac{a_1}{b_1}\right) = \frac{a_1^2}{4b_1} \tag{A1}$$

The horizontal summation of demand curves yield the equation

$$Q = \frac{a_1 - P}{b_1} + \frac{a_2 - P}{b_2} \tag{A2}$$

or

$$P = \frac{b_2 a_1 + a_2 b_1}{b_1 + b_2} - \frac{b_1 b_2}{b_1 + b_2} Q \tag{A3}$$

Setting marginal revenue along that segment equal to zero (cost), optimum quantity Q_F is given by

$$Q_F = \frac{b_2 a_1 + a_2 b_1}{2b_1 b_2} \tag{A4}$$

and

$$P_F = \frac{b_2 a_1 + a_2 b_1}{2(b_1 + b_2)} \tag{A5}$$

Profits at F are their product:

$$\pi_F = \frac{(b_2 a_1 + a_2 b_1)^2}{4b_1 b_2 (b_1 + b_2)} \tag{A6}$$

The negative second price discussed in the text, footnote 10, and Table 1 results from the following calculation. By pursuing D_2, the discriminating seller can secure profits equal to $2.CS_2$, or

$$\pi_1 = \frac{(a_2 - P_2)^2}{b_2} \tag{A7}$$

If P_2 is not equal to zero (cost), his gain or loss is given by

$$\pi_2 = P_2 \left(\frac{a_1 - P_2}{b_1} + \frac{a_2 - P_2}{b_2}\right) \tag{A8}$$

Total profits comprise the sum of π_1 and π_2:

$$\pi_T = \frac{(a_2 - P_2)^2}{b_2} + P_2 \left(\frac{a_1 - P_2}{b_1} + \frac{a_2 - P_2}{b_2} \right) \tag{A9}$$

Differentiation by P_2 yields

$$P_2 = \frac{a_1 b_2 - a_2 b_1}{2b_2} \tag{A10}$$

of generally indeterminate sign.