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ESTIMATING EXPECTED LOSSES IN AUTO INSURANCE

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Introduction

The prediction from economic models of competitive markets with full information is that the price of a commodity will be equal to the marginal cost of providing that commodity. In insurance markets this would be translated as the price of insuring any risk would equal the expected loss of that risk plus a loading for transaction costs. However, actual insurance markets do not have full information and the expected value of the loss from insuring a particular risk is not known with certainty. In this situation, competitive pricing of any risk becomes more complicated and in particular would require some statistical estimation and decision theory.

The risk assessment process for automobile insurance is generally based on the prior losses of individuals of the population of insureds. Characteristic such as garage location of car, age, sex, etc., are collected and used to specify a risk class. After adjustments for trend and loss development have been made, the mean of past losses from a risk class is an estimate of expected future losses for individuals with similar characteristics.[1] Ignoring the procedure for best choosing

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[1] See "The Role of Risk Classifications in Property and Casualty Insurance" [11] for a full discussion of the actuarial pricing procedures. This report is subsequently referred to as the SRI report.

the characteristics which define a risk class, this basic statistical approach is consistent with my economic intuition of the outcome of a profit seeking insurance industry.

Actual cell means, however, are not always used as the estimate of expected loss since some cells don't have enough observations to generate "credible" estimates. When this is the case, certain adjustments are made to the particular cell mean. The optimal adjustments required depend on the model believed to generate the cell observations and its relationship to other available information. A statistical theory called credibility theory has been developed which derives the necessary adjustments for particular models.[2] Actual insurance practice uses some of these adjustments in forming their estimates of expected losses, but it appears that the insurance industry doesn't use many of the sophisticated methods of statistical analysis.

Recently Chang and Fairly [2] criticized the traditional estimation procedure used in automobile insurance rate making. They proposed the use of an additive least-squares model with no interactions over the traditional multiplicative method. Additionally, the Massachusetts insurance commissioner required the use of this method to generate the appropriate values of price differentials in the state of Massachusetts. The analysis

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2 Jewell [5] surveys the results of credibility theory and relates them to general statistical theory. The papers of a conference on credibility theory appear in Kahn [6].

of Chang and Fairley indicated that the least-squares procedure was statistically better than the traditional method for the situation analyzed.

Economic theory would suggest that in a competitive environment, innovative and profit seeking firms would have found and eliminated the bias described by Chang and Fairley, although the insurance commissioner in Massachusetts suggested that it might never have been corrected without regulatory encouragement. It is hard to dispute either position and it may be that both positions are correct. One would expect that the most successful insurance companies would be the ones which used the most accurate estimation methods, but it may take a long time to reveal the best method and innovation may be accelerated by regulatory encouragement.

The Massachusetts results, however, will not end the debate about the appropriate model to use. The additive model is not theoretically superior to the multiplicative model and other evidence indicates the superior performance of the multiplicative model in certain situations.[3] This paper, therefore, presents some further analysis of the multiplicative model. It is suggested that even within the multiplicative framework that insurance companies have historically operated, the traditional estimating procedures yield some of the same biases found by

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[3] See the discussion starting on page 76 of Automobile Insurance Affordability [1].

Chang and Fairley. The traditional estimating procedures overcharge individuals in the higher rated territories and classes and undercharge those in lower rated territories and classes.

Model Specification

The two-way layout is the conceptual framework for the analysis. There are I levels of a factor A (territory) and J levels of a factor B (class plan) which classify all losses into an IXJ table. The parameters of interest are the cell means, i.e., the average loss for the ith territory and jth class combination, denoted by N_{ij}. In the classical analysis of variance, the cell means are not estimated directly but are factored into additive effects (row and column) which are specific to the levels of A and B plus an interaction effect of the i^{th} level of A with the j^{th} level of B. However, this factorization has imposed no restrictions on the original cell means and is not the only natural way to factor the original cell means. A multiplicative factorization with an intuitive interpretation can represent the cell means just as well as the additive form.

Using the notation of Scheffe [10], the classical analysis of variance represents the cell means by

(1) $N_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij}$

where

$$\sum_{i=1}^{I} \alpha_i = \sum_{j=1}^{J} \beta_j = \sum_{i=1}^{I} \gamma_{ij} = \sum_{j=1}^{J} \gamma_{ij} = 0$$

The parameter μ is the general mean, α_i is the main effect of the ith level of A, β_j is the main effect of the jth level of B, and γ_{ij} is the interaction of the ith level of A and the jth level of B.[4] In terms of the cell means, the parameters are defined as

$$(2) \quad \mu = \sum_i \sum_j N_{ij}/IJ = N_{..}$$

$$\alpha_i = \sum_j N_{ij}/J - N_{..} = N_{i.} - N_{..}$$

$$\beta_j = \sum_i N_{ij}/I - N_{..} = N_{.j} - N_{..}$$

$$\gamma_{ij} = N_{ij} - N_{i.} - N_{.j} + N_{..}$$

A natural multiplicative factorization of the cell means N_{ij} (assuming $N_{ij} > 0$), might be

$$(3) \quad N_{ij} = \mu\alpha_i \beta_j \gamma_{ij}$$

where

$$\sum_{i=1}^{I} \alpha_i = I \qquad \sum_{j=1}^{J} \beta_j = J$$

$$\sum_{i=1}^{I} \alpha_i \gamma_{ij} = I \qquad \sum_{i=1}^{J} \beta_j \gamma_{ij} = J$$

This formulation defines the parameters by

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4 As defined by equations (2), the general mean is the average over rows or columns, main effects are defined as the excess of the mean for the ith (jth) level over the general mean, and interactions are the remaining excess from the specific cell mean. See Scheffe [10].

$$(4)\quad \mu = \sum_i \sum_j N_{ij}/IJ = N_{..}$$

$$\alpha_i = \sum_j N_{ij}/JN_{..} = N_{i.}/N_{..}$$

$$\beta_j = \sum_i N_{ij}/IN_{..} = N_{.j}/N_{..}$$

$$\gamma_{ij} = N_{..}\, N_{ij}/N_{i.}\, N_{.j}$$

The parameters in (4) can be given interpretations similar to those in (2) by talking about the excess of the mean for the i^{th} level relative to the general mean etc. In the language of the insurance industry the α_i would be the territorial relativities and the β_j would be the class relativities.

The formulations in (1) and (3) are both equally general as either can represent any possible values for the cell means N_{ij}. And particularly in an insurance context where the cell means themselves are the interesting parameters,[5] there is no reason to use or prefer either formulation. If sufficient data were available to estimate each cell mean separately, there would be no reason to estimate either the additive or the multiplicative factorization. The cell means themselves would provide all information of interest to the insurance company. However,

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[5] There is debate about whether means are the only interesting parameters from a social point of view, Ferreira [3]. But competitive pricing would lead to the use of cell means as the cost basis for price.

sufficient data is generally not available to give precise estimates of all cell means. Estimates of some cell means are subject to sufficient sampling variability that they are not considered credible to use as the basis of insurance pricing.

The lack of sufficient observations to use in estimating cell means, however, is not peculiar to an insurance context. Most applications of the analysis of variance have this problem and it is one reason a factorization scheme has value. With either a multiplicative (equation 3) or an additive (equation 1) paramaterization (and assuming some of the parameters such as the interaction terms are zero) one can use information from all cells to obtain estimates of any particular cell mean. It is at the point where some parameters are specified a priori that choosing between formulations (1) and (3) becomes important. But this choice is an empirical question which could be resolved with the proper data.

Consider the typical additive model or the model with no interaction

(5) $N_{ij} = \mu + \alpha_i + \beta_j$

This assumed structure has imposed certain restrictions on the relationship between cell means, but it does not eliminate the model (3) as being the true model or representation of the cell means. The factorization in (3) is perfectly general in representing any IJ numbers (since (3) imposes no restrictions on the N_{ij}) and can represent the model in (5) for any values μ, α_i, and β_j. Using (3) to describe the structure (5) is not a parsimonous use of parameters but it is still theoretically correct. What would be a specification error would be to use the structure

(6) $N_{ij} = \mu\alpha_i \beta_j$

to represent the structure (5). It would also be a specification error if one tried to represent the model (6) by the model (5) because a different set of restrictions are imposed by (6) than are imposed by (5).

Looked at from this perspective, the focus of a choice of model to use in estimating expected losses should not be restricted to a choice between the model (5) and the model (6). Either formulation could be correct but it is also possible that both specifications are incorrect and a factorization other than (5) or (6) would be the correct model. The maintained hypothesis or the most general hypothesis should be the N_{ij} themselves. The choice objective should be to find a factorization of the N_{ij} which is more parsimonious in its use of parameters than is the use of the full IJ parameters embodied in the N_{ij}.

The estimation and testing of model (5) relative to (1) has been extensively developed. Since this is not true for the model (6), the next section will discuss its estimation.

Estimation

The stochastic structure of the data embodied in the multiplicative model must be the same as in the classical analysis of variance since the only difference is in the factorization of the cell means into primitive components. The observed loss for the kth exposure in the ith territory and jth class combination is therefore represented by

(7) $Y_{ijk} = N_{ij} + E_{ijk}$

where the E_{ijk} are random variables with mean zero (as a consequence of defining N_{ij} to be the true cell mean). If distributional assumptions are added to the structure (7), comparisons between alternative estimators could be made. However, for a broad class of assumptions, least-squares estimators have desirable properties[6] and are the estimators presented in this paper.

The least-squares estimates minimize

(8) $$\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K_{ij}} (Y_{ijk} - N_{ij})^2$$

where K_{ij} is the number of exposures in the ith territory and jth class combination. The values of μ, α_i, and β_j which minimize expression (8) given the actual losses Y_{ijk}, are the parameter values which satisfy the first order conditions for minimization. The first order conditions (the derivatives of expression (8) set equal to zero) subject to the specification (6) are

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6 See Malinvaud [7], chapter 9, for the properties of the least-squares estimators.

(9) $$\sum_{j=1}^{J} \sum_{k=1}^{K_{ij}} (Y_{ijk} - \mu\alpha_i\beta_j)\mu\beta_j = 0$$

(10) $$\sum_{i=1}^{I} \sum_{k=1}^{K_{ij}} (Y_{ijk} - \mu\alpha_i\beta_j)\mu\alpha_i = 0$$

(11) $$\sum_{i} \sum_{j} \sum_{k=1}^{K_{ij}} (Y_{ijk} - \mu\alpha_i\beta_j)\alpha_i\beta_j = 0$$

From the constraints on the parameters, we can by summing expressions (9) and (10) obtain

(12) $$\mu I = \mu(\beta) I = \sum_{i=1}^{I} \frac{\sum_{j} \sum_{k=1}^{K_{ij}} Y_{ijk}\,\beta_j}{\sum_{j} K_{ij}\beta_j^2}$$

(13) $$\mu J = \mu(\alpha) J = \sum_{j} \frac{\sum_{i=1}^{I} \sum_{k=1}^{K_{ij}} Y_{ijk}\alpha_i}{\sum_{i=1}^{I} K_{ij}\alpha_i^2}$$

After making the obvious substitutions we obtain

(14) $$\sum_{j} \sum_{k=1}^{K_{ij}} Y_{ijk}\beta_j = \alpha_i \ \mu(\beta_j) \ \sum_{j} K_{ij}\beta_j^2$$

(15) $$\sum_{i} \sum_{k=1}^{K_{ij}} Y_{ijk}\alpha_i = \beta_j \ \mu(\alpha_i) \ \sum_{i} K_{ij}\alpha_i^2$$

The expressions (14) and (15) suggest an iterative procedure to obtain the values of α_i, β_j, and μ which minimize (8). For given values of β_j, we get values for α_i from expression (14), and for given values of α_i expression (15) provides values for the β_j. My experience is that these can be iterated until mutually consistent values are obtained for the α_i and β_j.

Certain comparisons should be made between the α_i, ρ_j, and μ defined by (11), (14) and (15), and those defined in terms of the underlying population means given in (4). The least-squares parameter estimates are not the sample equivalents of the population means given in (4). Even in the case of a balanced design (equal observations per cell) where the sample equivalents are easily determined, the orthogonality properties of the linear model are not preserved by the multiplicative model. The importance of this is that the marginal distribution or the row sums and column sums are not sufficient to estimate the parameter values. The parameters have to be jointly estimated, and the first order conditions are not equivalent to the traditional procedures. These claims can be demonstrated by analyzing equations (9), (10) and (11), but a simple example is easier to follow.

Suppose we observe the cell means given in Table I, where there are K observations per cell. The predicted values and the parameter estimates for models (5) and (6) are as given. Using sample equivalents of (4) would yield the same predicted value for the multiplicative model as the linear model predicts. The sample equivalents are $\mu^1 = 5.0$ $\alpha_1^1 = .8$, $\alpha_2^1 = 1.2$, $\rho_1^1 = 1$, $\rho_2^1 = 1$, but these cannot be the minimizing parameter values as can be seen from the minimizing multiplicative estimates in Table I.

The sample equivalents for this example would also be the relativity estimates obtained from the traditional estimating technique. The traditional method begins with a given set of class relativities β_j, and estimates territory relativities by

$$(16) \quad \alpha_i = \frac{\sum_j K_{ij} \beta_j \hat{\mu}}{\sum_j \sum_{k=1}^{K_{ij}} Y_{ijk}}$$

where $\hat{\mu}$ is the statewide average loss. Then the class relativities would depend on the territory relativities and be estimated by

$$(17) \quad \beta_j = \frac{\sum_i K_{ij} \alpha_i \hat{\mu}}{\sum_i \sum_{k=1}^{K_{ij}} Y_{ijk}}$$

The SRI report claims that iterating (16) and (17) until a stable set of relativities is theoretically preferred, but most times a one-pass computation is deemed satisfactory.

Table 1 approximately here.

TABLE 1

Actual Means

2	6
8	4

Linear Estimate

4	4
6	6

$\mu = 5$ $\beta_1 = 0$
$\alpha_1 = -1$ $\beta_2 = 0$
$\alpha_2 = 1$

Residual sum of squares = 16K

Multiplicative Estimate

4.130	3.448
6.682	5.579

$\mu = 4.960$ $\beta_1 = 1.090$
$\alpha_1 = .764$ $\beta_2 = .910$
$\alpha_2 = 1.236$

Residual sum of squares = 15.280K
Sum of residuals = .161K

In general, the traditional method would not result in estimates which would be normalized as in the previous section, but this doesn't affect the predicted loss costs. The important point is that the marginal conditions 9-11 are not equivalent to having the predicted row and column sums equal the actual row and column sums.

Classical tests of hypothesis are best suited for situations where one is trying to choose between a general model and a specific model which is nested in the more general model. Or in the framework here, clasical tests of hypothesis are best designed to distinguish between the models (5) and (1) or to distinguish between the models (6) and (3). Although there is some statistical theory to provide guidance in comparing the model (5) with the model (6), there is no unique best procedure.[7] The appropriate procedure to use depends on the final use to be made of the model, the prior information that is available, the cost of making a wrong decision, etc. The important point is that rational individuals could still disagree about the specification of a model after having analyzed the same data. However, one would expect to see some divergence of opinion, and the use of alternative models within the insurance industry if the evidence was not overwhelmingly supportive of a particular model.

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7 Ramsey [9] and Gaver and Geisel [4] survey many of the proposed test procedures.

Classical procedures can be used to test the hypothesis of no interactions in either the multiplicative or the additive framework. Exact tests of hypothesis are not available for the multiplicative model, but if we use the approximate test from linear least-squares theory, (compare the percentage change in residual sum of squares to an F distribution), there is a value of K that would lead one to reject both the additive least-squares model (5) and the multiplicative specification (6). For some value of K, we would conclude that the specification (1) or (3) must be the correct model for this data set. However, the balanced design is not the data available in an insurance context, and the test that all interactions are simultaneously zero is not necessarily the interesting or useful hypothesis to test. The interesting question to answer is how to estimate the expected losses in those cells with "few" observations when we cannot be comfortable with the hypothesis that there are no interactions. Statistical methods cannot be the only guidance or procedures used to answer this question. The logical conclusion from rejecting the hypothesis of no interaction is that the restrictions embodied in (5) or (6) are inconsistent with the observed data and either more complicated restrictions are appr-priate or the cell means themselves are the correct parameterization. The use of a single data set to identify more complicated restrictions is inappropriate (as most statistical texts point out) and if processed simultaneously by different researchers, likely to lead to conflicting conclusions. The next section considers this problem more carefully using actual loss data.

Massachusetts Loss Data

Table 2 and 3 from Chang and Fairley represents observed average losses for a modified Massachusetts classification plan. Table 4 and 5 contain the corresponding exposure figures. Chang and Fairly in their analysis concluded that the additive least-squares model gave a better fit to the data than the traditional multiplicative model. My analysis also confirmed that the additive least-squares model fit the data better than a multiplicative model which was estimated by least-squares. However, the lack of fit was sufficiently great that one would predict in competitive insurance markets, all firms would not use these estimates as expected loss costs.

Tables 2, 3, 4, and 5 approximately here.

Cell means were the available observations which preclude exact tests of the models (5) and (6) with the full cell means parameterization, but the results of the analysis are still interesting. Table 6 presents the change in the residual sum of squares when the additive and multiplicative restrictions are imposed on the data. If the within cell variance is 1,000,000 or more for the collision experience and the within cell variance is 400,000 or more for the combined compulsory experience, the additive least-squares model with no interactions is consistent with the data at a 5 percent level of significance. The multiplicative model without interactions is consistent with the data

Table 2

Observed Claims Amounts by Territory and Driver Class
Combined Compulsory Coverages
1975 Massachusetts Private Passenger Auto
(Dollars)

	Driver Class							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Territorial Weighted Average
Territory	15	10&12	30&31	24&26	50	20&40	22&42	
1	25.05	26.28	44.03	40.97	48.35	65.48	121.45	35.40
2	18.15	25.66	30.70	50.94	32.89	60.43	97.65	33.39
3	30.63	30.92	40.19	54.69	66.24	79.50	114.12	41.68
4	28.93	30.48	37.86	52.55	48.02	72.92	117.69	40.01
5	27.81	35.11	42.00	52.98	63.51	94.35	126.56	45.06
6	29.41	36.15	46.43	57.40	75.56	81.20	143.75	47.26
7	36.28	39.50	42.50	60.49	71.35	86.67	156.11	51.31
8	34.59	40.61	53.41	60.31	81.41	93.19	133.87	51.71
9	40.62	42.77	67.34	60.91	64.62	93.87	162.96	55.13
10	43.71	48.77	59.30	71.54	75.35	103.53	152.65	59.77
11	37.03	42.19	63.93	49.61	64.26	111.02	129.92	52.75
12	33.56	49.70	58.82	82.28	63.69	112.90	127.49	58.56
13	47.12	49.67	99.76	82.52	101.06	108.88	158.01	65.11
14	70.69	55.64	58.51	77.90	126.98	116.72	160.38	69.07
15	38.68	69.74	76.87	82.08	103.33	116.55	162.71	75.98
Driver Class Weighted Average	32.95	38.87	47.91	58.91	70.40	89.06	134.20	49.24

Note: Entries in the body of the Table (cells) are cell total claims divided by cell total exposures. Weighted averages are weighted by exposures. Sources of claim and exposure data by territory and driver class: Massachusetts Automobile Rating and Accident Prevention Bureau, PDSRP330 of October 26, 1976 and LSUM50 of October 28, 1976.

Table 3

Observed Claims Amounts by Territory and Driver Class
Collision $200 Deductible
1974-75 Massachusetts Private Passenger Auto
(Dollars)

	Driver Class							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Territorial Weighted Average
Territory	15	10&12	30&31	24&26	50	20&40	22&42	
1	22.82	39.59	60.18	72.00	84.88	118.72	198.62	50.66
2	23.44	38.73	60.32	69.84	73.38	83.38	165.38	48.84
3	29.15	43.90	65.50	73.11	86.16	105.03	190.58	56.46
4	29.49	47.88	73.00	80.34	91.49	108.91	213.63	60.89
5	32.37	52.44	76.07	83.98	101.93	117.61	228.91	66.59
6	32.89	56.14	82.24	92.28	110.12	123.62	239.09	71.02
7	33.39	61.82	88.49	94.36	127.66	138.98	243.46	76.84
8	39.21	65.50	98.65	92.52	116.81	134.93	236.38	80.26
9	43.33	76.51	99.86	100.89	138.71	152.20	290.11	91.75
10	39.49	71.88	102.19	100.84	129.72	139.35	256.90	85.28
11	37.69	78.35	113.71	111.55	138.70	168.02	267.16	94.84
12	47.27	90.34	108.65	116.69	168.77	165.78	274.93	101.43
13	49.70	93.15	132.89	122.04	174.47	171.28	267.21	108.34
14	62.55	110.36	137.42	138.07	201.88	201.75	400.62	129.99
15	53.84	146.94	125.18	155.56	324.77	201.24	349.37	153.10
16	82.60	171.43	187.63	183.89	305.55	262.95	433.46	182.48
17	48.80	89.71	154.04	121.77	133.57	178.65	254.95	102.38
18	48.00	97.09	113.34	142.83	206.81	182.37	318.55	111.54
Driver Class Weighted Average	35.51	62.74	87.47	92.36	119.04	131.91	236.08	76.28

Note: Entries in the body of the Table (cells) are cell total claims divided by cell total exposures. Weighted averages are weighted by exposures. Sources of claim and exposure data by tarritory and driver class: Massachusetts Automobile Rating and Accident Prevention Bureau, PLSRP330 of October 26, 1976 and LSCM50 of October 28, 1976.

Table 4

Joint Distribution of Exposures by Territory and Driver Class
Combined Compulsory 1975 Massachusetts Private Passenger Auto
(Car Years)

Territory	Driver Class: 15	10&12	30&31	24&26	50	20&40	22&41	Territorial Total
1	6,967	44,738	3,309	5,781	1,181	1,638	4,107	67,721
2	6,103	50,974	3,682	7,329	1,457	2,225	4,792	76,562
3	17,744	192,369	13,624	28,210	6,584	8,531	19,738	286,800
4	14,076	157,357	13,939	22,038	4,324	7,485	14,448	233,667
5	19,552	217,426	19,293	31,470	6,721	10,075	20,688	325,225
6	27,858	195,661	16,408	28,287	6,227	9,145	17,883	291,469
7	17,485	201,263	16,704	30,501	6,561	9,578	19,427	301,519
8	22,417	233,416	21,719	36,338	7,903	12,395	22,818	357,006
9	5,284	49,283	3,692	7,810	2,007	2,315	4,499	74,890
10	13,375	110,071	9,150	15,954	4,282	5,840	9,894	168,566
11	2,733	27,629	1,821	4,341	909	1,632	2,877	41,942
12	2,036	24,837	1,716	2,791	699	969	2,149	35,197
13	1,323	16,718	666	2,310	645	744	1,751	24,157
14	1,350	16,091	838	2,290	562	904	1,394	23,429
15	8,209	91,947	5,305	9,995	2,656	3,857	6,734	128,703
Driver Class Total	156,512	1,629,780	131,866	235,445	52,718	77,333	153,199	2,436,853

Table 5

Joint Distribution of Exposures by Territory and Driver Class
Collision ($200 Deductible Basis)
1974-75 Massachusetts Private Passenger Auto
(Car Years)

Territory	Driver Class 15	10&12	30&31	24&26	50	20&40	22&41	Territorial Total
1	6,492	46,647	4,010	5,922	1,072	1,558	2,636	68,337
2	6,112	56,986	4,728	8,146	1,492	2,307	3,361	83,132
3	17,510	219,788	17,308	32,096	6,825	9,269	14,126	316,922
4	14,394	182,941	17,918	25,404	4,545	8,193	10,409	263,804
5	20,416	256,638	24,601	37,692	7,229	11,537	15,866	373,979
6	18,644	232,596	21,702	34,802	6,791	10,539	14,091	339,165
7	18,598	246,306	21,168	37,852	7,226	11,114	15,236	357,500
8	23,490	281,512	30,634	44,730	8,425	14,747	18,318	421,856
9	5,543	60,753	5,117	9,762	2,261	2,803	3,974	90,213
10	15,530	139,062	13,241	20,962	4,893	7,254	8,243	209,185
11	2,751	32,824	2,521	5,232	928	1,922	2,218	48,396
12	2,176	28,276	2,142	3,460	663	1,118	1,422	39,257
13	1,198	18,263	854	2,599	658	811	1,237	25,620
14	1,452	18,935	1,161	2,862	615	1,119	1,158	27,302
15	1,698	20,997	763	2,309	550	823	915	28,055
16	2,319	45,122	2,941	4,564	1,115	1,550	1,779	59,390
17	1,072	11,666	752	1,772	323	447	576	16,607
18	2,795	25,995	1,398	3,640	558	1,623	1,297	37,306
Driver Class Total	162,189	1,925,307	172,959	283,806	56,169	88,734	116,862	2,806,026

at a 5 percent level of significance when the within cell variance is 1,250,000 for the collision experience and 560,000 for the combined compulsory experience. The SRI study found the within cell variance for personal injury claims in Massachusetts in 1970 to be slightly over 400,000. But what does one conclude from this? The statistics really only imply that the evidence is not sufficiently contradictory of a noninteractive model to alter the opinion of someone who thinks a noninteractive model is a reasonable description of reality. However, even if the within cell variance for collision was 1,300,000 and the within cell variance for combined compulsory was 600,000, there is sufficient contradictions in the data to preaict that not all insurance companies would hold to the noninteractive additive or multiplicative model. For each data set there are 3 cells with 10,000 or more car years of exposure tor which the predicted value lies outside a 95 percent confidence interval using the individual cell mean. This is not sufficient evidence using traditional confidence levels to reject the null hypothesis of no interactions overall, but is sufficient evidence to predict that some entrepreneur would take a gamble on these particular cells and use individual cell experience as an estimate of expected losses.

Table 6 approximately here.

Table 6

	Combined Compulsory			Collision		
	Traditional Iterated (not Iterated) Multiplicative	Least-Squares Multiplicative	Additive	Traditional Iterated (not Iterated) Multiplicative	Least-Squares Multiplicative	Additive
Sum of Squared Residuals	68123968 (67567936)	58721360	42328722	190920624 (184743840)	155229792	127038370

The most interesting comparison, however, is between the traditional estimating procedure for the multiplicative model and the least-squares estimates of the multiplicative model. The evidence just presented is not supportive of the multiplicative model, but if one had strong priors for using the multiplicative model, the statistical evidence would also not contradict its use. However, in answer to the question in the National Underwriter, "Auto Rates: Do They Penalize The Young, The Single, The Male," I respond yes when comparing the traditional method of estimating rates to the least-squares estimating procedure. The least-squares procedure generally yields relativities which are larger than the traditional relativities when the traditional relativities (normalized as in equation (3)) are less than 1, and yields relativities which are smaller than the traditional relativities when the traditional relativities are greater than 1. Tables 7-10 present the relativities and estimated loss costs for the Massachusetts data using the traditional method including iterating and the least-squares estimates of the multiplicative model.

Tables 7, 8, 9, and 10 approximately here.

Table 7

Least Squares Multiplicative Estimates
Collision

Driver Class

Territory	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Territorial Relativities
1	25.92	46.74	64.03	67.03	88.46	94.58	172.03	.6076
2	23.48	42.35	58.49	60.73	80.14	85.69	155.86	.5505
3	26.73	48.19	66.57	69.12	91.21	97.52	177.38	.6265
4	29.25	52.74	72.85	75.64	99.81	106.72	194.12	.6856
5	31.58	56.94	78.65	81.65	107.75	115.21	209.56	.7402
6	33.58	60.56	83.65	86.85	114.61	122.54	222.89	.7873
7	35.77	64.51	89.11	92.51	122.08	130.53	237.43	.8386
8	36.30	65.45	90.41	93.86	123.87	132.44	240.89	.8509
9	42.17	76.04	105.03	109.05	143.90	153.86	279.86	.9885
10	39.27	70.81	97.81	101.55	134.00	143.28	260.61	.9205
11	42.54	76.71	105.95	110.00	145.16	155.21	282.31	.9972
12	46.28	83.45	115.26	119.67	157.92	168.85	307.12	1.0848
13	46.88	84.53	116.76	121.22	159.97	171.04	311.11	1.0989
14	59.13	106.63	147.28	152.91	201.79	215.76	292.44	1.3862
15	68.72	123.91	171.16	177.70	234.50	250.74	456.06	1.6109
16	82.16	148.16	204.65	212.47	280.38	299.79	545.29	1.9260
17	46.35	83.57	115.44	119.85	158.16	169.11	307.60	1.0865
18	51.76	93.33	128.92	133.85	176.63	188.86	343.52	1.2133
Driver Class Relativities	.3244	.5850	.8080	.8389	1.1070	1.1837	2.1530	

Normalized Adjusted Average = 131.4991

Table 8

Least Squares Multiplicative Estimates
Combined Compulsory

Driver Class

Territory	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Territorial Relativities
1	25.64	30.56	37.65	45.82	54.62	68.58	104.71	.7421
2	22.97	27.37	33.73	41.04	48.92	61.42	93.78	.6647
3	27.86	33.20	40.91	49.78	59.34	74.51	113.75	.8062
4	27.32	32.56	40.12	48.81	58.19	73.06	111.55	.7906
5	30.45	36.29	44.72	54.41	64.86	81.44	124.35	.8813
6	32.53	38.77	47.78	58.13	69.30	87.01	132.85	.9416
7	34.98	41.69	51.37	62.51	74.52	93.56	142.85	1.0125
8	33.89	40.39	49.76	60.55	72.18	90.63	138.37	.9807
9	37.31	44.47	54.79	66.67	79.48	99.79	152.36	1.0799
10	39.20	46.72	57.57	70.05	83.50	104.85	160.08	1.1346
11	33.85	40.34	49.71	60.49	72.10	90.53	138.22	.9797
12	37.60	44.81	55.21	67.19	80.09	100.56	153.53	1.0882
13	41.72	49.72	61.26	74.54	88.86	111.57	170.35	1.2073
14	44.22	52.71	64.94	79.02	94.20	118.28	180.58	1.2799
15	48.75	58.10	71.58	87.11	103.83	130.37	199.05	1.4108
Driver Class Relativities	.4883	.5820	.7171	.8726	1.0401	1.3060	1.9940	

Normalized Adjusted Average = 70.7597

Table 9

Traditional Iterated Multiplicative Estimates
Collision

Territory	Driver Class (1)	(2)	(3)	(4)	(5)	(6)	(7)	Territorial Relativities
1	24.61	43.08	61.40	64.19	82.06	90.44	165.96	.5602
2	23.20	40.63	57.90	60.53	77.38	85.29	156.50	.5283
3	26.26	45.98	65.53	68.51	87.58	96.52	177.12	.5979
4	28.63	50.12	71.44	74.68	95.47	105.22	193.08	.6518
5	30.99	54.25	77.32	80.83	103.34	113.89	208.98	.7055
6	33.12	57.99	82.65	86.40	110.46	121.74	223.38	.7541
7	35.71	62.53	89.12	93.17	119.11	131.27	240.88	.8131
8	36.98	64.75	92.29	96.48	123.34	135.93	249.43	.8420
9	42.49	74.40	106.04	110.85	141.71	156.18	286.60	.9675
10	40.06	70.14	99.96	104.51	133.60	147.24	270.19	.9121
11	43.60	76.33	108.79	113.73	145.40	160.24	294.05	.9926
12	48.58	85.06	121.23	126.73	162.01	178.56	327.66	1.1061
13	49.92	87.40	124.57	130.22	166.48	183.48	336.68	1.1365
14	60.21	105.42	150.25	157.08	200.80	221.31	406.10	1.3709
15	74.81	130.98	186.67	195.15	249.48	274.95	504.54	1.7032
16	88.85	155.55	221.70	231.77	296.29	326.55	599.21	2.0228
17	49.14	86.04	122.62	128.19	163.88	180.62	331.43	1.1188
18	53.44	93.56	133.34	139.40	178.21	196.41	360.40	1.2166
Driver Class Relativities	.3239	.5672	.8083	.8450	1.0803	1.1906	2.1847	

Normalized Adjusted Average = 135.5926

Table 10

Traditional Iterated Multiplicative Estimates
Combined Compulsory

Driver Class

Territory	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Territorial Relativities
1	24.30	28.56	35.53	42.64	51.59	65.29	100.08	.6940
2	22.54	26.49	32.95	40.48	47.85	60.56	92.83	.6438
3	27.62	32.46	40.38	49.60	58.63	74.21	113.74	.7888
4	26.87	31.58	39.28	48.24	57.03	72.18	110.64	.7673
5	30.13	35.41	44.04	54.09	63.94	80.93	124.05	.8603
6	31.72	37.28	46.37	56.96	67.33	85.22	130.62	.9059
7	34.16	40.15	49.94	61.34	72.51	91.77	140.67	.9755
8	34.33	40.34	50.18	61.63	72.86	92.21	141.34	.9802
9	36.97	43.45	54.04	66.38	78.47	99.32	152.23	1.0557
10	40.25	47.31	58.84	72.28	85.44	108.14	165.75	1.1495
11	34.66	40.73	50.67	62.23	73.57	93.11	142.72	.9898
12	39.88	46.87	58.30	71.61	84.65	107.14	164.23	1.1389
13	42.89	50.40	62.69	77.00	91.03	115.21	176.59	1.2247
14	46.30	54.41	67.68	83.13	98.27	124.38	190.65	1.3221
15	52.65	61.88	76.97	94.54	111.76	141.45	216.81	1.5036
Driver Class Relativities	.4875	.5729	.7126	.8753	1.0347	1.3096	2.007	

Normalized Adjusted Average = 71.8346

This is a difficult result to reconcile with profit maximizing behavior. The use of estimates of loss cost which are too high and which don't result in loss of market share is perfectly understandable. If one can charge a price higher than costs, one makes a larger profit. But to use an estimate of loss costs which are too low, implies one is losing money on those individuals. Monopoly positions or any other market phenomena would not produce this result in a profit maximizing environment. A possible answer is that some companies are more innovative than most, but because of government regulation they cannot expand as rapidly as might be predicted. Also, the phenomena of company specialization and underwriting might make these results less important if most data used to estimate losses come from only a few risk classes. But a full understanding of the behavior in competitive terms is still lacking. I will admit to being possibly missing something, but as of now, I would conclude that the insurance industry is not very innovative.

Conclusion

The analysis of this paper supports the decision and findings of the Massachusetts Insurance Commissioner. The traditional pricing procedures contain biases that result in overcharging individuals in the highest rated risk classes. The biases however are not necessarily a result of the multiplicative model. They are a result of the estimating techniques traditionally used by the insurance industry. But the most important conclusion to be drawn from the analysis concerns the operation of industry rating bureaus.

There are benefits to the statistical pooling of losses from many companies. More accurate results are obtained when more data go into the analysis. Small companies are able to viably compete with large companies when they have access to statistical analysis of data sets which are broader than their own company experience. But there are also benefits from divergent opinions. Any mechanism which permits the pooling of experience data from many companies should also provide for independent access and analysis by various technicians.

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