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Firm-Specific Information, Product Differentiation, and Industry Equilibrium

Jeffrey M. Perloff and Steven Salop

Research over the last three decades has shown that imperfect consumer information may enable even small firms to set their prices above marginal cost.[1] Much of the recent literature has assumed that consumers possess information about the general market, but lack information about specific firms. This paper presents a new model in which consumers have imperfect information about specific firms and lack information about the market. The resulting equilibrium has very different properties than in previous models.[2]

Consumers gather information in a number of diverse ways. One method is a personal inspection or search before purchase. This prepurchase inspection may be aided by the use of screening devices and signals. Prepurchase information may also be purchased from diagnostic and testing agencies, certifiers, newspapers, and brokers. Recommendations from friends may also be used. Finally, advertising by sellers and personal experience yield information that is more or less reliable.

Most attention has been paid to the information gathering role of search or inspection, perhaps because it contains both the result of *informational market power* and the possibility of nonexistence of equilibrium, as emphasized by Stiglitz (1979). Search or inspection has been studied by Wilde and Schwartz (1979) and a number of others since Diamond (1971).

At the same time, however, the other information gathering institutions have been analyzed in detail. For example, Phelps (1972) analyzes screening devices. Nelson (1974) examines the role of product market signals, particularly advertising and marketshare. The educational signalling literature of Spence (1973, 1975), Stiglitz (1975), Guasch and Weiss (1980) and others may be reinterpreted as product testing and certification. Leland (1979) analyzes the effect of licensing to ensure minimum quality standards. Plott and Wilde (1979) have studied diagnosticians both theoretically and experimentally. Newspaper information has been analyzed by Salop and Stiglitz (1977) and Varian (1980).

Recommendations from friends have been paid less attention, except to the extent that such information may be similar to that gained from using marketshare as a positive signal. The role of advertising in directly providing firm-specific information has been analyzed by Butters (1977). The behavior of brokers has been implicitly modeled in the agency literature. Moreover, the direct mailing advertising in Butters (1977) may be reinterpreted as an independent broker or salesman. The matchmaking role of brokers has been examined by Salop (1980). Personal experience has been analyzed by Phelps and Winter (1970); Grossman, Kihlstrom, and Mirman (1977); Smallwood and Conlisk (1979); and Shapiro (1980).

The model presented here might best be described as a newspaper model in that consumers are endowed with some imperfect information about each firm in the market, though the equilibrium in the market for information is not explicitly analyzed. Alternatively, it might be better described as an amalgam of all information gathering, past and present, about *specific* firms and brands, where the number of consumers perfectly informed about every firm is initially taken to be insignificant.

On the other hand, unlike the other search, newspaper, and signalling models, the consumers here are restricted to *firm-specific information*. Additional *general market information*, such as the range or density of actual prices in the market, is not known to the consumer. His general market information is limited to only that which may be inferred from firm-specific data, and is therefore redundant. This model has strikingly different properties from those of earlier models which were driven by their assumptions of perfect general market information. Indeed, in many ways, this firm-specific information model represents a retrenchment, for it has none of the strange and wondrous properties of search and other models. In a market restricted to firm-specific information gathering, if only an insignificant proportion of consumers are perfectly informed about all firms, market breakdown is far less likely; instead, equilibrium generally exists for the model presented here.[3]

Given firms' profit maximizing conditions hold, a unique single-price equilibrium does obtain; however, we have not ruled out the existence of additional multiple-price equilibria from pure or mixed strategies. Moreover, we show that price dispersion may occur if a significant number of consumers are perfectly informed. As the degree of information about all firms improves from perfect ignorance to perfect information, the equilibrium price falls continuously to the competitive price. In contrast, as Stiglitz (1979) discusses, most models have a discontinuity in that any imperfection of information causes price to be above marginal cost.

Finally, perhaps the most striking contrast with previous models occurs with respect to entry competition. In the search models, entry does not reduce price; if anything, it increases the equilibrium price by making discovery of the lowest price firm more costly on average. On the other hand, the firm-specific information model has the proerty that as the number of firms becomes sufficiently large, the equilibrium price falls to the perfectly competitive price.

These results are discussed below. Section II sets out the basic specific-firm information framwork, derives the equilibrium, and analyzes improvements in consumer information. Entry competition is examined in Section III, and multiple-price equilibria in Section IV.

In Section V, we show how the basic model may be reinterpreted and applied to industry equilibrium when products are differentiated. This product differentiation may be spurious, arising out of consumers' misperceptions, or it may be due to actual differences in product formulations and consumer preferences. As a model of product differentiation, the formal structure is a synthesis of the spatial approach of Hotelling (1929), Lancaster (1979), and others with the representative consumer approach of Spence (1976), Dixit and Stiglitz (1977), and Hart (1979). This model of product differentiation is analyzed in detail in the Addendum. Possible improvements and extensions are discussed in the conclusions.

II. Equilibrium with Imperfect Information

In this section, we analyze a model of industry equilibrium when consumers are imperfectly informed. As discussed in the Introduction, this model differs somewhat from other work in its conceptualization of information imperfections and consumer decisionmaking.

Two classes of price and quality data may be distinguished, firm-specific and general market information. By firm-specific information, we mean consumers' direct estimates of the prices and qualities of various commodities available from different firms. By general market information, we mean consumers' estimates of these parameters for the market generally. For example, in the case of price uncertainty, a consumer's firm-specific information may be a prior probability distribution $F_i(p_i)$ over the possible prices, p_i, of each firm, $i = 1,2,\ldots,n$; or it may simply be a point estimate s_i of each price. With respect to the market in general, the consumer may have a probability distribution $G(\underline{p})$ of the set of all prices charged for the commodity in question, or simply the range of prices charged.

These two classes of information are related, of course. The general market distribution $G(\underline{p})$ may be derived from the appropriate aggregation of the firm-specific distributions, $F_i(p_i)$. Similarly, in the absence of any additional firm-specific infromation, a consumer treats $G(\underline{p})$ as the firm-specific distribution as well.

Models of search equilibrium such as Diamond (1971) generally assume that consumers' general market information is rational; that is, the prior price distribution $G(\underline{p})$ is self-fulfilled by the actual equilibrium distribution of prices in the market. Additional firm-specific information is gathered from search; in particular, a consumer obtains perfect firm-specific information by sampling a store or product. For example, Butters's (1977) advertising model has a diffuse prior $G(\underline{p})$ and perfect firm-specific information if an advertisement is received. The newspaper model of Salop and Stiglitz (1977) has a rational $G(\underline{p})$ and, additionally, perfect firm-specific information for all firms, if the newspaper is purchased.

We take a different approach here. We assume that consumers have only imperfect firm-specific information and no additional general market information about prices, beyond that implied by the firm-specific distributions. This formalization is more in the spirit of estimation models, rather than the search literature.

Specifically, we assume each consumer j ($j = 1,2,\ldots,L$) enters the market armed with a point estimate s_i^j for each of the $i = 1,2,\ldots,n$ firms in the market and purchases from the firm estimated to have the lowest price, or $\min_i s_i^j$. For now we focus on the case in which products in the industry are homogeneous and known to be homogeneous (i.e., this general market information does exist).[4]

Consumers may form their estimates s_i^j by gathering information in a variety of ways, according to the costs and benefits of each. As discussed previously, inspection, reliable and unreliable experience, truthful and deceptive advertising, friends and neutral third parties are among the information gathering methods analyzed in the literature.[5] According to the exact structure of information gathering assumed, particular restrictions on the estimates are implied. For example, if a price is sampled, it will yield a perfect price estimate. For other information gathering methods, it is a difficult question exactly what sort of rationality restrictions to place on consumers' estimates.

In this model, we do not derive the structure of the estimates from an explicit information gathering technology. Instead, we begin with an exogenously generated set of estimates, satisfying certain plausible conditions. In particular, we assume that consumer j's estimates, $(s_1^j, s_2^j, \ldots, s_n^j)$, are generated as follows:

$$s_i^j = p_i + \beta\theta_i^j \tag{1}$$

where,

$$\theta_i^j \sim F_i^j(\theta), \quad \theta \in [a,b],$$
$$E(\theta_i^j) = 0, \text{ Var } (\theta_i^j) > 0,$$

and where $F_i^j(\theta)$ is a continuously differentiable distribution function with density $f_i^j(\theta)$.[6]

Thus, estimates are taken to be unbiased and, if $\beta > 0$, as imperfect.[7] The scale parameter β permits a range of information states from perfect information ($\beta = 0$) to perfect ignorance ($\beta \to \infty$). Those consumers who draw $\theta = 0$ have truthful estimates, while those who draw $\theta < 0$ have an under-estimates, and those with $\theta > 0$ have an overestimate of price. Estimates are related to the actual price p_i charged by the firm.[8] Finally, the support of θ, $\theta \in [a,b]$ may be finite or infinite. One natural restriction would be to assume price estimates must be non-negative; although, as will be demonstrated below, weaker restrictions will suffice.

Given his estimates, $(s_1^j, s_2^j, \ldots, s_n^j)$, each consumer j selects the firm with the lowest estimated price, $\min_i s_i^j$, and shops there. Further comparison shopping is not permitted, although the model could accommodate it; thus, we implicitly assume the cost of further search is prohibitive.[9] Instead, once at the selected store, the consumer observes the actual price, p_i, and purchases $d(p_i)$ units.

As a results of this formulation, a disproportionate share of each firm's sales are made to customers who underestimated its price. Comparison shopping would affect this proportion. Finally, in the static model analyzed here, no additional learning is permitted; every period is independent of the past. In contrast, a richer intertemporal model would include an analysis of the evolution of estimates over time as experienced consumers learn and eventually die, and new ignorant buyers enter the market.[10]

Given this formal structure, we may derive the form of the demand curves facing each firm in the market. It is apparent that for $\beta > 0$, these demand curves are downward-sloping, even though all products are homogeneous. Since consumers are not perfectly informed of the lowest price store, higher priced stores do obtain some unlucky customers.[11] Under these circumstances, demand is elastic for two reasons: a price reduction brings forth additional customers and each customer purchases additional units.

In the case of perfect information ($\beta = 0$), however, the lowest price store does obtain all the customers, and, thus, shading one's price below a common level $\bar{p}$ does yield a discontinuous demand increase (i.e., demand is perfectly elastic). In contrast, in the perfect ignorance case ($\beta \to \infty$), the flow of customers is unrelated to actual price; demand elasticity comes only from additional purchases from each customer obtained.

We now derive the exact form of firms' demand curves from the theory of order statistics. For a representative firm i, the probability that it is selected by consumer j is the probability that s_i^j is the lowest estimate. Dropping the superscript j for convenience and substituting from equation (1), we have[12]

$$Pr_i \equiv Pr(s_i \leq s_1, s_i \leq s_2, \ldots, s_i \leq s_n) = \int \prod_{k \neq i} \{1 - F_k\left(\frac{p_i - p_k}{\beta} + \theta\right)\} f_i(\theta) d\theta. \quad (2)$$

After selecting a firm, each consumer observes the actual price p_i and purchases $d(p_i)$ units there. If there are L consumers, with identical demand curves, then the expected demand of firm i is given by

$$Q_i(p_1, p_2, \ldots, p_n) = Ld(p_i)Pr_i. \tag{3}$$

Given these demand curves for each firm, the industry equilibrium for an exogenous number of firms n may be derived using conventional methods. If firm i has a constant marginal cost c_i, then its expected operating profits are given by

$$\pi_i(p_1, p_2, \ldots, p_n) = (p_i - c_i)Q_i(p_1, p_2, \ldots, p_n). \tag{4}$$

Each firm maximizes expected operating profits, taking the prices at other firms as given; that is, we derive a Nash-in-price equilibrium. Note that this approach assumes firms have perfect information regarding their competitors' prices, in contrast to consumers.[13] Differentiating equation (4) with respect to p_i under the Nash conjectural variation and rewriting, we have[14]

$$p_i = c_i - \frac{Q_i}{\partial Q_i/\partial p_i}. \tag{5}$$

We now derive a *symmetric, single price, Nash equilibrium*, given the structure of demand given by equation (3). By symmetry, we mean that the degree of imperfect information for all consumers and costs are identical for all firms, or

$$\begin{aligned} F_i^j(\theta) &= F(\theta), \\ c_i &= c. \end{aligned} \tag{6}$$

Moreover, we *assume* that equilibrium entails identical prices for all firms,[15]

$$p_i = p. \tag{7}$$

We derive the equilibrium as follows: Assuming that all firms except firm i charge an identical price p, then after substituting into equation (3), we have

$$Q_i(p,\ldots,p_i,\ldots,p) = Ld(p_i)\int\{1 - F\left(\frac{p_i-p}{\beta} + \theta\right)\}^{n-1} f(\theta)d\theta. \quad (8)$$

Differentiating (8) with respect to p_i under the Nash conjecture, the demand slope is given by

$$\frac{\partial Q_i}{\partial p_i} = \frac{d'(p_i)}{d(p_i)} Q_i - \left(\frac{n-1}{\beta}\right) Ld(p_i)\int\{1 - F\left(\frac{p_i-p}{\beta} + \theta\right)\}^{n-2} f\left(\frac{p_i-p}{\beta} + \theta\right) f(\theta)d\theta. \quad (9)$$

Substituting the equilibrium value $p_i = p$ into (8) and (9), we have

$$Q_i = Ld(p)\int\{1 - F(\theta)\}^{n-1} f(\theta)d\theta = \frac{L}{n} d(p),^{16} \quad (10)$$

$$\frac{\partial Q_i}{\partial p_i} = \frac{L}{n} d'(p) - \left(\frac{n-1}{\beta}\right) Ld(p) \int\{1 - F(\theta)\}^{n-2}\{f(\theta)\}^2 d\theta. \quad (11)$$

The individual consumer's demand elasticity is

$$\eta \equiv - \frac{\partial Q_i}{\partial p_i} \frac{p_i}{Q_i} = - \frac{p_i d'(p_i)}{d(p_i)} . \quad (12)$$

Substituting equations (10) - (12) into (5), the symmetric, single-price equilibrium price $p(n)$ is characterized as follows, when there are n firms in the market:

$$p(n) = c + 1/M(n), \quad (13)$$

where

$$M(n) = \frac{\eta}{p(n)} + \frac{n(n-1)}{\beta} \int\{1 - F(\theta)\}^{n-2}\{f(\theta)\}^2 d\theta. \quad (14)$$

Equations (13) and (14) define a single-price equilibrium between the competitive and monopoly prices. For example, if $\beta = 0$ (perfect information), then $M(n) \to \infty$ and $p = c$, that is, perfect competition obtains.[17] This result is analogous to the usual "Bertrand" equilibrium, of course. At the other extreme, if $\beta \to \infty$ (perfect ignorance), then $M(n) = \eta/p$ and the monopoly price p^m obtains, where p^m satisfies the usual Lerner markup condition

$$\frac{p^m - c}{p^m} = \frac{1}{\eta} .$$

Improved information is catured by decreases in the scale parameter β. If the elasticity η is non-decreasing in price, then it is easily shown that a firm's aggregate demand becomes more elastic; thus, the equilibrium price falls. Differentiating equations (13) and (14) with respect to β, we have $\partial p/\partial\beta > 0$. That is,

<u>Theorem 1</u>: A reduction in consumer information (in the sense of an increase in β) raises the equilibrium price.

Moreover, as information becomes perfect, the equilibrium price approaches the perfectly competitive price continuously. This result is in contrast to Diamond's that small but strictly positive search costs yield an equilibrium at the monopoly price. That is, in this model, a small degree of imperfect information gives only a small degree of informational market power.

This difference from Diamond's result is not difficult to explain. A small search cost does not, in fact, imply a low cost to becoming perfectly informed. In fact, Diamond's result obtains because at his monopoly price equilibrium, becoming perfectly informed entails sampling an infinite number of stores, and thus an infinite cost, if search costs are strictly positive.

It should be added that if decreased information is formalized as a general mean-preserving-spread of the density $f(\theta)$, the effect on the equilibrium price is indeterminate. This ambiguity arises because the firm's demand elasticity depends on the entire noise distribution, as discussed in Appendix I. This result takes on greater importance in the analysis of product differentiation in Section V.

III. Entry Competition

In this section, we examine the effect of entry competition (increases in the exogenous number of firms n) on the single price equilibrium. It is a property of even tradtional Cournot models of imperfect competition that entry may not lower the equilibrium price (Seade (1980)). We have not yet obtained a general entry result for small changes in the number of firms, but we have derived some asymptotic properties.

Although entry shifts each firm's demand curve inward, the elasticity of demand may not rise and, thus, equilibrium price may not fall. This ambiguity may be confirmed by differentiating the expression for $M(n)$ in equation (14) with respect to n.

On the other hand, for the limiting case of $n \to \infty$, a complete characterization does obtain. Of course, if each firm has strictly positive fixed costs, the market is unable to support an infinite number of firms. Instead, ignoring the integer problem, a zero profit equilibrium is characterized by the usual tangency of demand with average cost. Only if the level of fixed costs approaches zero (*perfectly free entry*) may the number of competitors become infinite. The following two theorems present conditions under which the perfectly free entry price equals the perfectly competitive price under full information. The proofs are contained in Appendix II.[18]

Theorem 2: If the support $[a,b]$ of the noise density $f(\theta)$ is bounded from below (i.e., if a is finite), then

$$\lim_{n\to\infty} p(n) = c.$$

Theorem 3: If the domain $(a,b]$ is unbounded from below (i.e., if

$a \rightarrow -\infty$), and if

$$\lim_{\theta \rightarrow -\infty} \frac{f'(\theta)}{f(\theta)} = \infty ,$$

then,

$$\lim_{n \rightarrow \infty} p(n) = c.$$

Intuitively, the Nash equilibrium price approaches the competitive price if firms' Nash demand curves become perfectly elastic. If so, then even the smallest price increase causes the loss of all customers. Recall that a representative firm obtains only those customers who most underestimate its price. Indeed for $n \rightarrow \infty$ and finite lower bound a, a firm obtains only those customers who draw the maximum underestimate $\theta = -a$, since each customer chooses a firm from an infinite sized sample from $f(\theta)$: That is, the first (lowest) order statistic equals the lower bound a. Similarly, since the sample is infinitely large, the second order statistic also approach the lower bound a. In other words, all of the firm's customers represent close wins, and each of these close wins is converted into a close loss if the firm raises its price even slightly. Thus, its demand is perfectly elastic and Theorem 2 holds.

On the other hand, if $a \rightarrow -\infty$, then the first two order statistics need not cluster together, and thus, demand may not be perfectly elastic. The elasticity depends on the speed with which the density converges to zero, as stated in Theorem 3. The exponential $f(\theta) = \lambda e^{\lambda\theta}$, $\theta < 0$, is one density for which the equilibrium price does not converge to perfect competition.[19]

Of course, if price estimates are restricted to be non-negative, then the condition of Theorem 2 is satisfied, and perfectly free entry implies perfect competition.[20] Although biased estimates have not been formally analyzed here, the reader may confirm that the theorems generalize to the case of a common biased distribution $F(\theta)$. In this sense, deceptive (biased) advertising does not destroy perfect competition in the perfectly free entry case, so long as the degree of bias is identical for all firms.[21]

IV. Uniqueness, Mass Points, and Multi-Price Equilibria

Thus far, we have restricted our attention to single price equilibria. In this section, we discuss the possible existence of multi-price equilibria, as well as the uniqueness of the single price equilibrium derived above. We turn first to the uniqueness issue.

In principle, there could be multiple single price equilibria; however, for the conventional case where the individual consumer's demand elasticity, $\eta(p)$, is nondecreasing in price, multiple single price equilibria cannot occur:

> Theorem 4: If $\eta(p)$ is nondecreasing in price, and if a single price equilibrium exists, then it is unique.

This result may be shown by rewriting (13) as follows:

$$\frac{p-c}{p} = \frac{1}{pM(n)} .$$

The left-hand side is monotonically increasing in p, while the right-hand side is monotonically decreasing. Since the left-hand side equals zero when $p = c$ and the right-hand side approaches zero as p becomes infinitely large, the two sides must intersect exactly once at a positive price markup ($\frac{p-c}{p} > 0$).

This result does not rule out the additional possibility of multiple price equilibria, even under the symmetric information and cost conditions set out in Section II. We do not have a general theorem on the nonexistence of multi-price equilibria; however, such equilibria can be rejected in a duopoly ($n = 2$) model, to which we now turn.

For simplicity, suppose that consumers have perfectly inelastic demands ($\eta = 0$). Normalizing $\beta = 1$, the probability that firm 1 obtains a representative customer is

$$Pr(s_1 \le s_2) = Pr(\theta_1 - \theta_2 \le p_2 - p_1). \tag{15}$$

The distribution of $\mu \equiv \theta_1 - \theta_2$, $H(\mu)$ is symmetric with mean equal to zero, so that $H(0) = \frac{1}{2}$.[22] Substituting the definition of μ into equation (15) and normalizing $L = 1$ so that expected sales equal the representative probability, we have

$$Q_1(p_1,p_2) = H(p_2-p_1), \tag{16a}$$

$$Q_2(p_1,p_2) = 1 - H(p_2-p_1). \tag{16b}$$

Calculating expected profits and substituting into the profit-maximizing condition, analogous to equation (5), we obtain

$$p_1 = c + \frac{H(p_2-p_1)}{h(p_2-p_1)}, \tag{17a}$$

$$p_2 = c + \frac{1 - H(p_2-p_1)}{h(p_2-p_1)}, \tag{17b}$$

where $h(\mu)$ is the density of $H(\mu)$. Subtracting (17a) from (17b), we have

$$p_2 - p_1 = \frac{1}{h(p_2-p_1)}\{1 - 2H(p_2-p_1)\}. \tag{18}$$

Since $H(0) = \frac{1}{2}$, equation (18) is only satisfied for $p = p_1 = p_2$, and the unique single-price equilibrium is given by

$$p = c + \frac{H(0)}{h(0)}\ .^{23} \tag{19}$$

Two price equilibria may be ruled out by examining (18). If $p_2 - p_1 > 0$, then $H(p_2-p_1) > \frac{1}{2}$ and, since $h(p_2-p_1) > 0$, the right-hand side of (18) is negative while the left-hand side is positive. A similar contradiction obtains for $p_2 - p_1 < 0$.[24]

Thus, if $n = 2$ and $\eta = 0$, only a single price equilibrium obtains. For $\eta > 0$, the result obtains if η is nondecreasing in price. This method of proof cannot be easily extended to the case of more than two firms, however.

Beginning from a single price satisfying the equilibrium conditions, suppose a deviant firm, say firm 1, sets its price at a level other than the common price p. In this case, letting $\mu_i = \theta_1 - \theta_i$, $i = 2,\ldots,n$, the n-firm equation analogous to (18) might be derived. Unfortunately, the marginal distributions of the μ_i's are not independent, complicating the calculations.

Until now, we have ruled out mass points. Mass points are important because they lead to the possibility of ties between the lowest estimates. These ties in turn lead to discontinuities in demand. Mass points may occur at $\theta = 0$ if some consumers are perfectly informed,[25] or they may occur elsewhere. The introduction of mass points greatly changes the analysis. In particular, we may prove the following theorems.

Theorem 5: If the distribution function $F(\theta)$ has a mass point, no single price equilibrium exists.

The proof is straight-forward and proceeds by first ruling out a single price equilibrium at $p > c$ and then by ruling out a single price equilibrium at $p = c$. For any $p > c$, one deviant firm could break all ties by shading its price slightly. Sales would jump discontinuously, if there were a strict proportion of ties, raising its profits.

For $p = c$, unless absolutely all consumers were perfectly informed about all firms, a deviant could earn positive profits by charging $p_i > c$, and relying on the occasional unlucky buyer. In contrast, nondeviants set $p = c$ and earn zero profits.

The presence of mass points also has implications for the nature of multi-price equilibria:

Theorem 6: If the distribution function $F(\theta)$ has a mass point, an equilibrium price vector cannot contain two or more prices which are equal.

If so, the previsou argument would apply. One of the firms could increase its sales and profits discontinuously be shading its price slightly.

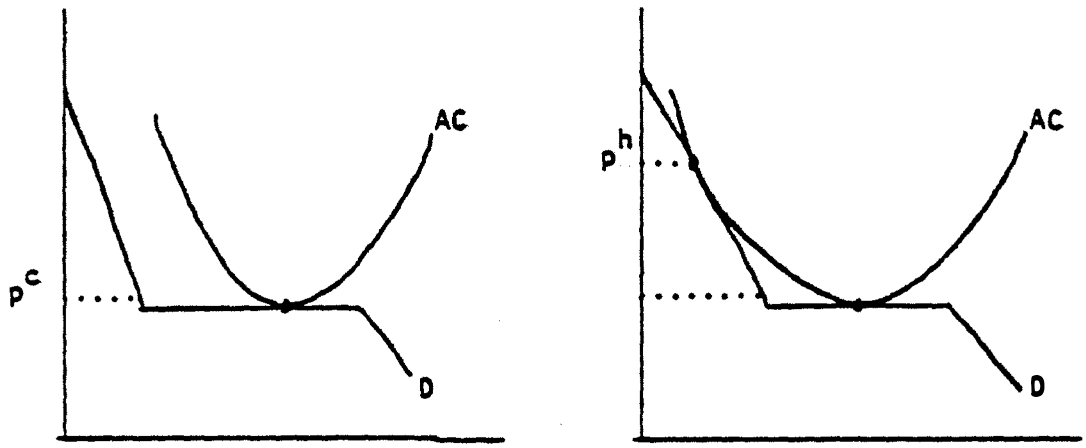
As yet, we have not been able to take the analysis much further. It appears possible for a multi-price equilibrium to exist with (given appropriate reordering of firms) $p_1 < p_2 < \ldots < p_n$. It is clear that $p_1 > c$ and $p_n \leq p^m$, the monopoly price. We have obtained no further restrictions beyond equal profitability.

Given masspoints, if average costs are U-shaped, however, either single price or two or more price equilibria may obtain. Figure 1 illustrates possible single price and two price equilibria for this structure. This result is similar to Salop and Stiglitz's (1977) newspaper model. The difference is that the uninformed consumers here purchase according to their different estimates, while in the newspaper model, they purchase randomly.

These results are possible because the demand discontinuities. Thus, common prices may only occur at the competitive price. There may still be a two-price equilibrium if there is only one high price (say at p^h in Figure 1) deviant. Three price equilibria require only two deviants, and so forth.

Although the existence of multi-price equilibria might cause an embarrassing non-unlqueness, they would enrich the model considerably. In particular, they would permit general market information to be more easily incorporated into the formal model, allowing the conventional search model to be more easily compared to this one. The existence of multi-price equilibria would remove the necessity of the restriction of only firm-specific information as follows: In the current model, where equilibrium entails only a a single price, a consumer with that general market information would purchase randomly, regardless of the actual estimates drawn. Further analysis along these lines must await a sequel.

Figure 1



Single-Price Equilibrium   Two-Price Equilibrium

V. Spurious and Actual Product Differentiation

As discussed earlier, the model may be reinterpreted to include both spurious and actual product differentiation. By spurious product differentiation, we mean that consumers mistakenly perceive brands to differ by more than they do actually, including the purely spurious differentiation case in which brands are actually homogeneous, but are perceived to differ.[26] By actual product differentiation, we mean the case in which consumers differ in their actual valuation of different brands.

The model may easily handle spurious product differentiation by interpreting θ_i^j as quality misperceptions rather than price misperceptions. Similarly actual product differentiation may be treated by reinterpreting θ_i^j as an actual (cardinal) brand preferences. In both cases, s_i^j is redefined as the negative of consumer surplus.

All of the previous theorems hold for these variants of the basic model. Interestingly, the addition of quality misperceptions to price misperceptions may not raise the equilibrium price. As is shown in Appendix I, a mean-preserving spread in the noise density may raise or lower the equilibrium price. The actual product differentiation model is examined in more detail in the acompanying note, "Equilibrium with Product Differentiation: An Addendum."

VI. Extensions and Conclusions

To recapitulate the main results of the firm-specific information model, if second-order conditions are satisfied, then at least one single price equilibrium obtains. There is a unique single price equilibrium if individual demand elasticities are nondecreasing in price. Multi-price equilibria appear to be possible as well, although more work needs to be done to rigorously establish existence and additional properties of such equilibria.

If a mass of consumers are well-informed, a single price equilibrium cannot exist if marginal costs are constant. If average costs are U-shaped, however, then single-price equilibria at the competitive price or multi-price equilibria may obtain.

If there are an insignificant number of well-informed consumers, then the single-price equilibrium has the following properties. Improved information, in the sense of the scaling parameter defined above, lowers the equilibrium price. Entry competition lowers price for sufficiently vigorous entry, and in the case of perfectly free entry, equilibrium price falls to the competitive price under certain fairly weak conditions.

Beyond these results, few other properties have been established. More work needs to be done here with respect to both symmetric multi-price equilibria and multi-price equilibria arising from differential costs and information endowments. The degree of information must be made endogenous. Particular distributions should be examined. The dynamics of the model must be analyzed.

Finally, and probably most important, search must be explicitly intoduced into the model. This modification may be done in either of two ways. First, having arrived at a store, a consumer will often find he has underestimated the price charged, so he may have a sufficient incentive to sample the firm with second lowest estimate. Such search will probably have little or no effect on the general qualitative properties of the model.

Of course, a more sophisticated or experienced consumer may infer that his lowest estimate tends to be an underestimate. This information will not alter his behavior significantly unless he also infers that all prices are identical, if in fact they are.[27] In that case, if consumers ignore their firm-specific estimates and choose firms randomly, price rises to the monopoly level. Of course, in this case, if a deviant lowers his price, and hence the firm-specific estimates of his price, will consumers rely on the information? This is the usual logical difficulty arising in search and newspaper models.[28] The problem can be avoided in the case of multi-price equilibria. At such an equilibrium, general market information corresponding to the full rational expectations hypothesis of the search and newspaper models can be well accomodated.

Appendix I

We rewrite the density as $f(\theta;\alpha)$ where α is a parameter representing the level of uncertainty: as α increases, uncertainty increases due to a mean preserving spread. Differentiating (13), it may be shown that the sign of $\partial p/\partial\alpha$ is the same as the sign of

$$\frac{\partial}{\partial\alpha}\int_a^b \{f(\theta;\alpha)\}^2 d\theta.$$

Figure 2 shows a symmetric density to which a mean preserving spread has been applied. Various size regions are shown and identified by capital letters: all regions with the same letter are of the same size.

If $f(\theta)$ is the original density and $h(\theta)$ is the density after two sections (labeled "A", which are e by x as shown in Figure 2) are removed from the center and added to the tails, then the change in the integral of the squared density is given as follows:

$$\int_a^b \{h^2(\theta) - f^2(\theta)\}d\theta = 2\Big(\int_0^x \{f(\theta)-e\}^2 d\theta - \int_0^x f^2(\theta)d\theta$$

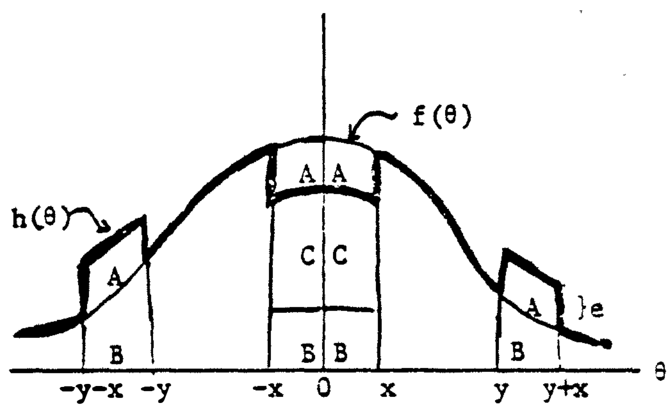
$$+ \int_y^{y+x} \{f(\theta)+e\}^2 d\theta - \int_y^{y+x} f^2(\theta)d\theta\Big)$$

$$= 4e\big(ex + \{(F(y+x) - F(y)) - (F(x) - F(0))\}\big)$$

This value may be either positive or negative. Graphically, it is positive if the areas A and B are greater than C; and negative if A plus B is less than C.

Heuristically, if the density is nearly uniform, this value is positive, so price rises as uncertainty increases. If the density is single peaked with a large mode, then the price will fall as uncertainty increases. Thus, the price effect depends on the density and the type of mean preserving spread used.

Figure 2



Appendix II

The proofs of Theorems 2 and 3 are given here. These proofs assume that the density function $f(\theta)$ has the following properties (which could be relaxed at the cost of greater complexity in the proofs):

1. $f(\theta) > 0$, $\theta \in (a,b)$.
2. $f(\theta)$ is as many times differentiable as needed.

We wish to prove that under the conditions given in Theorems 2 and 3, entry will drive the equilibrium price to marginal cost (even given limited consumer information). Since $p = c + 1/M(n)$, showing that

$$\lim_{n\to\infty} M(n) = \infty$$

is sufficient to show that

$$\lim_{n\to\infty} p = c.$$

The following lemmas establish that if a is finite or

$$\lim_{\theta\to a} \frac{f'(\theta)}{f(\theta)} = \infty,$$

then

$$\lim_{n\to\infty} M(n) = \infty.$$

Lemma 1: If $f(a) > 0$, then $\lim_{n\to\infty} M(n) = \infty$.

Proof of Lemma 1: By the continuity of $f(\theta)$, if $f(a) > 0$, then there exists an interval $[a,a+\delta)$ s.t. for $\theta \in [a,a+\delta)$, $f(\theta) \geq \xi > 0$. As a result,

$$M(n) = \int_a^{a+\delta} n(n-1)\{1-F(\theta)\}^{n-2}\{f(\theta)\}^2 d\theta + \int_{a+\delta}^{b} n(n-1)\{1-F(\theta)\}^{n-2}\ f(\theta)\ ^2$$

$$\geq \xi \int_a^{a+\delta} n(n-1)\{1-F(\theta)\}^{n-2} f(\theta)\, d\theta + K,$$

where

$$K = \int_{a+\delta}^{b} n(n-1)\{1-F(\theta)\}^{n-2}\{f(\theta)\}^2 d\theta.$$

Therefore,

$$\lim_{n\to\infty} M(n) \geq \lim_{n\to\infty} n\xi - \lim_{n\to\infty} n\xi\{1-F(a+\delta)\}^{n-1} + \lim_{n\to\infty} K = \infty$$

We know, however, that

1. $\lim_{n\to\infty} n\xi = \infty$

2. $\lim_{n\to\infty} n\xi\{1-F(a+\delta)\}^{n-1} = 0$, since $1 > \{1-F(a+\delta)\} > 0$

3. $\lim_{n\to\infty} K \geq 0$ since $n(n-1)\{1-F(\theta)\}^{n-2}\{f(\theta)\}^2 \geq 0$ for all $\delta \in [a+\delta,b]$.

   Indeed, it can be shown that $\lim_{n\to\infty} K = 0$.

Thus, $\lim_{n\to\infty} M(n) = \infty$

<u>Lemma 2</u>: If $f(a) = 0$, and $\lim_{\theta\to a} \frac{f'(\theta)}{f(\theta)} = \infty$, then $\lim_{n\to\infty} M(n) = \infty$.

<u>Proof of Lemma 2</u>: Since $f(a) = 0$, then by integrating by parts,

$$M(n) = \int_a^b n\{1-F(\theta)\}^{n-1} f(\theta)\left(\frac{f'(\theta)}{f(\theta)}\right) d\theta,$$

since $f(\theta) > 0$, $\theta \in (a,b)$. Further, since $f'(\theta)$ is continuous near a, $f'(\theta) \geq 0$ near a, so that $\frac{f'(\theta)}{f(\theta)} \geq 0$ near a. If $\lim_{\theta\to a} \frac{f'(\theta)}{f(\theta)} = \infty$, then for $\xi > 0$, there exists a δ such that if $\theta \in [a,a+\delta)$, $\frac{f'(\theta)}{f(\theta)} \geq \xi$. Then,

$$M(n) = \int_a^{a+\delta} n\{1-F(\theta)\}^{n-1} f(\theta)\left(\frac{f'(\theta)}{f(\theta)}\right) d\theta + \bar{K},$$

where,

$$\bar{K} = \int_{a+\delta}^{b} n\{1-F(\theta)\}^{n-1} f(\theta)\left(\frac{f'(\theta)}{f(\theta)}\right) d\theta.$$

Therefore,

$$M(n) \geq \xi\int_a^{a+\delta} n\{1-F(\theta)\}^{n-1} f(\theta) d\theta + \bar{K} = \xi\left(1-\{1-F(a+\delta)\}^n\right) + \bar{K}.$$

Then

$$\lim_{n\to\infty} M(n) \geq \lim_{n\to\infty} \left(\xi - \xi\{1-F(a+\delta)\}^n + \bar{K}\right) = \xi,$$

since

1. $1 > \{1-F(a+\delta)\} > 0$, $\lim_{n\to\infty} \{1-F(a+\delta)\}^n = 0$,

2. It can be shown that $\lim_{n\to\infty} \bar{K} = 0$.

Since ξ is arbitrary, we can make it arbitrarilly large. Therefore,

$$\lim_{n\to\infty} M(n) = \infty$$

<u>Lemma 3</u>: If a is finite and $f(a) = 0$, then $\lim_{\theta\to a} \frac{f'(\theta)}{f(\theta)} = \infty$

<u>Proof of Lemma 3</u>: Since $f(a) = 0$, $\lim_{\theta\to a} \frac{f'(\theta)}{f(\theta)}$ is of the form of A/0 (A is a positive constant), $+\infty/0$, or $0/0$. The first two forms are infinite. A necessary condition for the third form to be finite, by L'Hospital's rule, is that $\lim_{\theta\to a} f^{(i)}(\theta) = 0$, where $f^{(i)}$ is the i^{th} derivative of $f(\theta)$. Since a is finite, $f(\theta)$ may be written as a Taylor's expansion around a:

$$f(\theta) = \frac{f^{(i)}(a)}{i!} (\theta-a)^i.$$

But if $f^{(i)}(a) = 0$ for all i, then $f(\theta) \equiv 0$ for all θ--a contradiction.

Thus, $\lim_{\theta\to a} \frac{f'(\theta)}{f(\theta)} = \infty$.

Combined with our earlier discussion, Lemmas 1 through 3 establish Theorem 1. Lemma 1 shows the theorem is true if $f(a) > 0$, and Lemmas 2 and 3 show it is true if $f(a) = 0$. Theorem 2 follows from our earlier discussion and Lemma 2.

Footnotes

* The opinions expressed here are those of the authors and may not reflect the views of the Federal Trade Commission or any individual commissioners or other staff members. The authors wish to thank B. Allen, H. Beales, D. Cass, D. Crawford, J. Galambos, S. Grossman, M. Katz, T. Romer, M. Rothschild, D. Sant, D. Scheffman, and especially R. Willig for useful discussions and advice.

1. The concept that imperfect consumer information endows even small firms with informational market power was developed by Scitovsky (1950), Arrow (1958), and Stigler (1961) among others. The elegant modeling of this phenomenon by Diamond (1971) and the discovery of the lemons principle by Akerlof (1970) has stimulated research by economists and policy analysts on both the scope of and potential remedies for imperfect information. The policy implications are emphasized by Pitofsky (1972), Schwartz and Wilde (1979), and the Federal Trade Commission (1978, 1979).

2. Stiglitz (1979) surveys most of the major models and discusses their properties.

3. This assertion is true for those cases in which the usual second-order conditions for profit maximization hold for each firm. See Section II.

4. We might note here that the estimates s_i^j could easily be reinterpreted as estimates of expected consumer surplus, so that real or spurious product differentiation may be incorporated into this model. This extension is made below in Section V.

5. See Federal Trade Commission (1979) for a non-technical discussion of these different methods.

6. Mass points to $F_i^j(\theta)$ are discussed in Section IV. The other assumed properties of these functions are presented in Appendix II.

7. In fact, this restriction of unbiased estimates is not necessary for many of the results derived below. A weaker restriction of identical bias for all estimates would suffice.

8. There exists some evidence on the nature of $F_i^j(\theta)$. For example, the Progressive Grocer (November, 1974, p. 39) conducted a survey of 560 shoppers in four Providence and Boston area supermarkets in July, 1974. The consumers were asked to cite the selling price of 44 popular brand name and nationally advertised items. Only 24% of the shoppers tested knew the "correct" price (within five percent) for a specific product compared to 32% in a similar

study in 1963. Other evidence is provided by Gabor and Granger (1961) and Uhl and Brown (1972).

9. Further search would be induced if the actual price p_i exceeded the second lowest estimate, $\min_k s_k^j$, in excess of the consumer's search cost. This topic is discussed in more detail below.

10. Cf., Phelps and Winter (1970) and Smallwood and Conlisk (1979).

11. For example, if store 1 charges \$10 and estimates are (8, 10, 12) and if store 2 charges \$11 with estimates (9, 11, 13), then store 2 will obtain customers who draw the estimate pairs {(10, 9), (12, 9), (12, 11)}.

12. If $s_i \leq s_k$, then $\theta_k > \frac{p_i - p_k}{\beta} + \theta_i$. Thus, given θ_i, the probability that $s_i \leq s_k$ is

$$1 - F\left(\frac{p_i - p_k}{\beta} + \theta_i\right).$$

Since the θ_i are drawn independently, equation (2) follows.

13. This assumption may be justified on the grounds that the gains to gathering this information are higher for firms then for individual consumers.

14. We assume that the second-order conditions are fulfilled, an assumption that is not true in general for all $F(\theta)$ and $d(p)$. See also footnote 23.

15. It should be emphasized that we assume a single price equilibrium. Although this assumption may be easily proved for the case of $n = 2$, we have not ruled out multi-price equilibria for larger n. This issue is discussed in more detail in Section IV.

16. Since,

$$\int \{1 - F(\theta)\}^{n-1} f(\theta)d\theta = 1/n.$$

17. Of course, if $n \to \infty$, then $p(n) = c$ as well.

18. These proofs are due to Robert Willig and Janos Galambos. Any remaining errors are our own.

19. Cf. Wilson (1977) for a similar result in his competitive bidding model.

20. In Hart (1979), the ratio of customers to firms is the crucial issue. Here, an increase in this ratio would leave price unaffected.

21. Of course, if advertising is treated as a fixed cost, the perfectly free entry condition is not satisfied by a zero zero profit equilibrium.

22. Symmetry may be shown by deriving $h(\mu)$, the density of $H(\mu)$, using a convolution with substitutions $\mu = \theta_1 - \theta_2$ and $\zeta = \theta_1 + \theta_2$. With a little manipulation, it can be shown that $h(\mu) = h(-\mu)$.

23. For the second-order condition for profit-maximization to hold, we need

$$\frac{\partial^2 \pi}{\partial p_i^2} = 2\frac{\partial Q_i}{\partial p_i} + (p_i - c)\frac{\partial^2 \ _i}{\partial p_i^2} < 0.$$

The first term on the right-hand side must be negative. Therefore, a sufficient condition for the second-order condition to hold at the single price equilibrium is that the second term is also negative. Expressing the second term in terms of the usual $f(\theta)$ density, a fairly weak sufficient condition for the second term to be nonpositive is that $\{f(b)\}^2 - \{f(a)\}^2 \geq 0$.

24. A similar analysis can be used to analyze the case of differential costs. If $c_1 < c_2$, then it can be shown that $p_1 < p_2$, that $p_1 - c_1 > p_2 - c_2$, and that the low cost firm has a higher gross margin, $(p_1 - c_1)/p_1$.

25. By perfect information we mean that the vectors $\underline{s}^j = \underline{p}^j$ (e.g., $\underline{\theta}^j \equiv 0$ or $\beta \equiv 0$).

26. The classic story of spurious product differentiation concerns the consumer who forms a false belief that one aspirin brand is superior to another after it relieves a mild headache and the "inferior" brand does not relieve a more serious one. This story may not be too far fetched: Even a placebo achieves a relief rate of around 45% compared to a relief rate of around 80% for actual aspirin (Food and Drug Administration (1977)). Such spurious product differentiation has been suggested by a number of writers including Chamberlin and Galbraith with respect to a wide variety of consumer products such as beer, detergents, lemon juice, and even soft drinks. The experimental evidence is interesting on this point. Blind tests of consumers' preferences after use do not replicate market shares. In addition, they vary according to whether products are labelled with brand names. See Tucker (1964), McConnell (1968), Morris and Bronson (1969), and Monroe (1976) for evidence; Schmalensee (1979) for a related model; and Craswell (1979) for a good discussion of some of the policy implications of this phenomenon.

27. The level of the expected beneifts of search will be altered, of course.

28. Cf. the solutions of Salop and Stiglitz (1977) and Diamond and Rothschild (1979).

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EQUILIBRIUM WITH PRODUCT DIFFERENTIATION: AN ADDENDUM

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In the accompanying paper, we analyzed a model of informational market power for a homogeneous commodity. In this note, the formal structure of that model is reinterpreted and applied to market power arising from product differentiation. In particular, we show the formal equivalence of these two sources of market power.

Consider the following model of consumer preferences for differentiated brands in a product class. Suppose there are an unlimited number of distinct possible brands indexed by $i = 1,2,\ldots$. Each consumer attaches relative values to these brands according to his <u>preference vector</u> $\underline{\theta} = (\theta_1, \theta_2, \ldots\ )$ Suppose the market consists to a large number of (small) consumers and suppose that n brands $(i = 1,2,\ldots,n)$ are available. Denoting the joint density of brand preferences by $\bar{g}(\underline{\theta})$, where an element of $\bar{g}(\underline{\theta})$ is an individual preference vector $\underline{\theta}$, then the proportion of consumers who (weakly) most prefer some brand i is given by the order statistic,

$$\Pr(\theta_i \geq \max_{j \neq i} \theta_j) = \bar{G}(\theta_i, \theta_i, \ldots, \theta_i) \tag{1}$$

where $\bar{G}(\underline{\theta})$ is the cumulative distribution function to the density $\bar{g}(\underline{\theta})$.

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*/ University of Pennsylvania and Federal Trade Commission respectively. Perloff's work was partially supported by the FTC. The opinions expressed here are the views of the authors and do not necessarily reflect the views of The Commission, individual Commissioners of other staff members.

The joint density of preferences may be integrated to obtain the (marginal) density of valuations for every existing and potential brand denoted by $g_i(\theta_i)$. Integrating, for brand 1, we have

(2) $$g_1(\theta_1) = \int \cdots \int \bar{g}(\underline{\theta})\, d\theta_2 d\theta_3 \ldots d\theta_n$$

By specifying particular structures for the joint preference density $\bar{g}(\underline{\theta})$, particular demand specifications and interrelationships among brand preferences may be captured. For example, if consumers who highly value brand one highly also devalue brand two and vice versa, then this dependence can be built explicitly into the joint preference density. If two brands, say i and k, are identical, then the preference density would have $\theta_i \equiv \theta_k$. */

In this note, we analyze a particularly simple <u>symmetric preference</u> density. By symmetric, we mean that preferences for each particular brand are independent and identically distributed, or

(3) $$g_i(\theta_i) = g(\theta) \qquad i = 1, 2, \ldots$$

---

*/ Butters (1978) studies a similar model where every group of consumers has a different ordinal ranking of any n available brands, and every brand's ranking across consumers is identical. Corresponding cardinal valuations might range from $\theta_{max} = n$ for the most preferred brand to $\theta_{min} = 1$ for the least preferred; $\bar{g}(\theta)$ would then have the property that every $\underline{\theta}$ vector is integer valued only, $n \geq \theta_i \neq \theta_j > 0$ for i, j and $\Sigma\, \theta_i = n(n+1)/2$.

Thus, the joint density $\bar{g}(\underline{\theta})$ for available brands $i = 1,2,\ldots,n$ is given by

$$\bar{g}(\underline{\theta}) = g(\theta_1)\ g(\theta_2)\ldots g(\theta_n) \tag{4}$$

Under this specification, each brand is most preferred by an equal $(^1/_n)$ share of consumers in the market. As will be shown below, it also entails an identical demand function for each brand offered in the market.

We capture consumer demand as follows: Each consumer chooses the brand that maximizes his net surplus s_i among all brands offered, where

$$s_i = \theta_i - p_i \qquad i = 1,2,\ldots,n \tag{5}$$

and θ_i is an element of $\underline{\theta} = (\theta_1,\ldots,\theta_n)$ from $\bar{g}(\underline{\theta})$. We call this brand the <u>best-buy</u>; its surplus is given by

$$\max_i\ s_i \tag{6}$$

Of course, given prices $(p_1, p_2, \ldots, p_n)$, even the best buy may give negative surplus $(s < 0)$, or surplus less than some opportunity value $\bar{v}$, if "outside goods" are included in the analysis. Because outside goods complicate the analysis considerably, we take the more restricted approach here that exactly one unit is surely purchased. Of course, even if outside goods were included, this one unit purchase would occur if $\underline{\theta}$ were sufficiently large. */

*/ See Salop (1979) for an analysis of equilibrium with outside goods. Formally, if $\theta_{min} > p_{max} + \bar{v}$, where θ_{min} is the lowest possible value for each θ_i in $g(\theta)$ and p_{max} exceeds the highest price charged, then $\underline{\theta}$ is "sufficiently large" to allow us to ignore outside goods.

Thus, given prices $(p_1, p_2, \ldots, p_n)$ for the n available brands, the proportion of consumers who purchase brand-one is given by

$$\Pr\ (s_i \geq \max_{j \neq i} s_j) = \int \prod_{j \neq i} \{G(p_j - p_i + \theta_i)\}\ g(\theta_i)\ d\theta_i \tag{7}$$

where $G(\theta_j)$ is the distribution to $g(\theta_j)$. We examine only the special case in which each consumer purchases exactly one unit of his best-buy. */ In this case, the expected market share for brand i, which we denote by $Q_i(p_1, p_2, \ldots, p_n)$ equals the proportion given by equation (7), or

$$Q_i\ (p_1, p_2, \ldots, p_n) = \Pr\ (s_i \geq \max_{j \neq i} s_i) \tag{8}$$

The symmetric industry equilibrium for this structure is easy to characterize. If there are $i = 1, 2, \ldots, n$ brands available, and each has constant marginal costs c, then the expected profits of brand i are proportional to

$$(p_i - c)\ \ Q_i(p_1, p_2, \ldots, p_n) \tag{9}$$

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*/ Elastic demand could be permitted by having a consumer's demand for each brand be given by $d_i(s_i)$. This approach would allow the construction of a "representative consumer" model, following Spence (1976) and Dixit-Stiglitz (1977). This is discussed in more detail in the conclusions.

If the equilibrium concept is "Nash-in-prices," that is, if each firm maximizes expected profits given a conjecture of fixed prices for all other firms, then by differentiating equation (7) and rewriting, we have the MR = MC condition,

$$(10) \qquad p_i = c - \frac{Q_i(p_1, p_2, \ldots, p_n)}{\partial Q_i / \partial p_i} \qquad i = 1,2,\ldots,n$$

Following Perloff and Salop (1980), the following proposition is immediate.

Proposition I: If $g(\theta)$ is continuous (no mass points) on a finite support, and if the second-order condition is satisfied, then a unique single price equilibrium $p_i = p(n)$ obtains for an n-firm industry, where

$$(11) \qquad p(n) = c + \frac{1}{M(n)}$$

$$(12) \qquad M(n) = n(n-1) \int \{G(\theta)\}^{n-2} \{g(\theta)\}^2 \, d\theta$$

*/ To make (12) comparable to the notation in Perloff and Salop, let $G(\theta) = 1-F(\theta)$.

Preference intensity can be formalized as follows. Denoting the typical preference sector $\underline{\theta}$ as a scaled up form of a standard intensity vector $\underline{\theta}^{\circ}$.

(13) $$\underline{\theta} = \beta\underline{\theta}^{\circ}$$

where $\underline{\theta}^{\circ}$ is a standard vector drawn from $\bar{g}(\underline{\theta}^{\circ})$. Then, a larger β represents more intense preferences. Perfect substitutes are captured by $\beta = 0$. */

Substituting equation (13) into (5), it may be shown that under this formalization, <u>increased preference intensity raises the equilibrium price.</u> In particular,

(14) $$p(n) = c + \frac{\beta}{M(n)}$$

Entry by additional distinct brands crowds the implicit product space under the specification given by equation (3). However, as n increases, Nash demand curves may become more or less elastic so that the equilibrium price p(n) may fall or rise. In the limit, however, if the maximum valuation of each brand remains finite, Perloff and Salop (Theorems 2 and 3) have shown the following:

(15) $$\lim_{n \to \infty} p(n) = c$$

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*/ If outside goods were explicitly reckoned into the analysis, then the alternative form $\underline{\theta} = \beta\underline{\theta}^{\circ} + v^{\circ}$ (where $v_o > 0$) would maintain a positive cardinal valuation level even as $\beta \to 0$.

That is, as the level of fixed costs approaches zero, permitting an infinite number of brands each earning non-negative profits (perfectly free entry), the equilibrium price approaches the perfectly competitive price, even though consumers have distinct brand preferences. This corresponds to the results reported by Hart (1979).

It should be noted, however, that if the maximum valuation (the upper limit on the domain of $g(\theta)$) is unbounded and if $\lim_{\theta \to \infty} \frac{g'(\theta)}{g(\theta)}$ is finite, then this result does not obtain, as stated by Theorem 3 in Perloff and Salop. This corresponds to a situation in which the valuation of the most-preferred brand surely becomes unbounded ($\lim_{n \to \infty} (\max \theta_i) \to \infty$).

CONCLUSIONS:

By reinterpreting the variables used in Perloff and Salop, we have shown the formal equivalence of market power arising from product differentiation and market power arising from firm-specific imperfect information. Although the model has a number of significant limitations, particularly inelastic individual demands and the absence of outside goods, it is suggestive of the type of results that could be obtained in a more general framework.

A second appeal of this analysis is that it suggests a fairly general framework that might permit the synthesis of a number of competing models of monopolistic competition. In this context, the model here is Chamberlinian in nature; every brand competes with every other available brand. Although the model explicitly considers differences in the preferences of individual consumers, a "representative consumer" model of the sort analyzed by Spence (1976) and Dixit and Stiglitz (1977) may be obtained by treating the joint preference density $\bar{g}(\underline{\theta})$ as the "aggregate" preferences of a representative consumer.

The underlying product space of brand attributes is ignored in this analysis. In addition, the particular form of preferences here and the manner in which entry affects demand suggests a set of special assumptions on brand formulations and competition in product space. In particular, additional brands "crowd" product space, so that on an average consumers get additional utility when more brands are available. That the density of brand preferences $g_i(\theta)$ is not altered upon entry represents an assumption that brands are not reformulated or do not relocate after entry.

Although every consumer has some most preferred brand in this approach, the concept of "localized" competition (or, "linked oligopoly") in spatial competition is not captured by the treatment here. For that, a somewhat different structure must be placed on the preference density. In particular, in those models, every consumer has exactly one brand valuation equal to some θ_{max} and all other brands are less valued according to some compensation (transport cost) function. In addition, such models also make particular assumptions about formulation symmetry and reformulation after entry.

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