WORKING
PAPERS

CAPITAL-GOODS MARKET DEFINITION

Richard S. Higgins

WORKING PAPER NO. 156

September 1987

BUREAU OF ECONOMICS
FEDERAL TRADE COMMISSION
WASHINGTON, DC 20580

DRAFT
September 9, 1987

DO NOT QUOTE
WITHOUT PERMISSION

CAPITAL-GOODS MARKET DEFINITION

by

Richard S. Higgins
Federal Trade Commission

I. INTRODUCTION

According to the Justice Department Horizontal Merger Guidelines, a product market is relevant for purposes of antitrust analysis if joint profit maximization by all the producers of the product would lead to a significant price increase for one year. In the case of a capital good, which by definition provides services over several time periods, new capital-goods production alone may not comprise an antitrust market on account of the substitute product, the stock of used capital goods. This paper investigates the conditions under which used capital is in the relevant product market. It is discovered that contrary to the conventional wisdom, "used capital is almost always in the market," when demand growth is low. Moreover, with zero growth, a relevant market is unlikely even when new and used capital are combined. These findings apply under the most conservative of assumptions about the ability of users of capital to extend the life of capital through more expensive maintenance. However, when demand growth is significant, the presence of a relevant product market depends on the percentage rate of growth relative to the percentage rate of depreciation.

This paper addresses a problem very similar to that in a recent paper by Luke Froeb [3]. Both papers rely on "precommitment" models, choose a long-run competitive equilibrium benchmark, and focus on the joint dependence of market definition on demand elasticity and depreciation rate. The present paper differs in some important respects however. First, the present results do not depend on the assumption of endogenous depreciation. While a fixed depreciation rate, which is assumed here for simplicity and is generous towards market definition, may be seen as a shortcoming, it is certainly not true that "models with precommitment imply that durability limits market power only if depreciation is endogenous (see Froeb [3,p. 15])."

Second, the present paper shows how including used goods in the provisional market affects relevant-market definition. Finally, the present paper expands the inquiry to allow for demand growth. The role of market growth is very important; were it not for this factor virtually no capital-goods market would be relevant for antitrust analysis.

Section II provides an informal description of the analytical framework and presents the major implications for antitrust market definition. Section III more formally analyzes three illustrative cases: zero-growth and single ownership of the means of production, zero-growth and single ownership of the means of production as well as the capital stock extant at the creation of monopoly, and positive growth along with combined ownership of new and used capital. In each case, the effect of depreciation, growth and demand elasticity on market definition is evaluated. Section IV provides a brief digression on the likely demand elasticity for capital goods when the buyers are not final consumers. Section V concludes and looks ahead to an analysis of anticompetitive effect in capital goods markets.

II. INFORMAL DISCUSSION

Economists have certainly not been oblivious to the effect of durability on market power. Prompted by the Alcoa and the United Shoe Machinery cases economists have grappled with the effect of a used-good market on the ability of a single primary producer to exercise market power.[1] (See Gaskins [4] and Swan [11]). But not until Coase's (Coase [2]) seminal article was it clear how, given durability, the cost of commitment affects the value of the monopoly producer or, with durability variable, how commitment could be effected through the choice of durability. Recently, several papers have corroborated Coase's original analysis of the constant marginal-cost case and completed his analysis of increasing marginal cost. (See Bond and Samuelson [1], Kahn [8], Rust [9] and Stokey [10]). In all these papers the problem is to describe capital accumulation by a single producer out of long-run equilibrium under various assumptions about marginal cost and the monopoly producer's ability to commit to an output path. Depending on the circumstances there is a definite relationship between exogenous durability and the value of the monopoly.[2] For example, with constant marginal cost and no commitment the monopolist produces the competitive output, and regardless of durability the value of the monopoly is zero.

In the present paper the effect of durability on market power (or market definition) is analyzed in a different context. It is assumed that the merger, which prompts the market definition exercise, occurs after the

---

[1] See U.S. v Aluminum Co. of America, 148 F.2nd 416(1945), and U.S. v United Shoe Machinery, 110 F. Supp. 295 (1953).

[2] When perfect commitment is impossible the choice of durability is also affected, at least when marginal cost is increasing. In effect, commitment is partially effected through the durability choice.

market is in steady-state perfectly competitive equilibrium, with a constant percentage rate of demand growth (which may be zero).[3] In this context, the issue is whether or not the producers of capital as a group can profitably raise the price of capital 10% (the typical FTC price test) in one year. This problem is addressed under various assumptions about the exogenous percentage rates of depreciation and growth and about ownership of the outstanding stock of capital by the producers. (Assuming the producers as a group own the outstanding capital stock is tantamount to including used capital in the market.) Under all circumstances it is assumed that the producers of capital can perfectly commit to output plans. Thus, regardless of the assumption about the producers' ownership of the outstanding capital at the time of merger, the capital subsequently produced is effectively owned by the producers.

The market definition exercise would be altered significantly if precommitment were not assummed. For example, without precommitment when the producers own none of the outstanding capital stock and marginal cost is constant, merger to monopoly would result in a zero price increase regardless of the depreciation and growth rates. With increasing marginal cost the equilibrium time-rate of capital accummulation will generally exceed the rate of accummulation in precommitment models, which reduces the likelihood of a relevant market, ceteris paribus. These no-commitment models are not analyzed here, but see Rust [9].

---

[3] This assumption is stronger than necessary, but it greatly simplifies the analysis without driving the results. For example, in the case of zero growth the merger might occur in the midst of stock adjustment with the current stock in excess of the long-run stock desired by the monopolist. In this case, the producers as a group would exploit monopoly power by allowing the outstanding stock of capital to depreciate just as in the case assumed in the text.

New capital is demanded from the producers of capital either to replace worn-out capital or to build up the existing capital stock. In long-run equilibrium new capital is produced solely to replace depreciated capital (when there is zero growth), or new capital is produced both to replace capital and to accumulate at the equilibrium rate of growth. In the no-growth model, which is discussed first, net investment is positive only outside of long-run equilibrium. If the capital-goods market is initially in long-run equilibrium, any price increase by the producers of capital as a group would induce depreciation of the outstanding capital stock, since users would replace capital at a lower rate at a higher price, which rate initially just matches the rate of depreciation.

1. Zero Demand Growth

The value of monopoly over capital goods production derives from the ability of the capital producers to reduce the outstanding stock thereby raising the market price of capital. Clearly, the more long-lived the capital good the less power the capital-goods producers have to induce a price increase. Specifically, the ability of the producers of capital to raise price significantly in a one-year time frame depends critically on the elasticity of the demand to hold capital and on the physical rate of depreciation. The more durable the capital (i.e., the more capital like the product in question), ceteris paribus, the less likely capital goods production alone is a relevant market, and the more likely used capital is in the market.

If used capital were included in the market, the behavior of the monopolist as producer of capital would be altered. Specifically, the monopolist would have no incentive at all to produce new capital. Instead,

it would allow capital to depreciate without replacement thus driving up the price of capital. Such decumulation would persist until the monopolist held the joint-profit maximizing stock, which it would maintain through replacement.

The implied price increase, over time and in one-year would be higher when used capital is in the market. [4] Even so, depending on the maximum rate of physical depreciation and the stock demand (or equivalently, service demand) elasticity, there may be no relevant market comprised of both new and used capital. Specifically, when used capital is in the market the relationship between demand elasticity, b, and the minimum annual percentage rate of depreciation necessary for market definition, D, is $D=1-(1.1)^{-b}$. When b=1, which would small enough to define a market for non-capital goods, there is no market if D<9.5%; which is equivalent to a capital life of 10.5 years. When used capital is not in the market the analogous relationship is $D=2[1-(1.1)^{-b}]$. That is, the minimum annual percentage rate of depreciation is twice as large. Production of machinery with a lifetime of over 5 years would not comprise a relevant market, for example.[5]

---

[4] In the case in which the marginal valuation of capital services is less than zero at the long-run (competitive) capital stock, the owners and producers of capital would immediately withhold capital from the market to maximize the total income to capital. The result would be a (discontinuous) jump in price upon monopolization of the combined new and used capital market, and the new price would be held constant until the outstanding stock available had depreciated to the income-maximizing stock. This special case is analyzed by Coase in another context (see Coase [2]).

[5] If the group of capital goods producers whose market power is being investigated excludes some fringe competitors finding a relevant market will be less likely. However, the typical market definition exercise combines all producers of the product, in which case there is no fringe, leaving an investigation of the role of fringe suppliers to an analysis of anticompetitive effect. A major exception to this rubric arises when international or interregional trade is important. That is, when product and geographic market definition are combined--e.g., the issue may be whether or not U.S.

If used capital is necessary to define a relevant product market for a particular capital good, the conventional wisdom is that the likelihood of anticompetitive effect is thereby drastically reduced. This view is incorrect in general. The capital goods producers as monopolist have an incentive to purchase the outstanding capital stock from the users, who may be numerous. And, the price the users are willing -- acting independently -- to accept will be less than the price the monopolist (the producers only) is willing to pay. If transactions costs -- in particular costs associated with holding out -- are not prohibitive, the producers as a group would purchase the outstanding capital (if they did not already own it), thereby eliminating the need for collusion among current owners/users and producers.

Superficially, the hold-out problem appears significant for two reasons. First, the monopolist need not own all the outstanding capital in order for the present value of all capital (new and used) to exceed what this present value would be with zero monopoly ownership of the outstanding capital. This would appear to provide an incentive to hold-out. Second, there is some critical ownership percentage beyond which the present value of all capital is not increased with increased ownership by the monopolist. This would appear to increase the likelihood that no trade would occur.

A user/owner would prefer to retain ownership and have every one else tender their capital to the monopolist. This fact makes it uncertain that the monopolist could ever purchase any of the outstanding capital it does not already own at t=0. For example, suppose the monopolist need 80% of all outstanding capital to maximize the present value of all capital.

---

capital goods production is a relevant market--the role of fringe supply in the capital accumulation model would have to be analyzed explicitly. This subject is not addressed in the present paper.

Suppose it makes a tender offer that it will pay $p_K"$/unit for all units provided it gets at least 80% of the capital. (It may even declare that this is the first and last offer.) Unless $p_K"$ gives all the gains from trade to the user/owner it is unlikely that any would independently tender their assets. They would not because they each know that if they held out and the tender offer were successful their capital gain would exceed that implied by the monopolist offer price $p_K"$. Alternatively, suppose that the monopolist sought less than 80%. Conceptually, there is no difference. It still appears that there is no equilibrium without collusion among buyers.

The absence of an equilibrium may arise because the monopolist has not been permitted (by assumption) to commit to one and only one offer. There are two aspects of this commitment. First, suppose that the monopolist needs 100% to maximize present value. It declares one and only one offer. An individual will know that holding out in order to be in the fringe is futile because the offer allows for no fringe. However, he knows that if he holds out the tender offer will fail and the monopolist may come back with another, offering more than before. Except for the cost of the time entailed in transacting, it is individually rational to wait. It appears here that there would be no equilibrium price, with the result that the monopolist would solve its optimization problem without ownership.

There are other strategies available to the monopolist, however. Suppose the monopolist offers a 100% tender deal at a price that just concedes some gain to the current owners, and at the same time, it precommitts to forfeiting the entire gains from trade if the deal fails and a second offer is made. Now, the users/owners know that a second offer would not be made. The monopolist would acquire all of the capital. And

there are probably other contracts that would accomplish the same result. (See Grossman and Hart [3] for a discussion of similar problems in stock acquisitions.)

The upshot is that monopolization of capital goods production is likely to be accompanied by the purchase of the outstanding capital by the monopolist unless transactions costs are prohibitive. (For example, in addition to the aforementioned hold-out problems, monitoring the users' maintenance of the capital may be prohibitively costly). In any case, the full ownership case, which may be a realistic description of monopoly behavior, provides an upper bound to the predicted monopoly price increase. Thus, for practical purposes, it maybe useful for market definition. As stated above, even when new and used capital are combined finding a relevant market is not a foregone conclusion.

The foregoing discussion illustrates the role of the various key factors that determine the presence or absence of a relevant market in new capital production and in new and used capital combined, assuming that the market is initially in long-run competitive equilibrium. I next turn to the case in which there is steady-state growth, assuming for simplicity that the monopolist owns all the outstanding capital. [6]

2. Positive Demand Growth

The pre-monopoly steady-state equilibrium witnesses the capital stock growing at a constant percentage rate. Capital is produced at a rate just

---

[6] The reader may be concerned that the results to this point are driven by the assumption that initially the state is long-run equilibrium. It is true that if the monopoly is created in the midst of stock adjustment short of long-run equilibrium the analysis may be more realistic. It would also be more complicated with no essentially different implications.

sufficient to replace worn-out capital and to add capital so as to leave the price unchanged. (Constant marginal cost is assumed throughout.) When the monopoly is created, all production stops. There are two consequences. The extant stock declines and price moves up a given demand curve at a rate depending on the rate of depreciation. Simultaneously, demand continues to grow which implies a higher price even were the stock not depreciating. Thus, price rises on both counts at a rate positively related to the rate of growth and the absolute value of the rate of depreciation and negatively related to demand elasticity. If the applicable demand curve is a constant-elasticity demand curve, the monopoly steady-state capital stock is achieved when marginal revenue grows to such an extent that it equals marginal cost. Thereafter the achieved capital stock will be maintained and new growth accommodated so that marginal revenue -- instead of price -- remains equal to marginal cost. And, since for constant-elasticity demand curves price is a constant premium above marginal revenue, price remains constant in the steady-state monopoly equilibrium.

The predicted annual percentage price increase depends on the annual rates of growth and depreciation and on demand elasticity. Specifically, the minimum annual percentage rate of depreciation necessary to define as relevant market is $D=1-(G+1)/(1.1)^b$. For example, if $b=1$ there is no market (for new and used capital) if $D=9.5\%$ and $G=0$, and there is a market regardless of D if $G>10\%$.

3. A Caveat

There is one major caveat to the present analysis. The assumption that capital depreciates at a constant instantaneous percentage rate, is consistent

with infinitely long-lived capital that gradually suffers quality degradation or with uniform quality capital up to a fixed finite life. [7] In both cases, used capital is perfectly substitutable across vintages so long as the stock prices are equilibrium prices. If, on the other hand, the users of capital are variegated in their preferences for some characteristics of the capital good, used-capital prices may not be fully compensating. In the latter instance new and used capital (and even different vintages of used capital) are not perfect substitutes at their market-clearing stock prices. For example, suppose there are two types of users, one for which a breakdown is more costly than for the other, and suppose that beyond T/2 years (T is the "life" of the capital good) the breakdown rate is significantly higher than for machines less than T/2 years old. Specifically, suppose that the expected breakdown cost for "old" machines (age > T/2 years) is thrice that of "new" ones (age < T/2 years) for group A users and only twice for group B users. The equilibrium price differences between old and new machines may be less than the difference in the expected cost of breakdown for group A users. (It could not be more and be an equilibrium price difference.) If so, new and old machines are not perfect substitutes for the group A users.

Group A users only use "new" machines; when A's machines reach age T/2 they are sold to B. It appears possible for there to be a competitive long-run (no growth) equilibrium, in which the output of new capital just replaces group A's capital which is sold at age T/2 to group B and together with B's purchase from A just replaces B's depreciated capital. Here, the

---

[7] Actually, the latter case implies that the average depreciation rate depends on the average (weighted) vintage. In the formal analysis that follows, vintage effects have been ignored even though the age distribution is typically not uniform. Thus, in effect, the analysis implicitly assumes infinite capital with gradual quality degradation.

price differential that makes old and new machines less-than perfect substitutes for group A remains, and the relative stock of A's and B's capital remains fixed (along with the total outstanding stock). For this equilibrium to obtain Group A's stock must be smaller than group B's otherwise sales by group A of old machines would depress their value until the price difference grew to make old and new machines perfect substitutes.

A monopoly producer of capital would produce no capital at all (recall the simple example of a competitive long-run no-growth equilibrium prior to monopolization), and the group A stock would depreciate at a larger percentage rate than would group B's, provided A continued to sell old capital to B at age T/2. The effect would be to drive up new capital's price relative to old capital until the two were perfect substitutes at the equilibrium stock prices, at which time group A would cease to sell to B and the market would function as in the model with perfect substitutability at every point in time. Until such time, the monopoly producers of capital could drive up the price of new capital relative to old capital (the difference between A's evaluation of the difference between new and used capital and the market price difference is the maximum premium that could be exploited) in effect price discriminating, until price differences were fully compensating to all users. In this case, there could be a relevant market for capital production whereas there would not have been had the initial equilibrium reflected fully compensating price differences. But all this merely implies that the effective rate of depreciation for purposes of market definition may not be the average rate in the market.

Still, it makes sense to use the entire market as a benchmark, as if users were homogeneous. If the rate of growth is small and the rate of

depreciation is small, there is unlikely to be antitrust market -- especially if there is no evidence that some users specialize in recent-vintage machinery.

III. MARKET DEFINITION FOR CAPITAL GOODS

The following three sections present an analysis of monopoly pricing in capital-goods markets. In all cases it is assumed that capital-goods production is perfectly competitive up to time t=0 at which point production decisions are made jointly. To simplify the analysis, it is assumed capital is produced at constant marginal cost. Constant marginal cost implies no limits on the rate of capital accumulation; thus, the long-run desired capital stock is always held. The latter result would appear to make analysis trivial; however, since in the present context a profit-maximizing monopolist will reduce the outstanding capital stock the relevant cost of adjustment depends on the rate of depreciation not the marginal cost of new production.

In the first two sections zero (exogenous) growth is presumed. That is, long-run equilibrium entails a constant capital stock, which is continuously maintained. In Section A it is assumed that the monopolist owns none of the outstanding capital stock except that which if produces new after t=0. In Section B, the monopolist owns all outstanding capital. [8] Section C introduces exogenous demand growth for the case in which the monopolist

---

[8] Ownership is important in two ways. As an analytical device it is important because it provides a simple way of modeling perfect commitment with perfect foresight by buyers. Specifically, a monopolist who sold to buyers with perfect foresight could not price discriminate intertemporally, and a model of full ownership by the monopolist automatically removes any incentives to price discriminate in this manner. (See Coase [2]). Also, if the monopolist owns all capital, new and old, then the relevant market naturally contains both new and used capital. To investigate whether or not capital goods production alone is a market requires that the monopolist not own the outstanding stock accumulated prior to t=0.

owns all the capital including the stock accumulated prior to t=0. [9] The long-run perfectly competitive equilibrium, which is assumed to apply prior to monopolization at t=0, is in this case a steady-state equilibrium.

A. The Monopolist Does Not Own the Outstanding Capital Stock at t=0.

The monopolist chooses K(t) to maximize

$$(1) \qquad \int_0^{\infty} \{p_K^d(K)[K-K(0)e^{-dt}]-c\ k\}\ e^{-rt}dt,$$

where $dK/dt = k - dK$ and $K(0)=K_c^*$ which satisfies $p_K^d(K) = (r+d)c$. It is assumed that there is a constant instantaneous percentage rate of depreciation, - d. [10] The rate of production, k, contributes to net additions to capital and to capital replacement, dK. In the long-run perfectly competitive equilibrium, the rental demand price of capital is just equal to the full (including a depreciation expense) rental cost of capital, (r+d)c. The model assumes that used capital is traded at sufficiently low cost that price discrimination is unprofitable to the monopolist. A model of no trading and

---

[9] Analysis in Sections A and B show that the full-ownership case sets an upper bound on the predicted monopoly price increase. Thus Section C provides a conservative framework for market definition.

[10] The depreciation rate, d, has two interpretations. Either capital provides a constant-quality service flow up to its death date, and the age distribution of capital is uniform (or simply ignored), or capital provides a gradually-declining quality of service. In the former all still-useful capital goods are perfect substitutes. In the latter, all capital goods are perfect substitutes at time-adjusted asset prices.

price discrimination would not alter the conclusions except in obvious qualitative ways.

In (1) the monopolist ignores the effects of its pricing and output decisions on the capital already accumulated before it attained monopoly status. Thus, I am assuming that the monopolization event was fully unanticipated. For all capital produced after t=0 there is perfect foresight about the time path of price and the monopolist is able to commit perfectly to an output plan; so, in effect, the monopolist owns all capital produced after t=0. [11] Assuming that demand is linear, (p_K^d=A+BK), (1) has a fairly simple solution. The marginal condition (see Hadley and Kemp) [6]) is

$$(2) \quad (r+d)c-A + BK_c^*e^{-dt} - 2BK = 0,$$

which implies

$$(3) \quad K^*(t) = [A-(r+d)c-BK_c^*e^{-dt}]/(-2B) = K_c^*(1+e^{-dt})/2.$$

According to (3) the monopolist allows capital to depreciate at a declining rate. Initially, at t=0, $(dK^*/dt)/K^*=-d$ and as $t\to\infty$, $(dK^*/dt)/K^*\to 0$. That is, the monopolist gradually replaces old capital, which it does not own, with new capital which it does own. If the process were allowed to attain a new long-run equilibrium, the monopolist would hold $K_c^*/2$ units of capital which it would maintain by producing at instantaneously rate, $k=dK_c^*/2$.

---

[11] If some users of capital had a more urgent demand to hold capital in the near term than others, some intertemporal price discrimination would be possible even within the context of perfect foresight. Such differences in preferences are ignored here.

Rearranging (3) so that

$$(4) \quad K^*(t) = (K_c^* - K_c^* e^{-dt})/2 + K_c^* e^{-dt}$$

reveals a simple graphical representation. In Figure 1, the marginal rental cost function, (r+d)c, is drawn along with the rental demand price function, p_K^d, and the current stock of capital as of t=t', $K_c^* e^{-dt'}$. The current stock of capital at t', K*(t'), is found by equating marginal revenue given the outstanding stock owned by others, MR', to (r+d)c. Only when $K_c^* e^{-dt}$ has declined to zero, does MR' coincide with MR proper. [12]

In terms of this model of capital depreciation the factors influencing the determination of a market relevant for antitrust purposes are described. According to the Guidelines, capital-goods production is a relevant market if all the producers of capital could profitably raise price at least 10% within one year. [13] The above model predicts the one-year (percentage) price

---

[12] A continuous-time framework is adopted here to facilitate analysis. Unfortunately, the particular continuous-time model analyzed results in a major expositional shortcoming. Specifically,. the investment demand curve (the flow demand curve) is always flat. The presence of a relevant market depends on the profitability of reducing the rate of production (investment) sufficiently to influence the price of capital in the stock market by reducing the stock of capital outstanding. It is important to realize however that the implications of durabiity for capital-goods market definition does not depend on the analytical device adopted. Even in a discrete-time framework, high durability would make the annual flow demand sufficiently elastic that a significant price increase would be unprofitable, and perhaps even impossible since the rate of production can not be less than zero.

[13] Strictly speaking, the Guidelines market test may be inappropriate for capital goods because price does not increase maximally at once. Instead, price rises gradually as capital is allowed to depreciate. Thus, if the monopoly were expected to last for T years (perhaps, because after T years entry will have eliminated any monopoly power or because the collusive agreement lasts only T years) the T-years (percentage) price increase would exceed the one-year (percentage) increase. (See Froeb [3, p. 10].) The reader can easily determine for himself how the Guidelines market

increase. Specifically, suppose that the linear demand curve is approximated by a constant-elasticity demand curve. Thus, $K^d = p_K^{-b}$ is defined so that it is tangent to p_K^d in Figure 1 at $[K_c^*, (r+d)c]$.[14] Thus the profit-maximizing price path for the monopolist is

$$(5) \quad p_K^*(t) = [K_c^*(1+e^{-dt})/2]^{-1/b}$$

Since t is measured in years the predicted annual percentage price increase is $[p_K^*(1)-p_K^*(0)]/p_K^*(0)$, or

$$(6) \quad \% \Delta p_K = [(1+e^{-d})/2]^{-1/b} - 1.$$

One sees from (6) that the percentage price increase depends on b and d.

Based on (6) the annual percentage change is less than 10% -- and capital goods production is not a relevant market -- whenever

$$(7) \quad [(1+e^{-d})/2]^{-1/b} - 1 < 0.1.$$

Rearranging (7) one gets

---

definition test would have to be amended to accommodate prediction is about the expected life of the monopoly. Regardless of the criterion used the factors I cite is important within the Guidelines definition will remain so.

[14] Relative to the linear demand curve the constant-elasticity demand curve biases the market definition exercise in the direction of finding a relevant market. This is because, a linear demand curve gets more elastic at prices higher than (r+d)c.

(8) $1 - e^{-d} < 2[1-(1.1)^{-b}]$.

The left-hand side is the annual percentage rate of depreciation D. [15] Thus (8) says that whenever $D<2[1-(1.1)^{-b}]$, capital goods production is not a relevant market.

The relationship between the critical value of D and b is depicted in Figure 2. That is, given a value of demand elasticity, capital-goods production is not a market if the applicable D lies on or below the curve, D(b).

Some implications of the diagram are superficially counterintuitive. For example, for non-capital goods, b=11 is the maximum demand elasticity that will permit a profitable 10% price increase. [16] According to the diagram the annual percentage rate of depreciation must be greater than 130% to have a relevant market when b=11. Thus, the diagram appears to contradict the market-definition criteria for non-capital goods market. The contradiction is only apparent, stemming from the erroneous view that D>100% is impossible, or -- to put it differently -- that D=100% indicates noncapital. But recall that the model is based on continuous time, so that 1/D is the number of years the capital stock remains outstanding before turning over. Thus

---

[15] The annual percentage rate of depreciation is $\{\int_0^1 [d(K_c^* e^{-dt})/dt]dt\}/K_c^* = 1-e^{-d}$.

[16] Along a constant-elasticity demand curve higher percentage price increases are possible but they are not profit maximizing. The profit-maximizing output restriction equates marginal revenue (equals (b-1)/b times price) to marginal cost. Assuming constant marginal cost (which is generous towards finding a relevant market), marginal revenue is initially the competitive price, and, hence, $(p_1-p_0) = 1/b(b-1)$. Thus if b=11, $(p_1-p_0)/p_0=10\%$, and if b>11, $p_1-p_0)/p_0< 10\%$.

D=130% implies that the commodity stock in question turns over approximately once in .8 years. Thus, a box of cereal is a "capital good" which turns over, say, once in 1/52 of a year, that is, its D is 5200%.

With the foregoing prelude, reconsider Figure 2. An implication of Figure 2 is that if b=7.3 and D=100%, the annual percentage price increase is only 10%. Even though D=100% would permit the monopolist to produce at a zero rate for a year allowing outstanding capital to fall to zero at which point it could equate (immediately, since marginal cost of production is constant) marginal revenue to marginal cost (i.e., achieve its long-run desired stock in one year), this strategy is not profit maximizing. Instead, as indicated by (3), the monopolist begins producing immediately but at a rate that permits the stock to fall. However, at time t=1, the stock accumulated by the monopolist itself (since t=0) exceeds the long-run profit-maximizing stock. The latter implies that though b=7.3 permits a long-run (percentage) premium of $1/(7.3\text{-}1) \approx 16\%$, after one year, the (percentage) premium is only 10%. Thus, it is clear why critical D is 130% when b=11. If b=11 and D=100%, the implied price based on the end-of-the-year stock the monopolist would have accumulated, would be less than 10%. Ten percent only applies for b=11 when the monopolist holds its long-run desired stock. [17]

In light of Figure 2, it is unlikely that capital-goods production would be a relevant market for antitrust purposes -- for most goods commonly considered capital goods. For example, if the average life of a machine

---

[17] Definitions are instrumental. This is seen clearly here. For antitrust purposes a reasonable definition of non-capital is any good for which market definition is independent of D, that is, all goods for which D>130%. For example, for non Cajun families a bottle of Tabasco sauce is a capital good and hence, ever if its residual demand elasticity were equal to five, it may not be a market.

were at least 5.3 years, so that D<=1/5.3=19% then there would be no antitrust market for b=1. The conventional wisdom at the FTC is that "used machinery is almost never in the market." Instead, used machinery is almost always int he market -- at least when equilibrium growth is zero. [18] In the next section I investigate the conditions under which new and used capital is a relevant market.

B. The Monopolist Owns All New and Used Capital

A simple way to determine the conditions under which new and used capital goods comprise a relevant market for antitrust analysis is to allow the capital-goods monopoly producer to own all capital, including outstanding capital. In addition, modeling the monopolist's maximization problem premised on full ownership has the advantage of implicitly assuming buyers are not fooled by the monopolist into repeated capital losses through intertemporal price discrimination. As Coase (see Coase [2]) observes rational buyers will not purchase capital outright from a monopoly producer without some assurance from the monopolist that there will be no gains from postponing purchases. One guarantee entails short-term lease of the capital from the monopoly owners. [19]

Before the formation of the monopoly, the market is in long-run equilibrium. Specifically, the equilibrium capital stock, K_c^*, is [A-(r+d)c]/(-B), where p_K^d = A+BK and marginal rental cost is (r+d)c. Capital

---

[18] Although the above analysis does not deal with an initial state of short-run competitive equilibrium without long-run equilibrium, in which there is positive capital accumulation, the results are identical in kind.

[19] It will be shown further below that absent transactions costs the value of the monopoly is maximized when the outstanding stock of capital as of t=0 and all new capital is owned by the monopolist.

depreciates according to $(dK/dt)/K=-d$; so long-run production is dK_c^* which just maintains the capital stock. At t=0, the monopoly is formed, and the rate of production is reduced to zero, which allows the capital stock to depreciate to its long-run desired level $K_c^*/2$.

The time-path for the capital stock is given by

$$(9) \quad K^*(t) = K_c^* e^{-dt}$$

The capital stock declines at a declining rate until it reaches $K_c^*/2$ at time $\hat{t}$ satisfying $K_c^*/2=K_c^* e^{-d\hat{t}}$ which implies $\hat{t}=\ln(2)/d$. Unlike the case in which the monopolist does not own the stock K_c^* and produces at a positive rate over time (Section A), here, the monopolist produces nothing at all until time $\hat{t}$. At $\hat{t}$, the monopolist produces at rate $dK_c^*/2$ to just maintain the long-run monopoly capital stock.

The predicted annual percentage price increase is determined from knowledge of p_K^d and the capital stock path $K^*(t)$. [20] As in Section A, a constant-elasticity demand curve is used to approximate the linear demand initially posited. Thus,

---

[20] There is one exceptional case. When demand is inelastic at K_c^*, marginal revenue is negative. Thus, income would be maximized by the monopolist -- who owns all the capital, recall -- if K_c^*-K' were withheld from the market (K'is the capital stock that equates marginal revenue to zero). Price would jump immediately from (r+d)c to A+BK' and remain constant until K_c^* had depreciated to K', after which K would continue to depreciate at the maximal rate and price would gradually rise towards the long-run monopoly price, $A+BK_c^*/2$. In cases where the annual percentage rate of depreciation is less than $(K_c^*-K')/K_c^*$, the annual percentage price increase would be measured by $[A+BK' - (A+BK_c^*)]/(A+BK_c^*)=-B(K_c^*-K')/(A+BK_c^*)$, which may be greater or less than 10%, depending on how inelastic demand is at K_c^*.

(10) $\%\Delta p_K = (e^{-d})^{-1/b} - 1.$

When the right-hand side is less than 10% there is no relevant market. That is, there is no market when

(11) $D = (1-e^{-d}) < 1 - (1.1)^{-b}.$

The critical value of D is found by equating the right-and left-hand sides in (11).

Figure 3 depicts the graph of the relationship between b and the minimum value of D that defeats market definition. For purposes of comparison the analogous relationship from Section A is also depicted. The relationship implied by (11), D1, has half the height of that from Figure 2, D2. Consider the point on D1 when b=11. This point indicates that when 65% of K_c^* depreciates in one year the monopolist will exactly hold its long-run desired capital stock after one year. (The percentage is 65% and not 50% because the linear demand curve has been approximated by a constant-elasticity demand curve. The monopoly stock is not one-half of the competitive stock except for a linear demand curve.) If D were less than 65%, the monopolist would not reach its long-run optimal stock in one year and, thus, for b=11, the one-year price increase would be less than 10%. For b>11, there is no long-run optimal stock for which the implied percentage price increase is at least 10%; so for b>11, there is no market regardless of D -- just as in the noncapital goods case. Note however that when b=11, the capital good does not have to turnover but once every 1.5 years; that is the

good does not have to be a service proper for b=ll to imply a relevant market.

For values of b between zero and ll, new and used capital comprise a market depending on the value of D. For example, for b=l, if D< 9.5% there is no antitrust market even when used capital is included. That is, if the competitive stock/flow rate is 10.5 or greater (or if the average life of the capital good is of greater than 10.5 years) there would be no market.

In summary, it appears that not only is "used machinery almost always in the market", but even when the provisional market contains both new and used capital a relevant market is unlikely. This result follows because in a zero-growth context, the monopolist cannot raise price without reducing the outstanding capital stock, and the latter cannot be controlled through the rate of production beyond the limiting rate of depreciation. That is, the most the monopolist can do is reduce current output to zero which implies a time rate of change of -dK(t). When the monopolist owns all capital this maximal rate of stock reduction is optimal; when the monopolist only owns new capital produced after monopolization even this rate is too fast.

The upshot is that antitrust intervention is unlikely to raise consumer welfare when monopoly pricing in capital-goods markets is at issue, unless the growth rate of capital demand is significant. In the next section, I analyze the positive growth case, assuming steady-state equilibrium and assuming that the monopolist owns all the capital. The long-run equilibrium assumption merely simplifies the analysis; similar results follow otherwise. The ownership assumption does affect the market definition analysis but, in general, an antitrust market is more likely when all capital is owned by the

monopolist; thus, this assumption is conservative. Moreover, the ownership assumption might be the more realistic of the two alternatives.

Consider: If the monopolist owns all the capital the implied price path is $p_K^{*1}(t)=A+BK_c^*e^{-dt}$ and if it owns only new capital produced since monopolization, the price path is $p_K^{*2}(t)=A+BK_c^*(1+e^{-dt})/2$. At t=0, $p_K^*1=p_K^{*2}$. For t>0, $p_K^{*1}>p_K^{*2}$. Even if the monopolist owned all capital, its present value would be greater according to p_K^{*1} than according to p_K^{*2}. Thus, the value of the monopoly is greater when it owns all capital than when it does not. For this reason, the monopolist would be willing to pay at most the difference in present values to own the outstanding capital stock. The users of capital (if they are the owners of K_c^* at t=0) would experience capital value derived from time path $p_K^{*2}(t)$ were they to continue to own the capital. There are gains from trading ownership of the capital stock. It is predicted that monopolization of capital-goods production will be accompanied by acquisition of the outstanding capital stock, provided transactions costs are not prohibitive. (For example, monitoring maintenance may be prohibitive, or hold-out problems may scuttle the deal.) Thus, the cases analyzed in Section B (no growth) and to be analyzed below in Section C (positive growth) in which the monopolist owns new and used capital, besides providing an upper bound on the annual percentage price increase, may be reasonable descriptions of monopoly behavior.

C. Monopoly Ownership of Capital and Positive Growth

The demand price is $p_K^d = (Ke^{-at})^{-1/b}$; this specification incorporates exogenous demand growth at instantaneous percentage rate, a. Just prior to monopolization the market is in steady-state equilibrium, with constant marginal cost, c. Thus, the competitive rental price is (r+d)c, which equals p_K^d at t=0. That is, $(r+d)c = K^{-1/b}$ which implies $K_c^*(0) = [(r+d)c]^{-b}$. In the competitive steady-state equilibrium, output, k, is (a+d)K(t), which replaces depreciated capital and creates new capital so that rental price remains constant at (r+d)c.

Upon monopolization, the monopolist reduces output to zero. Price rises on two accounts. First, the stock, $K_c^*(0)$, depreciates moving up the demand price function and, second, the demand price function shifts to the right, which raises price for a given stock. The monopolist holds k=0 until marginal revenue equals marginal rental cost. Since stock demand is growing, marginal revenue grows, too. Specifically, $MR=[(b-1)/b]\ K^{-1/b}e^{at/b}= [(b-1)/b]p_K(t)$. When MR=(r+d)c, capital depreciation stops. The time lapse, $\hat{t}$, is found by solving

$$(12) \qquad [(b-1)/b][K(t)]^{-1/b}e^{at/b}=(r+d)c,$$

where $K(t)=K_c^*(0)e^{-dt}$. Thus,

$$(13) \qquad \hat{t}=[b/(a+b)][\ln b - \ln(b-1)].$$

The capital stock at $\hat{t}$ is $K_c^*(0)e^{-d\hat{t}}$. Price at $\hat{t}$ is b(r+d)c/(b-1). Beyond $\hat{t}$, capital is produced to maintain the equality between marginal revenue and marginal cost, and since $p_K=bMR/(b-1)$, price remains constant at $p_K(\hat{t})=b(r+d)c/(b-1)$. Production is positive: $k(\hat{t})=(a+d)\ K_c^*(0)e^{-d\hat{t}}$ and for $t>\hat{t}$, $k(t)=(a+d)K^*(t)$, where $K^*(t)=[K_c^*(0)e^{-d\hat{t}}]e^{a(t-\hat{t})}$.

The path of price from t=0 to t=t is found by inserting $K_c^*e^{-dt}$ into p_K^d:

(14) $$p_K^*(t)=[K_c^*e^{-dt}e^{-at}]^{-1/b}=(r+d)ce^{[(a+d)/b]t}$$

The predicted annual percentage price increase increase, assuming $\hat{t}>1$, is $e^{(a+d)/b}-1$.[21] There will be no relevant market whenever

(15) $$e^{(a+d)/b} < 1.1, \text{ or } 1-e^{-d} < 1-e^{a}(1.1)^{-b}.$$

Since the annual percentage rate of depreciation, D, is $1-e^{-d}$ and the annual percentage rate of growth is $G=e^a-1$, (15) can be rewritten as $D<1-(G+1)(1.1)^{-b}$. Thus, given b and G, the maximal value of D that will defeat market definition is

(16) $$D=1-(G+1)/(1.1)^b.$$

---

[21] If $\hat{t}<1$, the percentage price increase is $[b/(b-1)]-1=1/(b-1)$. Since $d\hat{t}/da<0$, $\hat{t}$ is more likely to be less than 1 the larger is a.

Figure 4 depicts the relationship between D and G given various values of b. For example, when b=1, new and used capital goods do not comprise a market if the applicable (G, D) combination falls on or below the line labeled b=1.

In summary, evidence about the expected annual percentage rate of growth and the annual percentage rate of depreciation is critical to market definition for capital goods. When growth is insignificant, new and used capital is unlikely to make a relevant market (and used capital is almost always in the market). However, when growth is significant, the Guidelines market definition criteria will be more easily satisfied. For example, if G>10% new and used capital is a relevant market even if capital is infinitely durable, when b=1.

IV. A DIGRESSION ON CAPITAL-STOCK DEMAND ELASTICITY

The above analysis shows that demand elasticity adjusted for depreciation and growth is critical for market definition. Typically, the determination of demand elasticity in antitrust analysis is rather impressionistic. That is, rarely are there econometric estimates of residual demand elasticity. Instead, the analysis relies on common sense about the closeness of available substitutes and on some theoretical rules of thumb. In the latter regard, for example, when market definition for an intermediate good is at issue it is commonly presumed that demand elasticity is inversely proportional to the input's percentage of final output value. Thus, an input that comprises only 1 percent of unit output cost will have a less elastic demand than one accounting for 10 percent of unit cost.

Like all rules of thumb, this one is too incomplete. As shown elsewhere (see Higgins and Shughart [7]), input demand elasticity depends on the elasticity of short-run output supply and on the elasticity of output demand. In particular, input demand elasticity is greater, ceteris paribus, the greater is output supply elasticity and the more cost increases are passed on to downstream customers. Furthermore, elasticity of output supply typically depends positively on the percentage of unit output cost that is sunk. The sunk-cost percentage depends on the degree to which other inputs (besides the input being analyzed for antitrust purposes) are specific to the output market(s) which uses the input question. Moreover, since capital is the quintessential sunk input factor, output supply elasticity depends negatively on the capital/variable account ratio. [22] This is all okay if the market definition exercise is for a noncapital input but what if a capital input factor is at issue?

If downstream output is flat -- perhaps, on account of imports -- the stock demand elasticity for a particular capital good will be negatively related to the ratio of the competitive rental cost of the capital good to the unit cost of output and to the ratio of the unit cost of all (other) sunk input to the unit cost of output. If, for example, all other inputs are "variable", then the demand for the capital good in question would be perfectly elastic. Alternatively, if there were a significant set of other sunk or specific inputs the demand for the capital good would still be very elastic if it were intensively used in production. If, on the other hand, the capital good were relatively unimportant in production, there would be a relatively

---

[22] In a Cobb-Doouglas production function the elasticity of shoort-run marginal cost depends negatively on the K/L ratio.

inelastic demand for its use. Note, however, that unless the user of capital owned the other specific inputs, which get exploited when capital's price is raised, there would be no "customer complaints."

If downstream output demand is not perfectly elastic, the demand for capital is more likely to be relatively inelastic thereby supporting an antitrust market. For example, even if there were no sunk inputs other than the capital in question, there would not be a perfectly elastic demand for capital on account of the ability of the capital users to pass on price increases. Note that in this case, the direct user of capital would not be harmed by a price increase at all; instead final customers would. The absence of direct purchasers' complaints about, say, an anticompetitive merger among capital-goods producers could then be misinterpreted.

Finally, the role of customer complaints is even more problematic when capital-goods markets are monopolized. Recall, that the analysis in Section III showed that the only way a monopoly capital-goods producer can raise capital's price is by reducing the stock of capital, which must await the depreciation of capital. For this reason, the users/owners of capital benefit from the monopolist's output restriction. Their capital increases in value. It is true that if the users also own other specific inputs they will lose more on this account than they gain. But, if the owners of the specific inputs whose presence accounts for the less-than-perfect elasticity of capital demand are not identical to the users/owners of capital, the users/owners of capital will benefit from a monopoly price increase. Thus, the management of the firms that use (and own the outstanding capital stock) the capital good would favor a merger that would result in an anticompetitive capital price increase.

V. CONCLUSION

The major finding in this paper is that relevant market definition for capital goods is much more problematic than for noncapital goods. Only when there is a large rate of growth of capital demand is there likely to be a relevant market for capital goods. This conclusion applies both to new and used capital and to a new capital only. The paper presents the combinations of the annual percentage rates of growth and depreciation and demand elasticity that yield a product market relevant for antitrust analysis.

An important remaining task is to analyze the likelihood of anticompetitive effect in capital-goods markets that pass the market definition test. For example, a dominant firm capital-goods producer will have to contend with fringe producers just as in the noncapital-goods case. But, for capital goods, the analysis would be somewhat more complex in light of the role for ownership of used capital discovered here. It was found that a monopoly capital-goods producer has higher value when the outstanding capital stock is also owned by the monopoly producer. It was also found that under some circumstances, the extant owners/users would sell to the monopoly producer. The existence of fringe competitors will have the effect of reducing the monopolists', and the existing owners,' willingness to trade capital. It is possible that there is no equilibrium exchange price for capital when there is a significant fringe.

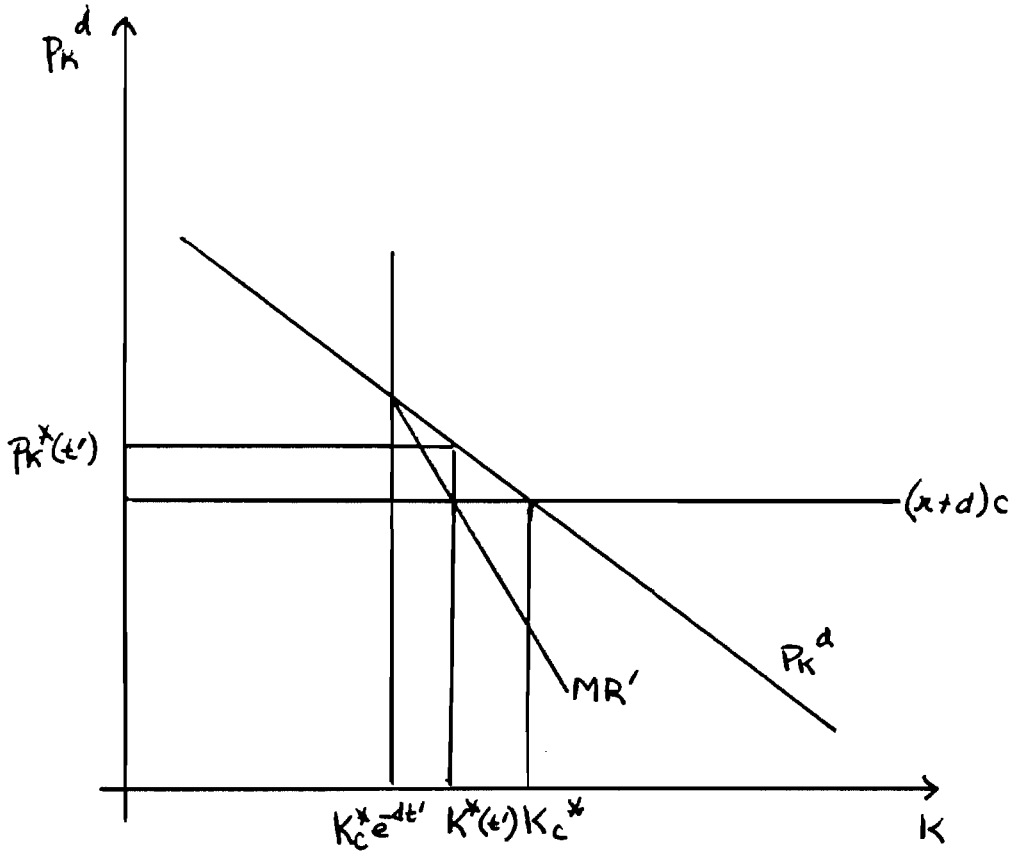

Figure 1

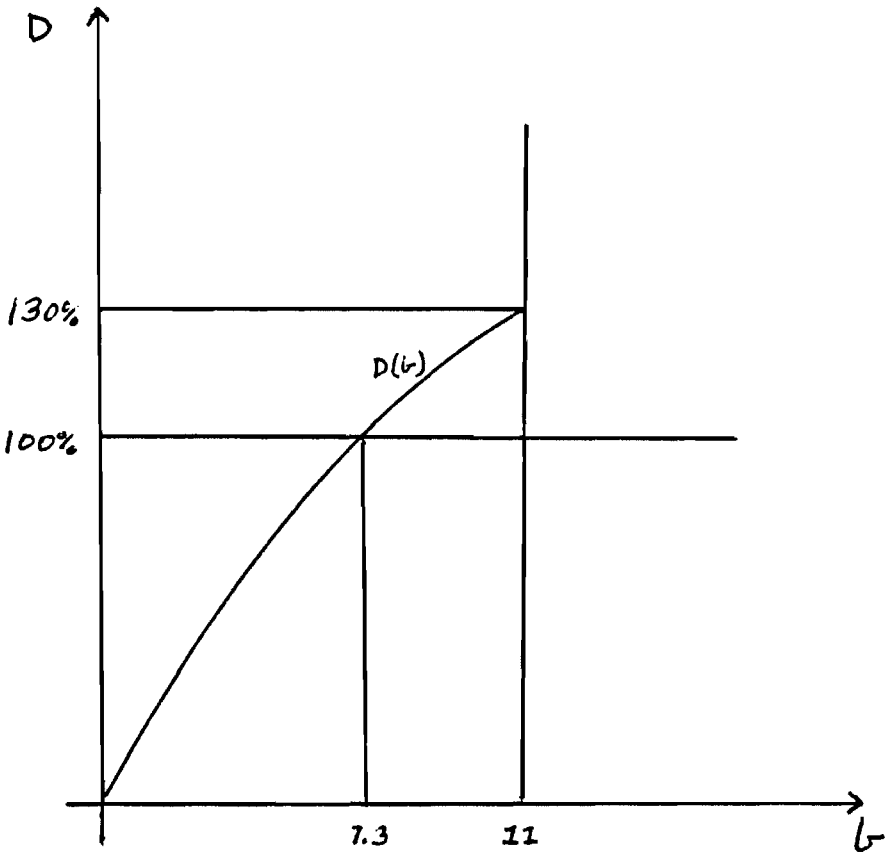

FIGURE 2

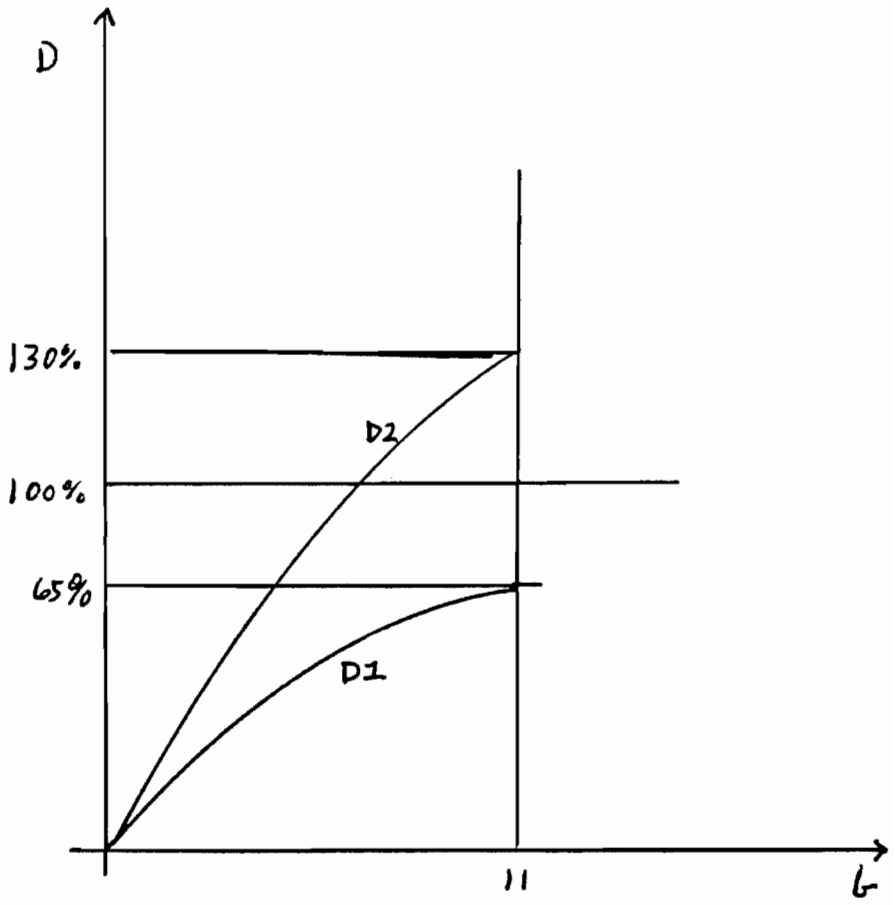

FIGURE 3

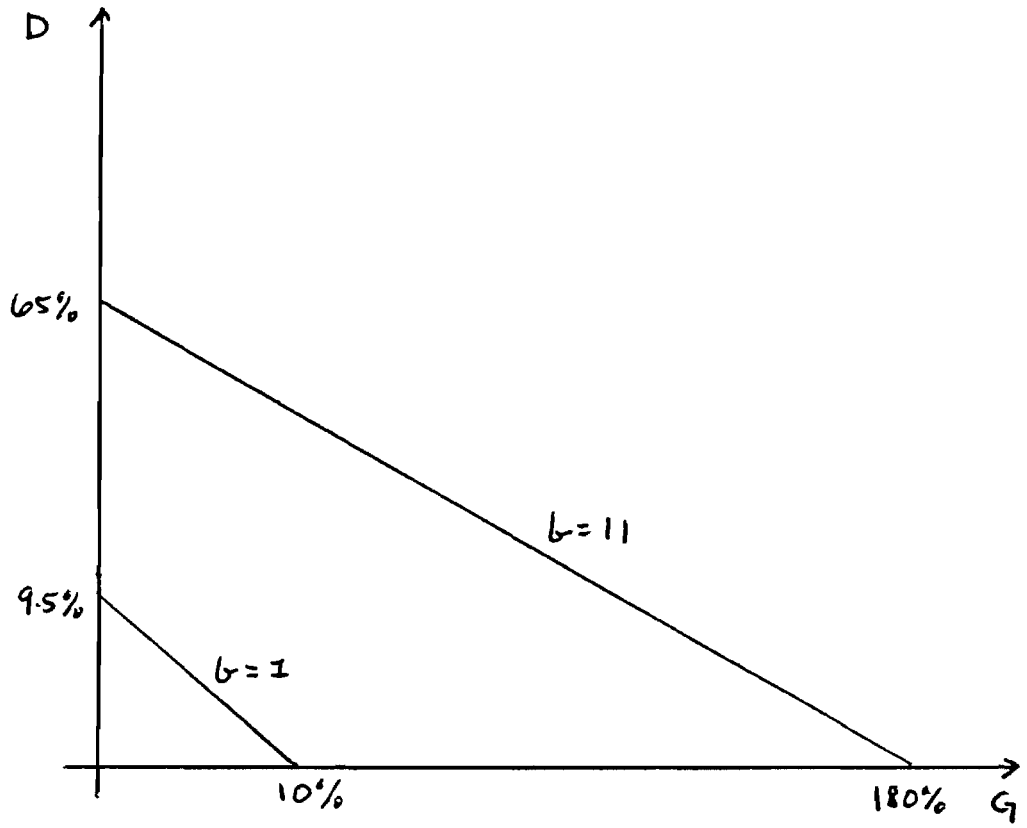

FIGURE 4

REFERENCES

1. Bond, E.W. and Samuelson, L. "Durable Goods Monopolies with Rational Expectations and Replacement Sales." Bell Journal of Economics. 15 (1984) 336-345.

2. Coase, Ronald, "Durability and Monopoly," Journal of Law and Economics, vol. xv, April 1972, 143-151.

3. Fudenberg, D. and Tirole, J. "Dynamic Models of Oligopoly." Technical Report #428, Institute for Mathematical Studies in the Social Sciences, Stanford University (1983).

4. Gaskins, D. "Aloca Revisted: The Welfare Implications of a Second-hand Market." Journal of Economic Theory 7 (1974) 254-271.

5. Grossman, Sanford and Oliver Hart, "Takeover Beds, the Free-rider Problem and the Theory of the Corporation," Bell Journal of Economics, vol. 11, Spring 1980, 42-65.

6. Hadley, G. and M. Kemp, Variational Methods in Economics, North-Holland, London, 1971.

7. Higgins, Richard and William Shughart, "Residual Demand Analysis of Input Market Definition," (mimeo, May 1987).

8. Kahn, C. "The Durable Goods Monopolist and Consistency with Increasing Costs" Econometrica 54 (1986) 275-290.

9. Rust, J "When is it optimal to kill of the market for used Durable Goods?" Econometrica 54 (1986) 65-86.

10. Stokey, N.L. "Rational Expectations and Durable Goods Pricing" Bell Journal of Economics 12 (1981) 112-128

11. Swan, P.L. "Alcoa: The Influence of Monopoly Power. Journal of Political Economy 88 (1982) 76-99.