HMG Review Project – Comment, Project No. P092900

This comment is in response to question 10.d (i.e., "The role of diversion ratios and price/cost margins in evaluating unilateral effects") in "Horizontal Merger Guidelines: Questions for Public Comment," FTC and DOJ, September 22, 2009. It suggests that revisions in that area could lead to a more accurate merger review process.

Cournot Competition and The UPP Test

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Introduction and summary

Farrell and Shapiro (2008) have proposed a test – called the Upward Pricing Pressure ("UPP") test – to evaluate potential unilateral effects of horizontal mergers.[2] The UPP test is based on the Bertrand model of price competition among suppliers of differentiated products.[3] Accordingly, the UPP test can be applied usefully in industries where suppliers first set prices and then supply the quantities that customers demand at those prices. Such Bertrand industries are characterized by relatively large amounts of excess capacity, so that *prices are driving quantities*.

This comment shows that the UPP test – suitably reinterpreted and properly implemented – can be applied usefully *also* in industries with Cournot competition where *quantities are driving prices*; that is, in Cournot industries where suppliers first set quantities or production capacities, and then prices adjust to ensure that demand equals supply.[4]

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[2] J. Farrell and C. Shapiro, "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition," working paper, 2008. http://faculty.haas.berkeley.edu/shapiro/alternative.pdf. *See* also G. Werden, "A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of Differentiated Products," *Journal of Industrial Economics*, 44 (1996), 409-413; D. O'Brien and S. Salop, "Competitive Effects of Partial Ownership: Financial Interest and Corporate Control," *Antitrust Law Journal*, 67 (2000), 559-614.

[3] For a formal treatment of Bertrand competition, *see*, e.g., M.R. Baye and D. Kovenock, "Bertrand Competition," in *The New Palgrave Dictionary of Economics*, 2nd Edition, Eds. S.N. Durlauf and L.E. Blume (Palgrave Macmillan), 2008. http://www.nash-equilibrium.com/baye/Bertrand_Palgrave2e.pdf.

[4] For a formal treatment of Cournot competition, *see*, e.g., J. Tirole, *The Theory of Industrial Organization*, section 5.7, The MIT Press, 1988. *See* also J. Farrell and C. Shapiro, "Horizontal Mergers: An Equilibrium Analysis," *American Economic Review*, 80 (1990), 107-26.

In Cournot industries with homogeneous products, the UPP test can be implemented correctly by using the formula developed by Farrell and Shapiro (2008) and assuming that the "diversion ratio" is equal to one.[5]

In Cournot industries with differentiated products, the UPP test also can be implemented using the formula of Farrell and Shapiro (2008) *provided* that one uses a definition of "diversion ratio" that is different from that used in the context of Bertrand industries. Specifically, the Farrell-Shapiro formula still applies if one uses the "price diversion ratio" instead of the "quantity diversion ratio."[6]

Interestingly, the price diversion ratio is equal to the quantity diversion ratio when there are only two firms. When there are more than two firms, I conjecture that the price diversion ratio is higher than the quantity diversion ratio. (I prove the result for the case with three firms.) Furthermore, I show that the price diversion ratio can be significantly higher than the quantity diversion ratio. This suggests that the Agencies would be conservative if they were to estimate the quantity diversion ratio and then apply the UPP test proposed by Farrell and Shapiro (2008) in both Bertrand and Cournot industries.

The general conclusion that seems to emerge from this comment is that the UPP test proposed by Farrell and Shapiro (suitably interpreted and properly implemented) could be useful for gauging potential unilateral effects in a variety of industries.[7] In particular, the UPP test – or other "price pressure indices" – could be used, possibly together with other considerations, to establish a "safe harbor" for unilateral effects and/or a presumption of potentially adverse unilateral effects.[8]

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[5] A diversion ratio equal to one and positive margins are not inconsistent with Bertrand price competition. For example, those characteristics arise in Hotelling's linear city model when there are two firms and the market is covered. *See* H. Hotelling, "Stability in Competition," *Economic Journal* 39 (1929), 41-57.

[6] In the Bertrand model, when a firm contemplates a unilateral price increase, it assumes that all the other firms will maintain their prices constant and expand output. Following a unilateral price increase by Firm 1, the quantity diversion ratio to Firm 2 is equal to the quantity of output gained by Firm 2 (given Firm 2's price) divided by the quantity of output lost by Firm 1.

In the Cournot model, when a firm contemplates a unilateral output reduction, it assumes that all the other firms will maintain their output levels constant and raise price. Following a unilateral output reduction by Firm 1, the price diversion ratio to Firm 2 is equal to the nominal price increase obtained by Firm 2 (given Firm 2's output) divided by the nominal price increase obtained by Firm 1. (In the special case with perfectly homogeneous products, the price increase is the same for both firms, and hence the price diversion ratio is one.)

[7] A companion comment shows that the UPP test (suitably reinterpreted and properly implemented) can be applied usefully also in industries with bidding competition. *See* S. Moresi, "Bidding Competition and The UPP Test," comment to HMG review process, 2009.

[8] This point is explained in more detail in S. Salop and S. Moresi, "Updating the Merger Guidelines: Comments," comment to HMG review process, 2009.

Cournot competition with homogeneous products

As explained in the Commentary, "in markets for homogeneous products, the Agencies consider whether proposed mergers would, once consummated, likely provide the incentive to restrict capacity or output significantly and thereby drive up prices."[9] The Agencies can address this issue by assuming a standard Cournot model of quantity competition and following the logic of the UPP test proposed by Farrell and Shapiro. This involves comparing the pre-merger first-order conditions of the two merging firms and the post-merger first-order conditions of the merged firm (both evaluated at the pre-merger equilibrium point).

The pre-merger first-order conditions of profit-maximization in the Cournot model with homogeneous products are:

$$P'(Q)q_i \;+\; P(Q) \;-\; C_i(q_i) \;=\; 0 \quad , \quad i = 1, \ldots, N \tag{1}$$

where:

N denotes the total number of suppliers in the industry under consideration;

q_i denotes the quantity supplied by supplier i (for $i = 1, \ldots, N$);

Q denotes the total quantity supplied by the N suppliers (i.e., $Q = q_1 + \ldots + q_N$);

$P(Q)$ denotes the inverse demand function faced by the industry (i.e., it gives the equilibrium price of the product as a function of the industry's total supply);

$P'(Q)$ denotes the derivative of $P(Q)$; and

$C_i(q_i)$ denotes the marginal cost function of supplier i.[10]

Hereafter, I will follow the Farrell-Shapiro notation and denote the pre-merger equilibrium values with an "upper bar" – e.g., $\overline{q}_i$ is the pre-merger value of q_i.

If suppliers 1 and 2 merge, the post-merger first-order condition for q_1 is:[11]

$$P'(Q)(q_1 + q_2) \;+\; P(Q) \;-\; (1-E)C_1(q_1) \;=\; 0 \tag{2}$$

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9 The Commentary suggests the U.S. Department of Justice used a merger simulation model based on the Cournot model in *United States v. Georgia-Pacific Corp. and Fort James Corp.* *See* U.S. Department of Justice and Federal Trade Commission, *Commentary on the Horizontal Merger Guidelines*, 2006, p. 27. http://ftc.gov/os/2006/03/CommentaryontheHorizontalMergerGuidelinesMarch2006.pdf

10 As usual, the demand and cost functions are assumed to be well-behaved. See footnote 4.

11 Equation (2) implicitly assumes an interior solution to the post-merger profit-maximization problem of the merged firm. The results described in this comment do not depend on this convenient assumption. *See* the discussion following the Corollary to Proposition 1.

where E denotes the "efficiencies credit" that the Agencies would be giving to the merging parties.[12]

Applying to the present Cournot setting the same approach as in Farrell and Shapiro (2008), one obtains the following result:[13]

Proposition 1 In the Cournot model of quantity competition with homogeneous products, a merger of firm 1 and firm 2 creates *upward pricing pressure* through a reduction in the quantity supplied by firm 1 *if and only if* the following condition is satisfied:

$$\bar{P} - \bar{C}_2 > E\bar{C}_1 \tag{3}$$

Proof Using (1) at $i = 1$, the left-hand side of (2) reduces to $P'(\bar{Q})\bar{q}_2 + EC_1(\bar{q}_1)$. Using (1) at $i = 2$, one finds $P'(\bar{Q})\bar{q}_2 = -P(\bar{Q}) + C_2(\bar{q}_2)$. It follows that the left-hand side of (2) is negative if and only if $-P(\bar{Q}) + C_2(\bar{q}_2) + EC_1(\bar{q}_1) < 0$. ■

Corollary In the Cournot model of quantity competition with homogeneous products, the appropriate UPP test is the same as in the Bertrand model of price competition with differentiated products, except that the diversion ratio must be set equal to one.

Proof Use (1) in Farrell and Shapiro (2008), set $D_{12} = 1$, and then compare with (3).[14] ■

A comment is in order. If the merging firms have different marginal costs of production, i.e., $\bar{C}_1 \neq \bar{C}_2$, then the merged firm can reduce total costs by reallocating output between the two firms' production facilities (while maintaining total output constant). Note that a similar production reallocation post-merger also can occur in Bertrand industries with differentiated products.[15] Thus, the same criticism would seem to apply to the UPP test of Farrell and Shapiro as well. However, this criticism is unlikely to be a serious one because the efficiencies from reallocating output can be included in the amount of the efficiency credit (E).[16]

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12 "The strength of the presumption established by the test can be adjusted—in the case of concentration measures, by choosing thresholds at which concentration evokes concern; in our case, by choosing how much credit to give for efficiencies." (Farrell & Shapiro, p. 3)

13 Proposition 1 can be extended to mergers involving more than two firms. *See* Proposition 1 in J. Farrell and C. Shapiro, "Horizontal Mergers: An Equilibrium Analysis," *American Economic Review*, 80 (1990), 107-26. Condition (3) in the text corresponds to condition (7) in their article.

14 In the standard Cournot model with homogeneous products, prices are identical, i.e., $\bar{P}_1 = \bar{P}_2 = \bar{P}$.

15 In *FTC v. H.J. Heinz Co.*, for example, the merging firms had plans to produce Beach-Nut babyfood products using the production facilities of Heinz.

16 This might require to use a different efficiency credit for each of the two merging firms.

The next section extends the above results to the case of Cournot quantity competition with differentiated products.

Cournot competition with differentiated products

The pre-merger first-order conditions of profit-maximization in the Cournot model with differentiated products are:

$$\frac{\partial P_i(q)}{\partial q_i} q_i \; + \; P_i(q) \; - \; C_i(q_i) \; = \; 0 \quad , \quad i = 1,\dots,N \tag{4}$$

where:

q denotes the quantity-vector supplied by the N suppliers (i.e., $q = (q_1,\dots,q_N)$);

$P_i(q)$ denotes the inverse demand function faced by supplier *i* (i.e., it gives the equilibrium price of product *i* as a function of the quantity-vector q supplied by all the suppliers); and

$\partial P_i(q) / \partial q_i$ denotes the partial derivative of $P_i(q)$ with respect to q_i.

If suppliers 1 and 2 merge, the post-merger first-order condition for q_1 is:

$$\frac{\partial P_1(q)}{\partial q_1} q_1 \; + \; \frac{\partial P_2(q)}{\partial q_1} q_2 \; + \; P_1(q) \; - \; (1-E) C_1(q_1) \; = \; 0 \tag{5}$$

This leads to the following result:

<u>Proposition 2</u> In the Cournot model of quantity competition with differentiated products, a merger of firm 1 and firm 2 creates *upward pricing pressure* through a reduction in the quantity supplied by firm 1 *if and only if* the following condition is satisfied:

$$\tilde{D}_{12}(\overline{P}_2 - \overline{C}_2) \; > \; E\overline{C}_1 \tag{6}$$

where $\tilde{D}_{12}$ denotes the "price diversion ratio" from firm 1 to firm 2:

$$\tilde{D}_{12} \; = \; \frac{\partial P_2(\overline{q})}{\partial q_1} \Bigg/ \frac{\partial P_2(\overline{q})}{\partial q_2} \tag{7}$$

<u>Proof</u> Using (4) at $i = 1$, the left-hand side of (5) reduces to $[\partial P_2(\overline{q}) / \partial q_1]\overline{q}_2 + EC_1(\overline{q}_1)$, which can be written as $\tilde{D}_{12}[\partial P_2(\overline{q}) / \partial q_2]\overline{q}_2 + EC_1(\overline{q}_1)$ using (7). Using (4) at $i = 2$, one finds $[\partial P_2(\bar{q})/\partial q_2]\bar{q}_2 = -\bar{P}_2(\bar{q}) + C_2(\bar{q}_2)$. It follows that the left-hand side of (5) is negative if and only if (6) holds. ■

Comments

(a) The UPP test for Cournot industries (i.e., Equation (6)) involves the "same" formula as the UPP test for Bertrand industries (i.e., Equation (1) in Farrell and Shapiro (2008)). That is, both tests compare the product of the diversion ratio and the margin (which gauges the anticompetitive effect from reduced competition) and the presumed reduction in variable costs (which gauges the procompetitive effect from presumed efficiencies).

(b) The margin of the merging partner (i.e., $\bar{P}_2 - \bar{C}_2$) and the presumed reduction in variable costs (i.e., $E\bar{C}_1$) are defined in the same way in both tests.

(c) The only difference is that each test uses a different definition of "diversion ratio" as explained next.

The UPP test for Bertrand industries, as developed by Farrell and Shapiro (2008), uses the concept of *quantity diversion ratio* because, when firm 1 unilaterally raises price and reduces output, firm 2 benefits by selling a higher quantity at the same price. The larger the quantity gained by firm 2 relative to the quantity lost by firm 1 – i.e., the larger the (quantity) diversion ratio from firm 1 to firm 2 – the stronger firm 1's incentive to raise price post-merger (*ceteris paribus*).

In sharp contrast, the UPP test for Cournot industries uses the concept of *price diversion ratio* because, when firm 1 unilaterally reduces output and raises price, firm 2 benefits by obtaining a higher price for the same quantity. The larger the price increase obtained by firm 2 (when firm 1 reduces output) relative to the price increase that firm 2 could obtain by reducing its own output directly – i.e., the larger the (price) diversion ratio from firm 1 to firm 2 – the stronger firm 1's incentive to reduce output post-merger (*ceteris paribus*).

The price diversion ratio is equal to the quantity diversion ratio when there are only two firms. When there are more than two firms, I conjecture that the price diversion ratio is higher than the quantity diversion ratio. The following proposition shows that the conjecture is true when there are three firms.

Proposition 3 When there are only two firms, the price diversion ratio is identical to the quantity diversion ratio. When there are three firms, the price diversion ratio is higher than the quantity diversion ratio.

Proof Let $q = d(P)$ be the demand system and let $(\bar{q}, \bar{P})$ be the pre-merger point, so that $\bar{q} = d(\bar{P})$. Assume that the Jacobian determinant of d at $\bar{P}$ is nonzero, i.e., $\left|J_d(\bar{P})\right| \neq 0$, so that $q = d(P)$ can be inverted to $P = P(q)$ in a neighborhood of $(\bar{q}, \bar{P})$. Form the Inverse Function Theorem, $J_P(\bar{q}) = \left[J_d(\bar{P})\right]^{-1}$. It follows that:

$$\tilde{D}_{12} = \frac{\partial P_2(\overline{q})}{\partial q_1} \Bigg/ \frac{\partial P_2(\overline{q})}{\partial q_2} = \frac{C_{12}}{C_{22}} \qquad (8)$$

where C_{ij} denotes a cofactor of $J_d(\overline{P})$. When there are three firms:

$$C_{12} = -\begin{vmatrix} d_{21} & d_{23} \\ d_{31} & d_{33} \end{vmatrix} \quad \text{and} \quad C_{22} = \begin{vmatrix} d_{11} & d_{13} \\ d_{31} & d_{33} \end{vmatrix} \qquad (9)$$

where $d_{ij} = \partial d_i(P)/\partial P_j$ evaluated at $P = \overline{P}$. Thus, (8) can be rewritten as:

$$\tilde{D}_{12} = \frac{-d_{21}d_{33} + d_{23}d_{31}}{d_{11}d_{33} - d_{13}d_{31}} = \frac{D_{12} + D_{13}D_{32}}{1 - D_{13}D_{31}} \qquad (10)$$

where $D_{ij} = -d_{ji}/d_{ii}$ is the quantity diversion ratio from firm *i* to firm *j*. Thus, when there are only two firms, $D_{32} = D_{31} = D_{13} = 0$ implies $\tilde{D}_{12} = D_{12}$. When there are three firms, $D_{ij} \in [0,1]$ implies $\tilde{D}_{12} \geq D_{12}$. ■

Roughly speaking, the term $D_{13}D_{32}$ in the numerator of (10) reflects the fact that, in the Cournot model, firm 3 raises its price (following an output reduction by firm 1) and thus the quantity diversion that firm 3 would have obtained in the Bertrand model (D_{13}) is in part "deflected" to firm 2 when firm 3 raises its price (D_{32}). Similarly, the term $D_{13}D_{31}$ in the denominator of (10) reflects the fact that the quantity diversion that firm 3 would have obtained in the Bertrand model (D_{13}) is in part "deflected back" to firm 1 when firm 3 raises its price (D_{31}).

Finally, (10) suggests that the price diversion ratio, $\tilde{D}_{12}$, may be significantly larger than the quantity diversion ratio, D_{12}. For example, if $D_{12} = D_{32} = D_{31} = D_{13} = 30\%$, then $\tilde{D}_{12} \cong 43\%$.