HMG Review Project – Comment, Project No. P092900

This comment is in response to the invitation from the FTC and the DOJ for public comments with regard to the possible update of the Horizontal Merger Guidelines. In particular, this comment is in response to questions 1, 10c, and 12 in "Horizontal Merger Guidelines: Questions for Public Comment," FTC and DOJ, September 22, 2009. The comment addresses issues that could lead to a more accurate and more efficient merger review process.

The HHI beyond the Cournot Model

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Introduction and Summary

An often-heard criticism against the use of the Herfindahl-Hirschman Index (henceforth, "HHI") for evaluating competitive effects (especially lessening of competition due to unilateral effects) is that it lacks rigorous analytic underpinnings, outside the context of the Cournot competition model. This comment describes a bargaining model, very different from the Cournot model, in which consumer surplus is directly linked to the HHI.

It should be emphasized upfront that the HHI emerges in the context of the specific bargaining model described below and, hence, this comment does not aim at making a general statement about the role or validity of the HHI in general bargaining environments. Rather, the comment's main purpose is to bring to the attention of the FTC and DOJ economists an economic model that could be fruitfully applied to some

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[1] Charles River Associates, ysarafidis@crai.com. This comment is based on my working paper, "A Model of Ordered Bargaining with Applications" (2009), jointly authored with Serge Moresi and Steven Salop. The paper is available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1287224. The analysis and conclusions in this comment and in the working paper do not represent the views of Charles River Associates.

real-life bargaining interactions in which the HHI emerges and is linked to consumer surplus. To the best of my knowledge, with the exception of the Cournot model, there is no other model in the economics literature which supports using the change in the HHI as an appropriate measure of unilateral competitive effects of a merger.

In particular, the bargaining model described in this comment is motivated by situations where a *lead negotiator* (buyer) bargains with several partners (sellers). This lead negotiator could be, for example, a health maintenance organization ("HMO") bargaining with several hospitals and clinics that it would like to include in its network, or a multiple system operator ("MSO") bargaining with several television channels and cable networks that it would like to include in its program offering to subscribers. The model's key features are that the buyer bargains with each seller sequentially, one-at-a-time, and wants to contract with as many sellers as possible. Under certain assumptions, it is shown that the buyer's equilibrium payoff is proportional to the sellers' HHI. Hence, following a merger the change in the buyer's welfare will be proportional to the change in the HHI.

A Model of Ordered Bargaining[2]

Consider the following bargaining set up. A large buyer bargains with n sellers of a good. The timing of events is as follows: i) the buyer bargains sequentially with each seller in a pre-determined order and one-at-a-time, ii) when bargaining with a seller ends, the buyer approaches the next seller in the bargaining queue, and iii) the buyer can approach each seller only once. Thus, we can index sellers by $i = 1, 2, ..., n$ according to their position in the bargaining queue.

Furthermore, suppose that each seller i has an inelastic supply of x_i units of the good. The buyer and each seller i engage in Nash bargaining over the transfer payment t_i that the seller requires from the buyer in exchange for his supply of the good, x_i. When

[2] Refer to Moresi, Salop and Sarafidis (2009) for more details and more results related to the model's application to merger analyses.

bargaining with seller *i*, the bargaining power coefficients are α_i and $1-\alpha_i$ for the buyer and seller *i*, respectively. Moreover, the sellers have no outside option and, hence, their disagreement payoffs are zero. The buyer's outside option when bargaining with seller *i* is also zero, but note that her disagreement payoff is determined endogenously in the game.

Finally, we assume that the buyer and the sellers have preferences that are given respectively by:

$$U = u(\sum_{i=1}^{n} x_i I_i) - \sum_{i=1}^{n} t_i I_i$$
$$\Pi_i = t_i I_i$$

where $u(\cdot)$ can be any non-decreasing function and I_i denotes the indicator function that takes the value 1, if the buyer and seller *i* reach an agreement.

One can establish the following intermediate result.

Proposition 1 The buyer's equilibrium payoff is equal to the expected utility of the lottery $L = \sum_{i=1}^{n} x_i y_i$, where each y_i is a Bernoulli random variable that takes the value 1 with probability α_i.

Proof

See proof of proposition 1 in Moresi, Salop and Sarafidis (2009). ■

In other words, we can give to the buyer's equilibrium payoff the following interpretation. Suppose that instead of bargaining with the buyer, each seller *i* were to flip a coin that landed on *HEADS* with probability $1-\alpha_i$. (Notice that $1-\alpha_i$ is the bargaining power coefficient of seller *i* when bargaining with the buyer.) If the coin lands *HEADS*, then the buyer and the seller do not trade; otherwise the seller surrenders his supply of x_i units to the buyer for free. Then, according to Proposition 1, the buyer's expected utility from this lottery is equal to the equilibrium payoff in our original bargaining game. Notice also that an immediate corollary of Proposition 1 is that the order of bargaining does not matter from the buyer's point of view.

Armed with this result, one can show that when all sellers have the same bargaining power coefficient and the buyer's utility function is linear-quadratic, then the buyer's equilibrium payoff is linear in the HHI.

Proposition 2 Let each seller's bargaining power coefficient be equal to $1-\alpha$, normalize each seller's supply x_i so that $\sum_{i=1}^{n} x_i = 1$ and suppose that the buyer's utility function is of the form: $u(z) = z - \frac{1}{2}z^2$. Then the buyer's equilibrium payoff is linear in the HHI.

Proof

By Proposition 1, we know that the buyer's equilibrium payoff is equal to the expected utility of the lottery $L = \sum_{i=1}^{n} x_i y_i$, where each y_i is a Bernoulli random variable that takes the value 1 with probability $\alpha_i = \alpha$ (where the last equality follows from the fact that all sellers have the same bargaining power coefficient). Given the buyer's utility function, the buyer's equilibrium payoff is a function of the lottery's expected value and variance, which are equal to α and $\alpha(1-\alpha)\sum_{i=1}^{n} x_i^2$, respectively. In particular, the buyer's payoff is equal to:

$$U = E[L] - \frac{1}{2}E[L^2] = E[L] - \frac{1}{2}(E[L]^2 + Var[L]) = \alpha - \frac{1}{2}\alpha^2 - \frac{1}{2}\alpha(1-\alpha)\sum_{i=1}^{n} x_i^2$$

The result follows once one notices that the HHI is equal to $\sum_{i=1}^{n} x_i^2$. ■

A corollary of this result is that following a merger between any two sellers *i* and *j* (and assuming that the bargaining power coefficient of the merged firm is also equal to α), the change in the buyer's payoff due to the merger will be proportional to the change in the HHI. That is: $\Delta U = \frac{1}{2}\alpha(1-\alpha)(2x_i x_j) = \frac{1}{2}\alpha(1-\alpha)\Delta HHI$.[3] In turn, this implies that the change in the buyer's payoff due to the merger is most pronounced (in absolute terms) when the buyer and each seller have the same bargaining power coefficient, i.e., when $\alpha = \frac{1}{2}$.[4] To see the intuition behind this result, notice that in the polar cases where either the buyer or the sellers have all the bargaining power, i.e., $\alpha = 1$ or $\alpha = 0$, respectively, the merger has no effect on the buyer's payoff. In the former case the buyer gets all the bargaining surplus, whereas in the latter the buyer' payoff is zero, irrespective of the number of seller.

[3] Notice that, because the supplies x_i are fixed, the merged firm's share in the post-merger equilibrium is equal to $(x_i + x_j)$. Hence, the difference between the post- and pre-merger HHI is equal to $2x_i x_j = (x_i + x_j)^2 - (x_i^2 + x_j^2)$. In contrast, in the Cournot model the expression $2x_i x_j$ does not coincide with the difference between the post- and pre-merger *equilibrium* HHI, because the merged firm's share in the post-merger equilibrium is not equal to $(x_i + x_j)$.

[4] This follows from the fact that the expression $\alpha(1-\alpha)$ is maximized when $\alpha = \frac{1}{2}$.