Exclusionary Discounts

Janusz Ordover[1] and Greg Shaffer[2]

August 25, 2006[3]

Abstract

We consider a two-period model with two sellers and one buyer in which the efficient outcome calls for the buyer to purchase one unit from each seller in each period. We show that when the buyer's valuations between periods are linked by switching costs and at least one seller is financially constrained, there exist plausible conditions under which exclusion arises as the unique equilibrium outcome. The exclusionary equilibria are supported by price-quantity schedules in which the excluding firm offers to sell its second unit to the buyer at a marginal price that is below its marginal cost of production. In some circumstances, the second unit is offered at a negative marginal price. Our findings have policy implications, and they contribute to the literatures on exclusive dealing, bundling, loyalty rebates, all-units discounts, and market-share discounts.

Keywords: Exclusive Dealing, Bundling, Loyalty Rebates, Market-Share Discounts.

[1]New York University and Competition Policy Associates; jao@snet.net.

[2]Simon School of Business, University of Rochester; shaffer@simon.rochester.edu.

[3]We thank seminar participants at the Fourth Annual International Industrial Organization Conference (Boston: 2006) and the Centre for Competition Policy at the University of East Anglia for helpful comments. We would like to thank Advanced Micro Devices for support of this research. The views expressed here are solely those of the authors.

1 Introduction

Manufacturers often encourage retailers to promote their products by offering discounts if the retailers' purchases meet or exceed certain quantity or share thresholds. Sometimes the discounts apply only to the incremental units purchased by a retailer (an incremental-units discount), while in other cases, the discounts apply to all the units purchased by a retailer (an all-units discount), once the target threshold is reached.[1] Generally these discounts are a sign of healthy price competition, and are often required by buyers as a price of doing business. But, as we will show, when implemented by a dominant manufacturer, who may have easier access to financing than its rival or rivals, they can exclude equally efficient rivals, misallocate resources, and lower consumer welfare.

Like a two-part tariff, incremental-units discounts and all-units discounts have the property that a buyer's *average* price is weakly decreasing in its purchases. Unlike a two-part tariff, however, they also have the property that a buyer's *marginal* price is lower with higher quantities, thus allowing a manufacturer, for example, to earn higher per-unit profit on its inframarginal sales than it earns on its marginal sales, and vice-versa for its retailers. It is this latter property that distinguishes these discounts and gives rise to a double-edged sword. On the one hand, this feature makes them potentially procompetitive because it creates incentives for a retailer to deploy market strategies that expand output. But, on the other hand, this feature also makes them potentially anticompetitive because it creates incentives for a retailer to promote the sale of products on which it is eligible to earn an additional discount potentially at the expense of other, substitute products, whose manufacturers may not feature similar discounts, or where the discounts offered are not of the same magnitude. Indeed, when the discounts apply to all units purchased by a retailer once the threshold is reached, the incentives for a retailer to promote a manufacturer's product exclusively when it is close to reaching a threshold are particularly strong because the marginal price it faces on the manufacturer's product actually turns negative at the break-point (due to the rebate it receives) before this price adjusts to its new lower rate for units purchased beyond the threshold.

Claiming that these discounts can be anticompetitive is one thing. Showing it in a fully-specified model where all relevant agents are acting rationally is another. It is not obvious, for example, how

[1] For example, in the former case, a manufacturer might specify a price of \$3 per unit for the first 999 units and a price of \$2.50 per unit thereafter, whereas in the latter case, the reduced price would also apply to the first 999 units on purchases of 1000 units or more. Thus, a retailer would receive a rebate of \$.50 x 999 on purchases of 1000 units or more when the discount applies to all units, but it would receive no rebate under an incremental-units discount.

or why a manufacturer would find it profitable to induce a retailer to switch some consumers away from a product or product line they might prefer (and therefore on which they would be willing to pay a higher price) toward a product or product line they like less, and even if it might be profitable to do so in some circumstances, it begs the question why cannot the manufacturer's rival offer the retailer an even more advantageous deal to stop it (after all, these consumers favor its product)? Indeed, in a first-best world, and in many actual market settings, the rival manufacturer likely will be able profitably to countervail the attempt at diversion.[2] But in many other realistic market settings, there may be impediments to competition that prevent efficient outcomes from emerging.

In this paper we seek to better understand how and why inefficient exclusion can arise even when competing suppliers are equally efficient in terms of their marginal production costs, and to identify critical factors that may prevent firms from competing on a level playing field. To this end, we consider a two-period model with two manufacturers and one retailer in which the efficient outcome calls for the retailer to buy from both manufacturers in each period. We show that when the retailer's—or his customers'— valuations between periods are linked by switching costs and at least one manufacturer is financially constrained, there are plausible conditions under which exclusion (the retailer buys from only one manufacturer) arises as the unique equilibrium outcome. Importantly, exclusive-dealing provisions in which a manufacturer requires the retailer to deal exclusively with it are not needed to obtain this outcome if quantity discounts are feasible.[3]

In addition to offering a new explanation for how and why inefficient exclusion can arise in equilibrium, our analysis breaks new ground in two other important ways. To our knowledge, we are the first to focus on the role of negative marginal prices in supporting equilibrium outcomes—when are they needed, when are they are superfluous, and when do they not support equilibria. We find, for example, that negative marginal prices can arise (under some conditions) in both efficient and exclusionary equilibria, but whereas they cannot always support efficient equilibria (when these equilibria exist), they can always support exclusionary equilibria (when these equilibria exist). More generally, we find that a common feature of all exclusionary equilibria is that they are supported by price-quantity schedules in which at some point along its schedule the excluding firm offers to

[2]Such countervailing strategies likely will benefit consumers.

[3]This challenges traditional antitrust analysis because, as Tom et al (2000) note, "The traditional analysis governing exclusive dealing arrangements has focused on a manufacturer's *requirement* that its distributors deal *exclusively* with it. In recent years, however, some manufacturers have begun to use subtler arrangements in which incentives replace requirements ...." They argue that these incentives should be analyzed like any other exclusionary conduct.

sell its product at a marginal price that is below its marginal cost of production. In some cases the excluding firm must offer a negative marginal price to induce exclusion; in other cases it need not.

We are also the first, to our knowledge, to examine the effects of potential remedies in a fully specified model in which the alleged harmful discounts can arise in both efficient and exclusionary equilibria. A vexing problem in antitrust is how to distinguish discounts that are procompetitive from discounts that are anticompetitive. If the dividing line is drawn too aggressively, the policy risks chilling price competition that would ultimately benefit consumers in the form of lower retail prices, but if the line is drawn too passively, there may still be room for anticompetitive behavior on the part of the dominant manufacturer, which ultimately leads to fewer product choices for consumers and higher retail prices. We consider these tradeoffs by considering first the consequences in our model of a ban on contracts in which marginal prices are below marginal cost. We then consider the effects of only a ban on negative marginal prices. We find that the former always yields a first-best outcome in the model; although the ban impacts both efficient and exclusionary equilibria, efficient equilibria survive (albeit with rent-shifting effects), whereas exclusionary equilibria do not survive. The latter ban, however, does not always eliminate the possibility of exclusion. It does, however, reduce the set of circumstances under which such equilibria can arise, and in that sense, it is welfare improving. The latter, of course, may also be easier to implement than the former when the excluding firm's marginal costs of production are not generally known or easily verified.

Our findings contribute to the literatures on exclusive dealing, bundling, loyalty rebates, all-units discounts, and market-share discounts. The majority of models of exclusionary conduct come from the first of these literatures, where a dominant manufacturer employs an explicit exclusive-dealing provision in its contracts with retailers.[4] In these models (as in our model), the dominant firm finds itself in an environment in which profits are linked across time or across markets and in which there are contracting externalities that keep the dominant firm from bearing the full cost of inducing exclusion. For example, Bernheim and Whinston (1998) show that exclusion can arise when there are 'noncoincident markets,' where the exclusive-dealing arrangements serve to extract rents from markets other than the ones in which they are employed. Rasmusen et al. (1991), and Segal and Whinston (2000a) show that exclusion can arise when there are economies of scale in production and multiple retail markets where the buyers' do not coordinate their decisions and no

[4] Although our focus is on exclusionary conduct, it is well known that exclusive-dealing arrangements may also have efficiency rationales. See, for example, Marvel (1982), Ornstein (1989), and Segal and Whinston (2000b).

one retailer is large enough to ensure the survival of the potential entrant. And, finally, Aghion and Bolton (1987) show that exclusion can arise when a dominant manufacturer leverages its first-mover advantage to extract rents from a more efficient entrant, when the entrant's costs are unknown.[5]

More recently, Nalebuff (2004) and Greenlee et al (2004) show how bundling by a multi-product firm can lead to the exclusion of an equally efficient rival, albeit one that produces a single-product.[6] In these models, as in the aforementioned models of exclusive dealing, the dominant manufacturer is assumed to have a first-mover advantage (or in the model of Bernheim and Whinston, it is assumed that retail markets develop sequentially, long-term contracts are feasible, and economies are such that a seller must compete in both markets). The current paper differs in that we assume the sellers' contracts are offered to the buyer simultaneously, and there are no economies of scale to exploit. In addition, there are no coordination issues as there is only one buyer, and the ability of the entrant to offer its own bundle does not ensure its survival as it does in the bundling models.

The literatures on loyalty rebates, all-units discounts, and market-share discounts have so far mostly focused on why these discounts might arise even in the absence of exclusionary motives. Papers by D. Spector, B. Kobayashi, and A. Heimler in "A Symposium on Loyalty Rebates" (2005) discuss anticompetitive uses of loyalty rebates but do not rigorously model the market settings in which the rebates arise in equilibrium of a well-specified game. These papers suggest various rule-of-reason screens that would delineate anti-competitive from permissable loyalty rebates. Greenlee and Reitman (2004) view loyalty rebates as a means of extracting surplus from heterogeneous buyers when other types of nonlinear pricing, for example, two-part tariffs, are infeasible. Kolay et al (2004) show how all-units discounts can be used by an upstream monopolist to induce self-selection downstream when retailers differ in size or there is uncertainty in demand. Marx and Shaffer (2004) show how discounts that are conditioned on a manufacturer's share of a retailer's purchases (market-share discounts) can enhance rent-extraction from a rival manufacturer. Mills (2004) shows that market-share discounts can be used as a means of rewarding ex-post retailers who offer promotional benefits, and thus argues that they can mitigate free-riding. Marvel and Yang (2006) find that market-share discounts can increase competition by putting all consumers

[5] Erutku (2006) extends Aghion and Bolton to the case of multiple competing retailers and shows how the dominant manufacturer can induce exclusion by offering rebates to retailers conditional on their signing exclusivity agreements.

[6] Whinston (1990) was the first to formalize the notion that a dominant firm could extend its market power from one market into another by conditioning the sale of one product to the sales of another. More recently, Carlton and Waldman (2002) examine how tying can be used to foreclose competitors in industries with network externalities. See also Choi and Stefanadis (2001), DeGraba (1999), and the survey by Whinston (2001) on exclusivity and tying.

"in play" as opposed to only those at the margin, which happens when firms employ linear pricing.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 derives the key condition that enables exclusion and provides illustrative examples. Section 4 characterizes necessary and sufficient conditions for equilibria to exist and discusses the role of switching costs and financing constraints in supporting exclusion. Section 5 focuses on the sellers' contract terms and considers the effects of various restrictions on the space of allowable contracts. Section 6 considers extensions to the model, and section 7 discusses the implications of our findings for antitrust policy.

2 Model

We consider a simple two-period model in which there is one buyer and two sellers. For ease of exposition, we call one seller 'the entrant' and the other seller 'the incumbent.' In each period, the buyer wants at most two units of the products sold by the sellers. It can purchase both units from the incumbent, one unit from each seller, one unit from one seller, or no units. For now, we assume the buyer cannot purchase both units from the entrant (we will relax this assumption in section 6). For example, one can think of the entrant as being constrained in its capacity, and hence, as not being able to supply a second unit, or one can think of the buyer's demand as consisting of both a captive unit and a contestable unit, where the captive unit can only be supplied by the incumbent.

Let the buyer's valuations for the set of possible combinations in period one be denoted by

$$V^1_{II},\;\; V^1_{IE},\;\; V^1_{IO},\;\; V^1_{EO},$$

and V^1_{OO}, respectively, where the superscript 1 denotes period one, and the subscripts II, IE, IO, EO, and OO denote what the buyer purchases.[7] For example, the subscript II denotes the case in which the buyer purchases both units from the incumbent, the subscript IE denotes the case in which the buyer purchases one unit from each seller, and the subscripts IO and EO denote the cases in which the buyer purchases one unit from only the incumbent or only the entrant, respectively. We assume the buyer's valuation is zero if it purchases no units from either seller, i.e., $V^1_{OO} = 0$.

We assume the sellers have complete information (i.e., they know each other's costs and the buyer's valuations) and can make take-it-or-leave-it offers to the buyer. We also assume they have the same constant marginal cost of production, which we denote by c. There are no fixed costs.[8]

[7]One can think of the buyer either as being a final consumer of the sellers' products or as being a downstream firm whose valuations are derived from the revenue it expects to earn from reselling the products in its retail markets.

[8]Our assumption of no fixed costs and constant marginal costs implies that there are no economies of scale, which,

The buyer's valuations in the two periods are linked by switching costs. In period one, before it becomes 'locked-in,' we assume the buyer is willing to pay more than c for each unit it purchases. Thus, for example, we assume its willingness-to-pay for two units of the incumbent's product exceeds by more than c its willingness-to-pay for only one unit of the incumbent's product, and its willingness-to-pay for one unit of each seller's product exceeds by more than c the maximum of its willingness-to-pay for only one unit of the incumbent's product or only one unit of the entrant's product. We assume the buyer prefers variety in the sense that its willingness-to-pay for one unit of each seller's product exceeds its willingness-to-pay for two units from the incumbent. Lastly, we assume the buyer's willingness-to-pay for one unit of each seller's product is weakly less than the sum of its willingness-to-pay for each unit alone. These assumptions can be summarized as follows:

$$V_{IO}^1 > c, \quad V_{EO}^1 > c, \quad V_{II}^1 > V_{IO}^1 + c, \quad V_{IE}^1 > \max\{V_{IO}^1 + c, V_{EO}^1 + c\}, \tag{1}$$

$$V_{IE}^1 > V_{II}^1, \quad \text{and} \quad V_{IE}^1 \leq V_{IO}^1 + V_{EO}^1.$$

In period two, the buyer becomes locked-in to the seller or the sellers from whom it purchased in period one, making switching between the sellers prohibitively costly.[9] Thus, for example, a seller's product has zero value to the buyer in period two if the seller did not sell to the buyer in period one. And, if the contestable unit was supplied by the incumbent in period one, then it can only be supplied by the incumbent in period two, and similarly if it was supplied by the entrant.

We can write the buyer's valuations for the set of possible combinations in period two as

$$V_{IO}^2(k), \ V_{EO}^2(k), \ V_{II}^2(k), \quad \text{and} \quad V_{IE}^2(k),$$

where $k \in \{IE, I, E, O\}$ denotes whether the buyer purchased one unit from each seller in period one, only from the incumbent in period one, only from the entrant in period one, or nothing. Our assumptions above imply that if the buyer purchased only from the incumbent in period one, then

$$V_{IO}^2(I) > c, \quad V_{EO}^2(I) = 0, \quad V_{II}^2(I) > V_{IO}^2(I) + c, \quad \text{and} \quad V_{IE}^2(I) = V_{IO}^2(I), \tag{2}$$

and if the buyer purchased only from the entrant in period one, then

$$V_{IO}^2(E) = 0, \quad V_{EO}^2(E) > c, \quad V_{II}^2(E) = 0, \quad \text{and} \quad V_{IE}^2(E) = V_{EO}^2(E), \tag{3}$$

as is well known, drives many of the results on exclusion (e.g., Rasmusen et al, 1990). Our assumption that marginal costs are the same implies that the entrant and the incumbent are equally efficient. Our assumption that the sellers have complete information allows us to abstract from the type of exclusion that arises in Aghion and Bolton (1987).

[9]There are any number of reasons why the buyer might become locked-in to a seller's product. We refer the interested reader to the literature on switching costs. See, e.g., Klemperer (1987a) and Farrell and Shapiro (1988).

and if the buyer purchased one unit from each seller in period one, then

$$V_{IE}^2(IE) = V_I^2 + V_E^2, \quad V_{II}^2(IE) = V_I^2, \quad V_{IO}^2(IE) = V_I^2, \quad \text{and} \quad V_{EO}^2(IE) = V_E^2, \tag{4}$$

where V_I^2 and V_E^2 are the utilities that the buyer receives in the second period from the captive unit and the contestable unit, respectively, when the buyer is purchasing one unit from each seller.[10]

We also make some additional assumptions which are analogous to those made in period one:

$$V_{IE}^2(IE) > \max\{V_{II}^2(I), V_{EO}^2(E) + c\}, \quad V_{IO}^2(I) \geq V_I^2, \quad \text{and} \quad V_{EO}^2(E) \geq V_E^2. \tag{5}$$

Given our assumptions in (2)–(5), the efficient outcome is for the buyer to purchase one unit from each seller in period one and, given that it has done so, to purchase one unit from each seller in period two (i.e., overall surplus over the two periods is maximized at $V_{IE}^1 - 2c + V_{IE}^2(IE) - 2c$).[11]

The sellers' offers, which are made simultaneously, consist of menus of price-quantity pairs. Let $T_I^i = (T_I^i(1), T_I^i(2))$ and $T_E^i = T_E^i(1)$ denote the incumbent and the entrant's offer, respectively, in period i, where the numbers in parentheses denote the number of units offered to the buyer. If the buyer purchases zero units, its payment is zero. Otherwise, the buyer must pay each seller according to its offer. For now, we place no restrictions on the allowable form of contracts (later, we will restrict the allowable form of contracts, for example, to rule out negative marginal prices).

We use subgame perfection as our solution concept, and we begin by solving for the equilibrium in period two as a function of the buyer's first-period choices. Given our assumptions, this is relatively straightforward to do. In any equilibrium in period two, the sellers' contracts will be chosen to extract fully the buyer's surplus in all possible subgames. If the buyer purchased only from the incumbent in period one, then the sellers' contracts in period two will be such that in equilibrium the incumbent earns $V_{II}^2(I) - 2c$, and the buyer and the entrant earn zero. If the buyer purchased only from the entrant in period one, then the sellers' contracts will be such that in equilibrium the entrant earns $V_{EO}^2(E) - c$, and the buyer and the incumbent earn zero. And, if the buyer purchased one unit from each seller in period one, then the sellers' contracts will be such that in equilibrium the incumbent earns $V_I^2 - c$, the entrant earns $V_E^2 - c$, and the buyer earns zero.

The more interesting interactions take place in period one. We want to know whether the equilibrium outcome will be efficient (the buyer purchases from both sellers) or exclusionary (the buyer

[10]The purpose of the assumptions in (2)–(5) is to link the periods in a tractable way, reducing the buyer's 'clout' in period two (the 'lock-in' effect) while allowing for a simple resolution of the second-period equilibrium payoffs. However, all that is essential is that the incumbent's second-period payoff be increasing in its first period sales.

[11]We assume there is no discounting across the two periods strictly for ease of exposition.

purchases from only one seller). To obtain our results, we must make three further assumptions.

First, we assume the buyer cannot purchase more units than it actually consumes. This assumption serves a practical purpose. It means, for example, that the buyer cannot purchase two units from the incumbent and then throw one unit away, and it is meant to capture in a simplified way the often heavy monitoring that goes on in some industries between sellers and buyers to prevent a buyer from taking advantage of a seller's quantity discount and then reselling the unused units in a secondary market. Equivalently, the assumption that the buyer cannot purchase more units than it consumes is tantamount to assuming that the buyer's disposal costs are prohibitively high.

Second, we assume the sellers cannot commit to their second-period contracts in period one, and hence the buyer knows that its second-period surplus will be fully extracted in period two regardless of its choices in period one. The main justification for this assumption is that contracts between buyers and sellers in reality are often incomplete (e.g., an unverifiable shock to a seller's ability to supply), allowing one side or the other to engage in opportunism. This will naturally make both sides wary of committing too early on contracts for fear of what might happen if the environment subsequently changes. Moreover, contracting today over prices on future periods which may be far off may be unrealistic, especially in fast-paced industries that are characterized by many innovations. In such cases, demand and cost forecasting may be fraught with uncertainty.

Third, we assume the entrant is financially constrained in its ability to bid for the buyer's patronage. As has been pointed out by Tirole (2006), Clementi and Hopenhayn (2006), and others, in settings such as ours, the availability of 'free' cash (e.g, from installed-base revenues) is key to the ability to finance projects, such as paying a retailer for access to second-period customers.[12] Indeed it is not implausible that the incumbent will have access to more free cash than the entrant, and thus, for ease of exposition, we assume the entrant faces a financing constraint (an upper bound on how much it can bid for the buyer's patronage) in period one whereas the incumbent does not. We model the entrant's constraint by assuming the entrant must have a minimum payoff of θ in period one, i.e., we require that $T_E^1(1) \geq \theta + c$, where $\theta \in (-(V_E^2 - c), 0]$. The upper bound, $\theta = 0$, implies that the entrant is unable to borrow money in period one; the lower bound, $\theta = -(V_E^2 - c)$, implies that the entrant can borrow in period one up to its maximum non-exclusionary payoff in period two. The former implies a tight constraint; whereas the latter is essentially no constraint.

[12] Clementi and Hopenhayn (2006) show that financial constraints arise endogenously as a feature of long-term lending contracts, and that "such constraints relax as the value of the borrower's claim to future cash flows increases."

3 Key condition and examples

In this section, we derive the key condition that enables exclusion, and we provide examples to illustrate the phenomenon. The examples include situations in which the sellers' products differ along quality dimensions (vertical differentiation) and non-quality dimensions (horizontal differentiation).

We begin by supposing that T_E^* is a best-response to T_I^*, and that, given these contracts, the buyer would purchase one unit from each seller in period one (and hence also in period two). We want to know whether and under what conditions this can be an equilibrium (when is T_I^* also a best response to T_E^*). Under the supposition, the joint payoff of the incumbent and the buyer is

$$V_{IE}^1 - c - T_E^*(1) + V_I^2 - c, \tag{6}$$

where $V_{IE}^1 - c - T_E^*(1)$ is their joint payoff in the first period, and $V_I^2 - c$ is their joint payoff in the second period. If instead the buyer were to purchase both units from the incumbent in period one, thereby excluding the entrant, the joint payoff of the incumbent and the buyer would be

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c, \tag{7}$$

where now their first-period joint payoff is $V_{II}^1 - 2c$ and their second-period joint payoff is $V_{II}^2(I) - 2c$. It follows that T_I^* is a best-response to T_E^* only if (6) is weakly larger than (7), or equivalently,

$$V_{IE}^1 - V_{II}^1 - (T_E^*(1) - c) \;\; \geq \;\; V_{II}^2(I) - V_I^2 - c. \tag{8}$$

Since the entrant's payoff in the first period, $T_E^*(1) - c$, is bounded below by θ, it follows that

$$\theta \;\; \leq \;\; V_{IE}^1 - V_{II}^1 - \Big(V_{II}^2(I) - V_I^2 - c\Big). \tag{9}$$

is a necessary condition for the existence of equilibria in which the buyer purchases one unit from each seller. In other words, if (9) were not satisfied, the incumbent would be able to profitably deviate by offering an alternative contract that would induce the buyer to exclude the entrant.

Now let us suppose that T_E^{**} is a best-response to T_I^{**}, and that, given these contracts, the buyer would purchase both units from the incumbent in period one (and hence also in period two). We want to know whether and under what conditions this can be an equilibrium (when is T_I^{**} also a best response to T_E^{**}). Under this supposition, the joint payoff of the incumbent and the buyer is

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c. \tag{10}$$

If instead the incumbent were to induce the buyer to choose the efficient outcome (purchase one unit from each seller in period one), the joint payoff of the incumbent and the buyer would be

$$V_{IE}^1 - c - T_E^{**}(1) + V_I^2 - c. \tag{11}$$

It follows that T_I^{**} is a best-response to T_E^{**} only if (10) is weakly larger than (11), or equivalently,

$$V_{II}^2(I) - V_I^2 - c \;\; \geq \;\; V_{IE}^1 - V_{II}^1 - \left(T_E^{**}(1) - c\right). \tag{12}$$

Since the entrant's payoff in the first period, $T_E^{**}(1) - c$, is bounded below by θ, it follows that

$$\theta \;\; \geq \;\; V_{IE}^1 - V_{II}^1 - \Big(V_{II}^2(I) - V_I^2 - c\Big). \tag{13}$$

is a sufficient condition for the existence of equilibria in which the buyer purchases both units from the incumbent. Comparing (9) and (13), we see that if (13) holds strictly, there exist equilibria in which the entrant is excluded and there do not exist equilibria in which the entrant is not excluded.

The inequality in (13) has an intuitive interpretation, which can be seen by rewriting it as

$$V_{II}^2(I) - V_I^2 - c \;\; \geq \;\; V_{IE}^1 - V_{II}^1 - \theta. \tag{14}$$

Condition (14) says that exclusionary equilibria can be supported when the incumbent's second-period gain from excluding the entrant exceeds the cost of compensating the buyer for the loss of the entrant's product, where $V_{IE}^1 - V_{II}^1$ is the foregone surplus that would have been added by the entrant's product in period one, and $-\theta$ is the maximum amount the entrant can 'pay' the buyer in period one not to give in to the exclusion. It is the key condition that enables exclusion to arise, and when it holds strictly, there is no equilibrium in which the buyer purchases from the entrant.

3.1 Illustrative examples

We can illustrate the importance of the key condition with the help of two examples. In the first example, the sellers' products are horizontally differentiated. In second example, the sellers' products are vertically differentiated. In both examples, we suppose the buyer is a retailer who purchases the sellers' products in an upstream market and then resells them to two final consumers.

Each consumer wants at most one unit per period. The first consumer values the incumbent's product at 100 and the entrant's product at 60. The second consumer values the incumbent's product at X and the entrant's product at Y, where $Y > 60$ and $55 < X < 100$. Note that if

$X > Y$, then the sellers' products are vertically differentiated (the consumers agree on which is the better product), but if $X < Y$, then the sellers' products are horizontally differentiated (the consumers disagree on which is the better product). The sellers' marginal cost of production is 10.

Given these primitives, the buyer's valuations in the upstream market are determined as follows. The willingness-to-pay of the buyer for one unit only from the incumbent is 100 (this is the revenue it can earn in the retail market by selling the incumbent's product to the first consumer). Its willingness-to-pay for two units from the incumbent is $2X$ (at any price higher than X, the second consumer does not buy). Its willingness-to-pay for one unit only from the entrant is Y (this is the revenue it can earn by selling the entrant's product to the second consumer). And, finally, its willingness-to-pay for one unit from each seller is $100 + Y$ (it can sell the incumbent's product to the first consumer at a price of 100 and the entrant's product to the second consumer at a price of Y). For convenience, we can summarize the buyer's valuations using our notation as follows:[13]

$$V_{II}^1 \;=\; 2X,\;\; V_{IE}^1 \;=\; 100+Y,\;\; V_{IO}^1 \;=\; 100, \quad \text{and} \quad V_{EO}^1 \;=\; Y.$$

Now suppose that the entrant is unable to borrow in the first period, so that it has to charge no less than its marginal cost, and that the buyer's valuations for the sellers' products would be constant over time in the absence of any lock-in. Then, substituting $V_{II}^2(I) = V_{II}^1 = 2X$, $V_I^2 = 100$, $V_{IE}^1 = 100 + Y$, $c = 10$, and $\theta = 0$ into (14), the key condition for exclusion holds if and only if[14]

$$2X - 100 - 10 \;\geq\; 100 + Y - 2X. \tag{15}$$

Example 1: Horizontal differentiation: the case of $X = 80$ and $Y = 100$

In this case, since (15) holds strictly, we know from our discussion above that only an exclusionary equilibrium exists. To see how it can be supported, suppose the entrant and the incumbent offer contract terms $T_E(1) = 10$, $T_I(1) = 100$, and $T_I(2) = 70$, respectively. Then, given these terms, it is straightforward to verify that the buyer is indifferent among purchasing only from the entrant, $Y - T_E(1) = 90$, two units from the incumbent, $2X - T_I(2) = 90$, or one unit from each seller, $100 + Y - T_E(1) - T_I(1) = 90$ (as it must be in any equilibrium). Hence, it is a best-response for the buyer to purchase two units from the incumbent. It is also straightforward to verify that the entrant has no profitable deviation (since its offer to the buyer is already at its lower bound). Lastly,

[13] It is straightforward to verify that the key condition for exclusion holds in the two examples below even if we assume the buyer can perfectly price discriminate, so that $V_{II}^1 = 100 + X$, $V_{IE}^1 = 100 + Y$, $V_{IO}^1 = 100$, and $V_{EO}^1 = Y$.

[14] The key condition would hold over a larger set of parameters if the buyer's valuations were increasing over time.

it is straightforward to verify that the incumbent earns strictly higher profit by selling two units to the buyer. The incumbent's payoff given these contracts and assuming equilibrium contracts in the continuation game is 70-20 = 50 in the first period and 160-20 = 140 in the second period, for a total payoff of 190. The buyer's payoff is 160-70 = 90 in the first period and zero in the second period. The entrant's payoff is zero. It should be noted that the incumbent's first-period contract effectively offers the buyer a negative marginal price for the second unit. It should also be noted that simply charging the same marginal price for each unit so that the total is 70 for the two units, i.e., $T_I(1) = 35$ and $T_I(2) = 70$, does worse for the incumbent (because the buyer would purchase one unit from each seller in each period, giving the incumbent a total payoff of only 25+90 = 115).[15]

Example 2: Vertical differentiation: the case of $X = 80$ and $Y = 75$

In this case, both consumers value the incumbent's product more highly than the entrant's product and, since (15) holds strictly, only an exclusionary equilibrium exists. Consider, for example, the contract terms $T_E(1) = 10$, $T_I(1) = 100$, and $T_I(2) = 95$. Given these terms, it is straightforward to verify that it is a best-response for the buyer to purchase only from the incumbent. It is also straightforward to verify that neither the entrant nor the incumbent has a profitable deviation. The incumbent's payoff given these contracts and assuming equilibrium contracts in the continuation game is 95-20 = 75 in the first period and 160-20 = 140 in the second period, for a total payoff of 215. The buyer's payoff is 160-95= 65. The entrant's payoff is zero. As in the example above, the incumbent effectively offers the buyer a negative marginal price for the second unit in period one.[16]

These examples illustrate how exclusion works in the model. In each case, the incumbent's second-period gain from 'lock-in' exceeds the cost of compensating the buyer for the loss in overall first-period surplus, and the financing constraint is such that the entrant is unable to 'pay' the buyer enough to overcome the difference. As a result, exclusion is the unique equilibrium outcome.

[15] The incumbent also does worse by charging a 'two-part tariff,' for example, if it charged a fixed fee of 70 and a per-unit price of 0 for each unit, the buyer would purchase one unit from each seller in each period, giving the incumbent a payoff of only $60 + 90 = 150$. Alternatively, bundling two units at 70 and refusing to sell just one unit is profit maximizing. See Proposition 1 below for a complete characterization of equilibrium contracts in this case.

[16] The interested reader might wonder whether the second unit must be offered at a negative marginal price in order to induce exclusion. The answer is yes in this example (and in the first example) but no in general. As a case in point, suppose that $X = 80$ and $Y = 65$. Then it can be shown that only exclusionary equilibria exist, and that an exclusionary equilibrium can be supported with contract terms $T_E(1) = 10$, $T_I(1) = 100$, and $T_I(2) = 105$. In this case, the incumbent is able to induce exclusion without having to offer the second unit at a negative marginal price.

4 Solving the Model

We have seen thus far that (9) is a necessary condition for an efficient equilibrium to exist, and (13) is a sufficient condition for an exclusionary equilibrium to exist. It can also be shown (see appendix A) that the converses hold—an efficient equilibrium exists if and only if (9) holds, and an exclusionary equilibrium exists if and only if (13) holds. Since conditions (9) and (13) cannot be simultaneously satisfied unless they both hold with equality, it follows that the equilibrium outcome will be unique unless by coincidence it just happens that $\theta = V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$.

This is a surprising result because both outcomes typically co-exist in models where multiple sellers sell their products to a single buyer. In these cases, the literature often employs a refinement to select among equilibria, usually ruling out the exclusionary ones on the grounds that they are Pareto dominated by the efficient equilibria (see O'Brien and Shaffer, 1997; and Bernheim and Whinston, 1998). Here, no selection criterion is needed as multiple outcomes typically do not exist.

It remains to characterize the equilibrium contracts. As we show in appendix B, this involves, among other things, placing bounds on the marginal price of the incumbent's second unit. In the case of exclusionary equilibria, these bounds do not affect the payoffs received by each party, as $T_E^{**}(1)$ and $T_I^{**}(2)$ are uniquely determined. But, we can show that in order to support these equilibria, the marginal price of the second unit must be weakly less than the cost of producing it minus the discount that must be paid to the buyer to induce exclusion (there is obviously room for creative discounting, for example, at the equilibrium $T_I^{**}(2)$, one can always choose $T_1^{**}(1) >> 0$),

$$T_I^{**}(2) - T_I^{**}(1) \ \leq \ c - \left(V_{IE}^1 - V_{II}^1 - \theta\right), \tag{16}$$

where, as we have seen, $V_{IE}^1 - V_{II}^1 - \theta$ is the discount that must be paid to induce exclusion.

In the case of efficient equilibria, the bounds do not affect the payoff received by the incumbent, as $T_I^*(1)$ is uniquely given by the value that its first unit adds to overall surplus, but they do affect the payoffs received by the entrant and the buyer, as $T_E^*(1)$ and $T_I^*(2)$ are related in equilibrium. We can show that, in any efficient equilibrium, the marginal price of the incumbent's second unit is bounded below by the price that would induce exclusion, $c-(V_{IE}^1 - V_{II}^1 - \theta)$. When $T_I^*(2)-T_I^*(1)$ is at this lower bound, it is a best response for the entrant to offer $T_E^*(1)$ such that it earns θ in the first period. We can also show that, in any efficient equilibrium, the marginal price of the incumbent's second unit is bounded above by the 'opportunity' cost of the incumbent's second unit (otherwise, the incumbent would be offering to sell its second unit at a positive mark-up, which cannot hold in any equilibrium in which it does not actually sell the second unit), $c - \left(V^2_{II}(I) - V^2_I - c\right)$. When $T^*_I(2) - T^*_I(1)$ is at this upper bound, it is a best-response for the entrant to offer $T^*_E(1)$ such that it earns $V^1_{IE} - V^1_{II} - (V^2_{II}(I) - V^2_I - c)$ in the first period, which is greater than θ since (9) holds.

We can now state our main result, which proves existence and characterizes the equilibria.

Proposition 1 *Equilibria in which the buyer purchases one unit from each seller exist if and only if $\theta \leq V^1_{IE} - V^1_{II} - (V^2_{II}(I) - V^2_I - c)$. In all such equilibria, contracts T^*_I and T^*_E are such that*

$$T^*_E(1) \;=\; V^1_{IE} - V^1_{II} + T^*_I(2) - T^*_I(1), \quad T^*_I(1) \;=\; V^1_{IE} - V^1_{EO},$$

and

$$c - \left(V^2_{II}(I) - V^2_I - c\right) \;\geq\; T^*_I(2) - T^*_I(1) \;\geq\; c - \left(V^1_{IE} - V^1_{II} - \theta\right).$$

In contrast, equilibria in which the buyer purchases both units from the incumbent exist if and only if $\theta \geq V^1_{IE} - V^1_{II} - (V^2_{II}(I) - V^2_I - c)$. In all such equilibria, contracts T^{**}_I and T^{**}_E are such that

$$T^{**}_E(1) \;=\; \theta + c, \quad T^{**}_I(2) \;=\; V^1_{II} - V^1_{EO} + \theta + c,$$

and

$$T^{**}_I(2) - T^{**}_I(1) \;\leq\; c - \left(V^1_{IE} - V^1_{II} - \theta\right).$$

There do not exist equilibria in which the buyer purchases one unit from one seller or no units.

Proof: See appendix B.

Proposition 1 implies that whether or not the incumbent wants to induce the buyer to exclude the entrant depends on the tightness of the entrant's financing constraint, the value added of the entrant's product, and the incumbent's second-period gain from doing so (i.e., the gains from lock-in). More specifically, it depends on whether the discount that must be paid to the buyer to induce exclusion, $V^1_{IE} - V^1_{II} - \theta$, is greater than or less than the incumbent's second-period gain from lock-in, $V^2_{II}(I) - V^2_I - c$. Exclusionary equilibria arise if and only if the latter weakly exceeds the former, and conversely, efficient equilibria arise if and only if the former weakly exceeds the latter.

The Chicago school regards inefficient exclusion as unlikely because it alleges that any benefit to an upstream seller from excluding its rival will be outweighed by the concomitant loss to the downstream buyer, and hence, the downstream buyer will not agree to participate in the exclusion.

To put it another way, it is alleged that mutually-beneficial exclusion will not occur because the amount an upstream seller would have to pay to the downstream buyer to induce exclusion would render such strategy unprofitable. This view (see, for example, Director and Levi, 1956; Bork, 1978; and Posner, 2001) obviously fails to hold when the key condition for exclusion is satisfied in our model. And when it fails, it is not because there are a multiplicity of outcomes, as in other models, it is because efficient equilibria do not exist—something has gone wrong in the intuition.

We can gain a deeper insight into why the Chicago-school view does not always hold in our model by considering first a scenario in which it does hold. Suppose, for now, that the entrant does not face a financing constraint and thus can rationally borrow in period one up to its maximum possible payoff in period two, $V_E^2 - c$. Then, substituting this for θ into condition (13), we have

$$-(V_E^2 - c) \;\;\geq\;\; V_{IE}^1 - V_{II}^1 - \Big(V_{II}^2(I) - V_I^2 - c\Big)\,,$$

or, equivalently,

$$0 \;\;\geq\;\; V_{IE}^1 - V_{II}^1 + \Big(V_{IE}^2(IE) - V_{II}^2(I)\Big)\,.$$

But this inequality, which is equivalent to the key condition for exclusion when $\theta = -(V_E^2 - c)$, is never satisfied because by assumption the efficient outcome calls for the buyer to purchase one unit from each seller in each period (recall that we have assumed $V_{IE}^1 > V_{II}^1$ and $V_{IE}^2(IE) > V_{II}^2(I)$). Hence, we can conclude that the entrant's financing constraint is critical for our exclusion results.

In deciding whether to induce exclusion, the incumbent compares the benefit of inducing exclusion to the cost of inducing exclusion, which is what it must pay in the first period to get the buyer to acquiesce. As we have just shown, sometimes the Chicago-school view is correct in that the compensation the incumbent must pay the buyer to participate makes exclusion prohibitively costly. However, the Chicago-school view need not hold if the entrant is financially constrained. The reason is that in this case the loss to the buyer if it allows exclusion will be truncated, so that the incumbent will only have to compensate the buyer for an amount equal to the value added of the entrant's product plus what the entrant is able to pay out of its own pocket not to be excluded. If the latter is truncated enough, then the incumbent may well find it profitable to exclude its rival. The tighter is the entrant's financing constraint, the more likely this will be the case. In other words, the benefit to the incumbent from excluding its rival (second-period gain from lock-in) may outweigh the concomitant loss to the buyer from allowing the exclusion (attractiveness of the entrant's product and offer) when the amount the entrant can finance is sufficiently constrained.

Switching costs also play an important role in supporting exclusion. Although they may appear to be innocuous (because they increase *both* sides' incentive to fight for the contestable unit in period one), by linking the two periods, they may cause the entrant's financing constraint to bind, which handicaps the outcome in period one in favor of the incumbent. Thus, the entrant's financing constraint and the buyer's switching costs go hand in hand. If the buyer incurs switching costs but the entrant does not have a financing constraint, then, as we have seen, exclusion will not be profitable for the incumbent. Conversely, if the entrant faces a financing constraint but there are no switching costs, then, as we now show, exclusion will also not be profitable for the incumbent. To see this, note that if the buyer does not incur switching costs, the second-period payoff to the incumbent will be independent of its market-share in the first period, and hence the $\left(V_{II}^2(I) - V_I^2 - c\right)$ term in (13) will be zero in the analogue to (13). In this case, the key condition for exclusion to arise becomes $\theta \geq V_{IE}^1 - V_{II}^1$, which is never satisfied given that θ is non-positive.

5 Contract Terms and Restrictions

We have thus far discussed when and why exclusion may arise in the model, and we have emphasized the joint role of the entrant's financing constraint and the buyer's switching costs in obtaining this outcome. In this section, we consider how exclusion may arise. We focus on the nature of the sellers' equilibrium contract terms, and we discuss how these terms may be implemented in practice.

Whether the buyer purchases one unit or two units from the incumbent depends on the marginal price of the incumbent's second unit. Obviously, if this price is low enough, the buyer will purchase both units from the incumbent and the entrant be excluded. But how low does this price have to be to induce exclusion? Does it have to be negative, as in our illustrative examples, or can the marginal price of the incumbent's second unit be positive and still induce exclusion? Relatedly, is it necessary for the incumbent to offer a loyalty rebate, a market-share discount, or an all-units discount (where the incumbent essentially offers a rebate to the buyer if the buyer purchases the second unit) if it wants to induce exclusion, or can exclusion be induced with no rebate? In the following Corollary to Proposition 1, we state what must be true in any exclusionary equilibrium.

Corollary 1 *In all exclusionary equilibria, contract T_I^{**} is such that*

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right).$$

- *Exclusion requires a negative marginal price if and only if* $c - (V_{IE}^1 - V_{II}^1 - \theta) < 0$.
- *The marginal price of the incumbent's second unit in equilibrium is less than* c.

Corollary 1 implies that exclusionary equilibria can always be supported with contracts that exhibit negative marginal pricing (this follows because there is no lower bound on $T_I^{**}(2) - T_I^{**}(1)$). Thus, for example, whenever conditions are such that exclusionary equilibria arise (i.e., when (14) is satisfied), there exists an exclusionary equilibrium in which the incumbent bundles its two units together and charges a price of $T_I^{**}(2) = V_{II}^1 - V_{EO}^1 + \theta + c$ (see Proposition 1). In this case, the price of the first unit effectively becomes $T_I^{**}(1) = \infty$. This is an example of multi-unit bundling, where the individual units are not offered for sale, and it closely parallels the kind of exclusionary bundling that arises in Nalebuff (2004) if one interprets the incumbent's first and second units as separate goods. Unlike in Nalebuff's model, however, the contracts here are chosen simultaneously.

Another strategy the incumbent could use is to announce that it will not sell to the buyer if the buyer also purchases from the entrant, i.e., the incumbent can achieve the same outcome by requiring that the buyer agree to exclusive dealing. As with multi-unit bundling, an exclusive-dealing arrangement would effectively eliminate from consideration the case in which the buyer purchases from both sellers. Notice, however, that what drives the exclusion result in the current paper is the inability of the entrant to commit to sell its product at cost to the buyer in period two as well as its inability (because of its financing constraint) to give the buyer its entire intertemporal surplus upfront, as opposed to the incumbent and the buyer overestimating the extent of the surplus that can be extracted from the entrant, as in the model of Aghion and Bolton (1987).[17] Notice also that our assumptions of a single buyer and no fixed costs of production distinguishes the model from that of Rasmusen et al. (1991), where coordination issues and scale economies are important.

In practice, negative marginal prices can also be implemented via a market-share discount, a loyalty rebate, or an all-units discount. As an example of the former, the incumbent can announce an artificially high price per unit if it has 50% of the market (sells one unit) and a substantially lower price per unit if it has 100% of the market (sells two units), where the discount is such that the second unit is effectively offered to the buyer at a negative marginal price. As an example of the latter, the incumbent can implement a negative marginal price on its second unit without explicitly referring to its rival, and without ostensibly foreclosing the possibility of selling just one unit, by

[17] In the absence of uncertainty over the entrant's production costs, exclusion does not arise in their model.

charging a constant price per unit for its product and then offering a loyalty rebate of more than 50% off to its 'best customers,' or, equivalently, by offering a discount per unit that applies to both units purchased by the buyer only if the buyer purchases two units, i.e., an all-units discount.[18]

We can strengthen our results and obtain additional insight by noting that Corollary 1 further implies that exclusionary equilibria *must* be supported with negative marginal prices if the discount needed to induce exclusion, $V_{IE}^1 - V_{II}^1 - \theta$, is greater than the marginal cost of production c. Consider some special cases. Suppose the marginal cost of production is zero, or arbitrarily close to zero, as it might be, for example, in software markets, or certain high-tech industries. Then the sufficient condition for negative marginal pricing to occur, $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right) < 0$, is satisfied, and we can conclude that only negative marginal prices can arise in an exclusionary equilibrium. Alternatively, suppose the entrant is able to finance all of its second-period production costs in the first period, so that $-\theta > c$, or suppose the value added by the entrant's product to overall surplus in period one is greater than the entrant's marginal cost of production, so that $V_{IE}^1 - V_{II}^1 > c$. Then, we can once again conclude that only negative marginal prices can arise in an exclusionary equilibrium.

If, on the other hand, the marginal production costs are relatively high, the value added by the entrant's product to overall surplus is relatively low, and the entrant's financing constraint is relatively tight, so that $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right) > 0$, Corollary 1 suggests that exclusionary equilibria can be supported even if the incumbent offers its second unit at a positive marginal price. This is surprising because it suggests that even with 'outlay' schedules that do not jump down, it may still be possible, under some circumstances, for an incumbent to exclude an equally efficient entrant. Thus, for example, we see that when $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right) > 0$, exclusion can be induced even when the incumbent offers a price-quantity schedule that employs an incremental-units discount. This has implications for the efficacy of remedies that aim to improve efficiency, which we discuss below.

Turning to efficient equilibria, the next corollary implies that the marginal price of the second unit *must* be positive in some cases and negative in other cases. Both implications are surprising.

Corollary 2 *In all efficient equilibria, contract T_I^* is such that*

$$T_I^*(2) - T_I^*(1) \ \geq \ c - \left(V_{IE}^1 - V_{II}^1 - \theta\right).$$

$$T_I^*(2) - T_I^*(1) \ \leq \ c - \left(V_{II}^2(I) - V_I^2 - c\right).$$

[18] As discussed in the introduction, these various strategies have individually received prior attention in the literature. We bring them together here in the context of a common property they each have (or could have if the per-unit discount is sufficiently large)—they exhibit negative marginal prices over some region of the price-quantity schedule.

- *Efficient equilibria require a positive marginal price if and only if* $c - (V^1_{IE} - V^1_{II} - \theta) > 0$.
- *Efficient equilibria require a negative marginal price if and only if* $c - (V^2_{II}(I) - V^2_I - c) < 0$.
- *The marginal price of the incumbent's second unit in equilibrium is less than c.*

Corollary 2 implies that the incumbent's second unit *must* have a positive marginal price in all efficient equilibria if and only if the discount that would have been needed to induce exclusion (if the incumbent had wanted to induce exclusion), $V^1_{IE} - V^1_{II} - \theta$, is less than the incumbent's marginal cost of production c, a condition that is 'more likely' to be satisfied when the incumbent's marginal cost of production is relatively high, the value added by the entrant's product to overall surplus is relatively low, and the entrant's financing constraint is relatively tight. This implication follows because the lower bound on the marginal price of the incumbent's second unit is $c - (V^1_{IE} - V^1_{II} - \theta)$.

It also follows from Corollary 2 that when conditions require a positive marginal price on the second unit, contract forms with declining-block tariffs, which are a subset of incremental-units discounts, will yield higher profit for the incumbent than contract forms with loyalty rebates or all-units discounts (because the latter typically give rise to outlay schedules that 'jump down' when the threshold is reached, whereas the former do not provided the discounted lower price is positive).

This result contrasts with the findings in Kolay et al. (2003), who consider the case of a bilateral monopoly (one seller and one buyer). In their model, menus of all-units discounts are always preferred by the seller to menus of declining-block tariffs.[19] The problem the incumbent would face in our model if it offered a negative marginal price on its second unit when the key condition for exclusion does not hold and $c - (V^1_{IE} - V^1_{II} - \theta) > 0$, however, is that the buyer would then buy the second unit, which the incumbent does not want given that its second-period gain from lock-in under these conditions is small. This concern does not arise in Kolay et al's model.

It would be wrong to conclude, however, that positive marginal prices will always arise in efficient equilibria because even though the second-period gain from lock-in may not be very large, it may still be larger than the incumbent's marginal cost. That is, it may be that $V^2_{II}(I) - V^2_I - c > c$, in which case Corollary 2 implies that efficient equilibria can *only* be supported by a negative marginal price on the incumbent's second unit. This condition is 'more likely' to hold when the marginal costs of production are relatively low, for example, it always holds when marginal costs are zero.[20]

[19] Kolay et al. (2003) show that when the buyer has private information about the state of its demand and the seller's aim is control of the agent, menus of all-units discounts are more efficient than menus of two-part tariffs at inducing the buyer to reveal the true state of demand, and hence can lead to greater surplus extraction for the seller.

[20] To continue the discussion from Section 3, let $X = 80$, $Y = 120$, $c = 10$, and $\theta = 0$. Then it is easy to show that

But this begs the question why would the incumbent ever offer to sell its second unit at a negative marginal price when the key condition for exclusion does not hold, given that it would lose profit if it actually sold the unit.[21] The reason is that as long as its marginal price is above the maximum price that would induce the buyer to exclude the entrant, the incumbent will not actually be selling the second unit, and so its marginal price for the second unit will be of no consequence to it (although it will affect the distribution of payoffs between the entrant and the buyer). Hence, a negative marginal price for the second unit is an off-equilibrium offer which will not be taken up by the buyer but which is required to support an efficient equilibrium, when such outcome is feasible.

5.1 Restrictions on below-cost pricing

We have thus far focused our attention on whether the marginal price of the incumbent's second unit will be positive or negative in equilibrium. In either case, however, we know from Corollaries 1 and 2 that it will always be less than c. Whether this is surprising depends on one's perspective. On the one hand, the notion that switching costs may lead to below-cost pricing in a dynamic setting in the period prior to lock-in is well known in the literature (see, e.g., Klemperer, 1987a and Farrell and Shapiro, 1988). That switching costs might also make it easier to deter entry in some cases is also well-known (see, e.g., Klemperer, 1987b), although the mechanism by which entry is deterred is different. On the other hand, this implication of the model is surprising because one might be tempted (erroneously) to call such pricing 'predatory' when in fact its existence does not allow one to infer whether the induced equilibria are good or bad for welfare. Below-cost pricing is bad for welfare if, when competing for the contestable unit, the entrant is not able to match the incumbent's offer to sell at below cost (adjusted for differences in valuations) because of its financing constraint. On the other hand, below-cost pricing is good for welfare when the entrant's financing constraint does not bind because then the buyer's choices are efficient and competition to supply the contestable unit simply transfers surplus from the sellers to the buyer. In this case, unlike in the first case, socially-efficient outcomes and below-cost pricing are mutually compatible.

It is not obvious, therefore, whether a ban on below-cost pricing would improve social welfare, as it would have consequences not only for the exclusionary equilibria in the model but also for the

only efficient equilibria exist and, because $c - \left(V_{II}^2(I) - V_I^2 - c\right) = 10 - (160 - 100 - 10) = -40 < 0$, the equilibria must be supported with negative marginal prices on the second unit, e.g., $T_E(1) = 20$, $T_I(1) = 100$, and $T_I(2) = 60$.

[21] It also begs the question why must the incumbent's marginal price be less than or equal to $c - \left(V_{II}^2(I) - V_I^2 - c\right)$. The reason is that in any efficient equilibrium, the incumbent must be offering to sell its second unit at a marginal price that does not allow it to earn positive profit because otherwise, it would lower its price and sell the second unit.

model's efficient equilibria. Nevertheless, as we now show, in our model, the consequences for the latter are limited to redistribution effects.[22] If the incumbent is prohibited from pricing at below cost, then all exclusionary equilibria cease to exist and only a unique, efficient equilibrium remains.

Proposition 2 *Suppose the incumbent is banned from offering to sell its second unit at below marginal cost. Then there is a unique equilibrium. The buyer purchases one unit from each seller, and the contract terms are* $T_I^*(1) = V_{IE}^1 - V_{EO}^1$, $T_I^*(2) = V_{IE}^1 - V_{EO}^1 + c$, *and* $T_E^*(1) = V_{IE}^1 - V_{II}^1 + c$.

Proof: See appendix C.

In equilibrium, each seller extracts only the value that its product adds to overall surplus. In the case of the incumbent, this is $V_{IE}^1 - V_{EO}^1$. In the case of the entrant, this is $V_{IE}^1 - V_{II}^1 + c$. Notice also that the marginal price of the incumbent's second unit is c. Although the incumbent knows that an additional sale in the first period will generate an additional profit of $V_{II}^2(I) - V_I^2 - c$ in the second period, and hence, that its 'true' opportunity cost of selling another unit in the first period is not c, but rather $c - (V_{II}^2(I) - V_I^2 - c)$, it is constrained from charging a lower price. Hence, the incumbent has no way of compensating the buyer for exclusion when below-cost pricing is banned.

Proposition 2 represents a first-best solution in the sense that if the sellers' costs of production are known to outside parties, and the restriction is costless to enforce, then social welfare is maximized, albeit with a redistribution of surplus from the buyer and the incumbent to the entrant.[23] Redistribution issues aside, however, a potential problem may arise if the sellers' costs are not observable to the parties charged with enforcing the restriction.[24] If this is the case, and if the restriction is enforced based on an erroneous belief that the incumbent's marginal cost is higher than it really is (a type II error), then the effect of the enforcement may be to chill price competition unnecessarily as both sellers will be charging marginal prices that are above their marginal costs (which in the model exacerbates the redistribution of surplus, but, more generally, could lead to a welfare loss if the buyer's demand is downward sloping). Conversely, if the restriction is enforced based on an erroneous belief that the incumbent's marginal cost is lower than it really is (also a

[22] In the model, the sellers' prices do not affect the volume of sales but they do affect the distribution of sales and the distribution of rents. An interesting extension may examine this result when usage is also sensitive to pricing.

[23] It can be shown that the entrant will always be better off, the incumbent will be weakly worse off, and the buyer will always be worse off, when the incumbent is prohibited from pricing its second unit at below marginal cost.

[24] There is also a potentially wide-range of market settings in which below-cost prices are consistent with 'innocent' profit maximization. See, e.g., Evans and Schmalensee (2006), which discusses pricing in industries with two-sided platforms.

type II error), then the enforcement may have little impact and exclusion may result. The possibility of these type II errors lead us next to consider whether there might be another, more easily enforced restriction, such as a ban on negative marginal pricing, that would not be susceptible to these errors and which could be imposed on the incumbent's terms to improve the market outcome.

5.2 Restrictions on negative marginal pricing

One reason for singling out negative marginal prices is that they can be easily identified when the incumbent uses them, and hence, restricting their use does not require knowledge of the incumbent's cost parameters, which may not be verifiable or easy to obtain. Unfortunately, however, the link between 'bad' market outcomes and negative marginal prices is not as transparent as one might have hoped for. As we have seen from Corollaries 1 and 2, there are circumstances in which exclusionary equilibria can be supported even when the incumbent offers its second unit at a positive marginal price, and there are circumstances in which efficient equilibria require a negative marginal price. This raises some concerns because it may suggest (erroneously) that a ban on negative marginal prices may not always be desirable. One concern is that it may not eliminate exclusionary equilibria in all cases. Another concern is that it may sometimes have adverse consequences for the existence of efficient equilibria. As we now show, however, the latter concern is unfounded, at least in the context of the current model. Although exclusionary equilibria are not eliminated, a ban on negative marginal prices does increase the incidence of efficient equilibria and therefore is welfare improving (although it would be a second-best solution if the incumbent's costs were observable).

Proposition 3 *Suppose the incumbent is banned from offering to sell its second unit at a negative marginal price. Then, equilibria in which the buyer purchases one unit from each seller exist if and only if $\theta \leq V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}$. In all such equilibria, T_I^* and T_E^* are such that*

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EO}^1,$$

and

$$\max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\} \;\geq\; T_I^*(2) - T_I^*(1) \;\geq\; \max\left\{0, c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)\right\}.$$

In contrast, equilibria in which the buyer purchases both units from the incumbent exist if and only if $\theta \geq V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}$. In all such equilibria, T_I^{**} and T_E^{**} are such that

$$T_E^{**}(1) \;=\; \theta + c, \quad T_I^{**}(2) \;=\; V_{II}^1 - V_{EO}^1 + \theta + c,$$

and

$$c - \left(V^1_{IE} - V^1_{II} - \theta\right) \;\geq\; T^{**}_I(2) - T^{**}_I(1) \;\geq\; 0.$$

There do not exist equilibria in which the buyer purchases one unit from one seller or no units.

Proof: See appendix D.

Proposition 3 implies that, except in the special case of $\theta = V^1_{IE} - V^1_{II} - \min\{c, V^2_{II}(I) - V^2_I - c\}$, the equilibrium outcome is unique. When θ is less than the right-hand side of this expression, only efficient equilibria exist, and conversely, when θ is greater than the right-hand side of this expression only exclusionary equilibria exist. This accords with what we found in Proposition 1, but with one major difference. In Proposition 1, the threshold for the entrant's financing constraint was $\theta = V^1_{IE} - V^1_{II} - (V^2_{II}(I) - V^2_I - c)$. Here, the threshold is weakly higher, and it is strictly higher if and only if $V^2_{II}(I) - V^2_I - c > c$. This means that efficient equilibria are 'more likely' to arise—and thus exclusionary equilibria are 'less likely' to arise—for a given θ when negative marginal prices are banned. Or, to put it another way, for a given set of cost and demand parameters, efficient equilibria exist over a wider range of θ when the incumbent is prohibited from offering negative marginal prices. It follows that such a ban is (weakly) welfare improving—it never makes welfare worse, and in some cases, e.g., when $V^2_{II}(I) - V^2_I - c > c$ and the incumbent would otherwise want to induce exclusion, it may make the difference between the entrant being excluded or not.

The mechanism by which exclusion occurs (if it occurs) in this case is more or less the same as it was in the absence of a ban on negative marginal pricing: the incumbent's second-period gain from 'lock-in' must exceed the cost of compensating the buyer for the loss in overall first-period surplus, and the entrant's financing constraint must be such that the entrant is unable to 'pay' the buyer enough to overcome the difference. Or, in other words, the discount that is needed to induce exclusion must be less than the incumbent's second-period gain from lock-in. But now, in addition to this, the discount needed to induce exclusion must also be less than the incumbent's marginal cost of production, so that the incumbent is able to induce the buyer to exclude the entrant while maintaining a positive marginal price on its second unit.[25] It is for this reason that the set of exclusionary equilibria is reduced, and it follows that the lower is the incumbent's marginal cost of

[25] Note that when the condition for exclusion in Proposition 3 is satisfied, the upper bound on the incumbent's marginal price, $c - \left(V^1_{IE} - V^1_{II} - \theta\right)$ is non-negative, which implies that exclusion only arises in this case when the discount that is needed to induce exclusion is less than both the incumbent's second-period gain from lock-in and c.

production, all else being equal, the less likely it is that exclusion will arise. If the marginal costs of production are zero or close to zero, for example, a ban on negative marginal pricing will be just as effective as the more encompassing ban on below-cost pricing in eliminating inefficient exclusion.

6 Extensions

We have thus far considered the case of an entrant who can produce at most one unit (equivalently, a situation in which only one unit is contestable). We now relax this assumption and assume the entrant can compete for both units. This is an important extension to consider because it is often alleged that competitive concerns arise only when an incumbent aggregates its discounts across several products (for example, offers bundled discounts), thereby handicapping smaller rivals who offer only one product. Indeed, according to one prominent antitrust commentator, "One might say that bundled discounts could not exclude an equally efficient firm, if we defined such a firm as one that was an efficient producer of every product that went into the bundled discount (Hovenkamp, 2005: 173)". He then goes on to say that "Bundled discounts exclude precisely because a dominant multiproduct firm is likely to face upstart single-product rivals—or at least, rivals who produce a smaller range of products than the dominant firm does." This view of bundled discounts is implicit in Nalebuff (2004), who finds that an incumbent's bundled discounts do not lead to exclusion when the rival firm can offer its own bundle, and, in fact, "bundle-on-bundle" competition is more intense as compared to unbundled competition in his model. As we shall see, however, when the upstart faces a financing constraint, this reasoning is flawed—the incumbent's discounts can be exclusionary even when all units are contestable and the entrant is equally efficient. Moreover, we will show that the competitive concerns in this case are, if anything, exacerbated—efficient equilibria are no more likely to arise and the threat of exclusion is omnipresent, as exclusionary equilibria always exist.

Let the buyer's valuation if it purchases both units from the entrant in the first period be V^1_{EE}, and assume that $V^1_{EE} > V^1_{EO} + c$ and $V^1_{IE} > V^1_{EE}$. Let the buyer's valuation if it purchases both units from the entrant in the second period be $V^2_{EE}(k)$, where $k \in \{IE, I, E, O\}$ denotes whether the buyer purchased from each seller, only the incumbent, only the entrant, or nothing in period one, and assume that $V^2_{EE}(I) = 0$, $V^2_{EE}(E) > V^2_{EO}(E) + c$, and $V^2_{EE}(IE) = V^2_E$. Assume also that $V^2_{IE}(IE) > V^2_{EE}(E)$ and that all other assumptions of the model remain the same. Then, it continues to hold that the efficient outcome is for the buyer to purchase one unit from each seller in period one and, given that it has done so, to purchase one unit from each seller in period two.

We further make the simplifying assumption that the incumbent's product is weakly more valuable to the buyer ex-ante. In particular, we assume that $V_{II}^1 \geq V_{EE}^1$ and $V_{II}^2(I) \geq V_{EE}^2(E)$. Then, we have the following proposition, which proves existence and characterizes the equilibria.

Proposition 4 *Equilibria in which the buyer purchases one unit from each seller exist if and only if* $\theta \leq V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$. *In all such equilibria, contracts* T_I^* *and* T_E^* *are such that*

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1),$$

$$c - \left(V_{EE}^2(I) - V_E^2 - c\right) \;\geq\; T_E^*(2) - T_E^*(1) \;\geq\; c - \left(V_{IE}^1 - V_{EE}^1\right) - \left(V_I^2 - c\right),$$

and

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \;\geq\; T_I^*(2) - T_I^*(1) \;\geq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right).$$

In contrast, equilibria in which the incumbent is excluded do not exist, but equilibria in which the entrant is excluded always exist. In all such equilibria, contracts T_I^{**} *and* T_E^{**} *are such that*

$$T_E^{**}(1) \;\geq\; V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right), \quad T_E^{**}(2) \;=\; \theta + 2c,$$

and

$$T_I^{**}(1) \;\geq\; V_{IE}^1 - V_{II}^1 + c, \quad T_I^{**}(2) \;=\; V_{II}^1 - V_{EE}^1 + \theta + 2c.$$

There do not exist equilibria in which the buyer purchases one unit from one seller or no units.

Proof: See appendix E.

Proposition 4 offers some surprises when compared with Proposition 1. For example, comparing the 'if and only if' condition for efficient equilibria to arise in these propositions reveals that it does not depend on whether the entrant can compete for both units. That is, efficient equilibria arise under exactly the same set of conditions whether one or both units are contestable. This is surprising because one might have thought that the entrant's ability to compete for both units would make the playing field 'more level.' And, in a sense, it does, but the beneficiary is not the entrant but the buyer, who benefits from the more favorable terms offered by the incumbent (the incumbent offers $T_I^*(1) = V_{IE} - V_{EO}$ when only one unit is contestable, and it offers $T_I^*(1) = V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1)$, when both units are contestable). Intuitively, it makes sense that the incumbent offers better terms of trade when both units are contestable (because then the buyer's outside option is more attractive). But why can the entrant not capture any of these gains, or, equivalently, why are efficient equilibria not more likely? The reason is that the entrant does not have the wherewithal to induce the buyer to exclude the incumbent (so, it is not helped in that sense), and having two units to offer the buyer does not help if it is only trying to induce the buyer to purchase one unit.

Having the ability to compete for both units not only does not help the entrant, it can also be harmful. Whereas in Proposition 1, exclusionary equilibria were confined to a subset of parameter space, here they always exist. To understand how this can happen, note that having the ability to offer two units expands the entrant's strategy space. Before, the incumbent could threaten to turn the game into an all-or-nothing proposition (effectively forcing the buyer either to purchase two units from it or nothing), and the entrant could only respond by offering an attractive deal on its single unit. Now if the entrant anticipates an all-or-nothing strategy by the incumbent, it can respond with its own attractive offer on two units and compete head to head with the incumbent (either way it knows it is going to lose). However, if it does so, only the buyer ends up gaining. The incumbent succeeds in excluding the entrant but it is costly, and the entrant ends up being excluded, which may not have happened if it could have committed ex-ante to being less aggressive by having the capacity to sell only one unit, as is consistent with the strictures of "judo economics."

In short, the buyer gains, the incumbent and the entrant lose, and welfare is weakly lower when both units are contestable because of the increased likelihood that resources will be misallocated (consumers get the wrong products). These results are in sharp contrast to conventional wisdom.

7 Conclusion

We have considered a simple model in which two sellers compete to sell their inputs to a single buyer. We fully characterized the set of equilibria and showed that, although the efficient outcome is for the buyer to purchase one unit from each seller, under some plausible conditions, exclusion is the unique outcome. We showed this despite there being no downstream externalities (and thus no coordination difficulties among buyers), complete information about cost and demand parameters, and no economies of scale in production. Moreover, we showed that this result holds even though both firms have the same marginal cost of production and thus are equally efficient. The result holds whether the products sold to consumers are horizontally or vertically differentiated, and it holds whether or not the entrant can compete on the full range of offerings to the buyer (i.e, whether or not the entrant can compete on both units). We found that the key assumptions are (a) the entrant is more financially constrained than the incumbent, and (b) the buyer incurs switching costs after its initial round of purchases, which are features of many real-world market settings.

A novel feature of the analysis is that it pays particular attention to the way in which the incumbent is able to support the exclusionary outcomes (when the equilibrium calls for exclusion). We found that while exclusionary equilibria were often but not always supported by negative marginal prices on the incumbent's second unit, all exclusionary equilibria were supported by below marginal-cost pricing on this unit. Moreover, we found that the same outcome could not be achieved by simply equating the incumbent's marginal price on each unit sold, i.e., by offering the same overall discount but with a linear price. Instead, we found that the discount had to be structured in the above-mentioned way, e.g., by offering a loyalty rebate or an all-units discount.

We considered two potential remedies that a policy maker might adopt to ease competitive concerns. We found that a ban on below-cost pricing everywhere along the incumbent's price-quantity schedule was sufficient to eliminate all exclusionary equilibria, and in that sense it made for a first-best welfare improvement. However, we also noted that such a prohibition might be difficult to enforce and could lead to egregious type-two errors unless the incumbent's costs were known and easily verifiable. This led us to consider a second potential remedy, a ban on negative marginal pricing (i.e., no jumps down in the incumbent's price-quantity schedule). We found that such a ban was also welfare improving, although in a perfect world it would be a second-best solution in the sense that it would not suffice to eliminate all exclusionary equilibria. Nevertheless, we argued that the latter ban might be preferable in the real-world because of its ease of implementation.

It has been suggested by some that competitive concerns will not be an issue when an equally efficient entrant is able to produce the full-range of products offered by the incumbent (or, equivalently, competitive concerns will not be an issue when all units are contestable). However, we showed that this intuition is flawed, and that the potential for inefficient exclusion persists when the entrant is financially constrained, whether or not it is able to offer the full-range of products.

As noted earlier, our results were derived in a stylized model in which discounting emerges as a competitive or anti-competitive strategy but in which there is no role for standard, unilateral reasons for discounting (e.g., to mitigate the effects of double marginalization). It will be important, in future research, to combine these two analytical strands in order to sharpen the policy prescriptions.

Appendix A

To see why (9) is sufficient for an efficient equilibrium to exist, assume it is satisfied, and let

$$T_I^*(1) = V_{IE}^1 - V_{EO}^1,$$

$$T_I^*(2) = \left(V_{IE}^1 - V_{EO}^1\right) - \left(V_{II}^2(I) - V_I^2 - c\right) + c,$$

and

$$T_E^*(1) = \left(V_{IE}^1 - V_{II}^1\right) - \left(V_{II}^2(I) - V_I^2 - c\right) + c,$$

be the incumbent and the entrant's respective contract terms (notice that (9) ensures $T_E^*(1) - c \geq \theta$). Then, given these contracts, the payoff to the buyer if it purchases one unit from each seller is

$$\begin{aligned} \Pi_B &= V_{IE}^1 - T_I^*(1) - T_E^*(1) \\ &= V_{EO}^1 - (V_{IE}^1 - V_{II}^1) + (V_{II}^2(I) - V_I^2 - c) - c, \end{aligned}$$

which is the same as what its payoff would be if it purchased both units from the incumbent, $V_{II}^1 - T_I^*(2)$, or one unit from the entrant, $V_{EO}^1 - T_E^*(1)$, and which is strictly greater than what its payoff would be if it purchased only one unit from the incumbent, $V_{IO}^1 - T_I^*(1)$, or zero units. Thus, given T_I^* and T_E^*, it is a best response for the buyer to purchase one unit from each seller.

The entrant's payoff under T_E^* (given the buyer's decision to purchase from each seller) is

$$\begin{aligned} \Pi_E &= T_E^*(1) - c + V_E^2 - c \\ &= (V_{IE}^1 - V_{II}^1) - (V_{II}^2(I) - V_I^2 - c) + V_E^2 - c, \end{aligned}$$

which is the best the entrant can do given T_I^*. If it were to decrease its asking price for one unit, it would be leaving surplus on the table for the buyer, and if it were to increase its asking price, the buyer would purchase both units from the incumbent. Intuitively, the entrant's payoff in the first period is equal to the value added of the entrant's product to overall surplus minus the second-period gain to the incumbent if it were to induce the buyer to exclude the entrant—the latter part being the amount the entrant must pay the buyer in the first period to keep from being excluded.

The incumbent's payoff under T_I^* (given the buyer's decision to purchase from each seller) is

$$\begin{aligned} \Pi_I &= T_I^*(1) - c + V_I^2 - c \\ &= (V_{IE}^1 - V_{EO}^1 - c) + V_I^2 - c, \end{aligned}$$

which is the best the incumbent can do given T_E^*. To see this, note that the incumbent's contract has an 'equilibrium' and an 'off-equilibrium' component to it. In equilibrium, the incumbent earns $V_{IE}^1 - V_{EO}^1 - c$ in the first period, which represents the value added of its product to overall surplus. Off equilibrium, the incumbent offers to sell its second unit at a marginal price of $T_I^*(2) - T_I^*(1) =$

$c-(V_{II}^2(I)-V_I^2-c)$, which represents the cost of the second unit minus the second-period gain to the incumbent if it were to induce the buyer to exclude the entrant. The incumbent is not willing to lower its marginal price for the second unit further (because then it would incur a loss if it sold the second unit), and it is not willing to raise its asking price for the first unit (because then it would lose the buyer's patronage altogether). Thus, T_I^* is the best the incumbent can do given T_E^*.

Having shown that efficient equilibria exist if and only if (9) is satisfied, we now turn our attention to the existence of exclusionary equilibria. In attempting to construct such equilibria, we begin by setting $T_I^{**}(1)=\infty$, which essentially forces the buyer to purchase from either the incumbent or the entrant (or zero). The other terms then follow because we know that the entrant can do no better than to offer to sell its product at $T_E^{**}(1)=\theta+c$ (this is the lowest it can go and still satisfy its financing constraint), and that, given this, the incumbent cannot charge more than

$$T_I^{**}(2)=V_{II}^1-V_{EO}^1+\theta+c$$

because otherwise the buyer would purchase only from the entrant and earn

$$\begin{aligned}\Pi_B &= V_{EO}^1-T_E^{**}(1)\\ &= V_{EO}^1-\theta-c.\end{aligned}$$

Hence, under $T_I^{**}(2)$, the incumbent's payoff in the proposed exclusionary equilibrium is

$$\begin{aligned}\Pi_I &= T_I^{**}(2)-2c+V_{II}^2(I)-2c\\ &= V_{II}^1-V_{EO}^1+\theta-c+V_{II}^2(I)-2c,\end{aligned}$$

where $V_{II}^1-V_{EO}^1+\theta-c$ is the payoff to the incumbent in the first period, and $V_{II}^2(I)-2c$ is its payoff in the second period. In contrast, the maximum payoff to the incumbent if it were to lower its price on $T_I^{**}(1)$ in order to induce the buyer to purchase one unit from it and the entrant is

$$V_{IE}^1-V_{EO}^1-c+V_I^2-c, \tag{A.1}$$

where now $T_I^{**}(1)$ is $V_{IE}^1-V_{EO}^1$, so that the buyer's payoff is $V_{IE}^1-T_I^{**}(1)-T_E^{**}(1)=V_{EO}^1-\theta-c$. It follows that the incumbent's proposed equilibrium payoff is higher than its payoff in (A.1) if and only if (13) holds. Hence, (13) is necessary and sufficient for the existence of exclusionary equilibria.

Appendix B

Proof of Proposition 1

Suppose there is an equilibrium in which the buyer purchases one unit from each seller in period one (and hence also in period two), and let T_I^* and T_E^* denote the equilibrium contracts. Then, the payoff to each party, taking into account the subsequent play of the game in period two, is

$$\Pi_I = T_I^*(1) - c + V_I^2 - c, \tag{B.1}$$

$$\Pi_E = T_E^*(1) - c + V_E^2 - c, \tag{B.2}$$

$$\Pi_B = V_{IE}^1 - T_I^*(1) - T_E^*(1), \tag{B.3}$$

where Π_I, Π_E, and Π_B are the overall payoffs of the incumbent, the entrant, and the buyer, respectively, $T_j^*(1) - c$ is seller j's payoff in period one, and $V_j^2 - c$ is seller j's payoff in period two.

It must also be the case that, given contracts T_I^* and T_E^*, the buyer weakly prefers to purchase one unit from each seller (i.e., the buyer's incentive-compatibility constraints must be satisfied):

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{II}^1 - T_I^*(2), \tag{B.4}$$

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{IO}^1 - T_I^*(1), \tag{B.5}$$

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{EO}^1 - T_E^*(1), \tag{B.6}$$

and

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq 0, \tag{B.7}$$

where (B.4) ensures that the buyer does not want to purchase both units from the incumbent, and (B.5) to (B.7) ensure that the buyer does not want to purchase only one unit or no units. These conditions place bounds on $T_E^*(1)$ and $T_I^*(1)$. For example, it follows from (B.4) to (B.6) that

$$T_E^*(1) \leq \min\left\{V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), V_{IE}^1 - V_{IO}^1\right\}, \tag{B.8}$$

and

$$T_I^*(1) \leq V_{IE}^1 - V_{EO}^1. \tag{B.9}$$

Since (B.7) is satisfied whenever $T_E^*(1)$ and $T_I^*(1)$ satisfy (B.8) and (B.9),[26] it follows that (B.8) and (B.9) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

Lastly, it must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. Among other things, this means that (B.8) and (B.9) must hold with equality (otherwise, one or both sellers could increase their asking price for one unit without causing the buyer to cease buying from them). It also means that each seller must earn non-negative payoff

[26] To see this, substitute (B.9) into the right-hand side of (B.5), and note that $V_{IO}^1 + V_{EO}^1 \geq V_{IE}^1$ from (1).

under the proposed contracts, and in the case of the entrant, it means that the entrant must earn at least θ in period one. And, finally, it means that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$, such that, given T_E^*, the incumbent can make itself and the buyer better off by inducing exclusion.[27]

The requirement that the incumbent earn non-negative payoff is satisfied when (B.9) holds with equality because then the incumbent's payoff simplifies to $\Pi_I = V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (5). The requirement that the entrant earn non-negative payoff and at least θ in period one is satisfied when $T_E^*(1)$ equals the second term on the right-hand side of (B.8) because then the entrant's first and second period payoff simplify to $V_{IE}^1 - V_{IO}^1 - c > 0$ and $V_E^2 - c > 0$. However, if $T_E^*(1)$ equals the first term on the right-hand side of (B.8), then the entrant's payoff is

$$\Pi_E \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c + V_E^2 - c,$$

which, since $V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c$ is its first period payoff, implies that the entrant earns non-negative payoff and at least θ in period one if and only if $T_I^*(2) - T_I^*(1)$ is bounded below by

$$T_I^*(2) - T_I^*(1) \;\geq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{B.10}$$

The requirement that the incumbent not find it profitable to exclude the entrant implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c \;>\; T_I^*(1) - c + V_I^2 - c, \tag{B.11}$$

$$V_{II}^1 - \tilde{T}_I(2) \;>\; V_{IE}^1 - \tilde{T}_I(1) - T_E^*(1), \tag{B.12}$$

$$V_{II}^1 - \tilde{T}_I(2) \;>\; V_{IO}^1 - \tilde{T}_I(1), \tag{B.13}$$

$$V_{II}^1 - \tilde{T}_I(2) \;>\; V_{EO}^1 - T_E^*(1), \tag{B.14}$$

and

$$V_{II}^1 - \tilde{T}_I(2) \;\geq\; 0, \tag{B.15}$$

where (B.11) ensures that the incumbent is better off excluding the entrant, (B.12) to (B.14) ensure that the buyer is better off excluding the entrant, and (B.15) ensures that the buyer earns non-negative payoff. Since (B.9) holds with equality, we can replace the right-hand side of (B.14) with $V_{IE}^1 - T_I^*(1) - T_E^*(1)$, and since $\tilde{T}_I(1)$ can be arbitrarily large, we know that (B.12) and (B.13) can be satisfied. Hence, we can reduce the set of conditions (B.11) to (B.15) to the equivalent set[28]

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c \;>\; T_I^*(1) - c + V_I^2 - c, \tag{B.16}$$

and

$$V_{II}^1 - \tilde{T}_I(2) \;>\; V_{IE}^1 - T_I^*(1) - T_E^*(1). \tag{B.17}$$

[27] Note that the buyer must also be made better off because it must agree to go along with the exclusion.

[28] We do not need to include (B.15) in the set because we know that the right-hand side of (B.17) is weakly positive.

It follows that for T_I^* and T_E^* to arise in an efficient equilibrium, it must be that

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c \;\; \leq \;\; V_{IE}^1 - T_E^*(1) - c + V_I^2 - c. \tag{B.18}$$

Once again, we have an upper bound on $T_E^*(1)$ if T_I^* and T_E^* are to arise in an efficient equilibrium:

$$T_E^*(1) \;\; \leq \;\; V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{B.19}$$

Comparing (B.19) and the fact that (B.8) holds with equality in an efficient equilibrium implies

$$T_I^*(2) - T_I^*(1) \;\; \leq \;\; c - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{B.20}$$

Hence, (B.10) and (B.20) imply the following upper and lower bound on $T_I^*(2) - T_I^*(1)$:

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \;\; \geq \;\; T_I^*(2) - T_I^*(1) \;\; \geq \;\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right), \tag{B.21}$$

which can be satisfied if and only if

$$\theta \;\; \leq \;\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{B.22}$$

To summarize, we have shown that, in any efficient equilibrium, (B.8) and (B.9) hold with equality, and (B.21) and (B.22) hold. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) \; = \; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \; = \; V_{IE}^1 - V_{EO}^1,$$

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \; \geq \; T_I^*(2) - T_I^*(1) \; \geq \; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right),$$

and

$$\theta \;\; \leq \;\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{B.23}$$

The necessity of (B.23) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (B.23) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Exclusionary Equilibria

Now suppose there is an equilibrium in which the buyer purchases both units from the incumbent in period one (and hence also in period two), and let T_I^{**} and T_E^{**} denote the equilibrium contracts. Then, the payoff to each party, taking into account the subsequent play of the game, is

$$\Pi_I \;\; = \;\; T_I^{**}(2) - 2c + V_{II}^2(I) - 2c, \tag{B.24}$$

$$\Pi_E \;\; = \;\; 0, \tag{B.25}$$

$$\Pi_B \;\; = \;\; V_{II}^1 - T_I^{**}(2), \tag{B.26}$$

where $T_I^{**}(2) - 2c$ and $V_{II}^2(I) - 2c$ are the incumbent's payoffs in periods one and two, respectively.

It must also be the case that, given contracts T_I^{**} and T_E^{**}, the buyer weakly prefers to purchase both units from the incumbent. In this case, the buyer's incentive-compatibility constraints are:

$$V_{II}^1 - T_I^{**}(2) \;\; \geq \;\; V_{IE}^1 - T_I^{**}(1) - T_E^{**}(1), \tag{B.27}$$

$$V_{II}^1 - T_I^{**}(2) \;\; \geq \;\; V_{IO}^1 - T_I^{**}(1), \tag{B.28}$$

$$V_{II}^1 - T_I^{**}(2) \;\; \geq \;\; V_{EO}^1 - T_E^{**}(1), \tag{B.29}$$

and

$$V_{II}^1 - T_I^{**}(2) \;\; \geq \;\; 0, \tag{B.30}$$

where (B.27) ensures that the buyer does not want to purchase one unit from each seller, and (B.28) to (B.30) ensure that the buyer does not want to purchase only one unit or no units. These conditions place bounds on the terms of T_I^{**}. For example, it follows from (B.27) to (B.29) that

$$T_I^{**}(2) - T_I^{**}(1) \;\; \leq \;\; \min\left\{V_{II}^1 - V_{IE}^1 + T_E^{**}(1), \;\; V_{II}^1 - V_{IO}^1\right\}, \tag{B.31}$$

and

$$T_I^{**}(2) \;\; \leq \;\; V_{II}^1 - V_{EO}^1 + T_E^{**}(1). \tag{B.32}$$

Since (B.30) is satisfied whenever (B.31) and (B.32) are satisfied, provided that $T_E^{**}(1) \leq V_{EO}^1$ (as it will be in any equilibrium in which the entrant is excluded), it follows that if $T_E^{**}(1) \leq V_{EO}^1$ then (B.31) and (B.32) are necessary and sufficient for the buyer's incentive-compatibility conditions to be satisfied in any equilibrium in which the buyer purchases both units from the incumbent.

Lastly, it must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. For the entrant, this means that there must not exist a profitable deviation in which it induces the buyer to purchase from it. For the incumbent, this means that there must not exist a profitable deviation in which it continues to sell both units or only one unit to the buyer. It also means that its payoff under the proposed contracts must be non-negative.

Consider the entrant's situation. For T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not be a deviation such that the entrant can make itself and the buyer better off. That is, for T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist $\tilde{T}_E(1)$ such that

$$\tilde{T}_E(1) \;\; \geq \;\; \theta + c, \tag{B.33}$$

$$\tilde{T}_E(1) - c + V_E^2 - c \;\; > \;\; 0, \tag{B.34}$$

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} \;\; > \;\; V_{II}^1 - T_I^{**}(2), \tag{B.35}$$

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} \;\; > \;\; V_{IO}^1 - T_I^{**}(1), \tag{B.36}$$

and

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} \;\; \geq \;\; 0, \tag{B.37}$$

where (B.33) ensures that the entrant earns at least θ in period one, (B.34) ensures that the entrant is better off under the deviation, (B.35) and (B.36) ensure that the buyer is better off under the deviation, and (B.37) ensures that the buyer earns non-negative payoff. Since (B.35) implies (B.36) and (B.37), and (B.33) and (B.34) are satisfied if $\tilde{T}_E(1) > \theta + c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (B.35) does not hold at $\tilde{T}_E(1) = \theta + c$ if they are to arise in equilibrium:

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; V_{II}^1 - V_{IE}^1 + \theta + c, \tag{B.38}$$

and

$$T_I^{**}(2) \;\leq\; V_{II}^1 - V_{EO}^1 + \theta + c. \tag{B.39}$$

The interpretation is that the terms of T_I^{**} must be bounded above by (B.38) and (B.39) in any exclusionary equilibrium because if (B.38) or (B.39) were not satisfied, there would exist $\tilde{T}_E(1) > \theta + c$ such that the entrant would be able to profitably induce the buyer to purchase from it.

Now consider the incumbent's situation. For T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist a profitable deviation in which the incumbent sells both units to the buyer. It follows from this that (B.32) must hold with equality (because otherwise, the incumbent could increase $T_I^{**}(2)$ and still sell both units to the buyer), and therefore, using this result, (B.33), and (B.39), it follows that in any exclusionary equilibrium the entrant's offer must be such that:

$$T_E^{**}(1) \;=\; \theta + c, \tag{B.40}$$

and hence the incumbent's offer must be such that

$$T_I^{**}(2) \;=\; V_{II}^1 - V_{EO}^1 + \theta + c, \tag{B.41}$$

and

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; V_{II}^1 - V_{IE}^1 + \theta + c,$$

or, equivalently,

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{B.42}$$

There must also not exist a profitable deviation in which the incumbent sells only one unit to the buyer. It follows from this that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that

$$\tilde{T}_I(1) - c + V_I^2 - c \;>\; T_I^{**}(2) - 2c + V_{II}^2(I) - 2c, \tag{B.43}$$

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} \;>\; V_{II}^1 - \tilde{T}_I(2), \tag{B.44}$$

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} \;>\; V_{EO}^1 - T_E^{**}(1), \tag{B.45}$$

and

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} \;\geq\; 0, \tag{B.46}$$

where (B.43) ensures that the incumbent is better off under the deviation, (B.44) and (B.45) ensure that (whether or not it is better off under the deviation) the buyer is induced to purchase only one unit from the incumbent, and (B.46) ensures that the buyer earns non-negative payoff. Since (B.45) and (B.40) imply (B.46), and since (B.44) can always be satisfied by choosing $\tilde{T}_I(2)$ to be arbitrarily large, we can reduce the set of constraints in (B.43) to (B.46) to the equivalent set

$$\tilde{T}_I(1) - c + V_I^2 - c \;>\; V_{II}^1 - V_{EO}^1 + \theta + c - 2c + V_{II}^2(I) - 2c, \tag{B.47}$$

and

$$V_{IE}^1 - \tilde{T}_I(1) - \theta - c \;>\; V_{EO}^1 - \theta - c, \tag{B.48}$$

where we have substituted (B.40) and (B.41) into (B.43) and (B.45), and simplified the left-hand side of (B.45). It follows that if T_I^{**} and T_E^{**} are to arise in an exclusionary equilibrium, then

$$\theta + c \;\geq\; V_{IE}^1 - V_{II}^1 - V_{II}^2(I) + V_I^2 + 2c,$$

or, equivalently,

$$\theta \;\geq\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{B.49}$$

Finally, it must be that the incumbent earns non-negative payoff under the proposed equilibrium contracts. Substituting (B.41) into Π_I, we have that the incumbent's payoff is

$$\Pi_I \;=\; V_{II}^1 - V_{EO}^1 + \theta + c - 2c + V_{II}^2(I) - 2c,$$

which, from (B.49), weakly exceeds $V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (5).

To summarize, we have shown that, in any exclusionary equilibrium, contracts are such that (B.40), (B.41), (B.42), and (B.49) hold. Thus, in any exclusionary equilibrium, we have that

$$T_E^{**}(1) \;=\; \theta + c, \quad T_I^{**}(2) \;=\; V_{II}^1 - V_{EO}^1 + \theta + c,$$
$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right),$$

and

$$\theta \;\geq\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{B.50}$$

The necessity of (B.50) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (B.50) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

It remains only to show that there do not exist equilibria in which the buyer purchases one unit from one seller or no units. Suppose there is an equilibrium in which the buyer purchases only from the entrant or no units. Then the incumbent can profitably deviate by offering $T_1(1) = c + \epsilon$, for ϵ sufficiently small but positive, a contradiction. Similarly, suppose there is an equilibrium in which the buyer purchases only one unit from the incumbent or no units. Then the entrant can profitably deviate by offering $T_E(1) = c + \epsilon$, for ϵ sufficiently small but positive, a contradiction. Q.E.D.

Appendix C

Proof of Proposition 2

Suppose there is an equilibrium in which the buyer purchases one unit from each seller in period one (and hence also in period two), and let T_I^* and T_E^* denote the equilibrium contracts. Then, the payoffs to all three parties, taking into account the subsequent play of the game in period two, are given by (B.1), (B.2), and (B.3), and the buyer's incentive compatibility constraints in this case are given by (B.4), (B.5), (B.6), and (B.7). It follows from (B.4), (B.5) and (B.6) that

$$T_E^*(1) \;\; \leq \;\; \min\left\{V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), V_{IE}^1 - V_{IO}^1\right\}, \tag{C.1}$$

and

$$T_I^*(1) \;\; \leq \;\; V_{IE}^1 - V_{EO}^1. \tag{C.2}$$

Since (B.7) is satisfied whenever $T_E^*(1)$ and $T_I^*(1)$ satisfy (C.1) and (C.2), it follows that (C.1) and (C.2) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither seller can profitably deviate given the other's contract. This means that (C.1) and (C.2) must hold with equality. It also means that each seller must earn non-negative payoff under the proposed contracts, and in the case of the entrant, it means that the entrant must earn at least θ in period one. And, finally, it means that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that the incumbent can make itself and the buyer better off by inducing exclusion.

The requirement that the incumbent earn non-negative payoff is satisfied when (C.2) holds with equality because then the incumbent's payoff simplifies to $\Pi_I = V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (5). The requirement that the entrant earn non-negative payoff and at least θ in period one is satisfied when $T_E^*(1)$ equals the second term on the right-hand side of (D.1) because then the entrant's first and second period payoff simplify to $V_{IE}^1 - V_{IO}^1 - c > 0$ and $V_E^2 - c > 0$. And when $T_E^*(1)$ equals the first term on the right-hand side of (D.1), then the entrant's payoff is

$$\Pi_E \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c + V_E^2 - c,$$

which, since $V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c$ is its first period payoff, implies that the entrant earns non-negative payoff and at least θ in period one given that $T_I^*(2) - T_I^*(1)$ is bounded below by c.

The requirement that the incumbent not find it profitable to exclude the entrant implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that conditions (B.11), (B.12), (B.13), (B.14), and (B.15) hold, and $\tilde{T}_I(1) \geq c$ and $\tilde{T}_I(2) - \tilde{T}_I(1) \geq c$.

Since (C.2) holds with equality, we can replace (B.14) with (B.17), and since $\tilde{T}_I(1)$ has an upper bound of $\tilde{T}_I(2) - c$, we can replace (B.12) and (B.13) with this upper bound. It follows that we can reduce the set of conditions (B.11) to (B.15), $\tilde{T}_I(1) \geq c$, and $\tilde{T}_I(2) - \tilde{T}_I(1) \geq c$ to the equivalent set

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c \;\; > \;\; T_I^*(1) - c + V_I^2 - c, \tag{C.3}$$

$$V_{II}^1 - \tilde{T}_I(2) \;>\; V_{IE}^1 - T_I^*(1) - T_E^*(1), \tag{C.4}$$

$$V_{II}^1 \;>\; V_{IE}^1 + c - T_E^*(1), \tag{C.5}$$

and

$$\tilde{T}_I(2) \;\geq\; 2c. \tag{C.6}$$

It follows that for T_I^* and T_E^* to arise in an efficient equilibrium, it must be that

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c \;\leq\; V_{IE}^1 - T_E^*(1) - c + V_I^2 - c, \tag{C.7}$$

or

$$V_{II}^1 - 2c \;\leq\; V_{EO}^1 - T_E^*(1), \tag{C.8}$$

or

$$V_{II}^1 \;\leq\; V_{IE}^1 + c - T_E^*(1), \tag{C.9}$$

where condition (C.7) comes from summing the left and right-hand sides of (C.3) and (C.4), condition (C.8) comes from (C.4) by evaluating $\tilde{T}_I(2)$ at $2c$, and condition (C.9) comes from (C.5). Once again, we have an upper bound on $T_E^*(1)$ if T_I^* and T_E^* are to arise in an efficient equilibrium:

$$T_E^*(1) \;\leq\; V_{IE}^1 - V_{II}^1 + c. \tag{C.10}$$

Comparing (C.10) and the fact that (C.1) holds with equality in an efficient equilibrium implies

$$T_I^*(2) - T_I^*(1) \;\leq\; c, \tag{C.11}$$

which, because of the ban on below-cost pricing, can be satisfied if and only if $T_I^*(2) = T_I^*(1) + c$.

To summarize, we have shown that, in any efficient equilibrium, (C.1), (C.2), and (C.11) hold with equality. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + c, \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EO}^1,$$

and

$$T_I^*(2) \;=\; V_{IE}^1 - V_{EO}^1 + c. \tag{C.12}$$

The necessity of (C.12) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (C.12) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Exclusionary Equilibria

Now suppose there is an equilibrium in which the buyer purchases both units from the incumbent in period one (and hence also in period two), and let T_I^{**} and T_E^{**} denote the equilibrium contracts. Then, the payoffs to all three parties, taking into account the subsequent play of the game in period two, are given by (B.24), (B.25), and (B.26), and the buyer's incentive-compatibility constraints in this case are given by (B.27), (B.28), (B.29), and (B.30). It follows from (B.27) to (B.29) that

$$T_I^{**}(2) - T_I^{**}(1) \quad \leq \quad \min\left\{V_{II}^1 - V_{IE}^1 + T_E^{**}(1),\;\; V_{II}^1 - V_{IO}^1\right\}, \tag{C.13}$$

and

$$T_I^{**}(2) \quad \leq \quad V_{II}^1 - V_{EO}^1 + T_E^{**}(1). \tag{C.14}$$

Since (B.30) is satisfied if (C.13) and (C.14) are satisfied and $T_E^{**}(1) \leq V_{EO}^1$ (as it will be in any equilibrium in which the entrant is excluded), it follows that if $T_E^{**}(1) \leq V_{EO}^1$ then (C.13) and (C.14) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. For the entrant, this means that there must not exist a profitable deviation in which it induces the buyer to purchase from it. That is, for T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist $\tilde{T}_E(1)$ such that (B.33), (B.34), (B.35), (B.36), and (B.37) hold.

Since (B.35) implies (B.36) and (B.37), and (B.33) and (B.34) are satisfied if $\tilde{T}_E(1) > \theta + c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (B.35) does not hold at $\tilde{T}_E(1) = \theta + c$, i.e.,

$$T_I^{**}(2) - T_I^{**}(1) \quad \leq \quad V_{II}^1 - V_{IE}^1 + \theta + c, \tag{C.15}$$

and

$$T_I^{**}(2) \quad \leq \quad V_{II}^1 - V_{EO}^1 + \theta + c. \tag{C.16}$$

The interpretation is that the terms of T_I^{**} must be bounded above by (C.15) and (C.16) in any exclusionary equilibrium because if (C.15) or (C.16) were not satisfied, there would exist $\tilde{T}_E(1) > \theta + c$ such that the entrant would be able to profitably induce the buyer to purchase from it.

But notice that $V_{IE}^1 > V_{II}^1$ and $\theta \leq 0$ imply that (C.15) is satisfied only if $T_I^{**}(2) - T_I^{**}(1)$ is less than c, which violates the ban on pricing at below marginal cost.[29] It follows that the entrant can always profitably deviate, and hence, there can be no equilibrium in which the entrant is excluded.

It remains only to show that there do not exist equilibria in which the buyer purchases one unit from one seller or no units. The proof of this is given in the last paragraph of Appendix B. Q.E.D.

[29] The reader may note that we are allowing the entrant to price at below marginal cost but not the incumbent. This has no effect on our results. If the entrant is likewise constrained, then (B.33) becomes $\tilde{T}_E(1) \geq c$ and $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (B.35) does not hold at $\tilde{T}_E(1) = c$. Otherwise, the proof continues as in the text.

Appendix D

Proof of Proposition 3

Suppose there is an equilibrium in which the buyer purchases one unit from each seller in period one (and hence also in period two), and let T_I^* and T_E^* denote the equilibrium contracts. Then, the payoffs to all three parties, taking into account the subsequent play of the game in period two, are given by (B.1), (B.2), and (B.3), and the buyer's incentive compatibility constraints in this case are given by (B.4), (B.5), (B.6), and (B.7). It follows from (B.4), (B.5) and (B.6) that

$$T_E^*(1) \leq \min\left\{V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), V_{IE}^1 - V_{IO}^1\right\}, \tag{D.1}$$

and

$$T_I^*(1) \leq V_{IE}^1 - V_{EO}^1. \tag{D.2}$$

Since (B.7) is satisfied whenever $T_E^*(1)$ and $T_I^*(1)$ satisfy (D.1) and (D.2), it follows that (D.1) and (D.2) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither seller can profitably deviate given the other's contract. This means that (D.1) and (D.2) must hold with equality. It also means that each seller must earn non-negative payoff under the proposed contracts, and in the case of the entrant, it means that the entrant must earn at least θ in period one. And, finally, it means that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that the incumbent can make itself and the buyer better off by inducing exclusion.

The requirement that the incumbent earn non-negative payoff is satisfied when (D.2) holds with equality because then the incumbent's payoff simplifies to $\Pi_I = V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (5). The requirement that the entrant earn non-negative payoff and at least θ in period one is satisfied when $T_E^*(1)$ equals the second term on the right-hand side of (D.1) because then the entrant's first and second period payoff simplify to $V_{IE}^1 - V_{IO}^1 - c > 0$ and $V_E^2 - c > 0$. However, if $T_E^*(1)$ equals the first term on the right-hand side of (D.1), then the entrant's payoff is

$$\Pi_E = V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c + V_E^2 - c,$$

which, since $V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c$ is its first period payoff, implies that the entrant earns non-negative payoff and at least θ in period one if and only if $T_I^*(2) - T_I^*(1)$ is bounded below by

$$T_I^*(2) - T_I^*(1) \geq c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{D.3}$$

The requirement that the incumbent not find it profitable to exclude the entrant implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that conditions (B.11), (B.12), (B.13), (B.14), and (B.15) hold, and $\tilde{T}_I(1) \geq 0$ and $\tilde{T}_I(2) - \tilde{T}_I(1) \geq 0$.

Since (D.2) holds with equality, we can replace (B.14) with (B.17), and since $\tilde{T}_I(1)$ has an upper bound of $\tilde{T}_I(2)$, we can replace (B.12) and (B.13) with this upper bound. It follows that we can reduce the set of conditions (B.11) to (B.15), $\tilde{T}_I(1) \geq 0$, and $\tilde{T}_I(2) \geq \tilde{T}_I(1)$ to the equivalent set

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c \quad > \quad T_I^*(1) - c + V_I^2 - c, \tag{D.4}$$

$$V_{II}^1 - \tilde{T}_I(2) \quad > \quad V_{IE}^1 - T_I^*(1) - T_E^*(1), \tag{D.5}$$

$$V_{II}^1 \quad > \quad V_{IE}^1 - T_E^*(1), \tag{D.6}$$

and

$$\tilde{T}_I(2) \quad \geq \quad 0. \tag{D.7}$$

It follows that for T_I^* and T_E^* to arise in an efficient equilibrium, it must be that

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c \quad \leq \quad V_{IE}^1 - T_E^*(1) - c + V_I^2 - c, \tag{D.8}$$

or

$$V_{II}^1 \quad \leq \quad V_{EO}^1 - T_E^*(1), \tag{D.9}$$

or

$$V_{II}^1 \quad \leq \quad V_{IE}^1 - T_E^*(1), \tag{D.10}$$

where condition (D.8) comes from summing the left and right-hand sides of (D.4) and (D.5), condition (D.9) comes from (D.5) by evaluating $\tilde{T}_I(2)$ at 0, and condition (D.10) comes from (D.6). Once again, we have an upper bound on $T_E^*(1)$ if T_I^* and T_E^* are to arise in an efficient equilibrium:

$$T_E^*(1) \quad \leq \quad \max\left\{V_{IE}^1 - V_{II}^1, V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\}. \tag{D.11}$$

Comparing (D.11) and the fact that (D.1) holds with equality in an efficient equilibrium implies

$$T_I^*(2) - T_I^*(1) \quad \leq \quad \max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\}. \tag{D.12}$$

Hence, (D.3), (D.12), and the ban on negative marginal pricing, which implies $T_I^*(2) \geq T_I^*(1)$, imply the following upper and lower bound on $T_I^*(2) - T_I^*(1)$:

$$\max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\} \quad \geq \quad T_I^*(2) - T_I^*(1) \quad \geq \quad \max\left\{0, c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)\right\}, \tag{D.13}$$

which can be satisfied if and only if

$$\theta + c \quad \leq \quad V_{IE}^1 - V_{II}^1 + \max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\},$$

or, equivalently,

$$\theta \quad \leq \quad V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}. \tag{D.14}$$

To summarize, we have shown that, in any efficient equilibrium, (D.1) and (D.2) hold with equality, and (D.13) and (D.14) hold. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EO}^1,$$

$$\max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\} \;\geq\; T_I^*(2) - T_I^*(1) \;\geq\; c - \max\left\{0, c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)\right\},$$

and

$$\theta \;\leq\; V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}. \tag{D.15}$$

The necessity of (D.15) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (D.15) follows because when it holds, the buyer's incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Exclusionary Equilibria

Now suppose there is an equilibrium in which the buyer purchases both units from the incumbent in period one (and hence also in period two), and let T_I^{**} and T_E^{**} denote the equilibrium contracts. Then, the payoffs to all three parties, taking into account the subsequent play of the game in period two, are given by (B.24), (B.25), and (B.26), and the buyer's incentive-compatibility constraints in this case are given by (B.27), (B.28), (B.29), and (B.30). It follows from (B.27) to (B.29) that

$$T_I^{**}(2) - T_I^{**}(1) \;\;\leq\;\; \min\{V_{II}^1 - V_{IE}^1 + T_E^{**}(1),\;\; V_{II}^1 - V_{IO}^1\}, \tag{D.16}$$

and

$$T_I^{**}(2) \;\;\leq\;\; V_{II}^1 - V_{EO}^1 + T_E^{**}(1). \tag{D.17}$$

Since (B.30) is satisfied if (D.16) and (D.17) are satisfied and $T_E^{**}(1) \leq V_{EO}^1$ (as it will be in any equilibrium in which the entrant is excluded), it follows that if $T_E^{**}(1) \leq V_{EO}^1$ then (D.16) and (D.17) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. For the entrant, this means that there must not exist a profitable deviation in which it induces the buyer to purchase from it. For the incumbent, this means that there must not exist a deviation in which it increases its payoff while continuing to sell both units or only one unit to the buyer. It also means that its payoff under the proposed contracts must be non-negative.

Consider the entrant's situation. For T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist a deviation $\tilde{T}_E(1)$ such that (B.33), (B.34), (B.35), (B.36), and (B.37) hold.

Since (B.35) implies (B.36) and (B.37), and (B.33) and (B.34) are satisfied if $\tilde{T}_E(1) > \theta + c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (B.35) does not hold at $\tilde{T}_E(1) = \theta + c$, i.e.,

$$T_I^{**}(2) - T_I^{**}(1) \;\;\leq\;\; V_{II}^1 - V_{IE}^1 + \theta + c, \tag{D.18}$$

and

$$T_I^{**}(2) \;\;\leq\;\; V_{II}^1 - V_{EO}^1 + \theta + c. \tag{D.19}$$

The interpretation is that the terms of T_I^{**} must be bounded above by (D.18) and (D.19) in any exclusionary equilibrium because if (D.18) or (D.19) were not satisfied, there would exist $\tilde{T}_E(1)$ $> \theta + c$ such that the entrant would be able to profitably induce the buyer to purchase from it.

Now consider the incumbent's situation. For T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist a profitable deviation in which the incumbent sells both units to the buyer. It follows from this that (D.17) must hold with equality (because otherwise, the incumbent could increase $T_I^{**}(2)$ and still sell both units to the buyer), and therefore, using this result, (B.33), and (B.39), it follows that in any exclusionary equilibrium the entrant's offer must be such that:

$$T_E^{**}(1) \;=\; \theta + c, \tag{D.20}$$

and hence the incumbent's offer must be such that

$$T_I^{**}(2) \;=\; V_{II}^1 - V_{EO}^1 + \theta + c, \tag{D.21}$$

and

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{D.22}$$

Using (D.22), it follows from the ban on negative marginal pricing that

$$0 \;\leq\; T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right), \tag{D.23}$$

and thus, if T_I^{**} and T_E^{**} are to arise in an exclusionary equilibrium, then

$$\theta \;\geq\; V_{IE}^1 - V_{II}^1 - c. \tag{D.24}$$

There must also not exist a profitable deviation in which the incumbent sells only one unit to the buyer. It follows from this that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that conditions (B.43), (B.44), (B.45), and (B.46) hold, and $\tilde{T}_I(1) \geq 0$ and $\tilde{T}_I(2) - \tilde{T}_I(1) \geq 0$. Since (B.45) and (B.40) imply (B.46), and since (B.44) and $\tilde{T}_I(2) \geq \tilde{T}_I(1)$ can always be satisfied by choosing $\tilde{T}_I(2)$ to be arbitrarily large, we can reduce the set of constraints in (B.43) to (B.46) to the equivalent set

$$\tilde{T}_I(1) - c + V_I^2 - c \;>\; V_{II}^1 - V_{EO}^1 + \theta + c - 2c + V_{II}^2(I) - 2c, \tag{D.25}$$

and

$$V_{IE}^1 - \tilde{T}_I(1) - \theta - c \;>\; V_{EO}^1 - \theta - c, \tag{D.26}$$

where we have substituted (D.20) and (D.21) into (B.43) and (B.45), and simplified the left-hand side of (B.45). It follows that if T_I^{**} and T_E^{**} are to arise in an exclusionary equilibrium, then

$$\theta \;\geq\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{D.27}$$

Finally, it must be that the incumbent earns non-negative payoff under the proposed equilibrium contracts. Substituting (D.21) into Π_I, we have that the incumbent's payoff is

$$\Pi_I \;=\; V_{II}^1 - V_{EO}^1 + \theta + c - 2c + V_{II}^2(I) - 2c,$$

which, from (D.27), weakly exceeds $V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (5).

To summarize, we have shown that, in any exclusionary equilibrium, contracts are such that (D.20), (D.21), (D.23), (D.24), and (D.27) hold. Thus, in any exclusionary equilibrium, we have

$$\begin{aligned} T_E^{**}(1) &= \theta + c, \quad T_I^{**}(2) \; = \; V_{II}^1 - V_{EO}^1 + \theta + c, \\ 0 \; &\leq \; T_I^{**}(2) - T_I^{**}(1) \; \leq \; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right), \end{aligned}$$

and

$$\theta \;\geq\; \max\left\{V_{IE}^1 - V_{II}^1 - c, V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right)\right\}. \tag{D.28}$$

The necessity of (D.28) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (D.28) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

It remains only to show that there do not exist equilibria in which the buyer purchases one unit from one seller or no units. The proof of this is given in the last paragraph of Appendix B. Q.E.D.

Appendix E

Proof of Proposition 4

Suppose there is an equilibrium in which the buyer purchases one unit from each seller in period one (and hence also in period two), and let T_I^* and T_E^* denote the equilibrium contracts. Then, the payoffs to all three parties, taking into account the subsequent play of the game, are given by (B.1), (B.2), and (B.3), and the incentive compatibility constraints are given by (B.4), (B.5), (B.6), (B.7) and the requirement that the buyer does not want to purchase both units from the entrant:

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \;\; \geq \;\; V_{EE}^1 - T_E^*(2). \tag{E.1}$$

It follows from (B.4), (B.5), (B.6) and (E.1) that

$$T_E^*(1) \;\; \leq \;\; \min\left\{V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), V_{IE}^1 - V_{IO}^1\right\}, \tag{E.2}$$

and

$$T_I^*(1) \;\; \leq \;\; \min\left\{V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1), V_{IE}^1 - V_{EO}^1\right\}. \tag{E.3}$$

Since (B.7) is satisfied whenever $T_E^*(1)$ and $T_I^*(1)$ satisfy (E.2) and (E.3), it follows that (E.2) and (E.3) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither seller can profitably deviate given the other's contract. This means that (E.2) and (E.3) must hold with equality. It also means that each seller must earn non-negative payoff under the proposed contracts, and in the case of the entrant, it means that the entrant must earn at least θ in period one. And, finally, it means that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that the incumbent can make itself and the buyer better off by inducing exclusion, and similarly there must not exist a profitable deviation for the entrant that induces exclusion.

The requirement that the incumbent earn non-negative payoff under the proposed contracts is satisfied when $T_I^*(1)$ equals the second term on the right-hand side of (E.3) because then the incumbent's payoff simplifies to $\Pi_I = V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (5). But if $T_I^*(1)$ equals the first term on the right-hand side of (E.3), then the incumbent's payoff is

$$\Pi_I \;=\; V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1) - c + V_I^2 - c,$$

which implies that the incumbent earns non-negative payoff if and only if $T_E^*(2) - T_E^*(1)$ is bounded below by

$$T_E^*(2) - T_E^*(1) \;\; \geq \;\; c - \left(V_{IE}^1 - V_{EE}^1\right) - \left(V_I^2 - c\right). \tag{E.4}$$

The requirement that the entrant earn non-negative payoff and at least θ in period one is satisfied when $T_E^*(1)$ equals the second term on the right-hand side of (E.2) because then the entrant's first and second period payoff under the proposed contracts simplify to $V^1_{IE} - V^1_{IO} - c > 0$ and $V^2_E - c > 0$. But if $T^*_E(1)$ equals the first term on the right-hand side of (E.2), then the entrant's payoff is

$$\Pi_E \;=\; V^1_{IE} - V^1_{II} + T^*_I(2) - T^*_I(1) - c + V^2_E - c,$$

which, since $V^1_{IE} - V^1_{II} + T^*_I(2) - T^*_I(1) - c$ is its first period payoff, implies that the entrant earns non-negative payoff and at least θ in period one if and only if $T^*_I(2) - T^*_I(1)$ is bounded below by

$$T^*_I(2) - T^*_I(1) \;\geq\; c - \left(V^1_{IE} - V^1_{II} - \theta\right). \tag{E.5}$$

The requirement that the incumbent not be able to make itself and the buyer better off by excluding the entrant implies that for T^*_I and T^*_E to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that conditions (B.11), (B.12), (B.13), (B.14), and (B.15) hold and

$$V^1_{II} - \tilde{T}_I(2) \;>\; V^1_{EE} - T^*_E(2), \tag{E.6}$$

where (E.6) ensures that the buyer does not want to purchase both units from the entrant. Since (E.3) holds with equality, either (B.14) is equal to (B.17) and (E.6) is satisfied, or vice-versa, and since $\tilde{T}_I(1)$ can be chosen arbitrarily large, we know that (B.12) and (B.13) can be satisfied. It follows that we can reduce the set of conditions (B.11) to (B.15) and (E.6) to the equivalent set[30]

$$\tilde{T}_I(2) - 2c + V^2_{II}(I) - 2c \;>\; T^*_I(1) - c + V^2_I - c, \tag{E.7}$$

and

$$V^1_{II} - \tilde{T}_I(2) \;>\; V^1_{IE} - T^*_I(1) - T^*_E(1), \tag{E.8}$$

It follows that for T^*_I and T^*_E to arise in an efficient equilibrium, it must be that

$$V^1_{II} - 2c + V^2_{II}(I) - 2c \;\leq\; V^1_{IE} - T^*_E(1) - c + V^2_I - c, \tag{E.9}$$

Once again, we have an upper bound on $T^*_E(1)$ if T^*_I and T^*_E are to arise in an efficient equilibrium:

$$T^*_E(1) \;\leq\; V^1_{IE} - V^1_{II} + c - \left(V^2_{II}(I) - V^2_I - c\right). \tag{E.10}$$

Comparing (E.10) and the fact that (E.2) holds with equality in an efficient equilibrium implies

$$T^*_I(2) - T^*_I(1) \;\leq\; c - \left(V^2_{II}(I) - V^2_I - c\right). \tag{E.11}$$

Hence, (E.5) and (E.11) imply the following upper and lower bound on $T^*_I(2) - T^*_I(1)$:

$$c - \left(V^2_{II}(I) - V^2_I - c\right) \;\geq\; T^*_I(2) - T^*_I(1) \;\geq\; c - \left(V^1_{IE} - V^1_{II} - \theta\right), \tag{E.12}$$

[30]We do not need to include (B.15) in the set because we know that the right-hand side of (B.17) is weakly positive.

which can be satisfied if and only if

$$\theta \;\leq\; V_{IE}^1 - V_{II}^1 - \Big(V_{II}^2(I) - V_I^2 - c\Big). \tag{E.13}$$

The requirement that the entrant not be able to make itself and the buyer better off by excluding the incumbent implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that the analogues of conditions (B.11), (B.12), (B.13), (B.14), (B.15) and (E.6) hold. Following our earlier reasoning, these conditions can be reduced to an equivalent set of conditions, i.e., the analogues of (E.7) and (E.8), and thus ultimately to the analogue of (E.12):

$$c - \Big(V_{EE}^2(I) - V_E^2 - c\Big) \;\geq\; T_E^*(2) - T_E^*(1) \;\geq\; c - \Big(V_{IE}^1 - V_{EE}^1\Big) - \Big(V_I^2 - c\Big). \tag{E.14}$$

To summarize, we have shown that, in any efficient equilibrium, (E.2) and (E.3) hold with equality, and (E.12), (E.13) and (E.14) hold. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1),$$

$$c - \Big(V_{EE}^2(I) - V_E^2 - c\Big) \;\geq\; T_E^*(2) - T_E^*(1) \;\geq\; c - \Big(V_{IE}^1 - V_{EE}^1\Big) - \Big(V_I^2 - c\Big).$$

$$c - \Big(V_{II}^2(I) - V_I^2 - c\Big) \;\geq\; T_I^*(2) - T_I^*(1) \;\geq\; c - \Big(V_{IE}^1 - V_{II}^1 - \theta\Big),$$

and

$$\theta \;\leq\; V_{IE}^1 - V_{II}^1 - \Big(V_{II}^2(I) - V_I^2 - c\Big). \tag{E.15}$$

The necessity of (E.15) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (E.15) follows because when it holds, the buyer's incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Exclusionary Equilibria

Now suppose there is an equilibrium in which the buyer purchases both units from the incumbent in period one (and hence also in period two), and let T_I^{**} and T_E^{**} denote the equilibrium contracts. Then, the payoffs to all three parties, taking into account the subsequent play of the game, are given by (B.24) – (B.26), and the buyer's incentive-compatibility constraints are given by (B.27) – (B.30) and the requirement that the buyer does not want to purchase both units from the entrant:

$$V_{II}^1 - T_I^{**}(2) \;\;\geq\;\; V_{EE}^1 - T_E^*(2). \tag{E.16}$$

It follows from (B.27), (B.28), (B.29) and (E.16) that

$$T_I^{**}(2) - T_I^{**}(1) \;\;\leq\;\; \min\Big\{V_{II}^1 - V_{IE}^1 + T_E^{**}(1),\;\; V_{II}^1 - V_{IO}^1\Big\}, \tag{E.17}$$

and

$$T_I^{**}(2) \;\;\leq\;\; \min\Big\{V_{II}^1 - V_{EO}^1 + T_E^{**}(1),\;\; V_{II}^1 - V_{EE}^1 + T_E^{**}(2)\Big\} \tag{E.18}$$

Since (B.30) is satisfied if (E.17) and (E.18) are satisfied and $T_E^{**}(2) \leq V_{EE}^1$ (as it will be in any equilibrium in which the entrant is excluded), it follows that if $T_E^{**}(2) \leq V_{EE}^1$ then (E.17) and (E.18) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. For the entrant, this means that there must not exist a profitable deviation in which it induces the buyer to purchase from it. For the incumbent, this means that there must not exist a deviation in which it increases its payoff while continuing to sell both units or only one unit to the buyer. It also means that its payoff under the proposed contracts must be non-negative.

Consider the entrant's situation. For T_I^{**} and T_E^{**} to arise in equilibrium, there must not be a profitable deviation for the entrant in which it sells only one unit. That is, there must not exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that the following conditions hold: (B.33), (B.34), (B.35), (B.36), (B.37) and

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} \;>\; V_{EE}^1 - \tilde{T}_E(2), \tag{E.19}$$

Since (B.35) implies (B.36) and (B.37), (E.19) can easily be satisfied by choosing $\tilde{T}_E(2)$ to be arbitrarily large, and (B.33) and (B.34) are satisfied if $\tilde{T}_E(1) > \theta + c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (B.35) does not hold at $\tilde{T}_E(1) = \theta + c$ if they are to arise in equilibrium:

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; V_{II}^1 - V_{IE}^1 + \theta + c, \tag{E.20}$$

and

$$T_I^{**}(2) \;\leq\; V_{II}^1 - V_{EO}^1 + \theta + c. \tag{E.21}$$

The interpretation is that the terms of T_I^{**} must be bounded above by (E.20) and (E.20) in equilibrium because if (E.20) and (E.20) were not satisfied, there would exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that the entrant would be able to profitably induce the buyer to purchase one unit from it.

For T_I^{**} and T_E^{**} to arise in equilibrium, there also must not be a profitable deviation for the entrant in which it sells both units. That is, there must not exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that

$$\tilde{T}_E(2) \;\geq\; \theta + 2c, \tag{E.22}$$

$$\tilde{T}_E(2) - 2c + V_{EE}^2(E) - 2c \;>\; 0, \tag{E.23}$$

$$V_{EE}^1 - \tilde{T}_E(2) \;>\; V_{II}^1 - T_I^{**}(2), \tag{E.24}$$

$$V_{EE}^1 - \tilde{T}_E(2) \;>\; V_{IO}^1 - T_I^{**}(1), \tag{E.25}$$

$$V_{EE}^1 - \tilde{T}_E(2) \;>\; V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), \tag{E.26}$$

$$V_{EE}^1 - \tilde{T}_E(2) \;>\; V_{EO}^1 - \tilde{T}_E(1), \tag{E.27}$$

and

$$V_{EE}^1 - \tilde{T}_E(2) \;\geq\; 0, \tag{E.28}$$

where (E.22) ensures that the entrant earns at least θ in period one, (E.23) ensures that the entrant is better off under the deviation, (E.24) – (E.27) ensure that the buyer would prefer to purchase both units from the entrant, and (E.28) ensures that the buyer earns non-negative payoff. Since (E.24) implies (E.25) and (E.28), (E.26) and (E.27) can easily be satisfied by choosing $\tilde{T}_E(1)$ to be arbitrarily large, and (E.22) and (E.23) are satisfied if $\tilde{T}_E(2) > \theta + 2c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (E.24) does not hold at $\tilde{T}_E(2) = \theta + 2c$ if they are to arise in equilibrium:

$$T_I^{**}(2) \;\; \leq \;\; V_{II}^1 - V_{EE}^1 + \theta + 2c. \tag{E.29}$$

The interpretation here is that the terms of T_I^{**} must be bounded above by (E.29) in equilibrium because if (E.29) were not satisfied, there would exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that the entrant would be able to profitably induce the buyer to exclude the incumbent and purchase both units from it.

Now consider the incumbent's situation. For T_I^{**} and T_E^{**} to arise in equilibrium, there must not be a profitable deviation in which the incumbent sells both units to the buyer. It follows that (E.18) must hold with equality (because otherwise, the incumbent could increase $T_I^{**}(2)$ and still sell both units to the buyer), and therefore, using this result, (B.33), (E.21), (E.22), (E.29), and noting that (E.29) implies (E.21), it follows that the entrant's equilibrium offer must be such that

$$T_E^{**}(2) \;\; = \;\; \theta + 2c, \tag{E.30}$$

and hence the incumbent's offer must be such that

$$T_I^{**}(2) \;\; = \;\; V_{II}^1 - V_{EE}^1 + \theta + 2c, \tag{E.31}$$

and

$$T_I^{**}(2) - T_I^{**}(1) \;\; \leq \;\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{E.32}$$

For T_I^{**} and T_E^{**} to arise in equilibrium, there must also not exist a profitable deviation for the incumbent in which it sells only one unit to the buyer. That is, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that the following conditions hold: (B.43), (B.44), (B.45), (B.46), and

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} \;\; > \;\; V_{EE}^1 - T_E^{**}(2), \tag{E.33}$$

where (E.33) ensures that the buyer does not want to purchase both units from the entrant. Since (E.33) and (E.30) imply (B.45) and (B.46), and (B.44) can be satisfied by choosing $\tilde{T}_I(2)$ arbitrarily large, we can reduce the set of constraints in (B.43) to (B.46) and (E.33) to the equivalent set

$$\tilde{T}_I(1) - c + V_I^2 - c \;\; > \;\; V_{II}^1 - V_{EE}^1 + \theta + V_{II}^2(I) - 2c, \tag{E.34}$$

and

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} \;\; > \;\; V_{EE}^1 - \theta - 2c, \tag{E.35}$$

where we have substituted (E.31) into (B.43) to get (E.34) and (E.30) into (E.33) to get (E.35). By summing the left and right-hand sides of (E.34) and (E.35) using the first argument in the left-hand side of (E.35) and then doing it again using the second argument in the left-hand side of (E.35), we can further reduce the set of constraints in (B.43) to (B.46) and (E.33) to the equivalent set

$$V_{IE}^1 + V_I^2 - T_E^{**}(1) \quad > \quad V_{II}^1 + V_{II}^2(I) - 2c, \tag{E.36}$$

or

$$V_{IO}^1 + V_I^2 \quad > \quad V_{II}^1 + V_{II}^2(I) - 2c. \tag{E.37}$$

Since $V_{II}^1 - V_{IO}^1 > c$ and $V_{II}^2(I) - V_I^2 > c$ (using (1), (2), and (5)), (E.37) fails to hold, and thus, it follows that the set of constraints in (B.43) to (B.46) and (E.33) hold if and only if (E.36) holds.

It follows that if T_I^{**} and T_E^{**} are to arise in equilibrium, then

$$T_E^{**}(1) \quad \geq \quad V_{IE}^1 - V_{II}^1 + 2c - \left(V_{II}^2(I) - V_I^2\right). \tag{E.38}$$

Finally, it must be that the incumbent earns non-negative payoff under the proposed equilibrium contracts. Substituting (E.31) into (B.24), we have that the incumbent's payoff is

$$\Pi_I \; = \; V_{II}^1 - V_{EE}^1 + \theta + V_{II}^2(I) - 2c, \tag{E.39}$$

which, given our assumption that $V_{II}^1 \geq V_{EE}^1$ and $V_{II}^2(I) \geq V_{EE}^2(E)$, implies that Π_I is nonnegative.

To summarize, we have shown that in any equilibrium in which the entrant is excluded, contracts are such that (E.30), (E.31), (E.32), and (E.38) hold. Thus, in any such equilibrium, we have

$$T_E^{**}(1) \quad \geq \quad V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right),$$

$$T_E^{**}(2) \; = \; \theta + 2c,$$

$$T_I^{**}(2) \; = \; V_{II}^1 - V_{EE}^1 + \theta + 2c,$$

and

$$T_I^{**}(2) - T_I^{**}(1) \quad \leq \quad c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{E.40}$$

The necessity of (E.40) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (E.40) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Now suppose there is an equilibrium in which the buyer purchases both units from the entrant in period one (and hence also in period two). Then, using the reasoning above, we obtain the analogue to (E.40): in any equilibrium in which the incumbent is excluded, the entrant offers

$$T_E^{**}(2) \; = \; V_{EE}^1 - V_{II}^1 + \theta_I + 2c, \tag{E.41}$$

where $\theta_I = -\left(V_{II}^2(I) - 2c\right)$ is the maximum amount that an unconstrained incumbent would be willing to pay in period one to avoid being excluded. Given our assumption that $V_{II}^1 \geq V_{EE}^1$ and $V_{II}^2(I) \geq V_{EE}^2(E))$, it follows that $T_E^{**}(2) \;\leq\; \theta_I + 2c$, which violates our assumption that the entrant has a financing constraint. Hence, there is no equilibrium in which the incumbent is excluded.

It remains only to show that there do not exist equilibria in which the buyer purchases one unit from one seller or no units. The proof of this is given in the last paragraph of Appendix B. Q.E.D.

REFERENCES

Aghion, P. and P. Bolton (1987), Contracts as a Barrier to Entry, *American Economic Review*, 77: 388-401.

Bernheim, D. and M. Whinston (1998), Exclusive Dealing, *Journal of Political Economy*, 106: 64-103.

Bork, R. (1978), *Antitrust Paradox: A Policy at War With Itself*, Basic Books: New York.

Carlton, D. and M. Waldman (2002), The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries, *Rand Journal of Economics*, 33: 194-220.

Choi, J. and C. Stefanadis (2001), Tying, Investment, and the Dynamic Leverage Theory, *Rand Journal of Economics*, 32: 52-71.

Clementi, G.L. and H. Hopenhayn (2006), A Theory of Financing Constraints and Firm Dynamics, *Quarterly Journal of Economics*, 121: 229-265.

DeGraba, P. (1999), Tying and Exclusion in the Presence of Scale Economies and Sources of Outside Revenue: An Application to Microsoft, working paper, Federal Trade Commission.

Director, A. and E. Levi (1956), Law and the Future: Trade Regulation, *Northwestern University Law Review*, 51, 281-296.

Erutku, C. (2006), Rebates as Incentives to Exclusivity, *Canadian Journal of Economics*, 39: 477-492.

Evans, D. and R. Schmalensee (2006), The Industrial Organization of Markets with Two-Sided Platforms, working paper, Law & Economics Consulting Group, Cambridge, MA.

Farrell J. and C. Shapiro (1988), Dynamic Competition with Switching Costs, *Rand Journal of Economics*, 19: 123-137.

Greenlee, P., Reitman, D., and D. Sibley (2004a), An Antitrust Analysis of Bundled Loyalty Discounts, working paper, U.S. Department of Justice.

Greenlee, P. and D. Reitman (2004b), Competing with Loyalty Discounts, working paper, U.S. Department of Justice.

Hovenkamp, H. (2005), *The Antitrust Enterprise*, Harvard University Press: Boston.

Klemperer, P. (1987a), Markets with Consumer Switching Costs, *Quarterly Journal of Economics*, 102: 375-393.

Klemperer, P. (1987b), Entry Deterrence in Markets with Consumer Switching Costs, *Economic Journal*, 97: 99-117.

Kolay, S., Shaffer, G., and J. Ordover (2004), All-Units Discounts in Retail Contracts, *Journal of Economics & Management Strategy*, 13: 429-459.

Marvel, H. (1982), Exclusive Dealing, *Journal of Law & Economics*, 25: 1-25.

Marvel, H. and H. Yang (2006), Group Purchasing, Nonlinear Tariffs, and Oligopoly, working paper, Ohio State University.

Marx, L. and G. Shaffer (2004), Rent-Shifting, Exclusion, and Market-Share Discounts, working paper, University of Rochester.

Mills, D. (2004), Market-Share Discounts, working paper, University of Virginia.

Nalebuff, B. (2004), Bundling as a Barrier to Entry, *Quarterly Journal of Economics*, 119: 159-187.

O'Brien, D.P. and G. Shaffer (1997), Nonlinear Supply Contracts, Exclusive Dealing, and Equilibrium Market Foreclosure, *Journal of Economics & Management Strategy*, 6: 755-785.

Ornstein, S. (1989), Exclusive Dealing and Antitrust, *Antitrust Bulletin*, 34: 65-98.

Posner, R. (2001), *Antitrust Law*, 2nd. edition, University of Chicago Press: Chicago.

Rasmusen, E., Ramseyer, J., and J. Wiley (1991), Naked Exclusion, *American Economic Review*, 81: 1137-1145.

Segal, I. and M. Whinston (2000a), Naked Exclusion: Comment, *American Economic Review*, 90: 296-311.

Segal, I. and M. Whinston (2000b), Exclusive Dealing and Protection of Investments, *Rand Journal of Economics*, 603-633.

Symposium on Loyalty Rebates (2005), *Competition Policy International*, 1: 89-173.

Tirole, J. (2006), *The Theory of Corporate Finance*, Princeton University Press: Princeton.

Tom, W., Balto, D., and N. Averitt (2000), Anticompetitive Aspects of Market-Share Discounts and Other Incentives to Exclusive Dealing, *Antitrust Law Journal*, 67: 615-640.

Whinston, M. (1990), Tying, Foreclosure, and Exclusion, *American Economic Review*, 80: 837-859.

Whinston, M. (2001), Exclusivity and Tying in U.S. v. Microsoft: What We Know, and Don't Know, *Journal of Economic Perspectives*, 15: 63-80.