Forward-looking bidding

in

online auctions

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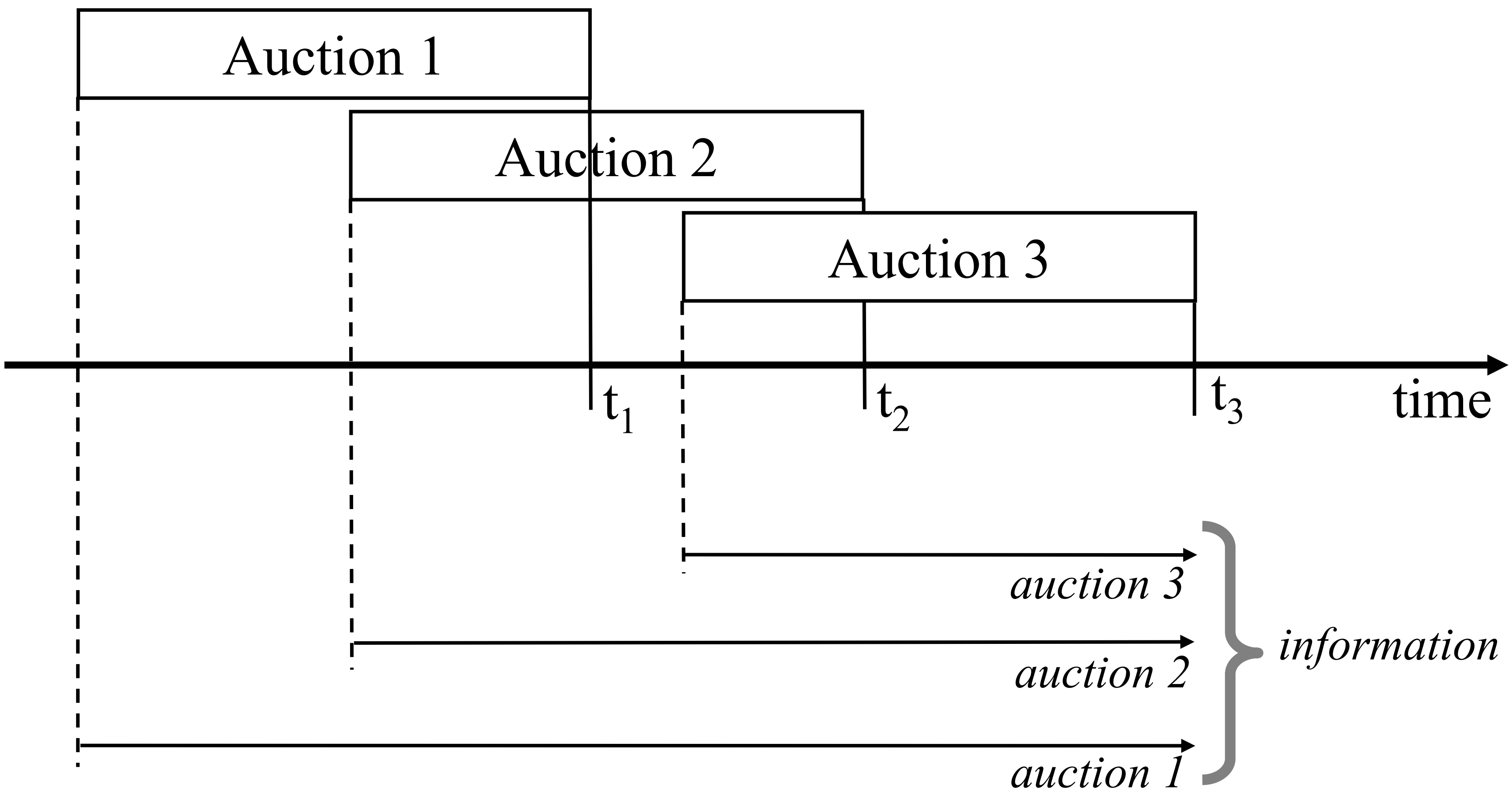
Let’s buy a digital camera on eBay...

Canon S30, *15 mins left*
Canon S40, *33 mins left*
Olympus D40, *45 mins left*
Canon S30, *47 mins left*
Olympus D40, *53 mins left*

- Electronics, movies, computers ... each buyer only wants one unit
- Population heterogeneity in preferences (I am shopping for Canon S30)
- Simultaneous? No, <u>sequential</u>, implicitly organized by end time
- Interlaced sequences of auctions for essentially identical objects

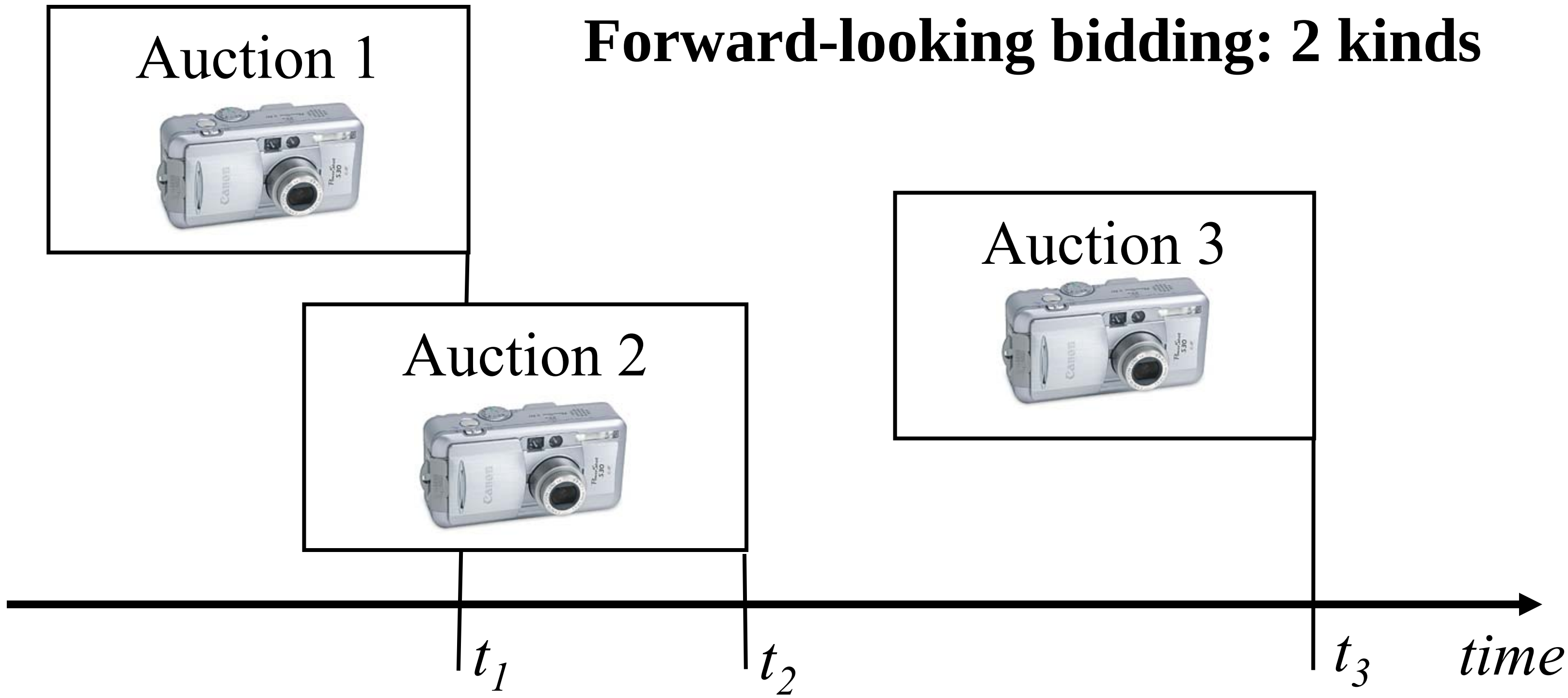
eBay: sequential auctions with overlapping information



Research questions:

1) How to bid while incorporating the available information?
2) Do eBay bidders bid consistently with the theory?

Forward-looking bidding: 2 kinds



unit-demand → **option-value of losing** → bid-shading (below isolated auction)

How to bid in auction 1?

- given the known ("**forward-seen**") auction 2
- given a potential ("yet unseen") auction 3 (Jofre-Bonet & Pesendorfer 03)

Some related work (all unit-demand bidders)

- **Milgrom & Weber (82b,99)** :
  - finite sequences , identical units
  - no use for information about future auctions (all the same)
  - finite → no bidder-replacement needed → elegant solution
- **Engelbrecht-Wiggans (94)** , **Jofre-Bonet & Pesendorfer (03)** :
  - finite sequences, stochastically equivalent units (different but *iid* units)
  - no information about future auctions → symmetric and independent future
- **Gale & Hausch (94)** :
  - two auctions, different and potentially correlated units
  - $(v_1, v_2) \sim$ continuous F, both (v_1, v_2) known at the start
  - units not necessarily identical → disposal issues
  - very hard to extend to many auctions
  - <u>Contrast</u>: I will only allow $v_i \in \{v, 0\} \approx \{\text{"desired"}, \text{"other"}\}$

Model: One-period look-ahead, 2-type example

Infinite sequence of second-price, sealed-bid auctions

- varying waiting-times ω between individual auctions
- each auction sells one unit of a type-k good, k:{1,2}, Pr(k =1) = ½
- no reserve

N_k bidders present in every period, live until win or exit (Pr(*exit*)=λ per hour)

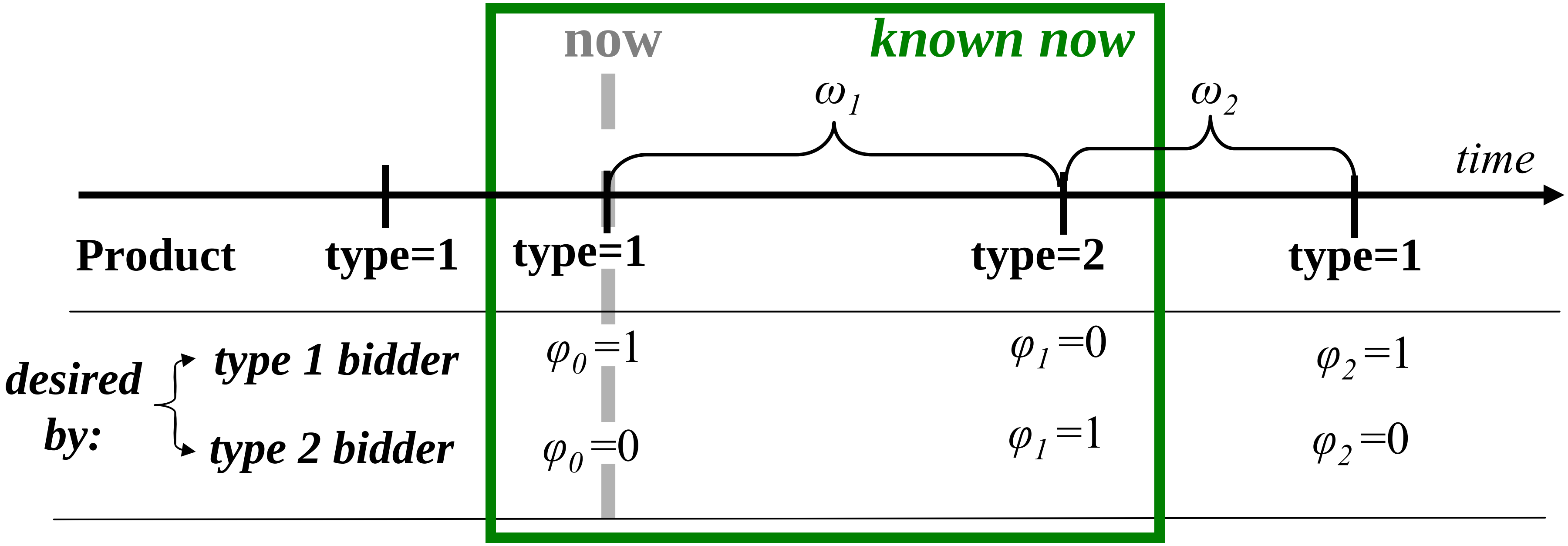
- unit-demand for only one type of good (“desired” type)
- IPV single-unit valuation of desired type, $v \sim F$ continuous
- **Info**: binary desirability of current unit φ_0 and next unit φ_1, waiting-time ω_1

Everyone discounts future δ per hour, no memory

Discussion of the assumptions

- Interlaced sequences of identical-goods auctions with non-overlapping pop.
- Some bidder-replacement essential (otherwise steady-state survivors v =0)
- Innovation: bids depend on forward-seen information (ω_1,φ_1)

Model: One-period look-ahead, 2-type example



$S(\varphi_0,\varphi_1,\omega_1,v_i\,|\,c_0)$:expected surplus given loss to current competitive bid $c_0 \sim G$

$$b(\varphi_0,\varphi_1,\omega_1,v) = \arg\max_{\beta} \int^{\beta} \underbrace{(\varphi_0 v - c_0)}_{\text{surplus if win now \& pay } c_0} dG(c_0) + (\delta\lambda)^{\omega_1} \int_{\beta} \underbrace{S(\varphi_0,\varphi_1,\omega_1,v\,|\,c_0)}_{\text{surplus if lose now to a bid } c_0} dG(c_0)$$

1(current desired)

1(next desired)

time till next

valuation of desired

surplus if win now & pay c_0

key tradeoff

surplus if lose now to a bid c_0

Optimal Forward-Seeing Bidding

$$b(\varphi_0,\varphi_1,\omega_1,v)=\arg\max_{\beta\geq 0}\int_0^{\beta}(\varphi_0 v-c_0)\,dG(c_0)+(\delta\lambda)^{\omega_1}\int_{\beta}S(\varphi_0,\varphi_1,\omega_1,v\,|\,c_0)\,dG(c_0)$$

FOC: $b(1,\varphi_1,\omega_1,v)=v-(\delta\lambda)^{\omega_1}S\big(1,\varphi_1,\omega_1,v\,|\,c_0=b(1,\varphi_1,\omega_1,v)\big)<v$

$b(0,\varphi_1,\omega_1,v)=0$

SOC: $\dfrac{\partial S(1,\varphi_1,\omega_1,v\,|\,c_0)}{\partial c_0}>-\dfrac{1}{(\lambda\delta)^{\omega_1}}$

Properties:

- can show FOC has a unique solution, and that SOC satisfied
- bid-shading (a benefit to losing compared to isolated 2PSB)
- “pivotal thinking” : bid as if about to lose in a tie to a bidder like you

Equilibrium

Bellman condition: In a symmetric pure-strategy Markov-Perfect equilibrium, the expected surplus function must be "correct":

$$S\left(\varphi_{0,1},\omega_1,v\,|\,c_0\right)=E_{\varphi_2,\omega_2}\left[\int^{b(\varphi_{1,2},\omega_2,v)}\left(v-c_1\right)dG\left(c_1\,|\,c_0,\varphi_{0,1,2},\omega_{1,2}\right)+\left(\delta\lambda\right)^{\omega_2}\int_{b(\varphi_{1,2},\omega_2,v)}S\left(\varphi_{1,2},\omega_2,v\,|\,c_1\right)dG\left(c_1\,|\,c_0,\varphi_{0,1,2},\omega_{1,2}\right)\right]$$

S exists when *F* has a continuous density on a compact interval.

For a given *F*, *S* can be obtained by value-function iteration.
Could this be a basis for a structural approach?

Bidders are not price-takers, take into account evolution of the pool of competitors.

Reduced-form test of model predictions

1) K+1 types, multi-period look-ahead with timing (**type-independent**) information Ω and product (**type-specific**) information Φ
   – eBay bidders usually see about a week ahead, could be many periods
   – Ω : auctions ending within the next hour marked in **red**, easy to see

2) Focus on a particular subset *x* of the state-variables (Ω,Φ) and integrate out the rest of the state, i.e. generate "on average" predictions given *x*:

   $\bar{b}(x,v) = E\left[b(1,\Phi,\Omega,v) \mid x\right]$ (example: x = # auctions ending within next hour)

3) If something is true for every valuation *v,* it will be true for the order-statistics of the valuations within each auction (keeping *N* constant)

4) Note that the first and second highest bids are observed in eBay data. => Regress bid order-statistics $b_{(j)}(x)$ on *x* (control for varying *N*)

Preliminary evidence for predicted behavior

- Most eventual winners won only one unit within the data-period (93% in MP3-players and 87% in movies).
- A substantial number of bidders participated in more than one auction (43% in MP3-players and 33% in movies) and those who did mostly stuck to bidding on one product-type.
- It does not seem that the multi-auction bidders simply submitted a very low bid initially to learn about the auction process or their true valuation, and only later raised their bid to their “full” willingness to pay. (Of the multi-bidders, 49% in movies and 59% in MP3 players submitted a higher second bid).