Auction Hosting Site Pricing and Competition*

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Abstract

This paper derives the analytics of seller and buyer participation in an auction hosting site, characterizes optimal hosting site pricing and investigates the nature of competition between two auction houses or hosting sites that are differentiated in the eyes of the bidders. The auction sites earn revenue by setting positive listing fees, trading off the increased revenue per seller from higher fees with the revenue reduction from the loss of sellers. The reduction in the number of sellers participating in a site has feedback effects, as it affects the number of bidders who would choose to visit that site. The sellers set the reserve in each sale to maximize their revenue. Unlike most prior models of auction site competition, buyers have preferences for bidding in the two sites. These preferences may be driven by interface layout, prior experience with a site (which makes it less costly in terms of transaction costs to buy from that site), customer service experience, transaction reliability and reputation of sellers, and — more generally — anything that would fall under the rubric of site loyalty. Different buyers have different preferences for site interface and different experiences, resulting in some measure of horizontal differentiation between the sites. This differentiation between the two sites implies that even if they both set the exact same rules and pricing policies, some buyers would strictly prefer to purchase from one, while others would strictly prefer to purchase at the rival site. Though preference for the two sites need not be symmetric, differentiation results in market power and positive economic profits for both sites. We start by the careful analysis of the monopoly case, in which the second of the two firms is absent. In the monopoly case, factors such interface layout and prior experience that would lead to site heterogeneity under duopoly, result in bidder heterogeneity with respect to participation costs. We then consider the special features involving the equilibrium under duopoly, noting that many of the features of the equilibrium under monopoly remain valid under duopoly as well.

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1 Introduction

Auction hosting sites, whether of the brick-and-mortar or Internet varieties, are firms whose product is the provision of a marketplace in which buyers and sellers can transact. They are thus intermediaries or market makers rather, rather than buyers or sellers of the products that flow through them.[1] Their product is the marketplace itself, or in the terminology of the recent literature, a platform that connects two sides of a market rather than a traditional physical product (see Rochet and Tirole, 2004, Armstrong, 2005, Hagiu, 2005, and references therein for a recent discussion on platforms and platform pricing). Much of the early literature on auctions abstracted from the presence of auction hosting sites and considered auctions to be an interaction between the owner of an item (the seller) and many potential buyers (the bidders).[2] Moreover, the bulk of the recent auction literature that focuses on the Internet auctions and on big-item art auctions, where competition between auction houses or Internet sites is likely very important, focuses primarily on seller vs buyer issues and the dynamics of prices (within a series of auctions or across time) and only minimally on auction site vs auction site competition issues (see Bajari and Hortacsu, 2004, and Ashenfelter and Graddy, 2003, for two recent excellent surveys of this literature and associated issues).

Some of the more recent literature recognizes that auctions are not merely interactions between a seller and the bidders, but rather that bidders often have a choice of which auction to attend, i.e., a choice of whose seller's item to bid for. Sellers, in other words, often compete with each other, and often do so indirectly, through the venue at which they choose to stage their auction.[3] Possibly the first contribution in this line of research is McAfee (1993) who considers competition between sellers through their choice of the reserve price. This work has been extended and generalized by Peters (1997), and Peters and Severinov (1997), who, as McAfee (1993), treat each seller as a "site," and by Hernando-Veciana (2005) who explicitly considers seller production/opportunity costs.[4] Recent literature has also developed in two additional and parallel directions. Some authors (Burguet and Sakovics, 1999 and Schmitz, 2003) examine equilibria in reserve competition when the number of sellers is small (in fact, equal to 2). Other authors, including Anderson, Ellison, Fudenberg (2004) and Ellison, Fudenberg, and Mobius (2004), explicitly recognize the platform nature of auction hosting sites. All of these papers, however, as well as recent work by Damianov (2005), do not consider

[1]See Spulber 1999 and 2006 for broad overviews on the literature on intermediaries and market makers.

[2]See McAfee and McMillan (1987a) and Klemperer (1999) for surveys of the early auction literature and Wolfstetter (1996) and Krishna (2002) for a textbook-style treatments of the subject.

[3]Of course, very often sellers who choose the same venue compete with each other directly and simultaneously, something that is typically modeled by auction theorists as a multi-unit auction.

[4]These papers consider seller competition to be in terms of the reserve being set; auctions are taken to be the chosen mechanism exogenously. Peters (2001) shows that second price auction will arise endogenously as the equilibrium mechanism in such environments.

the possibility that the auction hosting sites are differentiated in the eyes of the buyers and sellers, even when the number of buyers and sellers in each site is the same. Rather, hosting sites are only differentiated endogenously, i.e., by the equilibrium number of participants of each type that patronize them.[5]

This paper bridges the gap between the platform competition literature (which does not explicitly consider the structure of bidding competition) and the competing sellers and/or auction sites literature (which does not consider the possibility of intrinsic differentiation between the sites). Unlike the extant literature discussed above, we explicitly recognize that auction hosting sites are differentiated in the sense that some bidders would strictly prefer to browse and bid in one site while other would strictly prefer to browse and bid in another. We also recognize that some sites may be "better" that the other in the sense that they would attract more bidders (and sellers) if they offered the same terms for participation. Finally, unlike much of the existing literature, we carefully distinguish between strategic choices that are under the purview of sellers (who are short-run or one-shot players) and the hosting sites (who are long run players). Our aim is to understand the interplay between site differentiation, seller behavior, and site strategies, and how these elements affect the equilibrium in the market for auction hosting services.

We model horizontal differentiation of the auction hosting sites, from the point of view of buyers, using the standard linear city framework (Hotelling 1929). However, the two sites are also vertically differentiated in the sense that mean utility of shopping from one site is (potentially) higher than mean utility from shopping from the other site, even if the number of sellers and the posted reserve is the same for both sites.[6] With regards to site differentiation from the point of view of the sellers, we assume that differentiation is sufficiently strong that the sites have effective monopoly power on that side of the market. Potential sellers who are "priced-out" of one site due to excessively high listing fees would rather choose to keep the items for themselves rather than set-up an account with the competing site.[7] Such formulation allows us to identify the strategic effects from indirect, rather than direct, competition of sites for sellers. If a decrease in the listing fee attracts more sellers into a

[5]A distinct piece of research by Parlane (2005) considers differentiation of products offered by otherwise homogeneous sites.

[6]Economides (1989), Neven and Thisse (1990), and others, also develop extensions to the Hotelling model in which mean utility of the two brands differs across the two firms.

[7]In some respects, there is a resemblance between this work and research on endogenous bidder entry in auctions (see Engelbrecht-Wiggans, 1987, McAfee and McMillan, 1987b, and Engelbrecht-Wiggans, 1993). In our model both seller and bidder entry is endogenous, with the site plays the role of "seller" in setting entry fees, albeit for potential sellers not potential bidders, while the bidder entry is determined by expected surplus they would obtain by visiting a particular site. Given that from the point of view of the price-setting agent entry is probabilistic, the closest model of bidder entry are those of Levin and Smith (1994) and Samuelson (1985), though in the latter the bidders learn their valuations prior to the entry decision.

site, this affects the number of sellers who are attracted to the competing site not directly (through their diversion to the first site), but indirectly (through potential bidder site choices): As more sellers list in the first site, fewer buyers will frequent the competing site, thus reducing the value of that site to prospective sellers who would become more likely to abstain from selling there.

The premise of auction hosting site differentiation is based on (and consequently, this paper is related to) much of the recent literature on competition between online retailers. Competition between Internet retailers has been initially perceived as ushering in a era of marginal cost pricing for the retailers. Underlying this prediction was the presumption that online retailers were undifferentiated in the eyes of potential shoppers. In the last decade or so, however, it has become evident that this has not been the case. Though online margins may be lower than those of brick-and-mortar retailers, they are certainly non-negligible. Moreover, the lowest price online sellers are not the highest volume sellers (see Brynjolfsson and Smith, 2000). Not only are price/quantity more consistent with product markets with sellers that are both vertically and horizontally differentiated, but traditional marketing instruments, such as advertising, have the same qualitative effects on the prices and price dispersion of Internet retailers as they do on the prices of traditional retailers (see Clay, Krishnan, and Wolff, 2001). As the detailed, individual customer-level, study by Smith and Brynjolfsson (2001) has demonstrated, Internet retailers have apparently been able to differentiate themselves.[8] Much of this differentiation has been induced by the introduction of artificial informational frictions in online trading (see Ellison, G. and S. Ellison [2004a]). Not all of this differentiation, however, can be attributed to such "obfuscation" strategies: Internet book retailers, for example, appear to draw different types of clientele's and have standard looking demand curves and cross-elasticities (see Chevalier and Goolsbee, 2003).

Auction house competition is not limited to the online world. Brick-and-mortar auction sites actively compete with each other for sellers, with buyers following the sellers into the bidding hall. The severity of such competition is illustrated by the scope and magnitude of attempts by Christie's and Sotheby's to suppress it. Ashenfelter and Graddy (2004) provide a play-by-play discussion of the collusive agreement and the mode of competition between the auction houses, and also provide some anecdotal evidence that sellers have (to some extent) preferences for the two major auction houses.

There are, however, differences between online and physical auction houses. Physical houses, for example, may be better able to enforce rules on reserves than online sites.[9] Moreover, the extent to which bidder participation costs are driven by the cost of bidding rather than the cost of identifying

[8] A good discussion of findings on online markets and the implications for the broader Industrial Organization can be found in Ellison and Ellison (2004b).

[9] In an online site, a site-imposed on reserve may be easier to circumvent through phantom bidding from a different account.

items that they are interested in bidding in may vary between the online and physical world and also for different item categories within the online world. For this reason, we develop a number of variants to our basic modeling framework to better capture variations in the bidding and competitive environment. Thus, our framework is not meant to apply solely to online or physical auction sites, but rather provide a more general basis for analyzing competition issues.

2 The Basic Model

We consider two auction sites, A and B. There are M potential sellers for each site and $N \cdot M$ potential buyers in the market. Potential sellers of site A are indexed by j_A and potential sellers of site B by j_B (to economize on notation, when there is no possible ambiguity and no need to distinguish between the potential sellers of either site, we will index generic seller by j). Potential buyers are indexed by i. Each of the potential sellers has a single item to sell and a cost of parting with that item that is equal to c^S.[10] Denote the distribution of c^S by $G(\cdot)$. The items owned by the potential sellers differ within a site but are replicas of each other across sites: If a seller j'_A has a particular item, seller j'_B has that same item (though their costs of parting with that item are independent draws from $G(\cdot)$). Thus, the two sites are isomorphic in terms of the nature of their (potential) content.

Each potential buyer has an interest in purchasing a single unit of an item. The maximum willingness of bidder i to pay for a unit of the item owned by seller j, v_i^j, is an i.i.d. draw from the distribution $F_j(v)$ and is private information. We assume that the bidder observes v_i^j following his decision to visit one of the two sites. Implicitly, we assume that there is information about the details of the item and this information is observed only upon visiting the site. Alternatively, the process of browsing the site and bidding is necessary for the potential buyer to cognitively identify the maximum willingness to pay for that item.

For simplicity, we assume that in each of the two sites, there is only one potential seller who owns the item that a particular potential buyer is interested in. The willingness of that buyer to pay for the items owned by the other potential sellers is zero. Formally, $v_i^j > 0$ for $j = j'$ and $v_i^j = 0$ for $j \neq j'$. Moreover, we will also assume for simplicity that for each potential seller there are exactly N potential buyers that are interested in the item that he offers for sale. The other potential buyers have zero willingness to pay for it. Therefore, there is no substitutability between the items offered

[10] Costs of parting with the item can be thought of as transaction costs of putting it for sale. Alternatively, they can be thought of as the use value of the item for the seller. However, under this second interpretation, c^S would affect the optimal reserve the seller would prefer to post. When the seller posts no reserve or the reserve is set by the hosting site, it makes no difference as to the nature of c^S. When the seller chooses the reserve, we will treat c^S as being a transaction cost. This is done for simplicity, as the nature of the results would not be affected if c^S were thought of as the seller use value even when the reserve is set by the seller.

for sale within a site, and items do not differ in their "popularity." These assumptions are sufficient to allow us, without further loss of generality, to limit ourselves in the case of $M = 1$.

Sites are described by the costs that potential buyers must incur in order to transact there. These costs could include the costs of browsing the site to identify whether an item they are interested in is indeed there, the costs of finding the item, reading its description, setting up an account (if they do not have one), bidding, and obtaining the item. Sites differ in terms of their layouts and potential buyers differ in terms of their preferences for layouts and their prior experience with them. Therefore, the transaction costs for each site differ from bidder to bidder. Each bidder is characterized by their location x_i which indexes their relative preference for the layout of site A. The transaction costs of bidder i for site A are equal to $t_i^A = c^A + \theta x_i$ and the transaction costs of that same bidder for site B are equal to $t_i^B = c^B + \theta(1 - x_i)$. Notice that this is similar to linear "transport" costs in a Hotelling type of model, albeit one that augments them with a cost component that is that the same for all potential buyers but which is potentially different between the two sites. This second cost component (c^A or c^B) allows the two sites to differ vertically.

The sellers and buyers, and the associated participation costs to the sites, are schematically shown in Figure A, drawn for $M = 9$ and $N = 2$. Competition between potential buyers is schematically shown in Figure B. In this figure, the two potential that have positive valuations for a particular item are shown in boldface. The two sellers (one for each site) that own that item are also shown in boldface. Their choice to offer or not to offer the item for sale depends on the site fee (to be defined shortly), on their cost draw, and on how likely it is that the two potential buyers will attend site A or B. The participation decision of the buyers depends on their participation costs, the likelihood that the sellers will offer the item for sale, and how likely it is that the competing bidder will also attend.

The strategy space for each of the actors in this market is as follows. Sites decide on the listing fee, f, to charge to the sellers.[11] Potential sellers decide on whether or not to pay the listing fee and put their item for sale. They also decide on what reserve to place on their item. Buyers decide which auction site to visit and, if they identify an item they are interested in purchasing, how much to bid for it. Auctions are second price English format with a secret reserve or with a reserve that is not observed until after the prospective buyers have committed to a particular site. That is, sellers cannot commit to a particular reserve policy in advance of bidding for their item.

The timing of the actions is as follows. Auction sites set the listing fees, f^A and f^B, simultaneously. Potential sellers privately observe their transaction cost, c^S, and, given listing fees f, decide on whether to put their item for sale or keep it to themselves. The reserve, $\underline{v}_j$, is set, but is not observed by the

[11] In a future variation of the standard model, we allow the sites to also influence the reserve

buyers until after they commit to a site (or later). Potential buyers decide which site to attend, knowing the equilibrium fraction of potential sellers that choose to sell their items in each of the two sites (and anticipating the equilibrium reserve). They incur participation costs and identify whether their valuation exceeds the anticipated reserve.[12] They enter the English auction, determining as the price rises whether or not it exceeds their maximum willingness to pay. The winner is the last bidder to remain in the auction and payoffs are realized to all players. If a potential buyer is unsuccessful in a particular site, he cannot (or is assumed not to) attempt to browse and bid on the competing site.[13]

With regards to the nature of the equilibrium, we assume forward rationality and are looking for stable equilibria in an (infinite) replication of the above game in which the two sites are long-run players and sellers and buyers are short-run players who only participate in the one-shot version of the game (or are atomistic and anonymous so that their current actions do not impact future rounds of the game).[14]

A realization (or draw) of the equilibrium participation decisions to site A is shown in Figure C. The shaded potential buyers have location draws x_i that are relatively low and choose to browse site A (the remainder of the potential buyers either browse site B or browse neither site). The shaded potential sellers choose to offer their items for sale in site A (the other sellers shown below site A do not offer their products for sale at either site). The participation decisions of buyers are sellers are jointly determined. In equilibrium, the participating buyer with the highest value of x_i earns positive expected profits given the equilibrium seller participation, and the potential buyer with the next highest draw of x_i would have earned negative profits had he attended site A. Similarly, the participating seller with the highest cost would earn positive profits, given the site listing fee and the participation decisions of the buyers. The seller with the next highest cost would have earned negative surplus. Auction site A sets the fee optimally to maximize profits, taking the inter-relationship between seller and buyer participation decisions into consideration.

3 Bidder and Seller Participation

In this section we derive some general relationships and propositions concerning the participation problem that sellers and bidders face. Recall that whereas bidders choose which site (if any) to

[12] In a variation of the model, we considered the possibility that the participation costs are incurred after the bidders enter the auction, i.e., they are bidding costs. Many of the features of the two models appear to be similar, but this model variation has not been analyzed as thoroughly as the base model.

[13] One possibility is that each buyer has a limited time to browse and bid. Another possibility is that the auction in the competing site has concluded and that other similar items will not appear in an timely fashion. Of course, it is possible to relax this assumption at some increase in the model's complexity (though it is clear from the model development that much if not all of the development of the results will not be affected qualitatively.

[14] Stability is important because there are also equilibria in the entry decision of the sellers that are not stable to small random fluctuations to likelihood of seller participation.

attend, sellers are *a priori* associated with one of the two sites. Thus, for sellers, the analysis only concerns their listing decision.

As far as bidders are concerned, we for now consider the case of only a single listing site. This case is used to establish all the main insights into bidder behavior. Moreover, this analysis closely reflects the case of duopolistic competition when the entire market is not covered. We assume that the single auction hosting site is firm A. The results are applied to the covered market duopoly case with firm B also present in later sections.

3.1 Preliminaries

Suppose that there are N potential bidders distributed uniformly on the interval $[0,1]$. Let x_n denote the location of bidder n in $[0,1]$. And let $x^{(n)}$ denote the location of the n^{th} ordered bidder from the left on $[0,1]$. Thus, $x^{(1)}$ is the location of the bidder located most to the left (the smallest value of x_n) and $x^{(N)}$ is the location of the bidder farthest to the right (i.e., the largest value of x_n).

Let $x_c \in [0,1]$. Then the following analysis gives the respective probabilities of auctions with different number of bidders, given that bidders who are located to the left of x_c attend the auction. The probability that there are *exactly* $n \in \{0,1,\dots,N\}$ potential bidders in the interval $[0,x_c]$ is given by,

$$h(n|x_c) := \Pr\left\{x^{(n)} \in [0,x_c] \wedge x^{(n+1)} \notin [0,x_c]\right\} = \binom{N}{n} x_c^n (1-x_c)^{N-n}.$$

And the probability that there are no more than n bidders in $[0,x_c]$ is given by,[15]

$$\begin{aligned} H(n|x_c) :&= \Pr\left\{x^{(n+1)} \notin [0,x_c]\right\} = \sum_{m=0}^{n} \binom{N}{m} x_c^m (1-x_c)^{N-m} \\ &= (N-n)\binom{N}{n} \int_0^{1-x_c} t^{N-n-1}(1-t)^n dt. \end{aligned} \tag{1}$$

Notice that this is the relevant number of bidders from the perspective of the seller and host site. However, a bidder, in making the calculations on what to expect, will be interested not in the total number of bidders, but in the number of rivals that are there.

Given these preliminaries, we can study the bidders' and seller's problem using standard techniques from auction theory.

3.2 The Bidders' Attendance Decision

Suppose there are $n \in \{1,\dots,N\}$ bidders at an auction with (optimally chosen) reserve price $\underline{v}$. Suppose further that the bidders' values are i.i.d. draws from the distribution $F(y)$ on $[0,\overline{v}]$, that has

[15] The final derivation is obtained through successive integration by parts (see, e.g., Feller (1950), pg. 173).

the monotone hazard rate property. Then bidder i's expected payoff when having value $v_i(> \underline{v})$ is given by[16]

$$E\pi(v_i, n) := \int_{\underline{v}}^{v_i} [F(y)]^{n-1} dy. \tag{2}$$

In general, let a bidder's expected continuation payoff before the auction begins (i.e., before his value is revealed), but after he has incurred the transactions costs associated with going to and browsing the site, be defined as:

$$E[\pi(n)] := E_{v_i} E\pi(v_i, n) = \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{w} [F(y)]^{n-1} dy F(w) f(w) dw. \tag{3}$$

That is, $E[\pi(n)]$ is the expectation of Equation (2) with respect to the bidder's value, v_i.

Prior to going to the auction site, however, it is not yet known how many sellers are at the site and whether any of these have listed the item that the bidder in question is searching for. Letting $q \in [0, 1]$ denote the measure of sellers at the auction — or, having made the assumption that the analysis of one sellers' problem suffices under the assumptions made in the model section the probability that the seller is at auction — (given a listing fee of f), one can determine the threshold location of the critical bidder *ex ante*. That is, one can find the location, x_c, of the marginal bidder who is indifferent between browsing and not browsing the auction site.

Lemma 1 (Critical Distance to Auction Site) *Given the (linear) 'transportation' costs to the auction site θx and the cost to the bidder of browsing the site, c^A, an equilibrium in which participation is sensitive to bidder location, is characterized by a cut-off value for going to the auction $x_c \in (0, 1)$ that is implicitly defined by the following relationship:*

$$q \sum_{n=1}^{N} \binom{N}{n+1} x_c^n (1 - x_c)^{N-n-1} E[\pi(n)] = c^A + \theta x_c \tag{4}$$

Proof. The critical bidder is just indifferent between visiting the site in the hopes of finding and obtaining and item at auction and staying out of the auction, obtaining a reservation utility of zero. Thus, the critical bidder's expected payoff from going to the auction site must equal his expected costs of doing so. The right-hand-side of Equation 4 gives the cost of attending the site, whereas the left-hand-side denotes the bidder's *ex ante* expected payoff. This is given by the bidder's expected payoff in an auction with given number of bidders n, as derived in Equation 3; weighted by the distribution of the expected number of bidders at the auction; as given in Equation 1. Finally, this expected payoff must be conditioned on the presence of the seller, a probability given by q. □

[16] See Wolfstetter, 1999.

Looking at Equation 4 it is not immediately clear what the relationship between q^D and x_c is. Indeed, while one might intuitively think that the relationship is positive, this is not as straightforward as it may appear at first blush. To see this note that as the likelihood that a seller is present increases (an increase in the left-hand-side of Equation 4), *ceteris paribus*, going to the auction becomes more attractive to a given bidder (an increase in the left hand side). However, this is true also of the bidder's rivals so that an increase in q suggests an increase in auction competition, which leads to a marginal decrease in any bidder's expected payoff from the auction when it takes place (decreasing the left-hand-side). In principle, if the marginal increase in competitive pressure is sufficiently strong, it may offset the increase in the left-hand-side brought about by increases in q — in this case, Equation 4 does not hold for any $x_c > 0$; which we rule out by assumption. Specifically, the marginal competition effect is not too large whenever bidder density is not too large or the linear transportation costs are sufficiently high to discourage large entry effects.

After re-writing the bidder participation condition in terms of the minimum necessary seller participation, q, that assures bidder participation up to x_c, we can analyze the properties of this relationship. To this end, we attach the superscript D to the seller participation as a mnemonic for 'demand,' signifying that the implied seller participation rate is that necessary to induce the bidder participation given by x_c:

$$q^D(x_c) = \frac{c^A + \theta x_c}{\sum_{n=1}^{N} \binom{N}{n+1} x_c^n (1-x_c)^{N-1-n} E[\pi(n)]}. \tag{5}$$

Lemma 2 *The relationship between minimum seller participation and bidder attendance is increasing and convex. That is,*

$$\frac{d}{dx_c} q^D(x_c) > 0$$
$$\frac{d^2}{dx_c^2} q^D(x_c) > 0.$$

Proof. Note that

$$\begin{aligned} \binom{N}{n+1} x_c^n (1-x_c)^{N-1-n}\Bigg|_{x_c=0} &= \begin{cases} 1, & \text{for} \quad n = 0 \\ 0, & \forall n > 0, \end{cases} \quad \text{and} \\ \binom{N}{n+1} x_c^n (1-x_c)^{N-1-n}\Bigg|_{x_c=1} &= \begin{cases} 0, & \forall n < N-1 \\ 1, & \text{for} \quad n = N-1. \end{cases} \end{aligned} \tag{6}$$

So, for all $c^A \geq 0$,

$$0 \leq q^D(0) = \frac{c^A}{E[\pi(1)]} < \frac{c^A + \theta}{E[\pi(N)]} = q^D(1),$$

where the second inequality is assured by the fact that an interior solution to Equation 4 implies increase in q^D. Consider now the curvature of q^D.

The transaction costs of participating in the auction $t(x_c)$ are linear. Also, the increase in bidder participation $H(n|x_c)$ yields a linear increase in the number of expected bidders at the auction n through a first-order stochastic dominance relationship induced by changes in x_c.[17] However, the bidder's expected payoff as a function of the number of rivals n is decreasing and convex, rendering q^D convex. □

3.3 The Seller's Listing Decision

Suppose there are $n \in \{0, 1, \ldots, N\}$ bidders whose values are i.i.d. draws from the distribution $F(y)$ on $[0, \overline{v}]$. Then the expected revenue generated in an auction with (optimally chosen) reserve $\underline{v}$ is given by

$$E[R(n, \underline{v})] := \int_{\underline{v}}^{\overline{v}} \left(y - \frac{1 - F(y)}{f(y)} \right) d[F(y)]^n. \tag{7}$$

Thus, with the bidders' cut-off value for going to the auction be denoted by by x_c, the seller's expected *ex interim* revenue from going to the auction, before knowing the number of bidders present, is given by

$$E[R(x_c)] := E_n E[R(n, \underline{v})] = \sum_{n=1}^{N} \binom{N}{n} x_c^n (1 - x_c)^{N-n} E[R(n, \underline{v})].$$

This gives the seller's expected revenue once he lists with the site, but before it is known how many bidders are present. Clearly, since bidders' anticipated payoffs, given in Equation 3, are a function of the optimally chosen reserve $\underline{v}$. The critical bidder participation cut-off, x_c, given in Lemma 1, is a function of the optimally chosen reserve, i.e., $x_c = x_c(\underline{v})$.

Recall, however, that the seller is a 'short-run' player, either because he only participates on a one-shot basis (i.e., only once if at all) or because his standing is sufficiently anonymous that he is unable to acquire a reputation. An implication of this is that the seller cannot commit to an optimal reserve policy that would go against his best short-run interest in setting a reserve independent of the bidder participation decision. Hence:

Lemma 3 (Optimal Reserve) *Despite the fact that bidder participation is a function of the reserve chosen, the optimal reserve is that of a standard auction with deterministic bidder entry. That is, the*

[17] See Equation 1 and, e.g., Wolfstetter (1999), p. 223.

optimal reserve is implicitly determined independently of the number of bidders by

$$\underline{v} = \frac{1 - F(\underline{v})}{f(\underline{v})}.$$

Since $F(\cdot)$ has the monotone hazard rate property, the optimal reserve is uniquely determined.

Proof. The proof for the optimal reserve for a given number of bidders is standard. The fact that this reserve applies to the current setting follows from the above discussion.[18] $\square$

Letting c^S denote the seller's cost of going to auction, for given x_c, a sellers' expected *ex ante* payoff from going to the auction is

$$E\Pi^S = E[R(x_c)] - c^S - f, \tag{8}$$

where, again, f is the listing fee.

Letting the superscript S be a mnemonic for the analysis from the seller's perspective, we obtain:

Lemma 4 (Seller Participation) *Let the distribution of sellers' costs be denoted by $G(c^S)$. Then, for $E[R(x_c)] > f$, the probability that a seller attends the site is implied by:*

$$\begin{aligned} q^S(x_c) &= \Pr\left\{c^S \leq E[R(x_c)] - f\right\} \\ &= G\left(\sum_{n=1}^{N} \binom{N}{n} x_c^n (1 - x_c)^{N-n} E[R(n, \underline{v})] - f\right); \end{aligned} \tag{9}$$

otherwise $q^S = 0$.

Moreover, $q^S(x_c)$ is increasing and a non-increasing density of seller transaction costs, i.e., $g'(c^S) \leq 0$, is sufficient to render $q(x_c)$ concave.

Proof. The proof is largely analogous to the proofs of Lemmas 1 and 2. Define

$$\underline{x}_c := \max\left\{x_c \left| q^S(x_c|f) = 0\right.\right\}.$$

Then $\forall f > 0, \underline{x}_c > 0$ and $\underline{x}_c$ is the threshold for bidder participation that at a minimum must be assured to make any seller participation worthwhile. Since expected bidder participation is increasing in x_c and expected revenue is increasing in expected bidder participation, it follows that $q^S(x_c)$ is increasing. Increases in the probability of bidder participation increase the expected number of bidders linearly; and expected revenue as a function of the expected number of bidders is concave — a property that is preserved in q^S whenever $G'' = g' \leq 0$. $\square$

[18] If the seller can establish a reputation or can commit in advance to a reserve, a lower reserve is chosen in order to increase bidder participation (see Levin and Smith, 1994; or Wolfstetter, 1999).

Thus, the behavior of Equation (9) is such that it is constant at $q^S = 0$ for $0 < x_c \leq \underline{x}_c$. Thereafter, for $x_c \geq \underline{x}_c$, it monotonically increases and is concave. Finally, using Equation 6, $q^S(1) = G\left(E[R(N, \underline{v})] - f\right)$.

The assumption on the distribution of seller cost G—namely that it have non-increasing density—is a sufficient, but not a necessary condition. Many distributions that might naturally reflect seller cost have the property, in particular, the exponential distribution has a non-decreasing density. The assumption is made largely for convenience, for—as is shown below—it yields a unique stable equilibrium with activity at the site.

3.4 Equilibrium for given site fee f—Entry Equilibrium

Before conducting an equilibrium analysis, we first consider potential equilibrium configurations, given an exogenously given listing fee f. We refer to this as the "Entry Equilibrium," in contrast to the "Market Equilibrium" that emerges as the solution to the auction host site's pricing policy, given behavior in the Entry Equilibrium.

The Entry Equilibrium is implied by the bidders' and the seller's participation problems given by Equations (4) and (9). We denote the seller participation rate in the entry equilibrium by q^* and that of bidders by x_c^*. Thence,

Lemma 5 (Existence of Entry Equilibrium) *An entry equilibrium for given f exists for all $x_c^* \in [0, 1]$ such that*

$$q^* = q^S\left(x_c^* | f\right) \geq q^D\left(x_c^*\right) \geq 0,$$

with $q^S > q^D$ only if $x_c^* = 1$.

Proof. By Lemma 2, for given x_c, q^D determines the minimum seller participation necessary to induce bidders to the left of x_c to participate. Thus, for given x_c, in equilibrium, $q^S \geq q^D$, with inequality only if $x_c^* = 1$. □

Notice that it trivially follows from Lemma 5 that there is always at least one equilibrium constellation:

Definition 6 (No-Trade Equilibrium) *In a no-trade equilibrium (NTE) the auction hosting site is vacant, so there are no sellers and no bidders. We have $q^* = x_c^* = 0$.*

A No-Trade Equilibrium exists for all parameter values, since the existence of the market is part of a coordination game, with the no-trade outcome coinciding with the potential Pareto inefficient outcome. However, a NTE may also be the unique entry equilibrium. Indeed, the NTE is trivially unique whenever the listing fee, f is so large that sellers do not wish to participate, independent of potential bidder turnout. Similarly, if the cost of getting to the site (θ) or browsing the site (c^A) is prohibitively high, bidders do not visit the site, independently of seller participation rates. However, these extreme cases are not the only ones in which the NTE uniquely obtains. That is, it is possible that both sellers and bidder could be induced to attend the auction site, yet $q^S(x) < q^D(x)$, for all $x \in (0,1]$ so that Lemma 5 does not hold for any x but $x_c^* = 0$.

Throughout we assume parameter constellations in which the NTE is not the unique equilibrium. Nevertheless—and this we detail in the subsequent section—the NTE may be important as an auction site my suffer a NTE as a result of potential duopoly competition, yielding a monopoly market even when there are two potential sites. Yet, at this point, the primary focus is on equilibrium configurations with positive expected trade.

Definition 7 (Entry Equilibrium with Trade) *An entry equilibrium with trade (EWT), is an entry equilibrium with positive expected trade, in which it may occur that no trade takes place. Thus, in a EWT we have* $0 < q^*, x_c^* \leq 1$.[19]

- *If* $x_c^* = 1$*, we refer to an EWT with full bidder participation (FBP).*
- *If* $x_c^* < 1$*, we have an EWT with partial bidder participation (PBP).*

Assuming for convenience that G has a non-increasing density, the curvature properties of q^D and q^S imply:

Proposition 8 (Existence of FBP-Entry Equilibrium) *A necessary and sufficient condition for the existence of a FBP-Entry Equilibrium whenever* $g' \leq 0$*, is that*

$$q^D(1|N) = \frac{c^A + \theta}{E[\pi(N)]} \leq G\left(E[R(N, \underline{v})] - f\right) = q^S(1|f, N).$$

Proof. The proof of sufficiency follows trivially from the equilibrium condition given in Lemma 5. And the proof of necessity follows from the curvature properties of q^D and q^S, given in Lemmas 2 and 4. $\square$

[19]Note that if the support of seller participation costs is unbounded, i.e., $G(c^S) < 1, \forall c^S < \infty$, then a $q^* = 1$ can be ruled out. Moreover, however, even is c^S is bounded above, $q^* = 1$ can only emerge as a limiting case, since otherwise the host site could increase the listing fee without affecting seller participation (thus, also without affecting bidder participation), which results in strictly greater profit for the host site.

Of course, Proposition 8 is not necessary for the existence of other (non-FBP) entry equilibrium configurations with trade. Indeed, there may be multiple equilibrium points (see Figures D and E). Thus,

Corollary 9 (Multiple EWT) *If the inequality given in Proposition 8 is strict, then there exist three equilibrium configurations:*

- *an NTE (i.e., $x_c^* = 0$),*
- *a PBP-EWT (i.e., $0 < x_c^* < 1$),*
- *an FBP-EWT (i.e., $x_c^* = 1$).*

If the condition in Proposition 8 holds with equality, the latter two equilibrium configurations merge to the FBP-Entry Equilibrium.

Proof. The proof follows since $q^D(0) > q^S(0|f)$, and both q^D and q^S are continuous functions, so, given the condition in Proposition 8, there exists some $x_c^* \in (0,1)$ such that $q^D(x_c^*) = q^S(x_c^*|f)$. □

The possibility of multiple equilibrium configurations of the entry game calls for a qualitative understanding of these. Indeed, the entry equilibrium configurations can be characterized by standard stability criteria. Thus,

Proposition 10 (Stable Equilibrium) *An equilibrium with trade is stable whenever a small deviation in seller or bidder participation triggers a change in participation on the other side of the market that does not result in positive payoffs for the marginal agent on the side of the market with this initial change. Thus, stability occurs whenever,*

$$q^S(x_c^*|f) = q^D(x_c^*) \quad \textit{with} \quad \left(\frac{d}{dx_c}\right) q^S(x_c^*|f) < \left(\frac{d}{dx_c}\right) q^D(x_c^*),$$
$$\textit{or} \quad x_c^* = 1.$$

Proof. Stability in the case of $x_c^* = 1$ is obvious. For the other cases note that since both q^D and q^S are increasing, a marginal increase of $\Delta > 0$ beyond x_c^*, yields $q^D(x_c^* + \Delta) > q^S(x_c^* + \Delta)$, violating Lemma 5. □

In addition to the qualitative differences in entry equilibrium configurations, we can also make welfare comparisons between differing entry equilibrium points. We use this to identify the — for us — relevant entry equilibrium configuration.

Theorem 11 (Entry Equilibrium) *Let $\{x_c^*\}$ denote the set of critical location thresholds associated with all entry equilibrium configurations. Define*

$$X_c^* := \max\{x_c^*\},$$

then X_c^* is associated with the Pareto-optimal entry equilibrium and this entry equilibrium is stable.

Proof. Since both q^D and q^S are increasing, X_c^* is associated with the entry equilibrium with the highest seller participation. Since sellers list only if they expect positive payoffs and sine their payoffs are increasing in the expected number of bidders at the site, which, in turn, is increasing in x_c^*, no seller is better off in any other entry equilibrium than that associated with X_c^*. Moreover, by Lemma 2, increases in q^* yield overall increases in expected bidder payoffs at the auction that are more than sufficient to offset increased transportation costs t, so, for any given location, a bidder's expected payoff in increasing in q^*. Thus, X_c^* is the Pareto-efficient entry equilibrium.

Stability of the entry equilibrium associated with X_c^* follows directly from Proposition 10 for the case of a FBP-entry equilibrium (i.e., $X_c^* = 1$); and otherwise it follows from Proposition 8 and its Corollary in conjunction with Proposition 10. □

Having ruled out complete coordination failure (i.e, the NTE, which is always stable), we further assume that market participants coordinate on on the stable equilibrium with positive market participation and a high probability of trade occurring. That is, bidders and sellers coordinate on entry equilibrium configurations with "thick markets," i.e., those characterized by X_c^*, given in Theorem 11.[20] We now turn to the market equilibrium, given by the solution to the auction hosting site's problem of choosing a listing fee f.

4 Monopoly Equilibrium

We now consider the market equilibrium when there is only one auction hosting site, assuming throughout that the entry equilibrium is characterized by X_c^*. The auction site maximizes its revenue by choosing the listing fee, f, taking into account how this affects the entry equilibrium, i.e., accounting for how the site fee affects the probability of sellers coming to auction, and thus affects bidder participation. In order to make sure that equilibrium configurations that involve potential trade exist, we make the following assumption.

[20] Indeed, as shown below, it is in the interest of the hosting site to maximize participation, and the auction site can employ simple surplus reallocation strategies (e.g., participation bonuses) to render this configuration as unique equilibrium configurations.

Assumption 1 (Potential for Positive Trade) *When the auction host site operates free of any listing fee to the seller ($f = 0$) then there is potential for trade. That is,*

$$\exists\, x_c' > 0 \quad s.t. \quad q^S(x_c'|0,\cdot) = q^D(x_c').$$

We now consider how changes in f affect the equilibrium with trade. Specifically, changes is f affect the location of q^S, while leaving q^D unaffected. Given the entry equilibrium, we obtain,,

Lemma 12 *Assuming a non-decreasing density of seller costs, bidder participation (and thus seller participation) in the entry equilibrium is decreasing and concave in the listing fee f. That is,*

$$\frac{d}{df}X_c^* \leq 0,$$
$$\frac{d^2}{df^2}X_c^* \leq 0,$$

with strict inequality whenever $X_c^* < 1$.

Proof. First assume that $X_c^* < 1$, then Propositions 10 and 11 imply that X_c^* is implicitly defined as a function of f by

$$J\left(X_c^*, f\right) := q^S\left(X_c^*|f\right) - q^D\left(X_c^*\right) = 0.$$

Hence, with subscripts denoting partial derivatives,

$$\frac{d}{df}X_c^* = \frac{-J_f}{J_{X_c^*}} = \frac{g}{\frac{dq^S}{dx_c} - \frac{dq^D}{dx_c}} < 0,$$

where g is the density of seller cost and the inequality follows from Proposition 10. Moreover,

$$\frac{d^2}{df^2}X_c^* = \frac{d}{df}\frac{-J_f}{J_{X_c^*}} = \frac{g'\frac{dq^D}{dx_c}}{\left(\frac{dq^S}{dx_c} - \frac{dq^D}{dx_c}\right)^2} < 0$$

whenever $g' < 0$.

The case of $X_c^* = 1$ follows immediately. $\square$

As a consequence of the first part of Lemma 12, if f is too large, q^S will be so small that $q^D > q^S$ for all $x_c > 0$, and the NTE is the unique equilibrium. Otherwise, notice that as both q^D and q^S are increasing, for a given fee, the host sites payoff is greater at the equilibrium with the greatest bidder participation, since this implies the greatest seller participation consistent with the entry equilibrium.

Notice that there are two branches to the host site's objective function. For small f, the implied equilibrium is the one in which $X_c^* = 1$, for larger f (and, thus, decreased q^S) it entails $X_c^* < 1$. The host site's objective function is continuous in f, but not differentiable at the point where one switches from one to the other type of equilibrium.

Hence, letting f' denote the level of f at which one switches from one branch to the other, the host site's objective function is

$$q^S(1|f,N) \times f = G\left(E[R(N,\underline{v})] - f\right) \times f \quad \text{if} \quad f < f', \tag{10}$$

$$q^S(x_c|f,N) \times f \quad \text{s.t.} \quad q^S(x_c) = q^D(x_c) \quad \text{if} \quad f \geq f'. \tag{11}$$

Where the strict inequality limiting the domain of the first branch and the weak inequality on the second branch stem from the fact that the as one switches from an FBP entry equilibrium (at a fee of f') to a PBP entry equilibrium the elasticity of seller participation with respect to the listing fee f drops (possibly discontinuously).

An immediate implication of the second part of Lemma 12 is that the overall objective is concave whenever the density of seller cost is non-decreasing. The optimum can occur on either branch of the host site's objective function. We can classify four types of monopoly equilibrium constellations.

Theorem 13 (Monopoly Market Equilibrium) *The following monopoly equilibrium constellations can occur.*

- *A FBP monopoly equilibrium on the top branch of the objective function obtains whenever,*

$$0 < f^* \equiv \frac{G\left(E[R(N,\underline{v})] - f^*\right)}{g\left(E[R(N,\underline{v})] - f^*\right)} < f'$$
$$\textit{with} \quad f' \equiv E[R(N,\underline{v})] - G^{-1}\left(\frac{c^A + \theta}{E[\pi(N)]}\right).$$

- *A FBP monopoly equilibrium on the second branch is given by*

$$f^* = f' \qquad \textit{and} \qquad X_c^* = 1,$$
$$\textit{with} \quad \left[\frac{\partial q^S}{\partial f} + \frac{\partial q^S}{\partial x_c}\frac{dx_c}{df}\right]_{X_c^*=1} \times f' + q^S\left(1, f'\right) \geq 0$$

- *A (Type-I) PBP monopoly equilibrium on the second branch is given by*

$$f^* < f' \qquad \textit{and} \qquad X_c^* < 1,$$
$$\textit{with} \quad \left[\frac{\partial q^S}{\partial f} + \frac{\partial q^S}{\partial x_c}\frac{dx_c}{df}\right] \times f^* + q^S\left(X_c^*, f^*\right) = 0$$

- *A (Type-II) PBP monopoly equilibrium that has*

$$\frac{d}{dx_c}q^S(x_c^*) = \frac{d}{dx_c}q^D(x_c^*),$$
$$\textit{with} \quad \left[\frac{\partial q^S}{\partial f} + \frac{\partial q^S}{\partial x_c}\frac{dx_c}{df}\right] \times f^* + q^S(X_c^*, f^*) \geq 0$$

Proof. The proof follows from applying Lemma 12 to the two branches of the objective function, given by Equations 10 and 11. □

The Type-II PBP monopoly equilibrium is a knife-edge equilibrium. The functions q^D and q^S are tangent to one another, so a further increase in f leads to a market collapse, resulting in a NTE. In this case the first order condition for profit maximization of the host site need not be satisfied.

5 Duopolistic Host Site Competition

We now consider equilibrium configurations when there are two hosting sites for auctions, one — as in the single firm case — located at $x = 0$, the other located at $x = 1$. Thus, the sites are at the endpoints of a Hotelling-type linear city, that houses the bidders. However, in contrast to the standard Hotelling game, the two auction hosting sites do not directly compete for bidders. Rather, they choose listing fees that they charge from their sellers, which affects the seller participation rates, which, in turn, affects where bidders browse. Again, recall that sellers are pre-assigned to auction sites so that auction listing sites do not compete against each other for sellers.

Since the analysis is obviously symmetric, we continue to argue from the vantage point of the hosting site located at $x = 0$. Where we need to distinguish the two sites, we index them A and B, respectively. E.g., the bidder cost of browsing site A is as before c^A (see Lemma 1), but the cost of browsing at site B is given by c^B. Similarly, while we let x_c^A denote the critical location up to which bidders attend auction site A, x_c^B gives the critical location beyond which bidders attend auction site B.

With this in mind, similar to Assumption 1, we assume parameters that allow for two firms to exist in the market simultaneously. Thus,

Assumption 2 (Potential for Two Sites) *There exist combinations of site listing fees at both sites simultaneously that yield positive expected trade at both sites. That is, letting $x_c^{A\prime}, x_c^{B\prime}$ be defined as in Assumption 1, we have*

$$0 < x_c^{A\prime} < x_c^{B\prime} < 1.$$

Indeed, there may exist equilibrium listing fees under which both sites are open and active, yet there is a range of bidder locations on the interior of the location interval for which bidders do not attend either site. In these cases — where bidders furthest from the hosting sites do not attend either site — each auction site's optimal policy is independent of the existence of the rival site and the bidder and seller presence at one site is independent of the activity at the other site as well. For these cases, the analysis of Section 4 applies to both sites, with the understanding that labelling is adjusted to account for site B's location at $x = 1$.

For the remaining duopoly analysis, we henceforth consider only parameter values for which this 'disjoint' equilibrium — in which the market is not entirely covered — does not exist. That is, we restrict the parameter space so that both firms may, in fact, engage in competition with each other through their listing fees. Thus,

Assumption 3 (Competitive Market) *The duopoly market is competitive in the sense that not both firms can simultaneously achieve their monopoly equilibrium. That is, letting X_c^{A*} and X_c^{B*} be defined as in Theorem 11,*

$$0 \leq X_c^{B*} < X_c^{A*} \leq 1.$$

Having defined the relevant parameter space for the competitive market, we now turn to the equilibrium analysis. As before, we do so in two steps: first we characterize the entry equilibrium conditions, and second we characterize the possible market equilibrium configurations.

Notice that since, by assumption, any potential seller is tied to a particular auction site, the sellers' behavior is no different between the single-site and the two-site analysis. This, however, is not the case with bidders. While bidders' participation thresholds in monopoly are given by Lemma 1, this must here be modified. In particular, letting x_c denote the location of the critical bidder, who is just indifferent between browsing at site A or not given a seller participation rate of q_B^S at the other site, we have

$$\begin{aligned} & q_A^D \sum_{n=1}^{N} \binom{N}{n+1} (x_c)^n (1-x_c)^{N-n-1} E[\pi(n)] - c^A - \theta x_c \\ = \max & \left\{ 0, q_B^S \sum_{n=1}^{N} \binom{N}{n+1} (x_c)^{N-n-1} (1-x_c)^n E[\pi(n)] - c^B - \theta (1-x_c) \right\}. \end{aligned}$$

Thus, for the case that the market is covered and both sites have positive expected attendance, one has

Lemma 14 (Critical Distance to Site A, covered market) *Given the* *seller attendance rate at site B, the cut-off location for attending site A is implied by*

$$q_A^D(q_B) = \frac{q_B \sum_{n=1}^{N} \binom{N}{n+1}(x_c)^{N-n-1}(1-x_c)^n E[\pi(n)] - c^B + c^A - \theta(1-2x_c)}{\sum_{n=1}^{N} \binom{N}{n+1}(x_c)^n(1-x_c)^{N-n-1}E[\pi(n)]}.$$

An analysis similar to Lemma 2 follows *mutatis mutandis.* In particular,

Lemma 15 *For given seller participation at site B, and increase in bidder participation at site A requires and increase in seller participation at site A. That is,*

$$\frac{d}{dx_c} q_A^D(x_c|q_B) > 0. \tag{12}$$

Proof. Whenever the market is not covered, the proof is identical to the proof of Lemma 2. Otherwise, given the properties of the component functions of $q_A^D(x_c|q_B)$ as derived in the proof to Lemma 2 yields the desired result, when noting that the exponents in $H(n|x_c)$ are reversed in the numerator of $q_A^D(x_c|q_B)$. □

From this and the prior analysis, the entry equilibrium for a covered market in duopoly follow immediately.

Theorem 16 (Entry Equilibrium, covered market) *The entry equilibrium when the market is covered is characterized by a critical bidder threshold x_c^* with*

$$\begin{aligned} q_A^* &= q_A^S\left(x_c^*(q_B), f^A\right) = q_A^D\left(x_c^*(q_B)\right) > 0, \\ q_B^* &= q_B^S\left(x_c^*(q_A), f^B\right) = q_B^D\left(x_c^*(q_A)\right) > 0, \quad \textit{and} \\ q_A^* &\sum_{n=1}^{N} \binom{N}{n+1}(x_c^*)^n(1-x_c^*)^{N-n-1}E[\pi(n)] - c^A - \theta x_c^* \\ = q_B^* &\sum_{n=1}^{N} \binom{N}{n+1}(x_c^*)^{N-n-1}(1-x_c^*)^n E[\pi(n)] - c^B - \theta(1-x_c^*). \end{aligned}$$

Note that the condition that $q^S = q^D$ at both sites stems from the fact that we are only considering active competition with a covered market.

Having established these results, we can now consider the market equilibrium for the case of duopoly.

Theorem 17 (Market Equilibrium) *There exist covered-market equilibrium configurations, which are characterized by Theorem 16. However, these need not by unique and there also exist equilibrium* *configurations that entail one site to shut down. In this case, supposing that site B is shut out, we have*

$$q_B^S(X_c^*|f_B=0)\sum_{n=1}^{N}\binom{N}{n+1}(X_c^*)^{N-n-1}(1-X_c^*)^n E[\pi(n)]-c^B-\theta\,(1-X_c^*)$$
$$\le q_A^*\sum_{n=1}^{N}\binom{N}{n+1}(X_c^*)^{n}(1-X_c^*)^{N-n-1} E[\pi(n)]-c^A-\theta X_c^*$$

The equilibrium configurations are illustrated by the examples in the following section. The reason for the multiplicity result in Theorem 17 is because best-response functions need not be continuous and may have jumps in them. To see this, consider a stable constellation in which both sites have positive expected trade. Then a discrete downward jump in the listing fee at firm A results in a discrete increase in q_A^S. By Lemma 14, this yields a discrete increase in q_B^D, which can result in a tipping of firm B's market and forcing it to shut down.

This can be seen as a successful example of predatory pricing. The reason for this is that in order to operate, a hosting site must attract sellers and buyers. So long as one's pricing policy (here low fees) can shut off one side of the market of a rival, one has successfully driven a rival from the market. This outcome is stable because the site that is shut down cannot affect the market outcome on the margin by changing its policies. The result indicates that the network externalities from providing the platform may be sufficiently strong so that a second market — while potentially viable, given Assumption 2 — does not form, despite it being the only possible outlet for its sellers and despite it being preferred by the bidders on the far side of the market.

6 Three Examples

Most of the insights of this model can be obtained by the careful analysis of three examples, chosen so as to illustrate much of the range of possible equilibria that can arise. The first example considers a single auction hosting site, A. The participation costs of potential buyers are uniformly distributed in $[c^A, c^A+\theta]$. Effectively, this is the Hotelling model in which there is only one firm at the one end of the unit interval. The participation of potential sellers are uniformly distributed in $[0, 2\mu]$. One of the features of this example is that there are equilibria in which all the buyers and all the sellers participate in the website. In order to preclude these full-participation corner solutions, we develop a second example in which the costs of sellers are distributed exponentially with mean μ. Finally, the third example considers competition between two firms at opposite ends of the unit interval, with consumers being uniformly located on it. In this example, we retain the assumption of exponential distributed seller costs. In all three examples, buyers valuations are uniformly distributed on [0,1] and sellers post a hidden reserve that is optimally (for them) set at 1/2.

6.1 Example 1: Monopoly with bounded seller costs.

Consider first the case of a single auction hosting website at one of the end-points of the Hotelling interval. We refer to this site as site A and place it at $x = 0$.

6.1.1 Bidder Behavior

In Figure 1 we plot the "location" of the consumer who is indifferent between browsing site A and taking part in some other activity, which assumed to give constant utility (= 0). We denote the location of this critical consumer by x_c. That is, letting q denote the probability that a sought-after item will be found at the auction site and letting c and θ denote the costs to the bidder described above, x_c is implied by

$$qE\left[\pi|x_c\right] = c^A + \theta x_c$$

As x_c becomes larger, the number of consumers who browse site A increases in the first order stochastic dominance sense. We plot x_c for three sets of parameter values. The first (solid line) is for $c = 0.02$ and $\theta = 0.05$, the second (dotted line) for the same value of c^A but for $\theta = 0.010$, while the third (dashed line) is for $c^A = 0.04$ and $\theta = 0.010$. For Figures 1 through 32, an expression in the legend of the figures of $y_0 17$ means $y = 0.017$ and similarly for any other number and parameter.

As the figure shows, the set of participating consumers increases with the probability that they will find an item that they are interested in, but not linearly. This function is concave because (i) if the set of consumers was kept constant their utility would increase linearly as q goes up, but (ii) since with q going up the set of participating consumers goes up, the probability of each participating consumer winning the item goes up slower than linearly. With "transport" costs being linear in the set of consumers, the location of the critical consumer is concave in q.

An increase in c shifts this line to the right in a parallel fashion. This makes sense, since with linear "transport" costs an increase in c corresponds to a particular change in the distance from the origin, holding x_c constant. Because this argument suggests a shift to the right rather than a seemingly equivalent shift downwards, in general, the vertical distance between two x_c functions is not likely to be constant (though it is for this example). The reason is that, for a fixed value of q, the values of x_c are different and as x_c changes, the number of participating bidders changes as well. In fact, a change in c does not lead to a change in x_c of $\frac{\Delta c}{\theta}$ but by a smaller amount.

Similarly, an increase in θ tilts this function downwards, around the point that $x_c = 0$. With linear transport costs, a change in θ translates into a proportional loss of surplus to the buyers (of a particular location). In general, as θ goes up, the x_c function stretches to the right. Finally, notice that in Figure 1, for sufficiently low values of θ and c^A, the x_c function tops-off at 1, since consumers are located on the unit interval (to make a comparison with the duopoly meaningful). If consumers were located on the half-line (i.e., if there were always marginal buyers), then this line would be increasing monotonically in q.

6.1.2 Seller Participation

Figure 2 plots the revenue of a seller, gross of entry costs, as a function of the anticipated bidders' assessment of that any individual seller participates in the auction (in equilibrium perceptions will be correct). Seller revenue (gross of entry costs) responds to changes in θ and c in the same way as the location of the indifferent consumer, x_c.

$$\text{seller expected revenue:} \quad E\left[Rev\left(x_c(c^A, \theta, q)\right)\right] - f$$

The shape of this function with respect the probability that a seller is perceived by the buyers to be at the site, i.e., with respect of the perceived probability that the consumers will find an item that is a match for what they are looking for, deserves some discussion. Recall that x_c is concave in this probability, q. Then, if revenue is concave in x_c, it would also be concave is q. But as x_c goes up, the distribution function of the number of bidders shifts in a way that it puts more weight on a higher number of bidders and increases the mean number of bidders linearly. Given that revenue is concave in the number of bidders when the number of bidders is not stochastic (possibly for all cases, but certainly for well behaved ones), the expected revenue of the seller is concave in x_c, and hence in q. This argument is made more formally in the analysis of the generalized model in a later section of the paper.

The seller revenue function does not include the seller participation costs. These costs, which differ across sellers, are being plotted along with a revenue function, in Figure 3.

$$\text{seller cost:} \quad c^S(q) + f$$

While f is given for all sellers (and here assumed to be 0), c^S is distributed across sellers. Thus, the dashed line is the inverse cost distribution function (of seller costs). It is gives the entry cost of the seller in the q^{th} cost quantile. Because this function has been plotted for $\mu = 0.3$, the highest possible seller cost is equal to 0.6. Since in this example the distribution of costs has been assumed to be uniform, the inverse cost distribution function is linear with a slope of 0.6. In this example, this line lies everywhere above the revenue line, plotted for $c^A = 0.02$, $\theta = 0.05$, and listing fee $f = 0$. This means that for any fraction, q, of sellers that sell their product in the website, the revenue they would earn if consumers have correct expectations about q, will not be sufficient to cover the entry costs. Thus, given that the fee has been set to zero, for these parameter values, there will be no auction hosting site in operation.

6.1.3 Market Equilibrium

For a market to exist, either the costs of sellers or those of the buyers must be (stochastically) lower. In Figure 4, we assume that the buyer costs have fallen uniformly for all buyers by 0.001 (an improvement in browser technology?). Now, the inverse cost function intersects the revenue function in two points. Therefore, there are two equilibria in the entry game.

The low q equilibrium is unstable: any random increase in seller entry or any slight consumer misperceptions will either lead to a demise of the market, or to an increase in q. The high q equilibrium is stable. This figure is somewhat generic, with two entry equilibria often being present. In this paper, for the most part we restrict our attention to the stable, high q, equilibrium. Note that there is, in fact, a third equilibrium: that of $q = 0$. When there is a positive q equilibrium, we will ignore the $q = 0$ equilibrium. However, it is worth keeping in mind: these markets may need a "push" to get going. Else, if sellers expect to see no buyers and buyers expect to see no sellers, neither will have any incentives to participate in the auction hosting site — there is a complete coordination failure.

Is there any other possibility in terms of equilibrium? Yes. Suppose that through further improvements in technology or through experience, sellers have lower costs of selling through the website. In particular, suppose that the mean of seller costs drops from 0.3 to 0.2, resulting in a downward tilting of the inverse cost distribution function. Now, the stable equilibrium involves 100% seller participation. If all sellers participate in the website, even the marginal seller (the one with the highest possible costs) will earn positive profits.

This corner solution arises from the parametric assumption on the distribution of seller costs that assumes that the highest possible cost is finite. Nevertheless, it is instructive to consider this case (in example 2, we will turn to a parametric formulation with an unbounded cost distribution in a moment).

6.1.4 Auction Site Listing Fees

Given that the strategic variable of the auction hosting site is the listing fee, as a prelude to construction of the site profit function, it is instructive to show what the effect of the listing fee is on the entry equilibria shown in the preceding two figures. In Figure 6, it can be seen that a positive listing fee shifts downwards the revenue function in a parallel way. This result is not specific to this example and follows directly from the expression of the revenue function. If the initial equilibrium involved 100% seller participation, a small fee will not deter any seller, and will raise revenue for the firm. Eventually, however, as the listing fee raises beyond a threshold, some sellers will choose not to participate on the website. Further increases in the listing fee will trade off a reduction in the number of sellers that participate in the site with an increased revenue from each seller (the traditional monopoly trade-off). Since we assumed that the site's marginal costs are zero, this process will continue until marginal revenue is zero.

Or maybe not!!! Consider Figure 7, which is purposefully drawn for a higher value of consumer participation costs than Figure 6. The figure shows the entry equilibrium for zero listing fees, in which there is 100% for both sellers and buyers, then for listing fees of 0.02 and 0.03, in which range there is less than 100% participation of sellers, but still full buyer participation, and then for a listing fee of 0.04, for which there is less than full participation of both buyers or sellers. All these fee increases are profitable for the auction hosting site: the first one trivially so, since it results in no loss of sellers (The subsequent increase from 0.02 to 0.03 results in a loss of only about 5% of the sellers, with a gain of 50% per seller, and the final increase results in a less dramatic increase in site revenue).

Could the firm reach the optimum by further slight increases in the listing fee, as we often teach in intro micro? Consider now Figure 8, in which the auction hosting takes another small increase in the listing fee, to 0.05. Figure 8 shows that this increase, rather than resulting in a marginal change in site revenue, leads to a collapse of the market and zero revenue! Conceptually, the market collapse is due to the following negative feedback mechanism. The slight increase in the fee leads some sellers to leave the auction hosting site. This in turn reduces the number of potential buyers that visit the site, thus reducing the value of the site to the sellers, leading to further exit of sellers, and market implodes. A mistake in setting the listing fee, if not corrected sufficiently quickly, can lead to the demise of the site.

The negative feedback described here is present for all listing fee changes, but as Figure 7 shows, the site can reach an equilibrium membership short of a complete collapse. Indeed, the initial changes in the listing fee where profitable for the site. However, in Figure 8, we can see that the optimum listing fee may be in a knife edge point. A slight increase past the optimum may be very very costly. In other words, the profit function at the optimum may not be zero, but rather positive, with a discontinuous drop past the optimum.

One can piece together the seller participation probabilities from the entry equilibria of figures such as those above to construct the demand curve for the auction hosting site (notice that the demand curve is "flipped" from the normal way that we are used to seeing demand curves, with quantity being plotted on the vertical axis). Notice that very rapid transition from 100% seller participation to market shut-down as the fee increases.

To illustrate the extent to which the negative feedback in the entry game leads to this rapid transition, it is instructive to see what this demand curve would look like in its absence. In Figure 10, we plot using a dotted line the auction site demand function if the number of consumers participating in the website where fixed to 2 (100% consumer attendance). One can see that the corresponding demand would have been much flatter (or much steeper when looking at the inverse demand curve). For very low fees, when seller participation is 100%, the two demand curves coincide and are flat. For slightly higher fees, they still coincide because, even though the number of sellers drops below 100%, it is still high enough that the consumer participation is 100%. For even higher fees, the demand with the negative feedback diverges strongly from the demand with an exogenous website value. If instead we fixed the number of participating bidders to 1, the demand curve shifts in, but it's slope is unchanged (except at very low fees, because for these parameter values seller participation would not reach 100% even for a fee of zero if the number of bidders is equal to 1).

The stark observation is that this negative feedback reduces the pricing ability of the auction hosting site by flattening the inverse demand function the website. This is true even if the there is substantial heterogeneity in the opportunity costs of the demanders of the auction site's services. The features of the demand curves shown in Figure 10, may not exist for all parameter values (e.g. the two demand curves may not coincide for any segment, or they may not be flat at any segment). But the main conclusion above regarding the relative elasticity of the demand curve holds generally.

The optimum fee is at "an edge of the cliff" of the profit function, as shown in the profit function drawn with a solid line in Figure 11. At the optimum, the profit function is continuous, but not differentiable. Decreasing the mean consumer attendance costs leads to higher profits and a differentiable profit function peak (as shown by the dotted line). Note that for both profit functions, the initial segment is the 45-degree line (plotted as a dashed line), as initially increases in the fee do not result in any loss of sellers for this example.

Decreasing θ, i.e., decreasing the dispersion of buyer attendance/ browsing costs, increases the profits of the site. This is is because as long as there is no full consumer participation, an decrease in theta increases the number of participating consumers, and thus shifts the demand for site services outwards. However, a decrease in θ makes the number of participating consumers very responsive to changes in q, thus making the revenue function steeper. This, in turn, increases the propensity that the optimum will be at the point of non-differentiability (the "cliff"), as Figure 12 shows. In fact, it results in an optimum that is at 100% seller participation, if the market cannot be supported at less than 100%

seller participation at the optimal listing fee. [By changing the value of θ we change both dispersion and the mean of attendance costs, but this makes no difference in the qualitative conclusions here.]

One should point out, in passing, that "standard looking" profit functions are not precluded, as Figure 13 shows. In fact, in this example, even at a fee of zero, seller participation is less than 100%, and thus the profit function involves a trade-off between number of sellers and and profit per attending seller starting from the origin (see that the 45-degree line is everywhere above the profit function). The example differs from the previous ones in that the value of θ is larger, making the revenue function flatter, the value of c^A is smaller, shifting the revenue function up, and the mean of sellers costs is higher, making the inverse cost distribution function steeper. All of these changes reduce the likelihood of full participation corner solutions.

6.1.5 Optimal Site Fees

We now turn our attention to optimal auction hosting site pricing policy, i.e., to the optimal value of the listing fee. Figure 14 plots the optimal listing fee as a function of θ for three different values of the consumer browsing costs, c^A. The optimal fee is generally declining in the level of consumer participation costs, whether these manifest themselves in a terms of higher θ, as shown here, or in terms of higher c^A (figure omitted, but can be inferred by the three lines plotted here). There is a flat region for low values of θ, which is driven by the fact that for low θ there is 100% consumer participation. In fact, when θ drops to some threshold value which ensures 100% consumer participation, website optimal pricing and profits are not affected by further declines in θ. Such declines only increase the surplus of the bidders.

This may sound a bit paradoxical: Why can't the website take further advantage of the reduction in the costs of buyers? The intuition for this as follows: Buyer attendance provides revenue to the sellers, and it is this revenue that generates the demand for website's product (since the site charges the sellers). The sellers have no way to extract more surplus from the bidders as their participation costs drop (holding bidder participation levels constants). Therefore, their surplus (and demand) for site services is not sensitive to bidders costs once the market is fully covered. Since demand for site services is not responsive to θ for very low θ, then the optimal fee is not responsive either and profits are not affected.

More generally, if consumer participation costs were not bounded from above, but rather there was a progressively thinning set of consumers with higher and higher participation costs, the optimal fee would not have been completely flat for low θ. However, for low θ, when the you have gotten almost all the consumers attending and the density of marginal consumers drops, the optimal fee can be relatively unresponsive to further reductions in consumer costs.

One last thing to notice in Figure 14 is that in the downward-sloping part, even though revenue is concave in x_c, which is linear in θ holding everything constant, the optimal fee raises faster than linearly as θ drops: the reason is the feedback mechanism. As θ drops, more buyers attend, which increases the value of the site to the sellers, leading more of them to attend, thus increasing the value to the buyers, etc.

In Figure 15, we plot the optimal fee as a function of the mean of seller costs for three different levels of consumer participation costs. The optimal listing fee is generally declining in the mean of seller participation costs. However, for some range of μ, there is unresponsiveness of the optimal fee on μ, as can be seen by the lines drawn for $c^A = 0.00$ and $c^A = 0.01$. Moreover, the listing fee can be insensitive to buyer attendance costs, even though it generally declines with c^A (compare across the three lines).

In order to understand the shape of the optimal fee function, it is instructive to compare this function with the one that would have resulted had the level of bidder participation been held fixed at some level. If that were the case, the auction hosting site's inverse demand function would be linear (with a possible kink and vertical drop if at low fees there was 100% seller participation). In that case, the optimal fee is independent of μ, as a change in μ does not affect the y-axis intercept of the inverse demand function, but rather only its slope. The only exception is if the optimal listing fee is at the kink, in which case it declines with μ, as an increase in μ reduces the price that corresponds to the kink in the inverse demand function. Therefore, for the optimal fee function drawn for $c^A = 0.00$, the initial downsloping portion corresponds to pricing at the kink and the subsequent flat portion corresponds to pricing when there is full consumer participation. It is the right-most part of the optimal fee function that is "different" from the standard monopoly pricing case, as further increases in the listing fee reduce consumer participation, and thus the value of the website to sellers, leading to a shifting in of the inverse demand function and a reduction in the optimal price.

This flat region in figure 15 has to do with 100% buyer participation, not 100% seller participation, and hence could still occur if seller costs had unbounded support. One may think that the fact this non-responsiveness is caused by 100% (consumer) market coverage makes it less interesting of a finding. But the more general implication is that when you get the bulk of the consumers, and there are few more consumers to be had, the listing fee will be relatively unresponsive to changes in costs.

The above discussion also provides the intuition of why the optimal fee is not always sensitive to small changes in c^A. When there is full consumer participation, such small changes do not affect at all the web-site demand function, and hence the optimal price it charges. In Figure 16, the profit function is plotted directly, and it can be directly seen that the optimal fee can be the same for small changes in μ and c^A. The top of the profit function is either the same across lines (when c^A changes and μ is held constant) or shifts down in a parallel fashion (when μ goes up and c^A remains constant).

6.2 Example 2: Monopoly with unbounded seller costs

We now turn to the second example, in which the distribution of seller entry costs is unbounded. We focus on the changes that this unboundedness has on the results (and figures) discussed thus far. The starting point of any differences is that with unbounded support of seller participation costs, there can be no entry equilibrium with 100% seller participation: the inverse cost distribution function asymptotes to infinity as q goes to 1 (see Figure 17). However, there can be equilibria in which there is 100% consumer participation, as shown in this same figure: at the point of intersection of the inverse cost distribution function with the revenue function, the latter is flat.

Many of the dramatic features discussed earlier, such as the market implosion, do not depend on 100% participation of either seller or buyers, or in the linearity of the inverse cost distribution function, and hence can take place here as well, as Figure 18 shows. However, because 100% seller participation is no longer possible, the website demand does not have a flat portion close to the origin (i.e., for very low fees). However, the key feature of rapid reduction in demand is still present (see Figure 19). Increasing the dispersion and (slightly) reducing the mean of consumer costs leads to a "flatter" demand curve for website services.

As in the example with uniformly distributed seller costs, these demand curves are a lot steeper compared to the standard demand curves, that is the demand curves for which consumer participation is held constant (see Figure 20). They also have the feature (just as was the case with the uniform distribution) that the inverse demand functions (i.e., the functions that plot the price or fee versus seller participation) are flat at the top. Moreover, like in the uniform case, the website profit functions have the feature that the peak can be right at the edge of the market collapse threshold, or very close to it (see Figure 21).

The optimal fee functions, when plotted against θ, look similar to those under uniformly distributed seller costs. They are flat for low values of θ, corresponding to parameters values for which at the optimum fee there is 100% consumer participation, and slope downwards for higher values of θ (see Figure 22). However, there is some qualitative difference when the optimal fee functions are plotted against μ. As Figure 23 shows, optimal fee functions are no longer flat at any range because even when there is 100% consumer participation, a change in μ in an exponentially shaped demand curve affects the optimal price (the exponential shape refers to the cumulative distribution function which generates the demand curve). Also, as Figure 23 shows, for low seller costs, when the optimal fee is such that 100% of consumers attend, the optimal fee is independent of consumer attendance costs, as a change in such costs does not affect the website's demand function around the optimum (also see Figure 24 which shows that at the old entry equilibrium, a change in c^A does not affect the revenue function locally).

6.3 Example 3: Duopoly with unbounded seller costs

We now introduce a second auction hosting site at the other end of the unit interval (site B), maintaining the assumption of exponentially distributed seller costs. We consider parameters such that the market is fully covered, i.e., every consumer browses one of the two websites. We also generally consider parameters such that both websites attract potential buyers. This means that from the point of each of the two websites, there is never "full" consumer participation, and more consumers can be attracted if the value of the auction hosting site to them increases. We distinguish between the bidder cost component of browsing a site (which is common to all bidders) as c^A and c^B. Similarly, we will later distinguish between the listing fee of site A from that of site B. The distribution of seller costs is assumed to be the same for the two sites, and as discussed in the model preliminaries, sites have monopoly power over their respective set of sellers. To facilitate comparability with the monopoly examples, we consider the viewpoint of auction hosting site A with location at the left end-point of the unit Hotelling interval.

One of the first major differences for the duopoly case can be observed by considering the plots of x_c — the location of the consumer who is indifferent between browsing from site A and site B. As Figure 25 shows, in contrast to the monopoly case, θ both tilts and shifts the location of the indifferent consumer (c^A continues to give a parallel shift as before). The reason why θ rotates the location of the indifferent buyer is because the value of the potential consumer's alternative choice (the website B), is also affected by a change in θ. In fact, θ rotates the location of the critical consumer around the value of q_A that results in the two firms splitting the market. For that value of q_A, a change in θ does not lead to a change in x_c, because it affects the value of webiste A as much at it affects the value of the website B (to the consumer).

In Figure 25, the market is split when $q_A = 1$ (for $c^A = c^B$ and $q_B = 1$), so the dotted and solid lines pivot around $q_A = 1$. Effectively, a change in θ pivots the line around its intersection with the horizontal line $x_c = 0.5$, not the horizontal line $x_c = 0$ (which is the monopoly case). In Figure 26, for which we consider $c^A > c^B$ and $q_B = 0.5$, the market is divided when $q_A \approx 0.67$. Therefore, a change in θ pivots x_c around that point. Notice that the $x_c = 0.5$ line goes through the point of pivoting.

With regards to the response of x_c to the other parameters, shown in Figure 27, c^B results in a parallel shift, while q_B leads to both a tilt and a shift (the shift is really what dominates).

The fact that theta pivots x_c, and hence the revenue function, around the point in which the market is shared equally between the two firms, has direct implications on the effect of a change in the degree of differentiation between the two sites on the entry equilibrium. Consider Figure 28, which has been drawn for parameter values such that the market is equally shared between the two firms when $q_A \approx 0.8$. An increase in θ can allow a site that is not in business to enter business, even though it leads to uniformly higher consumer costs. In this figure, costly differentiation can help website A. Qualitatively, this finding is similar to traditional Hotelling models. What is different is that as θ increases, the seller (and buyer) activity at site A can increase discontinuously: Site A does not get a foothold, which it can expand as differentiation increases, but rather it attracts up to 40% of the maximal seller activity upon entry. The positive feedback between the number of sellers and number of buyers means that there is a minimum scale of activity for each of the two websites.

Figure 29 shows the effect on the entry equilibrium in site A as there is an increase in the seller activity in the competing. It can be seen that an increase in q_B shifts the revenue function of sellers at site A downwards. Threshold effects at which site A suddenly "shuts down" as the activity of site B increases can exist here as well.

We now turn to the equilibrium seller participation, for given fee levels. In Figure 30 we examine symmetric sites charging identical fees and shown that equilibrium seller participation declines with μ, as expected. Because these are symmetric sites, a change in θ has no effect, and neither does a (symmetric) change in c^A and c^B.

The entry equilibria that are plotted in Figure 31 are symmetric. There are often also asymmetric entry equilibria for these same parameter values! For example, set μ to 0.16, the fees to zero, and keep the other parameters the same as those in Figure 30. The symmetric equilibrium is shown in Figure 31, which plots the revenue and inverse cost distribution functions for the sellers in site A. It can be seen that we are in an equilibrium, as the site-equilibrium value of q_A given the value of q_B is equal to q_B (recall that the other parameter values for the revenue and cost function at site B are the same as those at site A).

Figure 32 shows that for these same parameter values, there also exists an asymmetric equilibrium, in which $q_B = 0$, and q_A is higher than in the symmetric equilibrium. Such asymmetric equilibria do not exist for low μ and low fees. If μ is so low that participation in one site is positive even when all consumers would go to the other site under monopoly and seller activity in the other site is at the monopoly level, then this precludes site shutdown in an asymmetric equilibrium. [Simply picture what would happen in Figure 32 if the cost function were to move downwards.]

7 Concluding Remarks and Future Work

This paper derives the analytics of seller and buyer participation in auction hosting sites and its implications for optimal auction hosting listing fee pricing. It also investigates issues involving the competition between auction sites. Throughout, we assumed that the reserve (when used) is chosen by the seller independently. However, auction hosting sites may be able to influence the reserve the sellers can charge, or set and (possibly) enforce reserve setting rules. The auction hosting site, unlike a seller, may benefit from establishing a reputation for low reserves, and thus attract more buyers. The auction hosting site has a stronger incentive to lower the reserve than does a single seller. A single seller is not likely to be able to establish a reputation for low reserves that attracts more customers to him or generate more traffic to the website. Even if successful in somewhat lowering the expected reserve in that site, thus generating more traffic, his action would generate positive spillovers for other sellers in that same site. The auction site owner would internalize these spillovers, as he would be able to charge higher fees. Thus, reserves set by auction hosting sites would be lower than those set by individual sellers. The investigation of whether reserves and listing fees are substitutes from the point of the website is one of the extensions we will consider.

A second important question to be addressed are the welfare implications of the a site's monopoly power (in the single site case) and of the welfare loss from a site merger (in the duopoly case). Unlike traditional markets, site mergers have an important positive welfare enhancing role: They increase the number of sellers in a site, and thus reduce the likelihood that customers will incur browsing costs in vain. On the negative side, a mergers has the standard negative effect in terms of price increases, which not only shifts surplus from consumers to the website, but also reduces surplus through the elimination of positive surplus exchanges between buyers and sellers. There is, of course, the standard variety reduction effect of a merger, since the mean transport cost of buyers will increase post-merger. However, because the demand functions for a site's services is very elastic, as shown in this paper, the monopoly pricing distortion may be very small. Therefore, it is possible that a merger from duopoly to monopoly may be welfare enhancing.

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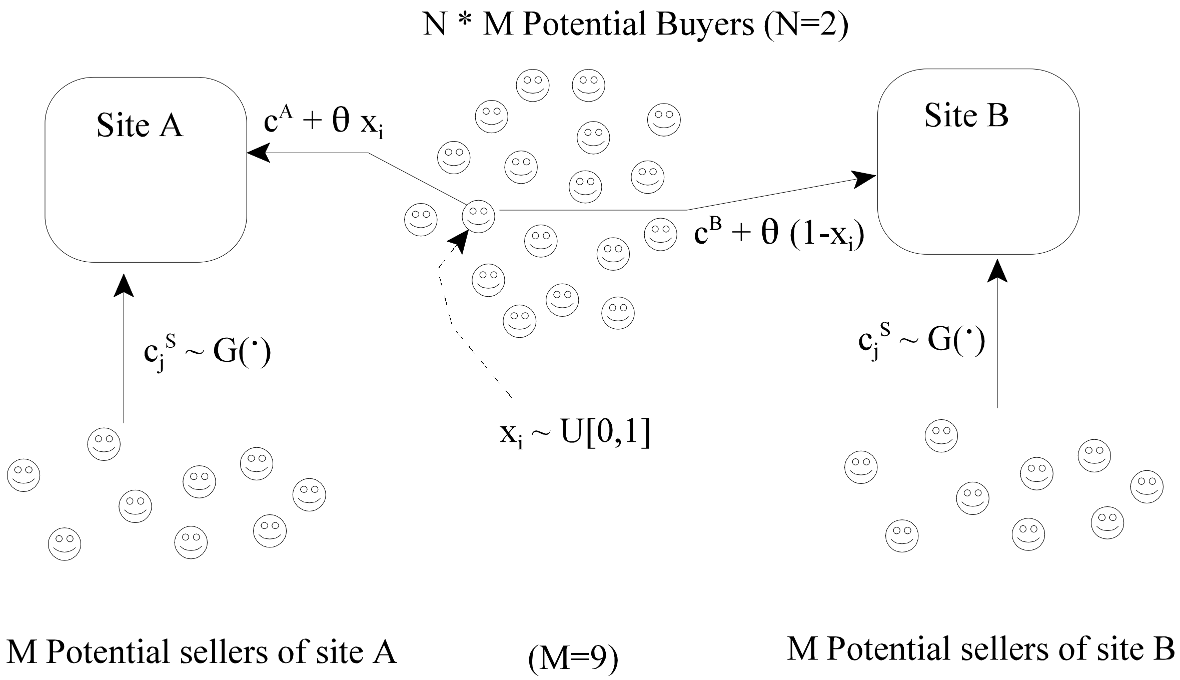


Figure A

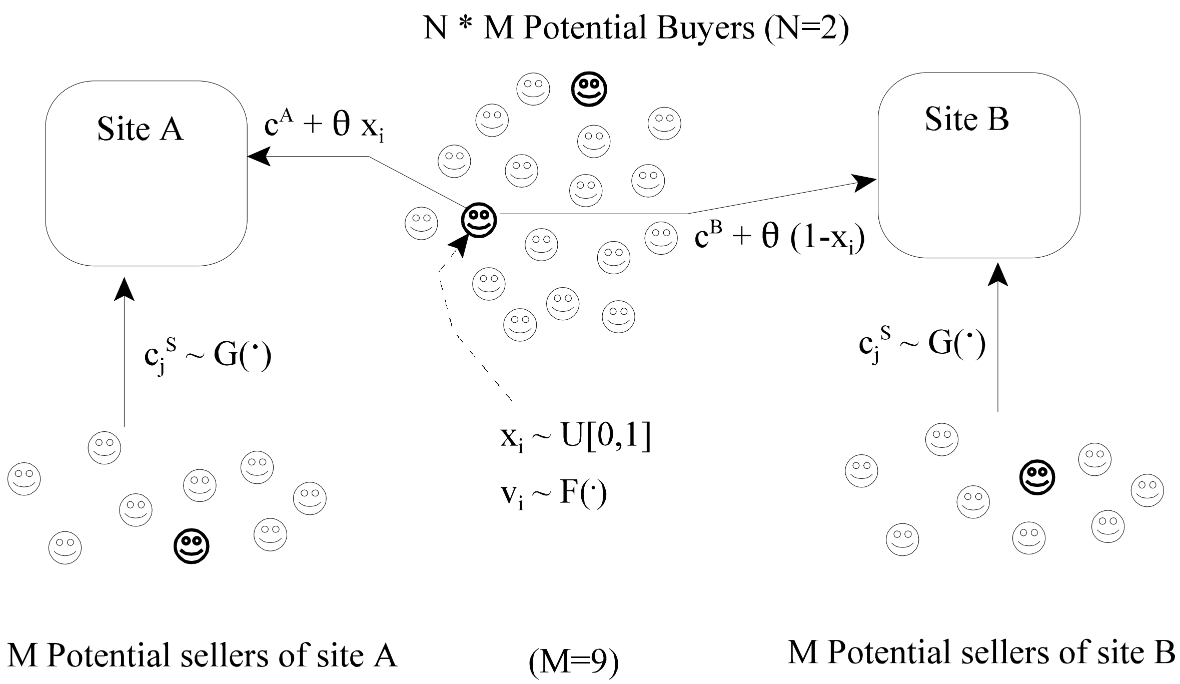


Figure B

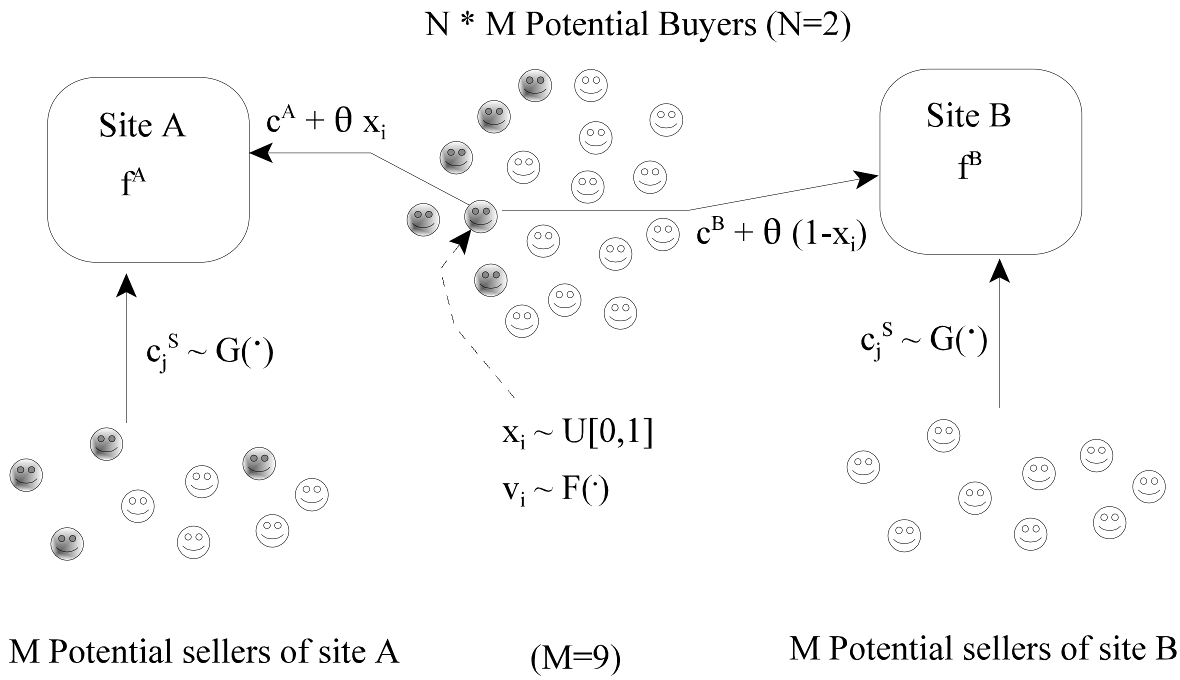


Figure C

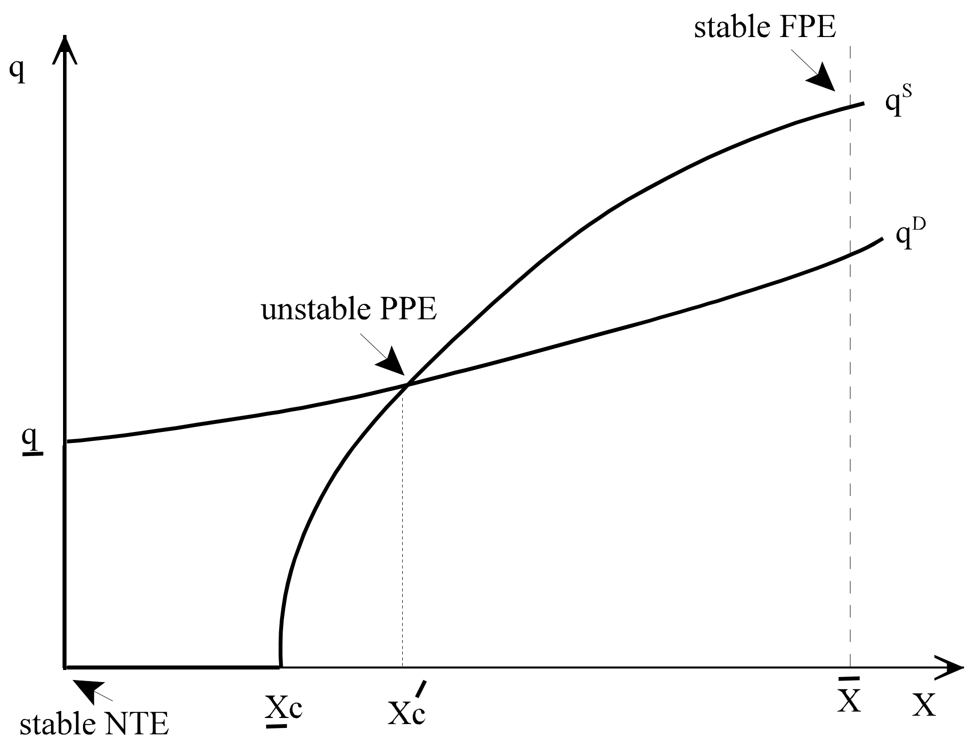


Figure D

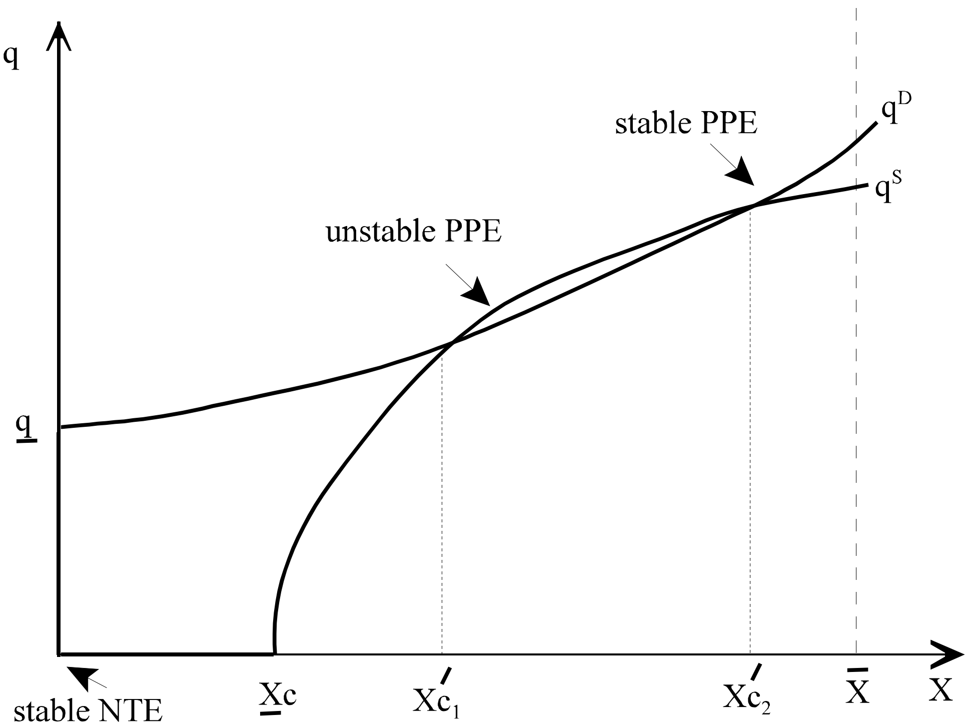


Figure E

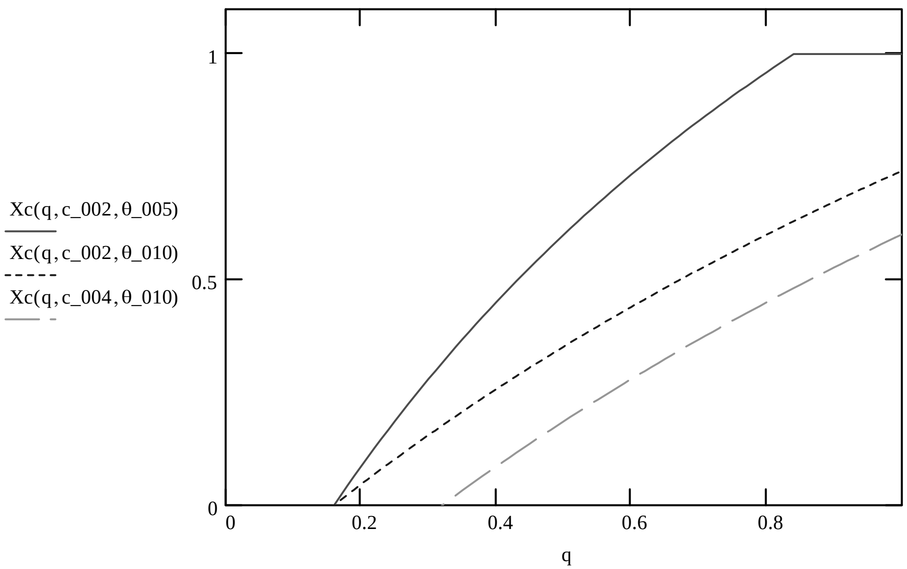


Figure 1. Location of critical consumer in monopoly.

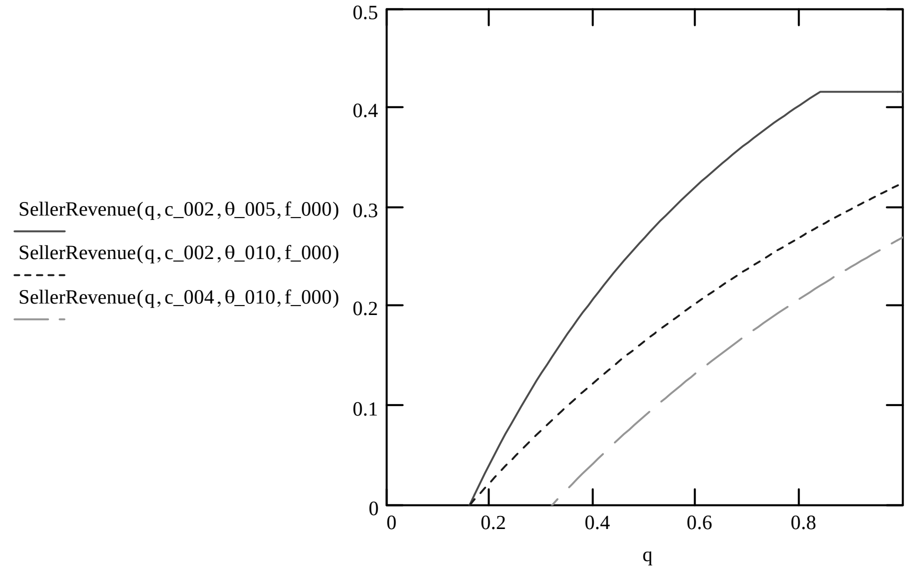


Figure 2. Seller revenue (gross of entry costs).

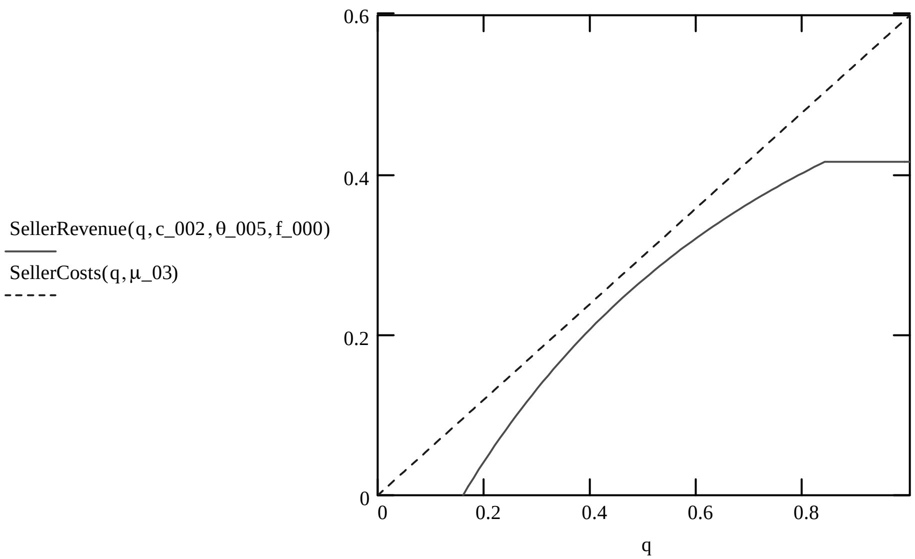


Figure 3. Market Equilibrium: Zero equilibrium supply (site shutdown).

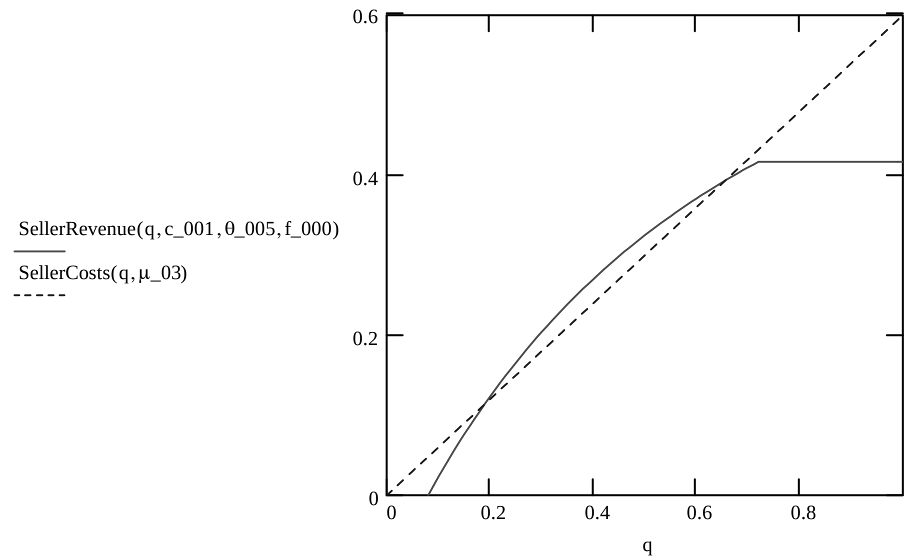
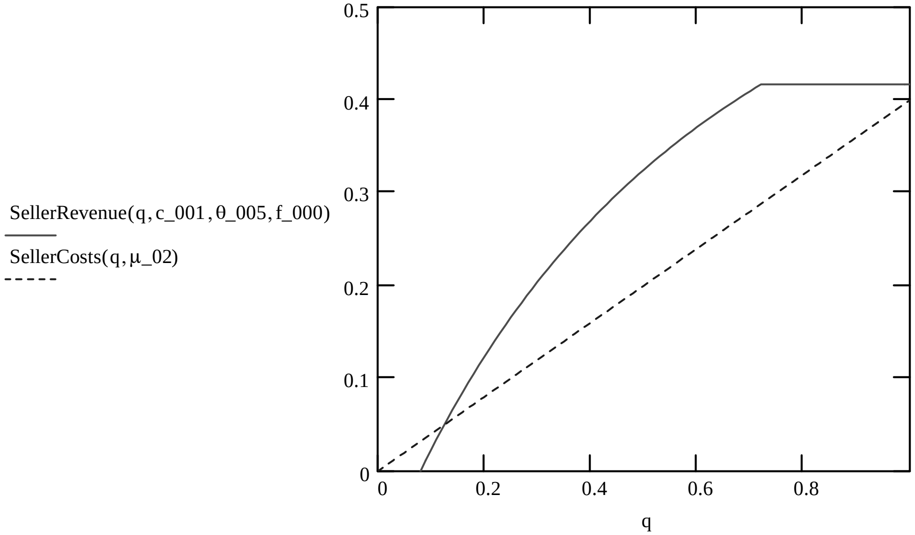


Figure 4. Market Equilibrium: Two interior entry equilibria (one stable).



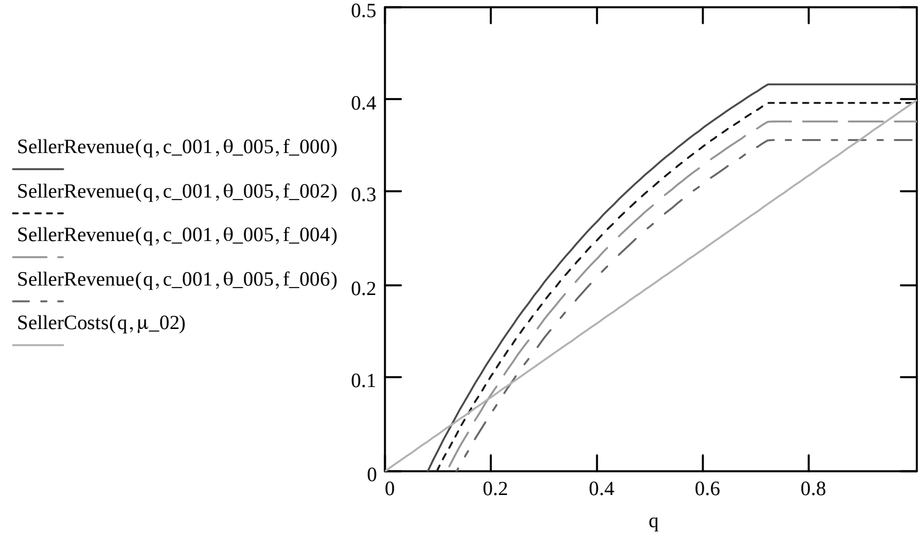
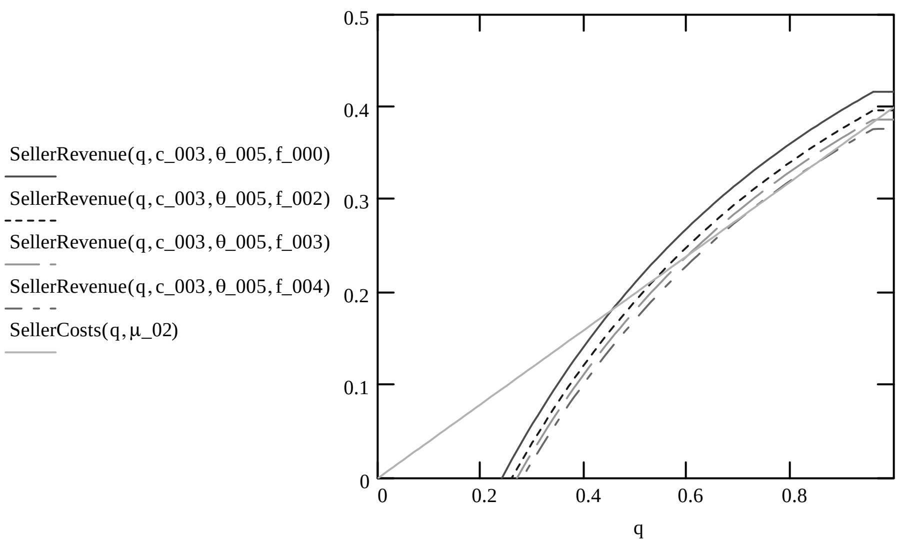


Figure 5. Market Equilibrium: One unstable interior entry equilibrium, stable full participation equilibrium.

Figure 6. Market Equilibrium: Effect of an increase in listing fees.



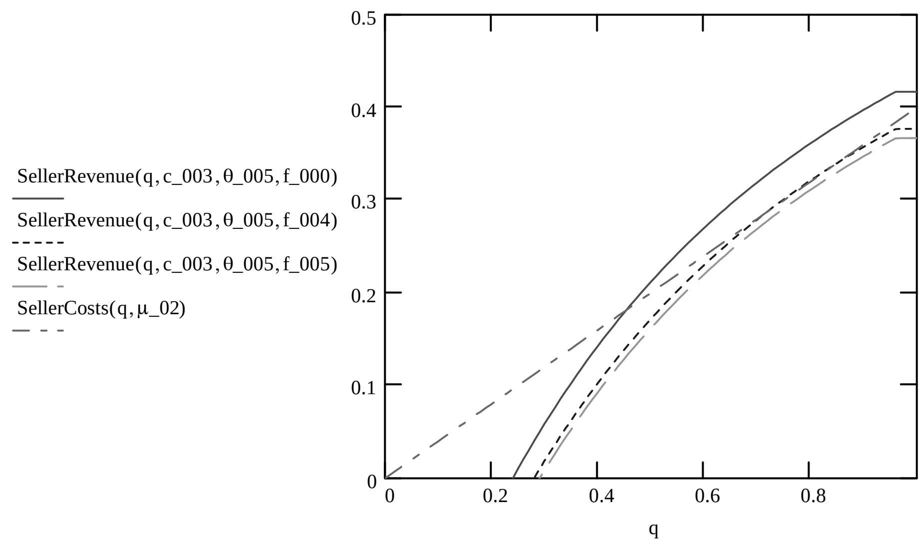


Figure 7. Market Equilibrium: Profitable increase in listing fees.

Figure 8. Market destroying effects of marginal changes in the listing fee.

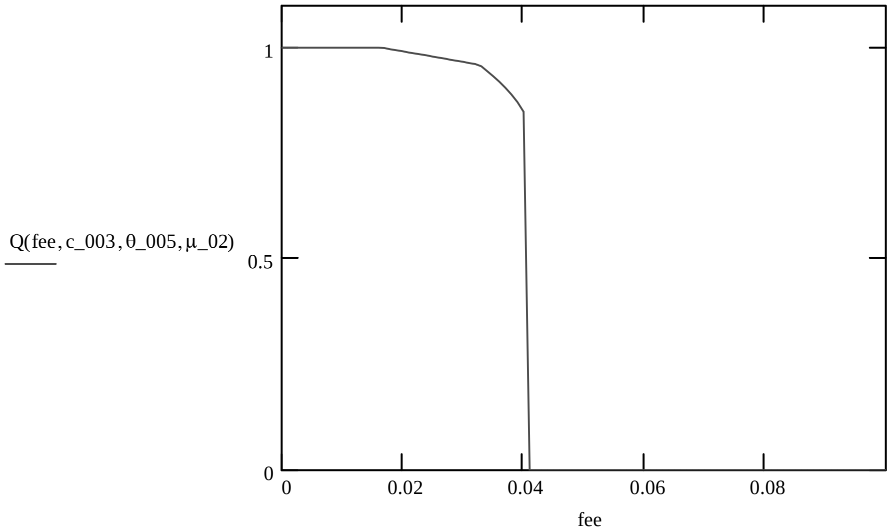


Figure 9. Auction hosting site demand curve.

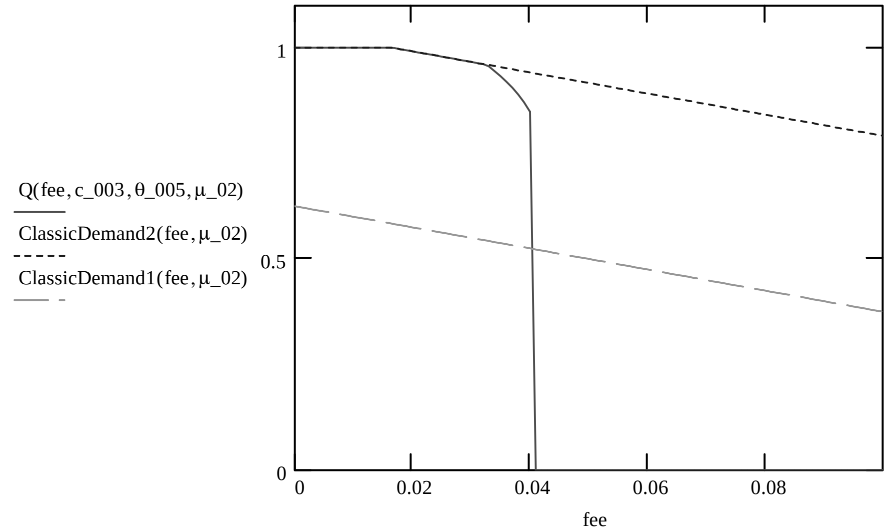


Figure 10. Comparison with "normal" (no feedback) auction site demand curves.

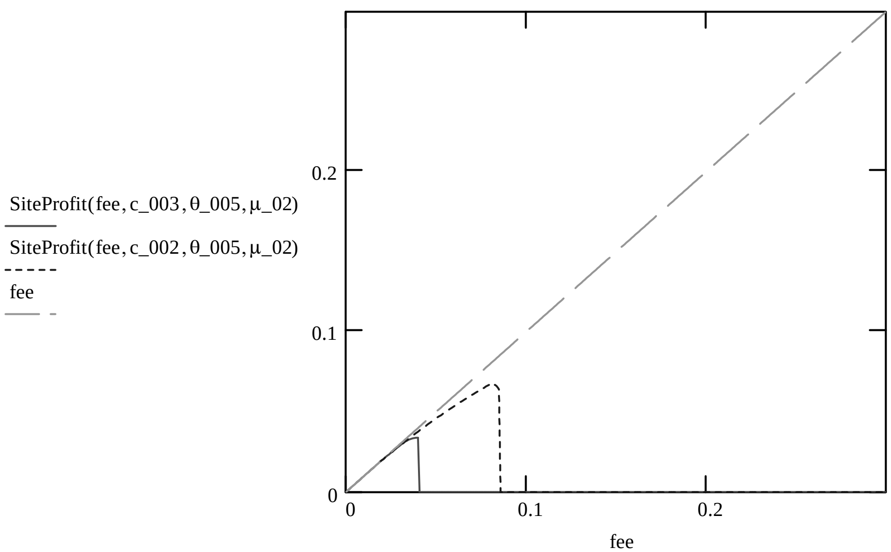


Figure 11. Site profit functions: changes in mean consumer attendence costs.

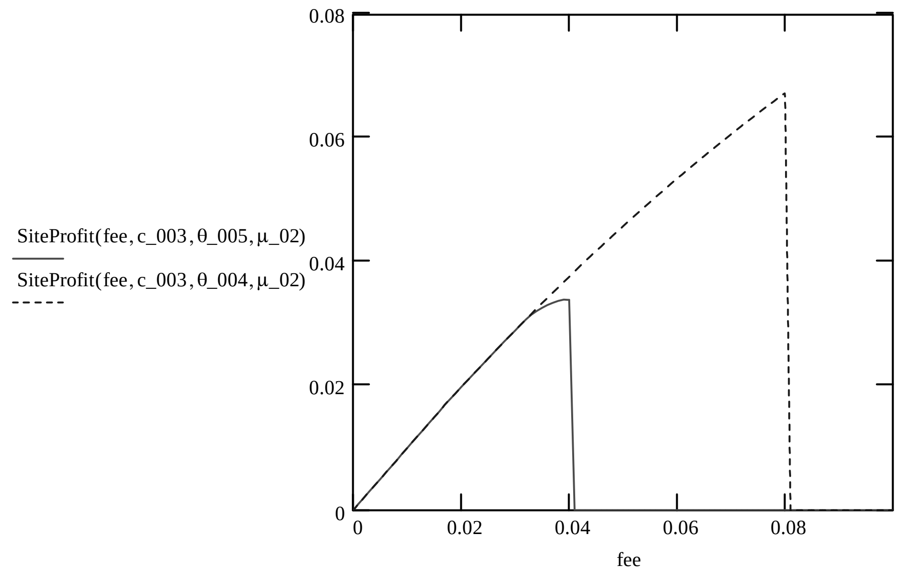


Figure 12. Site profit functions: changes in the dispersion of consumer attendence costs.

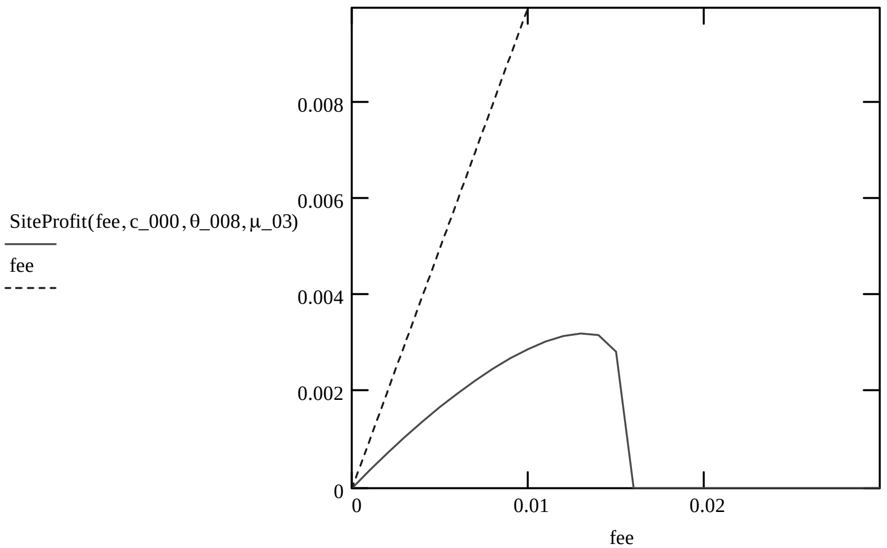


Figure 13. A site profit function with an interior optimum (q<100%, differentiable maximum).

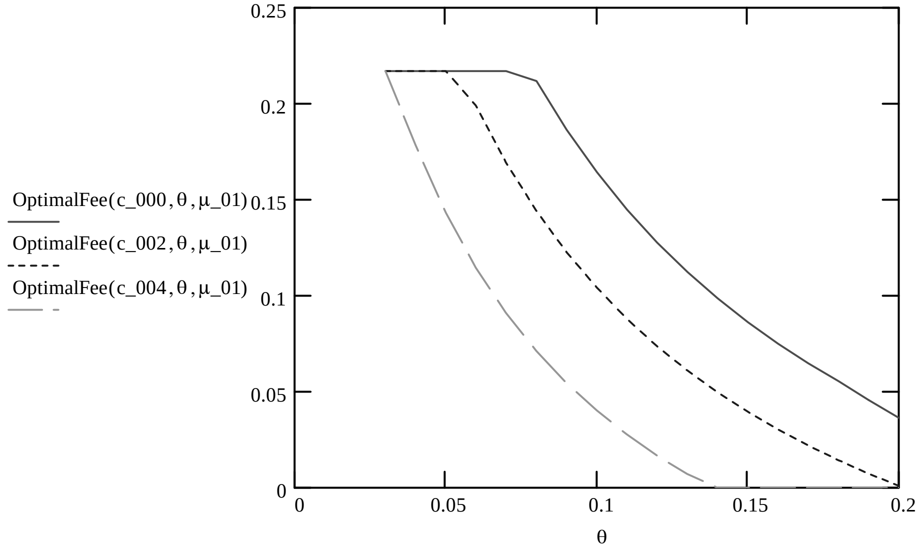


Figure 14. Optimal list fee as a function of the buyer participation costs.

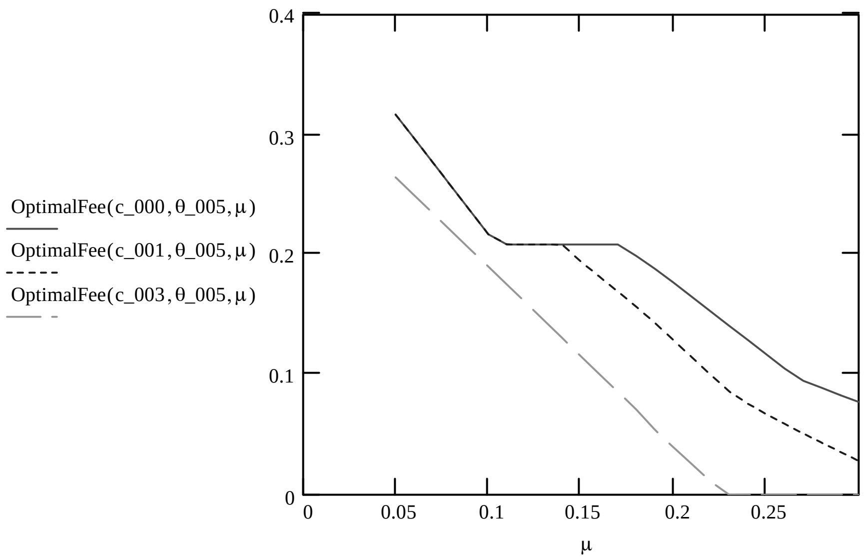


Figure 15. Optimal list fee as a function of the mean of seller participation costs.

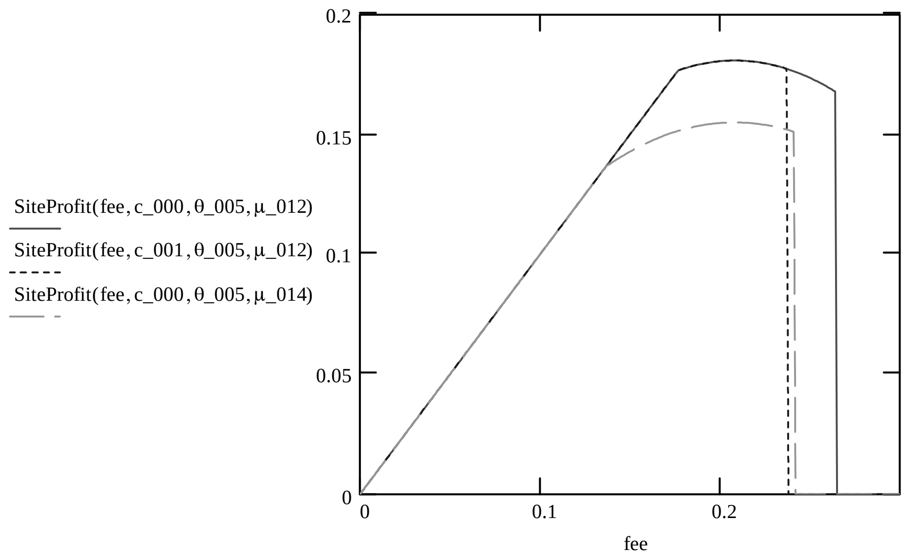


Figure 16. Possible local unresponsiveness of optimal fee to seller or buyer costs.

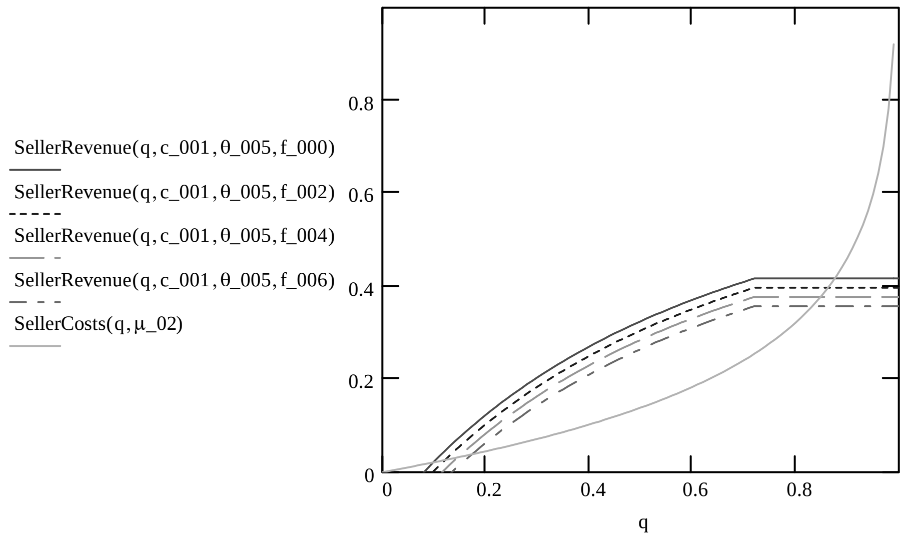


Figure 17. Market Equilibrium with exponentially distributed seller costs.

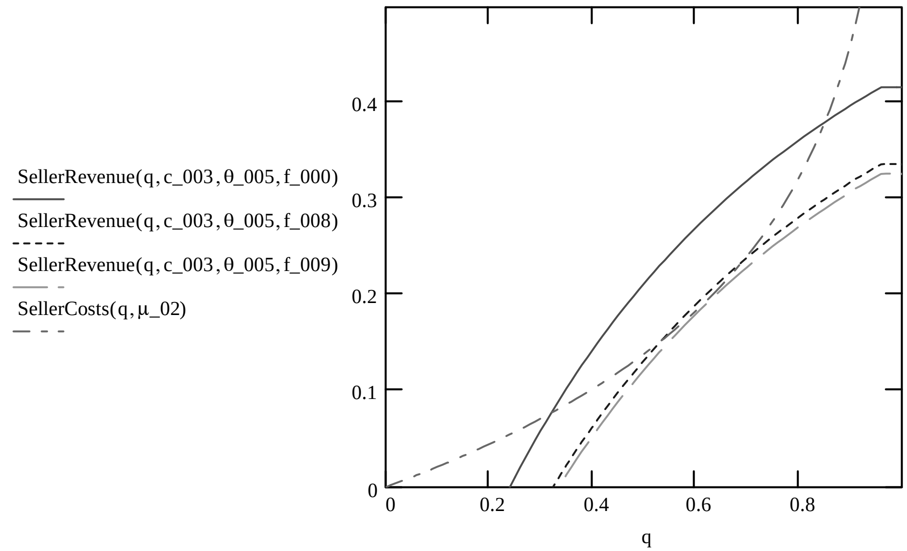


Figure 18. Market destroying effects of marginal changes in the listing fee with exponentially distributed costs.

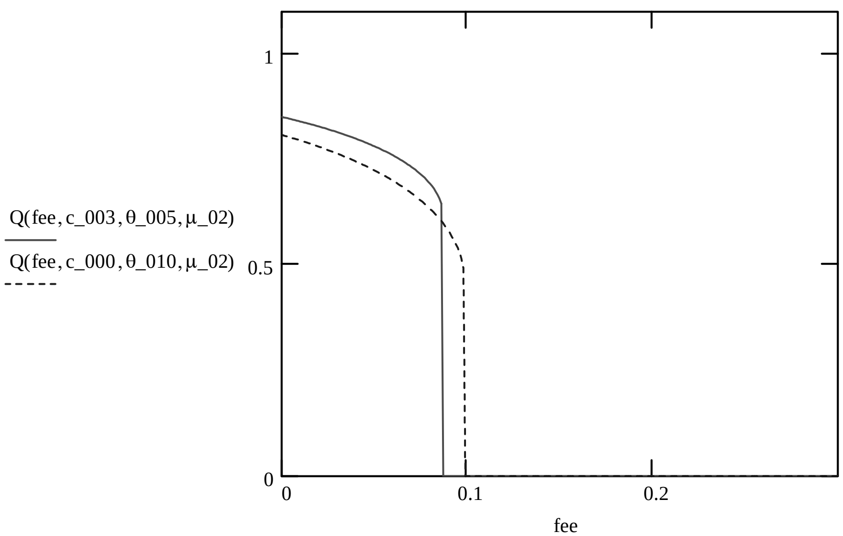


Figure 19. Auction hosting site demand curve with exponentially distributed seller costs.

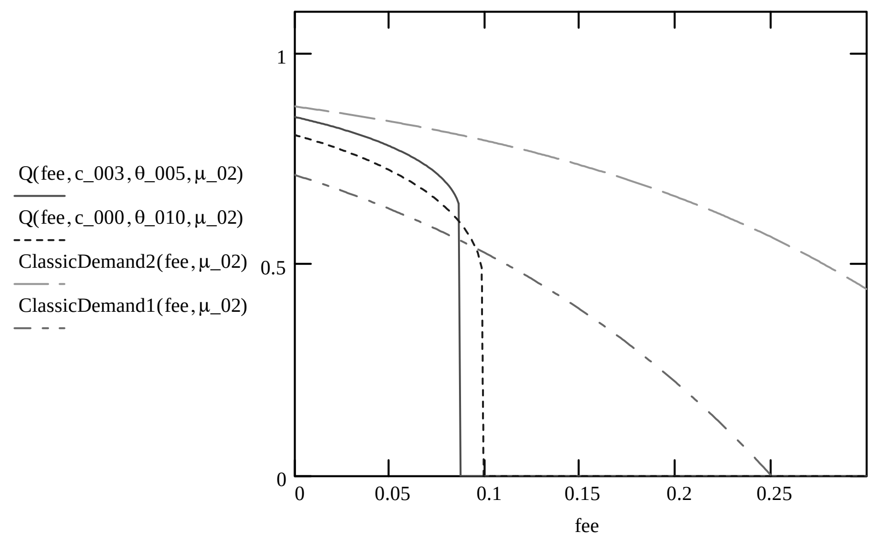


Figure 20. Comparison with "normal" demand curves with exponentially distributed seller costs.

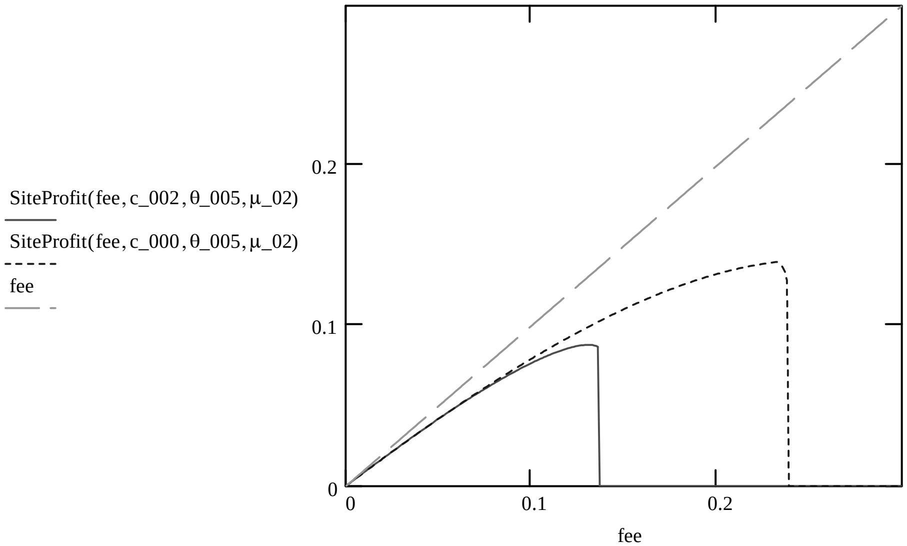


Figure 21. Site profit functions: changes in mean consumer attendence costs, exponentially distributed seller costs.

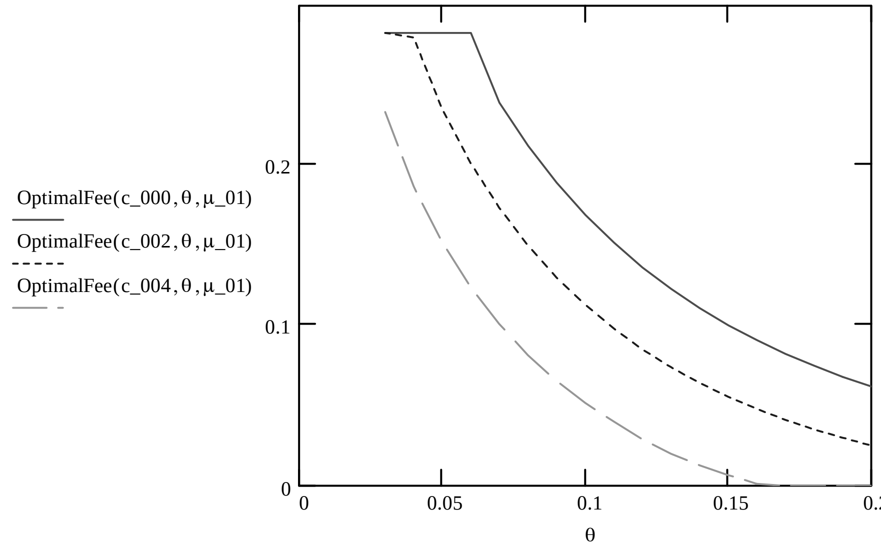


Figure 22. Optimal list fee as a function of the buyer participation costs: Exponentially distributed seller costs.

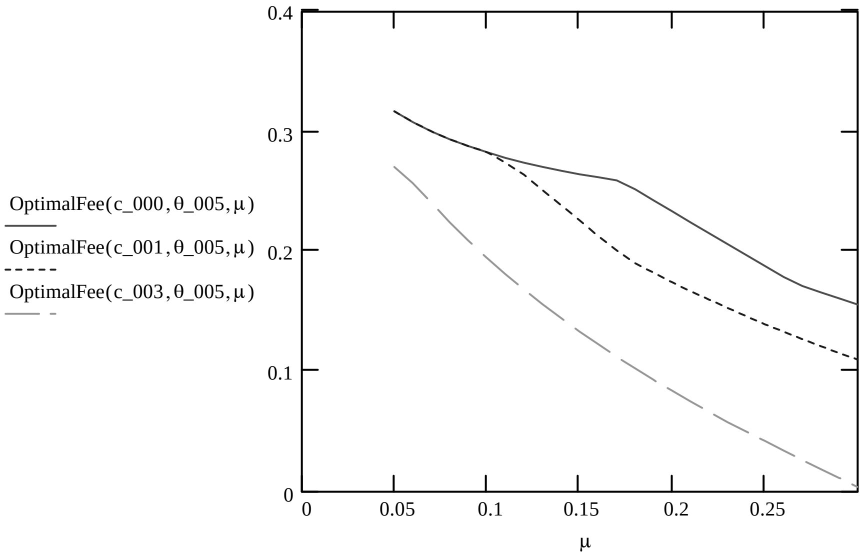


Figure 23. Optimal list fee as a function of the mean of seller participation costs: Exponentially distributed seller costs.

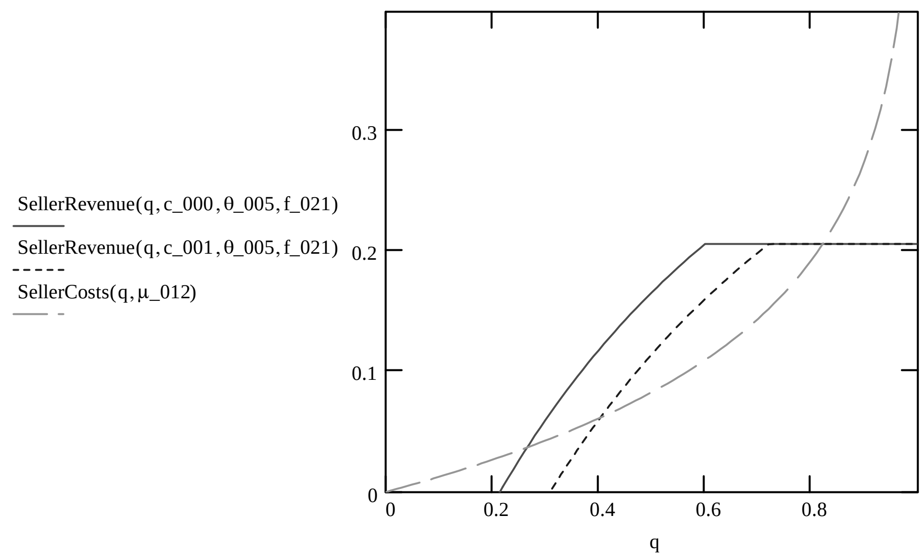


Figure 24. A closer look at the insensitivity of optimal fee to consumer attendence costs: Exponentially distributed seller costs.

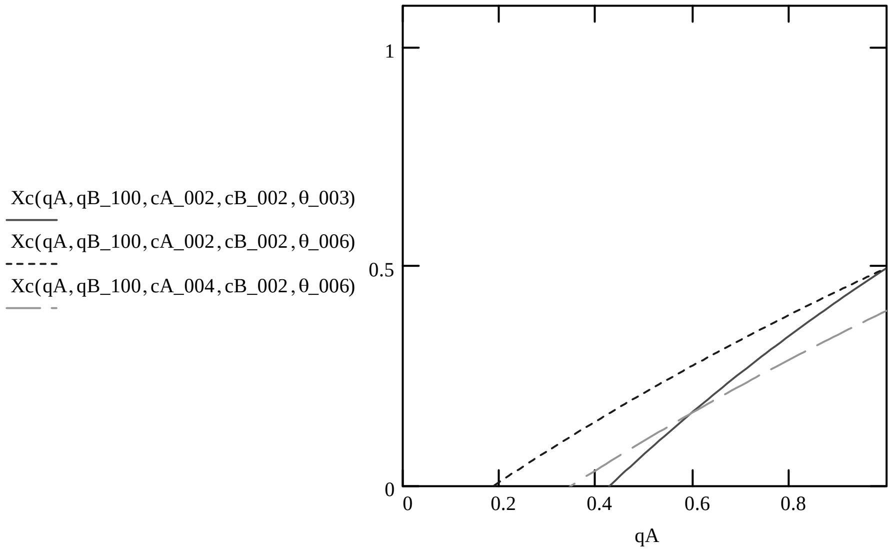


Figure 25. Location of critical consumer in duopoly.

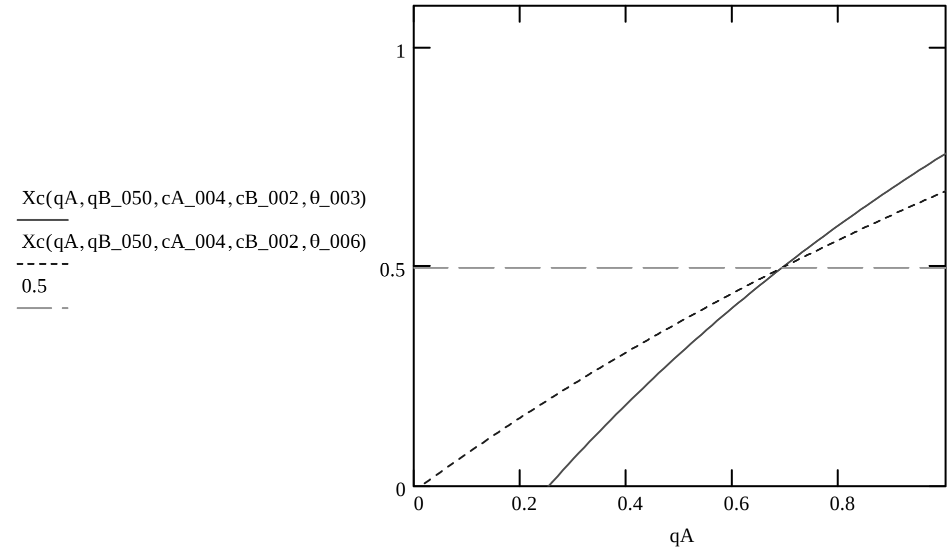


Figure 26. Location of critical consumer in duopoly: changes in theta.

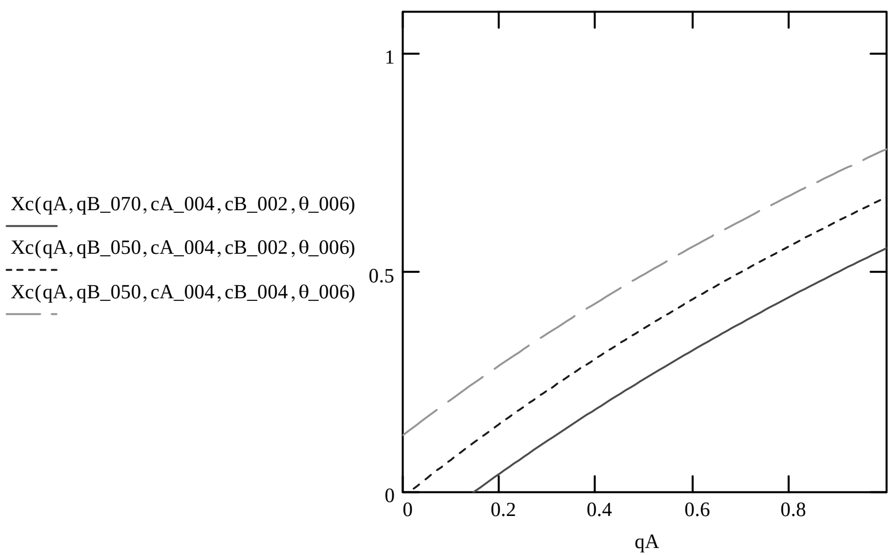


Figure 27. Location of critical consumer in duopoly: changes in other parameters.

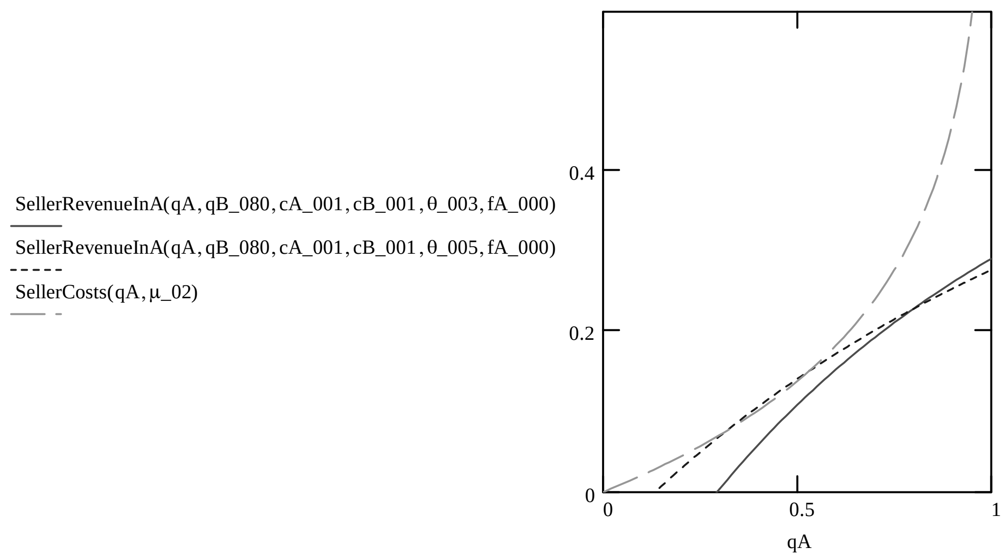


Figure 28. Site Equilibrium with exponentially distributed seller costs: Changes in theta.

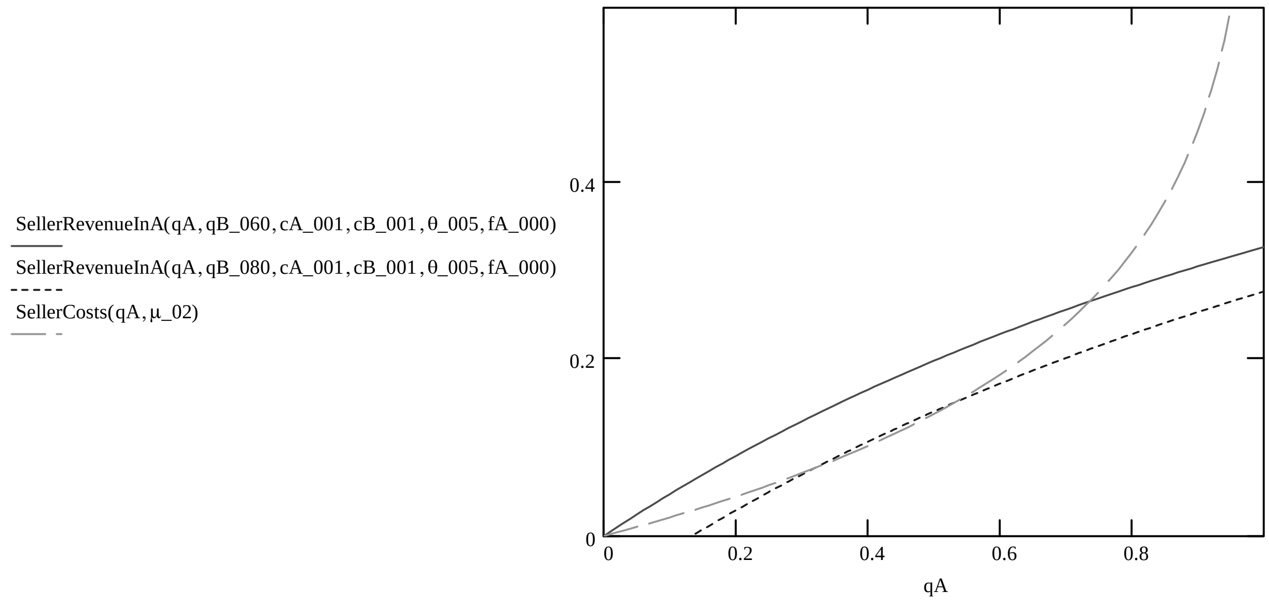


Figure 29. Site equilibrium with exponentially distributed seller costs: Changes in seller activity at competing site.

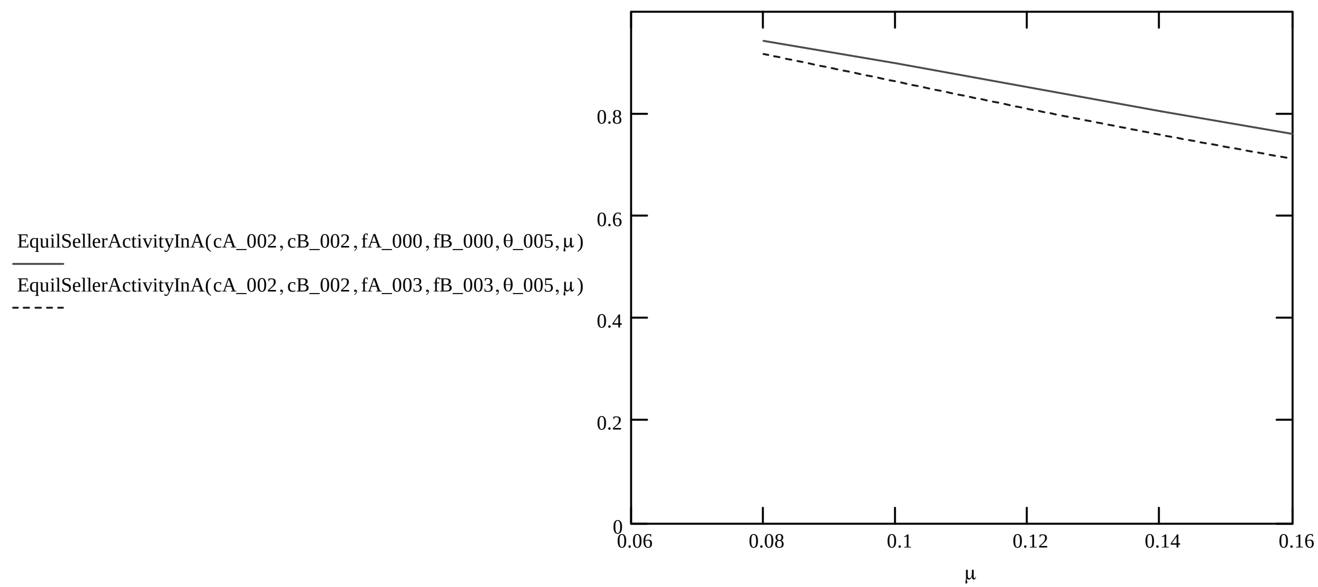


Figure 30. Equilibrium seller activity levels with exponentially distributed seller costs, as a function of mean seller costs: Symmetric sites.

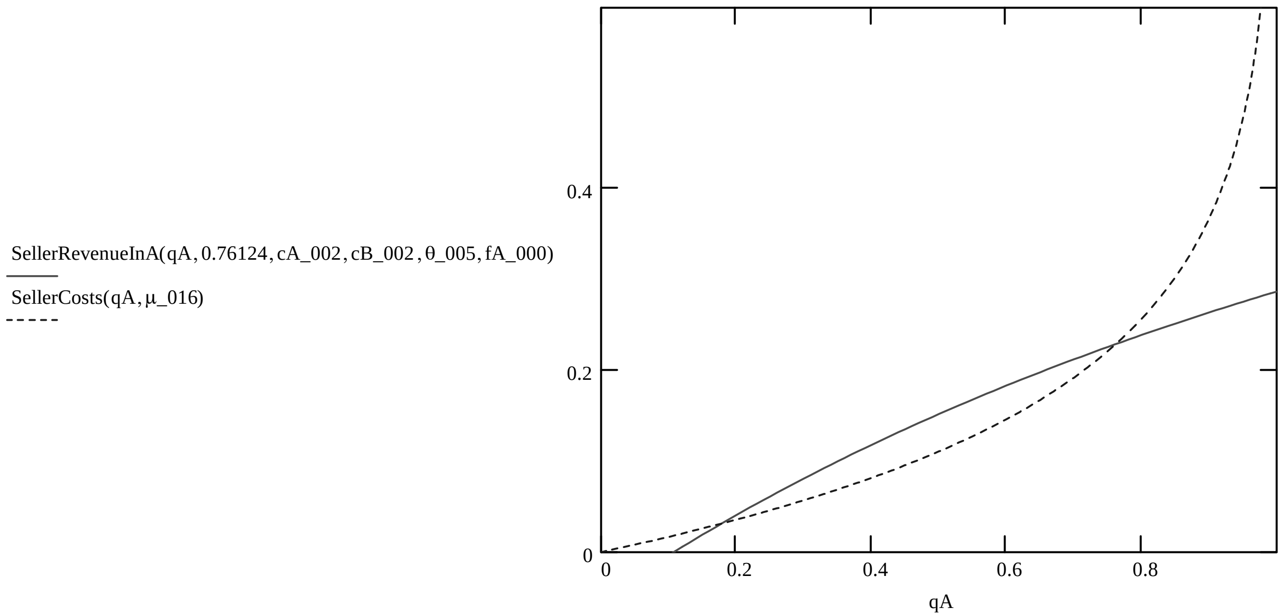


Figure 31. Symmetric equilibrium with exponentially distributed seller costs.

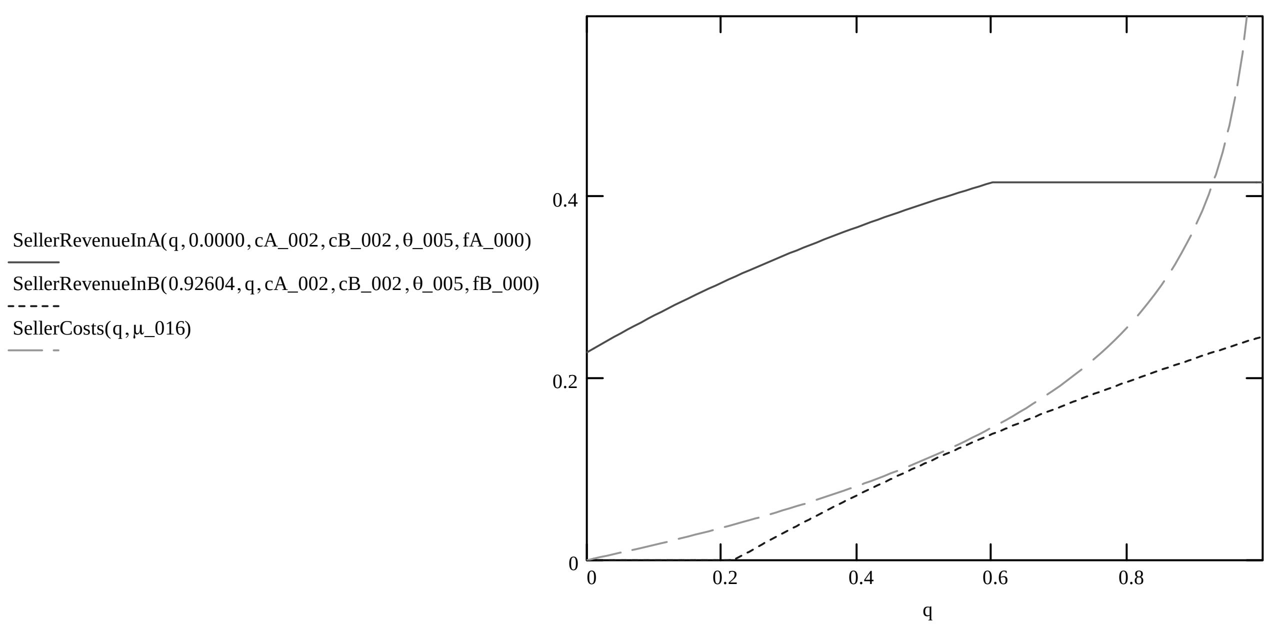


Figure 32. Asymmetric equilibrium with exponentially distributed seller costs.