Optimal Bidding in Sequential Online Auctions

Ashish Arora, Hao Xu[1], Rema Padman and William Vogt
The H. John Heinz III School of Public Policy Management
Carnegie Mellon University
Pittsburgh, PA15213
E-mail: {ashish;xhao;rpadman;wilibear}@andrew.cmu.edu

Abstract

Auctions are widely used online to conduct commercial transactions. An important feature of online auctions is that even bidders who intend to buy a single object frequently have the opportunity to bid in sequential auctions selling identical objects. This paper studies key features of the optimal bidding strategy, assuming rational, risk-neutral agents with independent private valuations and sealed-bid second-price sequential auctions. In contrast to previous work on this topic, we develop our theory using the concept of the "option value" of an upcoming auction – a measure of the expected payoff from being able to participate in a future auction. This option value depends, among other things, upon the mean and variance of the future number of bidders. We derive an optimal bidding strategy in sequential auctions that incorporates option value assessment. Furthermore, we establish that our optimal bidding strategy is tractable since it is independent of the bidding strategies of other bidders in the current auction and is only dependent on the option value assessment. We test and find support for our theory using data collected on 327 eBay auctions on digital cameras in first two months of 2001.

[1]Please address all correspondence to Hao Xu.

1. Introduction

There has been considerable interest in Online Auctions since they have become established mechanisms to conduct commercial transactions in consumer and business markets. Recent surveys (Pinker et al. 2001, de Vries and Vohra 2002) discuss alternative auction formats (English, Dutch, Yankee, Combinatorial Auctions) and survey the literature on a range of problems (prominent examples include winner determination, optimal bidding strategy, and lot sizing). These surveys complement the existing literature on standard auctions that have been extensively studied in the literature (Cassady 1967, Stark and Rothkopf 1979, Milgrom and Weber 1982, McAfee and McMillan 1987). As several recent surveys on online auctions note, the online environment generates a new set of requirements that challenge existing theories and models that have been analyzed to date. As Pinker, Seidmann, and Vakrat (2001) put it: ” This is not because the previous research has been flawed, but rather because there has been enormous change in the opportunities for the use of auctions.”

Of particular interest to this paper is the analysis of sequential auctions. Sequential auctions are auctions for the same good, ordered in time, and are commonly observed on consumer sites such as eBay (Kaiser and Kaiser 1999). Listings of sequential auctions and the ability to collect data on the bid history of past auctions using software agents are an important feature of the online environment, as shown in Figures 1, 2, and 3. This permits bidders to bid in multiple sequential auctions and to assess their likelihood of winning in an upcoming auction. These assessments could be used to formulate their bidding strategy in a current auction – in effect, bidding taking into account the “option value” of an upcoming auction. Developing and testing a theory of optimal bidding strategy in sequential auctions based on this observation is the focus of the paper.

Figure 1: Example of Ongoing Auctions

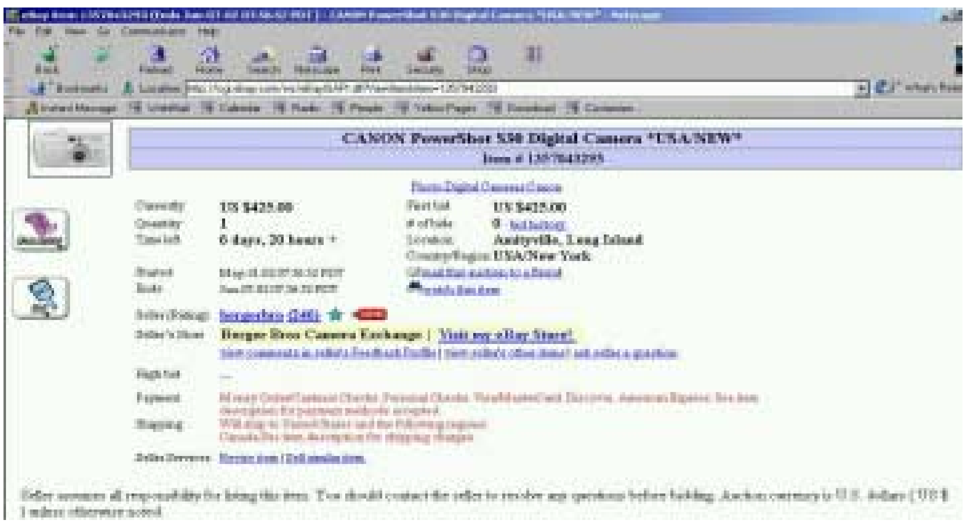

Figure 2: Item Information for a Specific Auction

Figure 3: Bid History for a Specific Item

This paper is organized as follows. Section 2 details background information and literature review. Section 3 studies bidders' optimal strategy and the impact of uncertainty on bidders' decisions. Section 4 presents our tests and results that explain the implications of our theory. We conclude the paper in Section 5 with a summary, the major contributions, and limitations of this research.

2.0 Background and Literature Review

2.1 Private Value Model and Online Bidder's Bidding Strategy

In a typical online auction, such as auctions conducted at eBay, Amazon, and Yahoo!, bidders are individual consumers, who buy items to satisfy their own needs. Although there is a chance that bidders might buy items in order to resell later for a profit (for example, bidders may buy stamps either for their own collection or for trading), experience tells us that many categories of goods selling on eBay, Amazon, and Yahoo! are not bought for resale. These goods include consumer electronics, computer parts, and digital equipments. For these "private-value" goods, each bidder knows her value and this value is independent of how other bidders value these items. On the contrary, "common-value" goods, such as public construction contracts, are believed to have consensus value and may be purchased for reselling. This paper will focus on the study of *"private-value"* goods.

The benchmark private-value model studies auctions of private goods. In this model, each bidder knows how much she values the object for sale, but her value is private information to herself. One conclusion of private-value model is that, in second-price auctions, a bidder's dominant strategy is to bid her reservation value. (Vickrey 1961, Wilson 1992, Matthews 1995, Thiel and Petry 1995). However, online bidders, particularly experienced online bidders, often appear to bid at a discount level of their reservation values (Sinclair 1999). In some online auction forums, bidders refer to this strategy as the optimal strategy. The data we collected from eBay.com website shows that many bidders bid in multiple auctions, with successively higher bids in later auctions. This type of bidding behavior seems to contradict the results of the private-value model. However, the private-value model implicitly assumes that bidders participate in only one auction. Online bidders often have the choice of bidding in a sequence of auctions selling the same type of item. Furthermore, as noted earlier, in contrast to conventional auctions, online auctions can be monitored using software agents. This enables bidders to estimate a variety of information such as the number of bidders, variability in the number of bidders, and how they vary as a function of when the auction is held (weekend vs. weekday), the type of product being auctioned, and so on. How should bidders incorporate these estimates into their bidding strategy in what is inherently an uncertain environment? These are the questions we address in this paper.

2.2 Previous Study about Sequential Non-Internet Auctions and Online Auctions

In this section, we review the research literature on non-Internet sequential auctions, and explain how Internet sequential auctions may differ from non-Internet auctions. Milgrom (1989) and Milgrom and Weber (1982) provide theoretical analyses of the price trend in a sequential auction of identical objects and conclude that expected prices should remain constant throughout the sequence of auctions within a sale. However, empirical evidence shows that actual price may either decline or increase sequentially. Ashenfelter (1989) reports that in sequential wine auctions, a downward price trend was observed. McAfee and Vincent (1993) examined data of Chicago wine auctions and attained similar results. Ashenfelter and Genesove (1992) also observed that “price-decline-anomaly” phenomenon occurs in real-estate market. Other empirical studies have found that prices may be increasing. Gandal (1997) studied sequential auctions of the cable TV licenses conducted in Israel and observed increasing prices. Jones, Menezes, and Vella (1998) noticed that prices may either increase or decrease in sequential wool auctions.

Much of the theoretical research on non-Internet sequential auctions focuses on the observance of “anomalous price trend” and its theoretical explanations. Von der Fehr (1994) explains the anomalous price trend using participation cost. Black and de Meza (1992), Krishna and Rosenthal (1996), Branco (1997), and Menezes and Monteiro (1999) consider synergies and their impact on price trend.

Pinker, Seidmann, and Vakrat (2001) provides an excellent review of current state of research on online auctions. Hence, we provide only a selective review of the earlier research that is most directly linked to this paper. Several studies present overviews of online auctions (Beam and Segev 1998, Klein and O’Keefe 1999, Lucking-Reiley 2000, Herschlag and Zwick 2000). These papers represent the earliest studies of key features of online auctions. Lucking-Reiley (2000) explores what types of goods are sold through online auctions and the various formats used in these auctions. Beam and Segev (1998) identify the defining characteristics of online auctions and major differences from standard auctions. Several studies (Lucking-Reiley et al. 1999), Bapna, Goes, and Gupta 2000a) have also focussed on overview study of eBay auctions. Bajari and Hortacsu (2000) give an overview of eBay auctions and provide empirical insights from them. Lucking-Reiley et al. (1999) conducted an analysis of determinants of prices in online auctions for collectible one-cent coins at eBay, and reports that seller's feedback ratings, particularly negative feedback ratings, have a measurable effect on her auction prices. Minimum bids and reserve prices have positive effects on the final price. Also, auctions with longer duration tend to be priced higher on average.

Another research focus is the study of multi-unit auctions. Bapna, Goes, and Gupta (2000b) identify three different types of bidders in multi-unit B2C auctions, indicating that bidders may adopt different bidding strategies. Easley and Tenorio (1999) study bidding strategies in multi-unit Internet Yankee auctions. Seidman, Pinker, and Vakrat (2001) study the optimal design of multi-unit and multi-period online auctions. Sandholm, Suri, Gilpin, and Levine (2002) investigate the winner determination problem of multi-unit combinatorial auctions. Arora, Cooper, Krishnan, and Padman (1999) design a e-market simulation environment, IBIZA, which allows researchers to experiment with different market mechanisms including various types of online auction formats.

Online sequential auctions are different from non-Internet sequential auctions. In most theoretical models of sequential auctions, the number of auctions is fixed; bidders enter and leave the auctions at the same point (or winners leave and the rest remain). So, the *k*th auction of bidder A is also assumed to be the *k*th auction of bidder B. However, in online sequential auctions, no such symmetry can be assumed. Each bidder faces a continuous stream of auctions. Bidders enter those auctions at different times and may have participated in different number of auctions. For example, the first auction of A may be the fifth auction of B. We develop a model where an auction may have bidders who have bid in earlier auctions, and other bidders for whom this is the first auction. Furthermore, in non-Internet auctions, the number of bidders is generally fixed while it is random in Internet auctions. The impact of this uncertainty on bidding behavior is an interesting issue that, to our knowledge, has not been studied before.

Thus, the principal contribution of this paper is a model of online sequential auctions that captures important features of these auctions neglected in earlier models of sequential auctions.

This model provides important and novel insights on how uncertainty affects bidding strategies. We test and find strong empirical support for these predictions.

3.0 Study of Optimal Bidding Strategy

In this paper, we consider a sale of two identical objects through second-price sealed-bid auctions. If a bidder wins an auction, she will leave the game. If she loses the first auction, she will bid in another auction with probability of 1.[2] Each bidder intends to buy at most a single object i.e., the utility of a second object is assumed to be zero. In future work, we intend to relax this assumption. However, we conjecture that this will not qualitatively change the results obtained here as long as marginal utility is diminishing i.e. the utility gained from the second object is smaller than that from the first. In this paper, we define *new bidders* to be those that have not bid in any auctions selling the same item before. *Seasoned bidders* are those for whom the current auction is their second auction.

It is easy to understand that a new bidder's expected utility comes from both auctions. The first component of a bidder's utility is the difference between her reservation value and price in the first auction; the second component is the corresponding difference in the second auction multiplied by the probability of losing in the first auction, because only if she loses in the first auction will she receive utility in the second auction. Since we use bidder's expected payoff as her utility function, we are assuming risk-neutrality here.

$$\text{Utility From First Auction} = \begin{cases} 0, \text{ if she loses;} \\ \text{reservation value - price, if she wins;} \end{cases}$$

[2] It is straightforward to generalize the model to the case where the bidder bids in a subsequent auction with some known probability, and to the case where there is more than one subsequent auction, and to allow for bidder valuations to be drawn from different distributions.

$$\text{Utility From Second Auction} = (\text{Probability of losing first auction}) \times \begin{cases} 0, \text{ if she loses;} \\ \text{reservation value - price, if she wins.} \end{cases}$$

In the rest part of this paper, we will assume independent, private valuation, as in the private-value model. We will also assume that within the same auction, there are two types of bidders – new bidders and seasoned bidders. We assume, mostly for ease of notation, that bidders of the same type adopt the same type of bidding function (strategy). From private-value model, we know that seasoned bidders would bid their reservation values. Hence, we use the following assumption.

Assumption: New bidders adopt the same type of bidding strategy $b(v)$, where v is a bidder's reservation value. Bidders' valuations are drawn independently from a distribution F(.).

3.1 Study of Bidding Strategy under Certainty

We start by assuming that bidders know the number of new and seasoned bidders that will bid in each auction. As in all private value models, the bidder is assumed to know the distribution of reservation values of other bidders but the actual value is private information to each bidder. A seasoned bidder *A* will bid his reservation value, since for him there are no future auctions. On the other hand, a new bidder, *B* has to consider how her first auction bid affects the likelihood of being able to participate in a second auction. Before formally deriving B's optimal strategy, a simple example provides some intuition for B's strategy.

Example 1:

B bids in two auctions. In either auction, B competes with another bidder whose bid is drawn from a uniform distribution U(0,6). B's reservation value is 4. We consider the following alternatives:

Alternative 1: B bids her reservation value in both auctions. Recall that since this is a second price auction, B wins at its opponents bid x when x <= 4.

$$E(u) = \int_0^4 (4-x).\frac{1}{6}dx + \frac{6-4}{6}\int_0^4 (4-x).\frac{1}{6}dx = 1.78$$

Alternative 2: B bids less than her reservation value in the first period and reservation value in the second period. If she bids 2 in the first auction, her expected utility is:

$$E(u) = \int_0^2 (4-x).\frac{1}{6}dx + \frac{6-2}{6}\int_0^4 (4-x).\frac{1}{6}dx = 1.89$$

Table 1 shows the expected utility of B for different bid amounts in the first auction. The increments in her bid will increase her expected utility initially. Once it reaches 2.67, her expected utility begins to drop. Simple calculations show that the optimal bid for bidder B in the first period is 2.67.

Table 1: Expected utility for a new bidder as a function of first period bids

Bidder B's bid in first period	Expected Utility
1	1.69
2	1.89
2.67 (Optimal Bid)	*1.93*
3	1.92
4(Reservation Value)	1.78

Optimal bidding strategy for B

We define B's expected utility if she bids *x* given her reservation value of *v* as

$$E(u(x;v)) = \int_0^x (v-z)d\Phi(z) + (1-\Phi(x))\int_0^v (v-z)d\tilde{\Phi}(z) \tag{1}$$

The first integral is the bidder's expected payoff in her first auction, where $\Phi(z) = \Pr(b(x^{(1)}) < z \,\&\&\, y^{(1)} < z)$, and where $b\ (x)$ is the other new bidders' bidding strategy, and $x^{(1)}$ is the largest reservation value among new bidders (excluding bidder B) in auction 1.

Hence the largest bid of new bidders is $b(x^{(1)})$. Similarly, $y^{(1)}$ denotes the largest reservation value, and hence, also the highest bid among seasoned bidders in auction 1. Similarly, we define $\tilde{x}^{(1)}, \tilde{y}^{(1)}$ for the second auction as the highest bids among the new bidders and seasoned bidders (excluding B). Therefore, $\Phi(x) = \Pr(b(x^{(1)}) < x \,\&\&\, y^{(1)} < x)$ is the probability of B winning her first auction. The second term in equation (1) is B's expected payoff in her second auction.

Solving the first order condition for an interior optimum with respect to *x*, we have the following optimal first period bid x*:

$$x^*(v) = v - \int_0^v \tilde{\Phi}(z)dz \tag{2}$$

where $\int_0^v \tilde{\Phi}(z)dz$ is bidder B's expected payoff if she bids in her second auction.

(Proofs and technical details of all the results are available in the Appendix.)

From the above equation, we may derive the following properties of bidder B's optimal strategy.

Property 1: x^* is B's dominant strategy in the sense that it is independent of the strategies of other bidders in the first auction.

$\Phi(z)$ includes all the information about the remaining bidders in the first auction. In B's optimal strategy, there is no $\Phi(z)$, meaning that her first period bid is independent of the strategies of all other bidders. The conclusion may seem surprising but it is easy to understand. If we let v_2

represent B's expected payoff from period 2 auction, B's "real" valuation of the first auction is $v - v_2$. Hence, according to the private-value model, B should bid her "real" valuation.

Property 2: $x^* < v$ (3)

This property explains why some experienced bidders would like to bid less than their reservation values in the first auction. Simply put, the option of being able to bid in a second auction is valuable. The smaller the value of this option, the greater is the probability of winning in the first auction. Thus the first period bid must trade-off the payoff from the first auction against the option value.

Property 3: x^* increases when N_2 or M_2 increases.

Here N_2 and M_2 denote the number of new bidders and seasoned bidders respectively. Intuitively, when the number of bidders increases, bidder B faces more competition in the second auction, which makes the second auction less valuable to her. Hence, B increases her bid in the first auction.

3.2 Bidding Strategy under Uncertainty

It is likely that B may only observe the distribution functions of the number of bidders. In this section, we will relax the certainty assumption made in Section 3.1 and check if the properties still hold.

Under uncertainty, we may define the expected utility function of B as the following:

$$E(u(x)) = \iint \int_0^x (v-z)\,d\Phi(z)\,d\xi(N_1)\,d\psi(M_1) +$$

$$(1 - \iint \Phi(x)\,d\xi(N_1)\,d\psi(M_1)) \iint \int_0^v (v-z)\,d\,\tilde{\Phi}(z)\,d\tilde{\xi}(N_2)\,d\tilde{\psi}(M_2) \qquad (4)$$

where N_1 and M_1 are the number of new bidders and seasoned bidders in the first auction, respectively. $\xi(N_1)$ and $\psi(M_1)$ are the corresponding distribution functions, respectively. We similarly define these parameters and distribution functions for the second auction, with the tilda characterizing second auction variables.

As before, we differentiate with respect to x and set equal to zero to derive B's optimal bidding strategy:

$$x^* = v - \iiint_0^v \tilde{\Phi}(z)dzd\tilde{\xi}(N_2)d\tilde{\psi}(M_2) \tag{5}$$

B's bidding strategy has the following properties:

Property 1: x^* is B's dominant strategy in the sense that it is independent of N_1, M_1,and $b_1(x)$.

Property 2: $x^* < v$ (6)

Both properties are similar to the case under certainty and have the same intuition.

Property 3: If $N_2 \succ_{F.S.D} \tilde{N}_2$, then $x^*(N_2) > x^*(\tilde{N}_2)$ (7)

$N_2 \succ_{F.S.D} \tilde{N}_2$ denotes that N_2 first order stochastic dominates $\tilde{N}_2$. $N_2 \succ_{F.S.D} \tilde{N}_2$ implies that $E(N_2) > E(\tilde{N}_2)$.

Property 3 implies that rational bidders would bid higher if they perceive larger expected number of bidders in auction 2. This is similar to property 3 in section 3.1.

The next property does not have a counterpart under the certainty case and addresses uncertainty about the number of bidders in auction 2. Suppose B has two alternatives for her second auction.

The mean numbers of bidders are the same, but they have different variances. Which alternative is more valuable to B?

Property 4: If $N_2 \succ_{S.S.D} \tilde{N}_2$, then $x^*(N_2) < x^*(\tilde{N}_2)$ (8)

where $N_2 \succ_{S.S.D} \tilde{N}_2$ implies that $\tilde{N}_2$ is more "uncertain" than N_2, or more formally, that the distribution of N_2 *second order stochastically dominates* the distribution of $\tilde{N}_2$.[3]

Property 4 implies that a risky or uncertain situation, at least as far as the number of bidders is concerned, is more valuable to B, hence B would bid less in the first auction if the second auction is more risky. This may contradict intuition. Why would a bidder prefer a risky situation even if she is assumed to be risk-neutral? Let us look at an example first.

Example 2:

B's reservation value is 4. For simplicity, we assume that her competitors would bid either 2 or 6 with 50% probability.

Alternative 1: She will compete with 2 bidders. In this case, the variance of the number of bidders is 0.

Alternative 2: The number of her competitors is either 1 or 3 with 50% probability for each. The mean number of bidders is also 2, but the variance of the number of bidders is 1. So this alternative features a more "risky" situation.

If she picks alternative 1, B wins only if both of her competitors bid 2. Hence, her expected payoff in the second period is: 0.5*0.5*(4-2)=0.5. If she picks alternative 2, her expected payoff in the second period is as follows: If there is 1 competitor, her expected payoff is: 0.5*0.5*(4-

[3] For a formal definition of first and second order stochastic dominance see Rothschild and Stiglitz, 1970.

2)=0.5. If there are 3 competitors, her expected payoff is: 0.5*0.5*0.5.0.5*(4-2)=0.125. So the total expected payoff for bidder B is 0.625. Obviously, alternative 2 is more valuable to B.

Why is a risk-neutral bidder better off in a more uncertain situation? The key insight is that she has the option to bid in the second auction. It is a standard result in option theory that the greater the uncertainty, the greater the value of the option (Dixit and Pindyk, 1994). This implies a lower first period bid, since the difference between the reservation value, v, and the bid, x, is the value of the option. [4]

4.0 Data Analysis

In this section, we present our approach for collecting eBay data and summarize key characteristics of the data sets. Then we examine the testable implications drawn from the theory.

3.1 Data Collection

Online auction firms, like eBay, provide rich information about the bidding history of completed auctions. However, there is no well-organized database for public use. This information is generally retrieved from web pages. We noticed that some researchers collect data from eBay website manually, but this approach is infeasible for collecting large data sets. In this study, we developed a software agent to collect data from eBay website on all the auctions of digital cameras during January and February of 2001. Since eBay posts all bidding information at the end of the auction, our software agent downloaded, parsed, and created data sets for each auction

[4] Note that property 3 and property 4 apply to parameter M_2, too. Thus, if the ratio of the new bidders to seasoned bidders is fixed and the only uncertainty is about the number and variance of the total number of bidders, these properties also apply to the total number of bidders.

monitored during the data collection period. To the best of our knowledge, this is the earliest software agent developed for large scale data collection from eBay.

For each auction, two files were retrieved: the first is a file of descriptive information about this auction, including what features the item has, when the auction begins and ends, who is the auctioneer, how much is the shipping and handling, and so on; the second is a file of the history bids of this auction: including who bid, when, and what amount.

Considering the requirements of the theory, we picked digital camera as the study object for the following reasons: First, we believe bidders buy these cameras for their own use (in our collected data, we did not see any example that a prior winner sells his or her purchase on eBay). In fact, most auctioneers of digital cameras sell many items at eBay and eBay actually offers a storefront for these merchants. The relationship between sellers and buyers in these auctions can be categorized as merchant to consumer. Therefore, these auctions may be treated as private-value auctions. Second, a digital camera is an expensive item on eBay. It is reasonable to assume that bidders make serious decisions. Furthermore, digital cameras can be defined by its make and model precisely. In other words, two digital cameras with the same make and model can be considered identical in quality.

The software agent for collecting data from eBay includes three components: Index Parser, Data Fetcher, and Raw Data Parser. The following is a demonstration of how the three components work together. The first step of this program is to search all the index pages about digital cameras. eBay maintains index pages listing all the currently completed auctions. These index pages are html files with links pointing to the detailed descriptions and historical bids of each auction. The index parser is designed to download these index pages and extract two kinds of links: description and historical bids. The data fetcher is used to download all the raw data files from the extracted links. The local parser analyzes the two pages for each auction and extracts useful data items, and then writes data in local files.

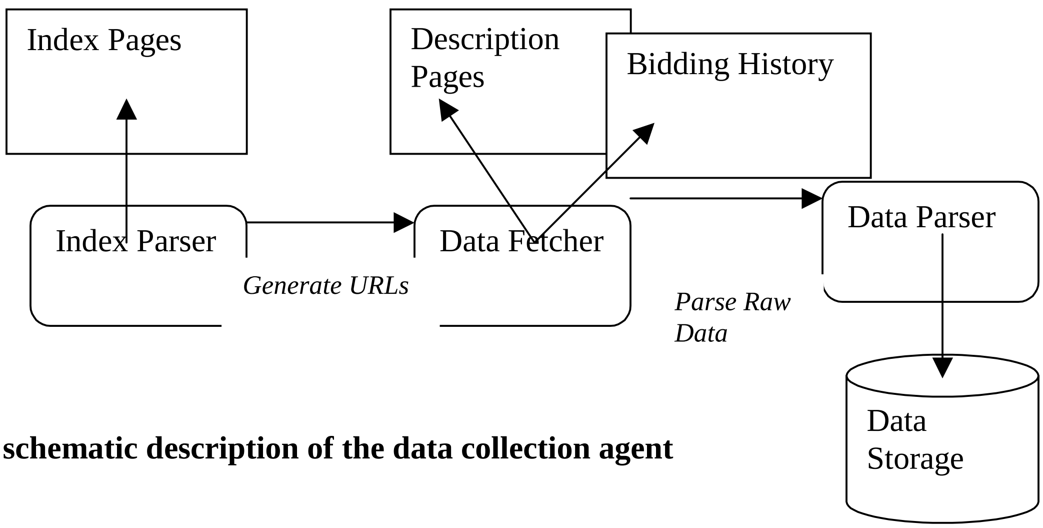

Figure 4: A schematic description of the data collection agent

For each model of digital cameras, we have two corresponding files. The first is a file with the descriptions about each auction. The second is a file with the bids in each auction. The description file includes the following variables:

EBayAuctionID: this is a 10-digit unique number used by eBay to identify each auction;

Evaluation: eBay maintains the credit rating for each auctioneer. Each eBay buyer is allowed to comment on the seller after the fulfillment of the purchase. eBay judges if these comments are positive, neutral, or negative. The net number of positive comments is called evaluation point. These files also record the beginning date, time and ending date, time, and the duration of the auction. The second file documents the historical bids in each auction. Each bidding record tells who bid, when, in what auction, and at what amount.

3.2 Summary of Data

In the following tests, we used a data set of all the eBay auctions selling Canon S10 cameras during the first two months of 2001. The following are some simple stats about this data set.

Auctions:

There were 420 such auctions conducted at eBay. For some of the tests, it is important to identify if a bidder was a new bidder or not and if the current auction was the last auction of a particular bidder. Therefore, in these tests, we omitted the auctions conducted during the first and last weeks, leaving 327 auctions in total. The average number of bidders in each auction, excluding auctions that generated no bids and auctions conducted in the first and last weeks was 10.51 and the average number of new bidders per auction, was 4.73. The average number of bids was 16.9 and the average price was $414.7.

Bidders:

Among the 327 auctions, there were 1599 unique bidders. Of these, 1547 did not bid in previous auctions of Canon S10 cameras conducted during the first week of 2001, which means that 52 bidders were seasoned bidders from the previous period. We note that this does not account for bidders who may have changed their user ID during the data collection period. There are 593 bidders who bid more than one auction during the sample period (Bidders who bid in another auction before January 1, 2001 or after February 28, 2001 are not considered.) Of these, 323 bidders bid in more than two auctions. The most aggressive bidders bid in 44 auctions. The average number of auctions that each bidder attended was 2.2. Table 2 shows the probability that a bidder bids in a second auction after the current auction. It shows that about 40% of the bidders who bid the first auction also participated in a second auction and that a little over half of those bidding in a second auction bid in a third auction.

4.3 Testable Implications from Theory

4.3.1 Test 1: Ascending bids

The first test of our theory is if bidders bid in an increasing order from earlier auctions to later auctions. Our theory is based on a two-auction model. However, in real-world auctions, bidders may bid in as many auctions as they would like to. The logic of the two auction model should apply as long as the number of auctions is finite, and suggests that a given bidders bid in the k+1th auction should be higher than in the kth auction.

Since a bidder is allowed to make more than one bid in an eBay auction, we picked the last bid of each bidder. As mentioned earlier, we used 327 auctions in the data set, cutting off the first week and last weeks in the time frame. The cutoff auctions are used as comparison groups.

There are two ways to test the theory. The first is to look at new bidders, using data from the 327 auctions. We call a bidder a new bidder only if she did not bid in previous auctions for the particular digital camera (including the first week of auctions not included in the analysis). For these individuals, we test whether their bid in their second auction is greater than their bid in the first auction. The problem with this approach is that since we may incorrectly classify as a new bidder someone who has in fact bid for the same object in an earlier (but unobserved by us) auction. However, the time interval between two auctions that a bidder participates is usually short, and therefore we do not expect the measurement errors to be large.

A somewhat different way to test the theory is to start at the end and ask if the bid in the bidders last auction is higher than her bid in the auction before. We used the same approach as before to identify if a bidder is a seasoned bidder. Hence, for each bidder, we compute the bid differences between her second and first auction, last and second-last auctions, and second-last and third-last auctions.

Table 3 reports the results. It shows that every sub-test shows significantly positive difference as the theory predicts. On average, bidders bid in an increasing order. However, some bidders may bid equal amounts or even in a decreasing order. Thus not all bidders act like a rational risk-neutral bidder.

4.3.2 Test 2: Bidding under uncertainty

In this paper, we assumed bidders only perceive the distribution functions of the number of bidders. These are key variables that condition the bid. In this test, we consider the impact of the mean number of bidders and its variability on a new bidder's strategy. Our theory implies the following points:

1) According to property 1, new bidder's bid is independent of the mean number of bidders and its variability in her first auction;

2) According to properties 3 and 4, both the mean number of bidders and its variability in the subsequent second auction affect a new bidder's decision in the first auction. Larger mean number of bidders in the second auction means more competition, therefore less value to new bidders, hence new bidders bid higher in the first auction. Similarly, property 4 implies that smaller variability also leads to a higher first auction bid.

To test these predictions, we estimate a regression equation having bid amount as the left-hand side variable with expected number of bidders and the variance in the number of bidders in the current and next auctions on the right-hand-side. However, we are not able to observe the market participants' perceived expected number of bidders (on an auction-by-auction basis) nor are we able to observe the perceived variance in the number of bidders. We are able to observe realizations of the number of bidders. The realization of the number of bidders in an auction is (by definition of expectation) the expected number of bidders plus a zero-mean error. We enter the actual number of bidders in the regression in place of its expectation. Of course, that means that we have a measurement error problem. However, this problem may be corrected by the use of two-stage least squares.

Similarly, we are unable to observe the variance of the number of bidders. Consider equation (3-1) which we assume predicts the number of bidders in an auction. For each auction, the variance in the number of bidders is just the variance of the error term. The residual from this regression squared (scaled by N-1/N to eliminate sampling bias) has expectation equal to the variance of the error term. So, just as above, using this squared residual in the equation leaves us with an equation properly specified, except for measurement error, which, again we can correct via two stage least squares.

$$NoOfBidder = \alpha + X^T \beta + \varepsilon \tag{9}$$

X^T, which provides the instrument vector for both the number of bidders and its variance, is a vector that includes the following:

Time Variables: T1_8, T8_12, T12_16 and T16_20 are dummy variables showing the duration within which the auction is completed. The auction is completed between 1am-8am if T1_8 is 1;

Saturday or Sunday: The auction is finished on Saturday, if SATURDAY is 1. We use this variable under the assumption that each auction may attract more bidders during weekends;

Length of the Auction: Days1_2, days3_4, days5_6 and days7_8 are dummy variables referring to the length of the auction. If the length of the auction ranges from 1 to 2 days, DAYS1_2=1. Intuitively, auctions that last longer may have more bidders;

Evaluation points: EV_10 is 1 if the evaluation points of this auctioneer are below 10, which means this auctioneer is inexperienced; EV_1000 is 1 if the evaluation points of the auctioneer are above 1000, indicating that the auctioneer is an experienced one with good reputation.

Tables 4a and 4b show the estimates of equation (9). The results are mostly in line with intuition. Auctions with high evaluation points (larger than 1000) tend to get 4.1 more bidders for each auction. Auctions with evaluation points less than 10 tend to get 1.75 fewer bidders. Auctions that last only one or two days get 4.49 fewer bidders on average. Auctions completed on Saturday get 1.75 more bidders. Intuitively, auctions that completed late at night or early in the morning should have fewer bidders. However the estimate does not seem to support it. Online auction business operates around the clock, conducting business with nation-wide and some international customers. Therefore it is reasonable to conclude that the time effect is weakened in online auctions. This argument may also explain why the number of bidders seems not to increase on Sunday.

Since some of the bids were ridiculously low, we also conducted regression using a partial data set.[5] We deleted 92 bids that were below \$200. Table 5 shows the results, where Bn_1st and Bn_2nd refer to the number of bidders in the first and second period, respectively. Var_1st and Var_2nd refer to the squared residual of the numbers in the first and second auction, respectively. We use two additional control variables. The first, *days* is the number of days between the ending date of the auction and Jan. 1st, 2001. Since the value of digital cameras drops over time, we include this variable to allow its impact on bidders' decision. To control the effect of credit rating system on bidders' valuation, we also use EV_10 and EV_1000 as regressors. EV_10 is 1 if the evaluation points of the first auction are less than 10. EV_1000 is 1 if the evaluation points of the first auction are more than 1000. Since the key right hand side variables are measured with error, we use two stage least squares estimates.

According to our theory, we anticipate that the coefficients of Bn_1st and Var_1st should be zero and that of Bn_2nd positive and Var_2nd negative. A larger predicted number of bidders in second auction causes new bidders to raise their bids in the first auction. Recall that the intuition is that a larger number of bidders implies more competition in the second auction, lowering the option value of the second auction. On the contrary, a larger variance of the number increases the option value, causing new bidders to decrease their bids in the first auction.

Table 5 shows that the coefficient of Bn_2nd = 4.88 and that of Var_2nd = -5.20, and both are statistically significant. To get some sense of the quantitative impact, these estimates imply that if the expected number of bidders in the second auction increases by 1, a new bidder's bid in the first auction increases by \$4.88. If variance increases by 1, the bid decreases by \$5.20. The

[5]We also conducted regression on complete data set, which shows similar result

estimates of Bn_1^{st} and Var_1^{st} are not significantly different from 0, which is as we expected. However, note that the estimated coefficient of Bn_1^{st} is fairly large. If the expected number of bidders in the first auction increases by 1, the bid in the first auction decreases by $2.95. One explanation is the weakness of the data. In the test, we assume that bidders' last bid show the maximum bid that they wanted to bid truthfully. However, some bidders may submit a very low bid, well below the maximum she wants to bid in the first period, and then leave the auction for some time. When she returns to the auction, the ongoing bid may already have surpassed not only her recorded bid but even her reservation value. In this case, our data will only show her attempted bid, not the actual desired first auction bid. Moreover, this outcome is naturally more likely the greater the number of bidders in the auction.

Since bidders' experiences may affect bidders' bidding behavior, we also conducted test on a partial data set, which includes only experienced bidders' bids. eBay maintains a rating for each bidder, just as for auctioneers. Bidders are awarded star rating if they achieve more than 10 net points. Table 6 shows regression of star bidders. During our data collection period, we only captured a fraction of bidders' ratings, hence this test only represents results on the data we have. Table 6 shows that experienced bidders do not act very differently from average bidders, indicating that our test results are robust.

5.0 Conclusions

In this paper, we study optimal bidding strategy for a rational, risk-neutral bidder in sequential online auctions. Our theoretical approach is premised on two key differences between online and non-online sequential auctions. Specifically, the number of bidders in online auctions is likely to vary stochastically, and any given auction is likely to have bidders who have bid in earlier auctions and others who have not. We derive the optimal strategy and show that this is consistent with an appropriately modified version of the standard private value second price auction model.

The major contribution of this paper is the study of how the uncertainty may affect bidders' bidding strategy. Volatility in the number of bidders in the second period lowers the first period bid. To test the implications of the theory, we collected data from eBay, using a software agent. The results support the implications of the theory. However, bidders seem to act rationally only on average. Some bidders, as we noticed, adopt different strategies, which contradict our theory. This is worth further study. Further development of the model should allow bidders to be risk-averse, or risk loving. Extending this model to multiple sequential auctions is another interesting research question, as is allowing more general utility functions where the utility of the second object is not set to zero.

Table 2: Probability of bidding in subsequent auctions

1st→2nd Auction	2nd →3rd Auction	3rd→More than 4 auctions
38.33%	54.46%	52.25%

Table 3: Difference between bid amounts in successive auctions.

Test	Bid Difference	Standard Error	Number of individuals
Second-First	$10.54	3.04	593
Last-Second Last	$10.89	3.45	541
Second Last-Third Last	$6.19	5.29	311

Table 4a: Variables used in regression analysis

Variable	Description
T1_8	= 1 if auction completed between 1 AM and 8 AM, = 0 otherwise
T_8_12	= 1 if auction completed between 8AM and 12 PM, = 0 otherwise
T12_16	= 1 if auction completed between 12 PM and 4 PM, = 0 otherwise
T16_20	= 1 if auction completed between 4 PM and 10 PM, = 0 otherwise
SATURDAY	= 1 if auction completed on a Saturday, = 0 otherwise
SUNDAY	= 1 if auction completed on a Sunday, = 0 otherwise
DAYS1_2	= 1 if auction lasts between 1 and 2 days, = 0 otherwise
DAYS3_4	= 1 if auction lasts between 3 and 4 days, = 0 otherwise
DAYS5_6	= 1 if auction lasts between 5 and 6 days, = 0 otherwise
DAYS7_8	= 1 if auction lasts between 7 and 8 days, = 0 otherwise
EV_10	=1 if seller has fewer than 10 evaluation points, =0 otherwise
EV_1000	=1 if seller has more than 1000 evaluation points, =0 otherwise
Bn_1st	= Predicted number of sellers in first auction
Var_1st	= Predicted variance in number of sellers in first auction
Bn_2nd	= Predicted number of sellers in second auction
Var_2nd	= Predicted variance in number of sellers in second auction

Table 4b: Determinants of the number of bidders in an auction: OLS estimates
Dependent variable: Number of bidders

Variable	Estimate	Std. error	*T statistic*
Constant	7.31	1.34	5.46
T1_8	-0.95	0.95	-0.99
T8_12	0.24	1.16	0.21
T12_16	0.00	0.82	0.00
T16_20	0.14	0.83	0.17
Saturday	1.75	1.25	1.40
Sunday	0.04	0.86	0.04

Days1_2	-4.49	1.44	-3.11
Days3_4	1.57	1.27	0.98
Days5_6	1.49	1.40	1.07
Days7_8	1.13	1.29	0.99
Ev_10	-1.75	0.71	-2.46
Ev_1000	4.10	0.81	5.06

Number of observations: 420

Table 5: First auction bid amounts of new bidders - Two Stage Least Squares estimates. Dependent variable: Final bid amount of bidder *i* in first auction, bids greater than $200 only.

Model	Coefficients	Std. Error	T-Statistic
Constant	318.76	19.63	16.24
Days	0.08	1.33	0.06
Bn_1st	-2.95	1.89	-1.56
Var_1st	1.32	2.20	0.60
Bn_2nd	4.88	0.93	5.25
Var_2nd	-5.20	2.03	-2.56
Ev_10	-7.18	8.62	-0.83
Ev_1000	31.05	12.86	2.42

Number of observations: 501

Table 6: First auction bid amounts of new bidders, star bidders only - Two Stage Least Squares estimates.
Dependent variable: Final bid amount of bidder *i* in first auction

Model	Coefficients	Std. Error	T-Statistic
Constant	316.25	18.98	16.66
Days	0.08	1.58	0.14
Bn_1st	-2.82	2.20	-1.28
Var_1st	1.42	2.61	0.54
Bn_2nd	5.27	1.08	4.87
Var_2nd	-5.88	2.19	-2.68
Ev_10	-12.52	10.19	-1.23
Ev_1000	33.87	15.15	2.24

Number of observations: 117

References

[1] Arora, A., Cooper, G., Krishnan, R. and Padman R., 1999, “ IBIZA: E-market Infrastructure for Custom-built Information Products", *Information System Frontiers*, forthcoming.

[2] Ashenfelter, O., 1989, “How Auctions Work for Wine and Art,” *Journal of Economic Persepectives 3*, 23-36.

[3] Ashenfelter, O. D. Genesove, 1992, “Testing for Price Anomalies in Real-Estate Auctions,” *American Economic Review 82*, 501-505.

[4] Bajari, P. and A. Hortacsu, 2000, “Winner’s Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions,” *Stanford University,* Working Paper.

[5] Bapna, R., Goes, P., and Gupta, 2000a, “On-Line Auctions: Insights and Analysis,” *Communications of the ACM.*

[6] Bapna, R., Goes, P., and Gupta, 2000b, “A Theoretical and Empirical Investigation of Multi-item On-line Auctions,” *Information Technology and Mangement 1,* 1-23.

[7] Beam, C., and A. Segev, 1998, “Auctions on the Internet: A Field Study”, *Haas School of Business, University of California Berkeley,* Working Paper.

[8] Black, J. and de Meza, D., 1992, “Systematic Price Differences Between Successive Auctions Are No Anomaly,” *Journal of Economics and Management Strategy* 1, 607-628.

[9] Branco, F, 1997, “Sequential Auctions with Synergies: An Example,” *Economics Letters 54,* 159-163.

[10] Cassady, R. 1967, “Auctions and Auctioneering”, *University of California Press,* Berkeley, CA.

[11] de Vries, S. and Vohra, R.V., 2002, “Combinatorial Auctions: A Survey”, *Working Paper*, Northwestern University.

[11] Dixit, A. K., and R.S. Pindyk, 1994, “Investment Under Uncertainty,” *Princeton University Press,* Princeton, New Jersey.

[12] Easley, R. and R. Tenorio. 1999, “Bidding Strategies in Internet Yankee Auctions,” *Notre Dame University, Working Paper.*

[13] Gandal, N. 1997, “Sequential Auctions of Interdependent Objects: Israeli Cable Television Licenses,” *Journal of Industrial Economics 45(3),* 227-244.

[14] Herschlag, M. and R. Zwick, 2000, “Internet Auctions: A Popular and Professional Literature Review.” *Quarterly Journal of Electronic Commerce.* 1(2), 161-186.

[15] Jones, C., F.M. Menezes and F. Vella, 1998, “Auction Price Anomalies Evidence from Wool Auctions in Australia, “Australian National University.

[16] Kaiser, L. and M. Kaiser. 1999, “The Official eBay Guide, “ *Firside,* New York.

[17] Krishna, V. and R. W. Rosenthal, 1996, “Simultaneous Auctions with Synergies”, *Games and Economic Behavior*, 17, 1-31.

[18] Klein,W. and R.O’Keefe, 1999, “The Impact of the Web on Auctions: Some Empirical Evidence and Theoretical Considerations,” *International Journal of Electronic Commerce.* 3(3), 7-20.

[19] Lucking-Reiley, D. 2000, “Auctions on the Internet: What is Being Auctioned, and How?” *Journal of Industrial Economics*, 48(3), 227-252.

[20] Lucking-Reiley,D., Bryan, D., Prasad, N. and D. Reeves, 1999, “Pennies from eBay: the Determinants of Price in Online Auctions”, *Vanderbilt University*. Working Paper.

[21] McAfee R. P. and McMillan J., 1987, “Auctions and Bidding,” *Journal of Economic Literature.* 25, 699-738.

[22] Milgrom P. 1989, “Auctions and Bidding: A Primer.” *Journal of Economic Perspectives,* 3, 58-73.

[23] Milgrom P. and Weber, R., 1982, “A Theory of Auctions and Competitive Bidding.” *Econometrica.* 50(5).

[24] Steven A. Matthews, 1995, “A Technical Primer on Auction Theory I: Independent Private Values: *Teaching Notes, Northwestern University.*

[25] Menezes, F. M. and P.K. Monteiro, 1999, “Synergies and Price trends in sequential Auctions”, W*orking paper,* Australian National University and EPGE/FGV.

[26] Pinker, E. J., Seidmann, A., and Vakrat, Y., 2001, “The Design of Online Auctions: Business Issues and Current Research”, W*orking paper*, [27 The University of Rochester.

[27] Sandholm, T., Suri, S., Gilpin, A., and Levine, D, 2002. “Winner Determination in Combinatorial Auction Generalizations”, *First International Joint Conference on Autonomous Agents and Multiagent Systems (AAMAS), Bologna, Italy.*

[28] Seidmann, A., Pinker, E., and Vakrat, Y., 2001, “The Optimal Design of Multi-Period Online Auctions,” *East Coast Round Table in Operations Research: Operations Research Models in e-commerce*, University of Rochester

[29] Stark, R. and M. Rothkopf, 1979, “Competitive Bidding: A Comprehensive Bibliography.” *Operations Research*, 27, 364-390.

[30] J.T. Sinclair, 1999, “eBay, The Smart Way”, published book.

[31] Thiel, S. and G. Petry, 1995, "Bidding Behavior in Second-Price Auctions: Rare Stamp Sales: 1923-1937," *Applied Economics 27(1), 11-16*

[32] Vickrey, W., 1961, "Counterspeculation, auctions, and competitive sealed tenders". Journal of Finance, 16, 8-37.

[33] Wilson, R. 1992, "Strategic Analysis of Auctions", *Handbook of Game Theory, Volume1*.

Appendix : Proofs for Section 3

1. Proofs for Section 3.1

Assuming constant parameters, bidder B's optimal bidding strategy is:

$$x^* = v - \int_0^v \tilde{\Phi}(z)dz$$

Proof:

$$E(u(x)) = \int_0^x (v-z)d\Phi(z) + (1-\Phi(x))\int_0^v (v-z)d\tilde{\Phi}(z)$$

Here $\Phi(z) = \Pr(b(x^{(1)}) < z \;\&\&\; y^{(1)} < z)$

$x^{(1)}$ denotes the new bidders with largest reservation value in auction 1, excluding bidder B; $y^{(1)}$ denotes the seasoned bidders with largest reservation value in auction 1. Therefore, $\Phi(z)$ is the winning probability of bidder B in auction 1. Similarly, we define $\tilde{\Phi}(z)$ as the winning probability of bidder B in auction 2, given that she loses the first auction.

To solve bidder B's optimal strategy, we consider the first order condition:

$$\frac{\partial E(u(x))}{\partial x} = (v-x)\Phi'(x) - \Phi'(x)\int_0^v (v-z)d\tilde{\Phi}(z)$$

Therefore, $x^* = v - \int_0^v (v-z)d\tilde{\Phi}(z)$

Using integration by parts, we have the simplified form:

$$x^* = v - \int_0^v \tilde{\Phi}(z)dz$$

Property 3: x^* increases when N_2 or M_2 increases.

Proof:

$\tilde{\Phi}(x) \equiv \tilde{F}_w(x)\tilde{F}_z(x)$

$\tilde{F}_w(x)$ denotes the probability that bidder B beats all the new bidders; $\tilde{F}_z(x)$ denotes the probability that bidder B beats all the seasoned bidders.

$\tilde{F}_w(x) = F^{N_2}(b_2^{-1}(x))$

$\tilde{F}_z(x) = G^{M_2}(x)$

Here $b_2(x)$ is new bidder's bidding function in auction 2. N_2 and M_2 are the number of new and seasoned bidders in auction 2, respectively. G(x) denotes the distribution function of seasoned bidder's reservation value. (The detailed derivation of $\tilde{F}_w(x)$ and $\tilde{F}_z(x)$ is skipped here for simplicity of reading.) Therefore, $\tilde{\Phi}(x) = F^{N_2}(b_2^{-1}(x))G^{M_2}(x)$. Since $0 < F\ (b_2^{-1}(x)) < 1$ and 0<G(x)<1, $\tilde{\Phi}(z)$, as a function of N_2 and M_2, is monotone decreasing. Hence x^* increases when N_2 or M_2 increases.

2 Proofs for Section 3.2

Under uncertainty about N and M, bidder B's optimal bidding strategy is:

$$x^* = v - \iint\int_0^v \tilde{\Phi}(z)dzd\tilde{\xi}(N_2)d\tilde{\psi}(M_2)$$

Proof:

$$E(u(x)) = \iint\int_0^x (v-z)d\Phi(z)d\xi(N_1)d\psi(M_1) +$$

$$(1 - \iint \Phi(x)d\xi(N_1)d\psi(M_1))\iint\int_0^v (v-z)d\ \tilde{\Phi}(z)d\tilde{\xi}(N_2)d\tilde{\psi}(M_2)$$

As before, we solve bidder B's optimal bidding strategy:

$$\frac{\partial E(u(x))}{\partial x} =$$

$$\iint (v-x)\Phi'(x)d\xi(N_1)d\psi(M_1) - \iint \Phi'(x)d\xi(N_1)d\psi(M_1) *$$
$$\iiint_0^v (v-z)d\tilde{\Phi}(z)d\tilde{\xi}(N_2)d\tilde{\psi}(M_2)$$

Let $\frac{\partial E(u(x^*))}{\partial x^*} = 0$

$$x^* = v - \iiint_0^v \tilde{\Phi}(z)dzd\tilde{\xi}(N_2)d\tilde{\psi}(M_2)$$

Property 3: If $N_2 \succ_{F.S.D} \tilde{N}_2$, then $x^*(N_2) > x^*(\tilde{N}_2)$

$N_2 \succ_{F.S.D} \tilde{N}_2$ denotes that N_2 first order stochastic dominates $\tilde{N}_2$. $N_2 \succ_{F.S.D} \tilde{N}_2$ implies that $E(N_2) > E(\tilde{N}_2)$.

Proof:

Note that $x^* = v - \iiint_0^v \tilde{\Phi}(z)dzd\tilde{\xi}(N_2)d\tilde{\psi}(M_2)$. We define $H(N_2) = v - \int\int_0^v \tilde{\Phi}(z)dzd\tilde{\psi}(M_2)$. It is obvious that $x^* = E(H(N_2))$. As a function of N_2, $\tilde{\Phi}(z)$, is monotone decreasing.

Note that $H(N_2)$ is monotone increasing in N_2.

Given that $N_2 \succ_{F.S.D} \tilde{N}_2$, we will prove that the following is also true: $H(N_2) \succ_{F.S.D} H(\tilde{N}_2)$

Note that for any z, we have:

$$\Pr(H(N_2) > z) = P(N_2 > H^{-1}(z)) \geq P(\tilde{N}_2 > H^{-1}(z)) = \Pr(H(\tilde{N}_2) > z)$$

Thus, for any z, $\Pr(H(N_2) > z) \geq \Pr(H(\tilde{N}_2) > z)$ and so $H(N_2) \succ_{F.S.D} H(\tilde{N}_2)$

It follows that $x^*(N_2) > x^*(\tilde{N}_2)$.

Property 4: If $N_2 \succ_{S.S.D} \tilde{N}_2$, then $x^*(N_2) > x^*(\tilde{N}_2)$

$N_2 \succ_{S.S.D} \tilde{N}_2$ implies that auction 2 with parameter $\tilde{N}_2$ is more risky than auction 2 with parameter N_2.

Proof:

Note that $\tilde{\Phi}(x) = F^{N_2}(b_2^{-1}(x))G^{M_2}(x)$

$$\Rightarrow \frac{\partial \tilde{\Phi}(x)}{\partial N_2} = \tilde{\Phi}(x).LOG(F(b_2^{-1}(x))) \leq 0$$

(Note that F(.) is a probability distribution function, hence $0 \leq F(b_2^{-1}(x)) \leq 1$.)

$$\Rightarrow \frac{\partial^2 \tilde{\Phi}(x)}{\partial N_2^{\,2}} = \tilde{\Phi}(x).LOG^2(F(b_2^{-1}(x))) > 0$$

We define the following function: $K(N_2) = \int\int\int_0^v \tilde{\Phi}(z) dz d\tilde{\xi}(N_2) d\tilde{\psi}(M_2)$. Since $\tilde{\Phi}(x)$ is convex in N_2, it follows from the properties of second order stochastic dominance that $K(N_2)$ will increase as the distribution of N_2 becomes riskier (while keeping the mean the same). Therefore, $x^* = v - K(N_2)$ will decrease. Thus, the bidder bids less under risky situations. Hence, $x^*(N_2) > x^*(\tilde{N}_2)$.