Loopholes

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February 26, 2008

Abstract

We use a dynamic game with two players (a legislator who is to write the Bill of Rights, and a judge who is to interpret it 200 years later) to formalize the argument of those American founding fathers who opposed the inclusion of the Bill of Rights into the American Constitution. Under some parameter range, there is a unique equilibrium where the legislator, who is not sure whether or not there are still rights that he is unaware of, optimally chooses not to write the Bill of Rights. That is, he optimally chooses not to enumerate even those rights that he is aware of. The reason is that, in equilibrium, how the judge treats those rights not in the Bill depends on how elaborated the Bill is. The more elaborated the Bill is, the less likely that the judge would protect those un-listed rights. We also show that, even if the legislator adds the sentence "any other rights not listed in this Bill are equally sacred and the government should not infringe them either" to the Bill, the equilibrium will stay the same. We then discuss the relations between this analysis and the incomplete contracts literature.

Keywords: loopholes, awareness of unawareness, incomplete contracts

1 Introduction

This paper revisits an old debate among Americas founding fathers, namely whether or not the Bill of Rights should be included in the Constitution. Some founding fathers, e.g. James Iredell, subsequent Supreme Court Justice, strongly opposed to the inclusion. Iredell told his fellow constitution ratifiers in North Carolina that it would be "not only useless, but *dangerous*, to enumerate a number of rights which are not intended to be given up; because it would be implying, in the strongest manner, that every right not included in the exception might be imparied by the government without usurpation." The goal of this paper is to examine the logic behind Iredell's argument, and explore its relations with incomplete contracts.

As is now well known, Iredell's argument did not prevail, and the Bill of Rights was eventually included in the American Constitution as a series of amendments. One may argue that Iredell's argument did not prevail because it contained a serious logical hole: if Iredell was worried that any omitted rights would be made more vulnerable by a detailed but inevitably incomplete Bill of Rights, then a better way to address his concern would be to write explicitly in the Bill that any omitted rights should be deemed as equally sacred as those rights in the Bill. Why would Iredell's recommended action (i.e., not to write the Bill at all) ever be optimal?

Understanding Iredell's logic can teach us something that goes beyond this isolated historic event. In particular, Iredell's argument resembled many modern-day arguments why it is sometimes optimal to write incomplete contracts. For example, merger agreements usually contain a material-adverse-change (MAC) clause that allows either party in a merger to opt out before completing the deal. The language of the clause is typically vague, leaving it to the courts to decide what it means by a "material" adverse change (that damages one party's business enough to justify the other party's pulling out). Why is it not a good idea to make the MAC clause less vague? *The Economist* magazine explains: "If a clause is too specific, factors that are not cited explicitly may be assumed by the courts to be excluded."[1] Note the resemblance between this argument and that of Iredell.

According to an urban legend among economists, once upon a time there was a very intelligent economist called Sanford G., who signed a contract with a subcontractor to build a new house. Being a non-trusting home owner, Sanford wrote an extremely detailed contract, painstakingly enumerating many requirements for his new house. When the house was finished, he found the house to be defective—perhaps the roof was leaking—something somehow slipped his mind when he wrote his otherwise very detailed contract. He refused to pay the subcontractor because of this defection, but lost the case in court. The judge explained that a leaky roof would typically be considered as unacceptable. However, since Sanford's contract was so detailed and yet did not require a good roof, the judge reasoned, he

[1] *The Economist*, December 8th, 2001, p.58.

must have thought of the possibility of a leaky roof and decided that it was acceptable. Sanford's story is one of economists' favorite classroom stories to explain the merit of incomplete contracts.

In light of the apparent logical hole in Iredell's argument, we can similarily challenge these arguments for incomplete contracts. Would not companies do even better if they replace those vague MAC clauses with detailed lists of opt-out excuses, capped with an extra clause saying that any excuses not listed should be treated as equally valid as those listed? Would Sanford have saved himself from the unnecessary agony if he had added to his already very detailed contract an extra sentence saying that any requirements not listed should be treated as equally important as those listed? Why and why not?

This paper formally examine Iredell's argument by studying a dynamic game with two players: a legislator who is to write the Bill of Rights, and a judge who is to interpret it 200 years later. The legislator is aware of some rights, which he can include into the Bill, but is also unaware of some rights, which he cannot. He is aware of his own unawareness, but is uncertain about the number of rights he is unaware of. We model such a mental state of the legislator using an object-based unawareness (OBU) structure recently axiomatized by Board and Chung (2007). An OBU structure has an advantage over first-generation unawareness models (e.g., Modica and Rustichini (1999), Li (2004), Halpern and Rego (2005), Li (2006), Heifetz, Meier, and Schipper (2006), Heifetz, Meier, and Schipper (2007a), and Heifetz, Meier, and Schipper (2007b)) in that it explicitly allows for agents who are aware of their own unawareness. It has an advantage over second-generation unawareness models (e.g., Halpern and Rego (2006)) in that it allows for agents who are uncertain about whether there exist things that they are unaware of without the undesirable implication that agents do not know what they are aware of.[2,3]

We show that, under certain parameter range, there is a unique equilibrium where the legislator optimally chooses not to write the Bill of Rights at all—that is, not even to enumerate those rights that he is aware of. The reason is that, in equilibrium, how the judge treats those rights not in the Bill depends on how elaborated the Bill is. The more elaborated the Bill is, the less likely that the judge would protect those un-listed rights.

More importantly, we also prove that, even if the legislator adds the sentence "any other rights not listed in this Bill are equally sacred and the government should not infringe them either" to the Bill, the equilibrium will stay the same (Theorem 1).

These two results, combined together, suggest that the logical hole in Iredell's argument was purely illusionary. Other founding fathers might disagree with Iredell on the values of certain parameters, but it would be wrong to dismiss his argument as illogical.

[2]Halpern and Rego (2006) also explicitly allows for agents who are aware of their own unawareness. However, in their model, whenever an agent is not sure whether there exist thigns that he is unaware of, he necessarily does not know what he is aware of, and hence violates certain introspection axioms.

[3]For other economic applications of OBU structures, see Board and Chung (2008).

Any one who tries to model and predicat a judge's behavior must make assumptions on the judge's preferences and constraints. Here, in our model, a crucial assumption is that judges are not constrained to strictly adhere to the legal interpretive doctrine of *expressio unius est exclusio alterius* (the expressio doctrine). This is probably the most controversial assumption in this paper, and hence we shall devote a whole section (Section 2) to defend it. Indeed, we would even take the stand that debunking the myth of the expressio doctrine, and formally modelling how judges deviate from it, holds the key of understanding how the common law system works.

Translated into English, the expressio doctrine says "expression of the one is exclusion of the other." For example, if a law says "children below 16 are not allowed to drive", then we don't need to ask whether a 17-year-old girl is allowed to drive or not. She is. However, as we argue in Section 2, the expressio doctrine is merely a myth, and judges do not strictly adhere to it. Section 2 also informally suggests why and how judges deviate from it, which we subsequently incorporate into our formal model.

Roughly speaking, judges deviate from the expressio doctrine by first performing an "awareness check": instead of immediately jumping to the conclusion that a 17-year-old girl is allowed to drive, they first ask whether the legislators had likely thought of the case of a 17-year-old girl. If not, then the judges's ruling is no longer bound by the expressio doctrine. In Section 2, we use two famous Supreme Court cases to illustrate this systematic deviation from the expressio doctrine.

When judges decide that they are not bound by the expressio doctrine, they exercise their professional judgment to rule on the case. Exercising ones' professional judgment is costly, and judges are willing to do so only when the expected improvements in the quality of their rulings are big enough. Since expected improvements depend on whether the legislators have high or low awareness types, judges perform an "awareness check" before deciding whether to exercise their professional judgment. Understanding this behavior of the judges, when the legislators want to incentivize the judges to exercise their professional judgment, they write a more incomplete law and credibly signal to the judges that they have low awareness types. This is the gist of Iredell's argument.

It also explains why Iredell might not be able to address his concern by merely adding an other-rights-are-also-sacred clause to the Bill of Rights, or why companies may not gain extra flexibility by merely adding an other-opt-out-excuses-are-also-valid clause to a merger agreement, or why Sanford might not be able to better protect himself by merely adding an other-requirements-are-also-important clause to his already very detailed contract. The purpose of these clauses, if they are ever effective, is to affect the results of the judges' "awareness check," and increase their incentives to exercise their professional judgment. But these clauses are not credible signalling devices, and do not affect the results of "awareness check."

The rest of this section reviews the related literature. Section 2 defends our assumption

that judges do not strictly adhere to the expressio doctrine. Section 3 presents the model and our main result. Sections 4 discusses the relation with the literature of incomplete contracts.

1.1 Related Literature

Our paper is related to the legal literature on rules versus standards. In Ehrlich and Posner (1974), rules are understood as precise boundaries between good and bad behaviors, whereas standards correspond to noisy boundaries. The noisiness inherent in a standard is costly to risk-averse citizens, but the precise boundary described in a rule may also be a poorer approximation the society's ideal boundary. So a choice between rules versus standards depends on this tradeoff.

Kaplow's (1992) analysis of rules versus standards is closer to our paper. In Kaplow (1992), standards are similar to what we will call "barebone laws" in our model, whereas rules are more elaborated descriptions of good and bad behaviors. Writing rules incurs more ex ante legislative costs, but save on ex post litigation costs. Kaplow (1992) differs from our paper in that, once his legislators have incurred the effort costs and carefully thought through which behaviors are good and which are bad, they would have no reasons to refrain from enumerating them in the law. Hence Kaplow (1992) is not an adequate framework to study the question of, for example, whether the Bill of Rights should be included into the constitution.

2 The Myth of the Expressio Doctrine

As mentioned in the Introduction, a crucial assumption in our model is that judges are not constrained to strictly adhere to the legal interpretive doctrine of *expressio unius est exclusio alterius* (the expressio doctrine). Since this assumption can be controversial, we shall devote this whole section to defend it. In particular, we shall use two famous Supreme Court cases to demonstrate (*i*) not only that Justices have no problem deviating from the doctrine, moreover, (*ii*) they deviate from it in a systematic way. That systematic way is to perform an "awareness check" first, and to adhere to the doctrine only if it deems likely that the legislators have thought of cases similar to the one the Justices are considering. In Section 3, this idea of "awareness check" will be incorporated into our formal model.

2.1 Maryland v. Craig, 497 U.S. 836 (1990)

This is a case involving a prosecution for sexual abuse of a young child. When the case was tried in the trial court, the judge decided that that the child would be too frightened to testify in the presence of the presumed abuser, and hence allowed her to testify in a separate

room, with only the prosecutor and defense counsel present, while the defendant, the judge, and the jury watched over closed-circuit television. The defendent appealed to the Supreme Court, challenging the constitutionality of this procedure.

According to the Confrontation Clause of the Sixth Amendment of the American Constitution, "[i]n *all* criminal prosecutions the accused shall enjoy the right ... to be confronted with the witnesses against him" (emphasis added). The defendent claimed that the procedure used in the trial court violated this clause. The Supreme Court split 5 to 4 on this issue, showing that this was by no means a clear-cut case. It is illuminating to examine in detail where the disagreement laid.

The disagreement was not over what "confrontation" meant. Both sides agreed that it means "face-to-face," in particular not "watching from another room." There was also no disagreement on why there was such a provision in the Constitution in the first place. It was agreed that the major purpose of this provision was precisely to frighten the witnesses so as to deter them from lying.

More relevant to our claim that adherence to the expressio doctrine is merely a myth, there was also agreement that "all" did not literally mean "all"! The majority opinion summarized the core question of this case as follows: "[The question is] whether any exceptions exist to the irreducible literal meaning of the Clause ... [i]t is all but universally assumed that there are circumstances that excuse compliance with the right of confrontation." Since "all" did not literally mean "all," the Justices were now left to debate on whether a child-sexual-abuse case belonged to the set "all" or to its complement—and this *was* where the disagreement laid.

How did the Justices decide whether a child-sexual-abuse case belonged to the set "all" or to its complement? The answer is: by an "awareness check." For example, Justice Scalia, who wrote the minority opinion, emphasized the following observations when he defended his opinion later: "Sexual abuse existed [in 1791, the time of the Sixth Amemdment], as it does now; little children were more easily upset than adults, then and now; a means of placing the defendant out of sight of the witness existed then as now (a screen could easily have been erected that would enable the defendant to see the witness, but not the witness the defendant)." (Scalia (1998)) After highlighting these observations, Justice Scalia asserted that a child-sexual-abuse belonged to the set "all," and hence the procedure used in the trial court was unconstitutional. Why were these observations important? Although Justice Scalia did not explain, it is only natural to guess that he considered them important because they increased the likelihood that *the authors of the Sixth Amendment were aware of the cases of sexual abuse of young child*. The higher is that likelihood, the more plausible is the hypothesis that a child-sexual-abuse case belongs to the set "all."[4]

[4]Given this logic, Justice Scalia found the majority opinion unfathomable. Rhetorics aside, he should not have. After all, his observations only increase the likelihood that a child-sexual-abuse case belongs to the set "all"; reasonable people can still disagree on whether that likelihood is big enough after the increase.

2.2 Church of Holy Trinity v. United States, 143 U.S. 457 (1892)

The Church of Holy Trinity, located in New York City, contracted with an Englishman to come over to be its pastor. The government claimed that this agreement violated a federal statute that made it unlawful for any person to "in any way assist or encourage the importation or migration of any alien ... into the United States ... under contract or agreement ... made previous to the importation or migration of such alien ... to perform labor or service of any kind in the United States ..." The fifth section of the statute makes specific exceptions, among them professional actors, artists, lecturers, singers and domestic servants; but the exceptions notably do not include pastors.

The case went all the way to the Supreme Court, and the Court ruled in favor of Holy Trinity. Had the Justices strictly adhered to the expressio doctrine, this would have been a clear-cut case: Holy Trinity violated the law. However, in a now famous quote, the Court in effect announced that it did not plan to strictly adhere to the expressio doctrine: "It is a familiar rule, that a thing may be within the letter of the statute and yet not within the statute ..." The Court then, famously, spent seven pages of its opinion to a lengthy discussion of how America is a religious nation. From that discussion, it concluded that "[i]t is a case where there was presented a definite evil, in view of which the legislature used general terms with the purpose of reaching all phases of that evil, and thereafter, unexpectedly, it is developed that the general language thus employed is broad enough to reach cases and acts which the whole history and life of the country affirm could not have been intentionally legislated against."

How should we make sense of this ruling? There is a malign interpretation: the Court had simply abused its power. According to this interpretation, the Justices were religious people, and they argued in seven pages that every other citizen was as religious as they were, and hence they insisted that granting churches preferential treatments was more important than following the law.

The malign interpretation may well be true, but there also exists a benign interpretation. According to this interpretation, the Court was entertaining two competing hypotheses: the one that pastors truly do not belong to the exception list; and an alternative one that legislators were simply too absent-minded as to have forgotten examples such pastors—had someone brought these examples to their attention they would have included them into the list. Since "America is a religious country," the Court regarded the first hypothesis as less likely. And given this result of their "awareness check," the Justices decided that they were no longer bound by the expressio doctrine.

3 The Model

There are two players in the game: the legislator (L, who we assume to be male), and the judge (J, who we assume to be female). For concreteness, we can think of the legislator is to write the Bill of Rights, and the judge is to interpret it 200 years later. In period 1, L decides how to write a law. He dies at the end of period 1. In period 2, nature randomly chooses a action, which we can think of as the government infringing a particular right. In period 3, that randomly chosen action is in front of J, and she has to rule whether or not it is illegal.

3.1 Actions: the Good, the Bad, and the Fair

Being the representative of the people, L's own personal preferenes are supposed to be the law of the land, provided that these preferences can be communicated well to the others. One reason why communication is imperfect is that L may not be aware of every possible action at the time of communication (i.e., at the time when he writes the law).

For every action, if L were aware of it, he would have regarded it as either good or bad. Among all possible actions, L would have regarded n of them good, and m of them bad. So there are totally $n + m$ possible actions.

Nature picks one of these $n + m$ actions using the following conditional probability distribution. With probability 1/2, nature picks a good action; and with probability 1/2 a bad action. Among the good actions, each has equal probability of being picked; similarly for bad actions. So every good action has probability $1/2n$ being picked, and every bad action $1/2m$.

The sets of good and bad actions are asymmetric: n is a fixed number which we assume is very large (or, equivalently, $1/n$ is vanishingly small); while m is random and is either 1 or 2 with equal probability. This asymmetry is necessary to explain the phenomenon that real-life laws are often lists of bad behavior instead of lists of good behavior.

J's objective is to serve the people: if she knows L's personal preferences and hence which actions are good and which are bad, she would like to rule a good action as `legal` and a bad action `illegal`. To formalize this, we define the "loss from judicial errors" as 1 if J rules a good action as `illegal` or a bad action as `legal`; and the loss is 0 otherwise.

In general, J does not know L's personal preferences. However, we assume the existence of an exogenous technology, using which J can figure out whether a particular action is fair or unfair. We call this technology the "fairness test.". We will see very soon why we chose this terminology, but it is meant to model nothing more than a judge exercising her professional judgment to fill in any gap she conceives in the law. A good action will be found fair by the test with probability p, where $1/2 < p < 1$. Symmetrically, a bad action will be found unfair

with probability p as well.

When an action is in front of J, she can exercise one of the following three options. She can rule it as `legal`, or as `illegal`; both of them are costless to J. She can also employ the `fairness test` and rule according to the result of the test. This third option is costly, however, and the effort cost of which is e.

J's objective is to minimize the expected sum of (1) the loss from judicial errors and (2) the effort cost (if any); where the expectation is taken with respect to her belief given L's strategy.

Due to the symmetry of the `fairness test`, the total cost of subjecting any action (regardless of how likely it is a good or a bad action) to it is $(1-p)+e$, where $(1-p)$ is the expected loss from judicial errors (given that J rules the action as `legal` if she finds it fair, and `illegal` if unfair), and e is the effort cost. We use l to denote the sum $(1-p)+e$, where l stands for the litigation cost associated with the `fairness test`.

Apparently, the `fairness test` will be redundant in this model if $l > 1/2$, because the judge can guarantee that the expected loss from judicial error is no more than $1/2$ by choosing only between `legal` and `illegal`. Therefore we shall focus on the case where $l < 1/2$.

3.2 Legislator: High and Low Types

A blank piece of paper is *not* the minimal form of law, because it does not give the court jurisdiction over actions, and in that sense it is not even a law. A barebone law contains at least one sentence: "All actions that are unfair are hereby declared illegal." This gives the court jurisdiction over actions. Given such a barebone law, and given any action randomly chosen by nature in period 2, J is not bound by this law to employ the costly `fairness test`. She can still short-circuit the costly `fairness test` and hold the action `legal` or `illegal` right away. So the only difference between the barebones law and a blank piece of paper is that the former gives the court jurisdiction. Any law that L may choose to write in period 1 must be at least barebones.

The barebones laws best known to economists is perhaps the U.S. FTC Act, which states that: "The [Federal Trade Commission] is empowered and directed to prevent persons, partnerships, or corporations ... from using unfair methods of competition in or affecting commerce ..." The word "unfair" is famously left undefined in the FTC Act.

On top of a barebones law, L can choose to add exceptions; e.g., "Congress shall make no law abridging the freedom of speech." or "Price fixing is hereby declared *per se* illegal." Formally, an exception is a pair of the form $(\texttt{action}, \texttt{good})$ or $(\texttt{action}, \texttt{bad})$. The former (latter) reads as: "Action `action` is hereby declared *per se* legal (illegal)."

Adding each exception incurs a cost of c, which we assume is small, but larger than $1/n$, so L still has the incentive to omit good actions from the law. We assume that whenever an

exception (`action`, `good`) is added, and if the action `action` is indeed picked by nature in period 2, then J would hold it `legal`. Similarly for any exception (`action`, `bad`).

However, in order to add the exception (`action`, `good` or `bad`), L must be at least aware of action `action`. If L were aware of all $m + n$ actions, he could have listed all of them as exceptions (with appropriate `good` or `bad` labels), and then there would be no guesswork left for J to do in period 3. But no legislator can have that extreme level of awareness.

Imagine that L is writing the law under some time pressure. After thinking really hard within certain time limit, L can still only think of a small portion of all possible actions. He is painfully aware of his limitation, and in particular he is fully aware of the fact that there are still a lot more actions out there that he has not yet thought of. But since time is up, he has to start writing the law now. His choice of exception list is hence limited by the number of actions that he has thought of. We assume that L can be of either high or low type. A high-type L will be able to think of two actions when time is up, and a low-type L will be able to think of only one. The exact probabilities that L is of high or low types are not important, as long as both are positive.

For a low-type L, the single action that he will be able to think of is a random selection from the set of all actions, and the distribution is the same as the distribution with which nature picks an action in period 2. I.e., conditional on m and n, each good action has a probability of $1/2n$ being thought of, and each bad action $1/2m$. The assumption that these two distributions are the same is not important. It is made only to ease notation.[5]

For a high-type L, the two actions that he will be able to think of are drawn from the same distribution *without replacement*.

L's objective is to minimize the expected sum of (1) period-1 law-writing cost, and (2) period-3 loss due to judicial errors; where the expectation is taken with respect to his belief given J's strategy. There is no discount. In this model, thinking is not costly to L, writing is. The former is not important, and introducing thinking costs adds little new insight. But the latter is important. Without writing cost, laws would be lists of both good and bad actions, instead of list of bad actions only.

3.3 The Game

L's (pure) strategy σ_L is a mapping from his awareness types to different exception lists. L has five possible awareness types: being aware of two bad actions (BB), being aware of one good and one bad actions (BG), being aware of two good actions (GG), being aware of one bad action (B), and being aware of one good action (G). His exception list can contain either two bad actions (BB), one good and one bad actions (BG), two good actions (GG),

[5]One possible justification of this assumption is that it may be more likely that a legislator is aware of an action if that action is taken more frequently by the general citizens.

one bad action (B), one good action (G), or nothing at all ($\emptyset$). The restriction is that he cannot list two actions if he is aware of only one.

Later on, we shall also consider an expanded message space for L, which allows him to also announce his type. Expanding the message space is important, because allowing L to announce his type will serve as the formal equivalence of allowing Iredell to add an other-rights-are-equally-sacred clause to the Bill of Rights. However, as a benchmark, we shall study the case with a more limited message space first.

J's (pure) strategy σ_J is a mapping from different exception lists to different ways to rule on an action not in the exception list. (Remember that we assume she has no discretion on how to rule on an action in the list.) J's belief β_J is a mapping from different exception lists to different distributions of L's awareness types.

The solution concept is (pure strategy) sequential equilibrium satisfying the intuitive criterion. We shall abuse terminology and refer to a (pure) strategy profile σ^* as an "equilibrium" if, for any c and n such that $1/n$, c, and $1/cn$ all close enough to 0, there exists belief β_J^* that makes the strategy-belief pair (σ^*, β_J^*) a (pure strategy) sequential equilibrium satisfying intuitive criterion. The requirement that $1/n$, c, and $1/cn$ close enough to 0 captures the idea that the number of good actions n is large, and the cost c of writing exceptions into the law is small, while not too small relative to $1/n$.

3.4 The Loophole Equilibria

In the Appendix, it is shown that $\sigma_L^*(\text{BB}) = \text{BB}$, $\sigma_L^*(\text{GG}) = \sigma_L^*(\text{G}) = \emptyset$, $\sigma_J^*(\text{BB}) =$ `legal`, and $\sigma_J^*(\emptyset) =$ `fairness test` in any equilibrium. This is very intuitive. Given that there are a lot of good actions, and each will appear in the court with vanishingly small probability, being able to think of one or two good actions is pretty useless as far as law making is concerned. So a GG- or G-type legislator may as well write a barebones law, and let the judge exercise her professional judgment on the bench. On the other extreme, a BB-type legislator has already thought of all bad actions, so there is no reason not to list all of them in the law, and let the judge rule every other action as `legal`.

So different equilibria differ from each other only in the strategies of the BG- and B-type legislators. These two types are similar in the sense that they both would have preferred to write a B law, at least if the other types did not exist. But these two types are also very different in terms of their assessments of how likely there is a second bad action that they are not yet aware of. The BG-type legislator is a high-type legislator. He has thought hard, and yet could only thought of one bad action. So he puts a lower probability on the existence of a second bad action. He does not mind that much if the judge simply rule all actions that are not against the law as `legal`. The B-type legislator is a low-type legislator, and he knows he has a low type. The fact that he has only thought of one bad action does not mean much. Hence he puts a higher probability on the existence of a second bad action.

He is more inclinded to have the judge apply the `fairness test` on all actions that are not against the law.

The full characterization of all equilibria is contained in the Appendix. Here in the text we want to highlight one particular equilibrium, called the loophole equilibrium. Let θ^{**} and θ^{*} be the legislator's beliefs that there exists a second bad action that he is not aware of, conditional on his awareness type being BG and B, respectively. Let q^{**} and q^{*} be the judge's beliefs that any action that is not against a B law being a good action, conditional on the events that that B law is written by a BG-type and a B-type legislator, respectively. Our earlier argument suggests that $\theta^{**} < \theta^{*}$ and $q^{**} > q^{*}$. Let's consider the parameter range where $\theta^{**}/4 < 1 - p < \theta^{*}/4$ and $1 - q^{**} < l < 1 - q^{*}$.

Suppose the legislator believes that the judge would rule any action that is not against a B law as `legal`, but would apply the `fairness test` to any action if the law is a barebones law. Consider a BG-type legislator who is choosing between a B law and a barebones law. The B law induces a loss of 1 when there is indeed a second bad action (with probability θ^{**}) and when that second bad action is indeed picked by nature (with probability 1/4). So the B law induces an expected loss of $\theta^{**}/4$. On the other hand, the barebones law induces an expected loss of $1-p$ regardless which action is picked by nature. In our assumed parameter range, a BG-type legislator therefore would choose the B law over the barebones law. On the other hand, by similar argument, a B-type legislator would instead choose the barebones law over the B law.

Suppose the judge sees a B law, and is pondering how to handle an action that is not against this law. If she believes that the law is written by a BG-type legislator, then ruling that action as `legal` would result in an expected loss of $1 - q^{**}$, while applying the `fairness test` would result in an expected loss of l. In our assumed parameter range, she therefore would choose to rule that action as `legal`. On the other hand, if she believes that the law is written by a B-type legislator, she would instead choose to apply the `fairness test`.

Therefore, in our assumed parameter range, it is an equilibrium that a BG-type legislator writes a law that lists the single bad action he is aware of; such a law prompts the judge to rule any action that is not against the law as `legal`, believing that the legislator has a high type and likely has exhausted of all bad actions; a B-type legislator, anticipating that, and being aware of that there is likely a second bad action that he is not yet aware of, hence refrains from listing the bad action he is aware of, and writes a barebones law instead; and this in turn justifies the judge's belief.

To see why it is appropriate to call such an equilibrium a loophole equilibrium, let's try to see things through the eyes of a B-type legislator. He is aware that there is likely a second bad action that he is not yet aware of. If he write a shorter and simpler law, that second bad action will be ruled as `illegal` with probability more than 1/2 whenever it appears in court. But if he makes the law longer and more complicated, that second bad action will be

ruled as `legal` with probability 1. The fact that that second bad action survives the trial with probability 1 is reminiscent to a loophole, which would not have existed had the law been short and simple.

Theorem 1 at the end of this section identifies the parameter range where the loophole equilibrium is the unique equilibrium. In the theorem, $\bar{q}$ is the judge's belief that any action that is not against a B law being a good action, conditional on the event that *both* a BG-type and a B-type legislator would have written that law. See the Appendix for more precise definitions.

3.5 Expanding L's Message Space

Here comes a very important question: is the loophole equilibrium an artifact of our assumption that the legislator can only write exceptions into the law? Since we have been assuming that writing cost is vanishingly small, would it not be natural for a B-type legislator to write not only a B law, but on top of that also add an extra sentence announcing that he has a low type? If he is allowed to do so, would he not be able to differentiate himself from a BG-type legislator? Would the loophole equilibrium hence unravel with this richer message space?

To answer the question formally, we shall expanded the message space. Consider an alternative game with one extra possible way to write the law, namely Bl, which can be read as containing two sentences: (1) "action `action` is hereby declared *per se* illegal," and (2) "by the way, this legislator has a low type, and hence there are likely other bad actions that this legislator is unaware of." Theorem 1 says that: whenever the loophole equilibrium exists, it remains an equilibrium in this alternative game; and whenever it is the unique equilibrium, it remains the unique equilibrium in this alternative game.

Why is it so? The reason is that, in the loophole equilibrium, even the BG-type legislator would love to have any action not against the law being subject to the `fairness test` as well. If writing a Bl law could have achieved that, he would deviate to writing such a law as well, as the writing cost of that extra sentence is vanishingly small anyway. Hence a B-type legislator cannot credibly convince the judge that he indeed has a low type even if he announces so in the law.

Theorem 1 *The loophole equilibrium is an equilibrium where* $\sigma_L^*(\mathrm{BB}) = \mathrm{BB}$, $\sigma_L^*(\mathrm{BG}) = \mathrm{B}$, $\sigma_L^*(\mathrm{GG}) = \sigma_L^*(\mathrm{B}) = \sigma_L^*(\mathrm{G}) = \emptyset$, $\sigma_J^*(\mathrm{BB}) = \sigma_J^*(\mathrm{B}) =$ `legal`, *and* $\sigma_J^*(\emptyset) =$ `fairness test`. *The loophole equilibrium is the unique equilibrium when* $\theta^{**}/4 < 1 - p < \theta^*/4$ *and* $1 - \bar{q} < l$. *Moreover, whenever the loophole equilibrium exists (respectively, is the unique equilibrium), it remains an equilibrium (respectively, remains the unique equilibrium) even if a low-type legislator is allowed to add a sentence into the law announcing that he has a low type.*

4 Relations with Incomplete Contracts

What does a model of Iredell's argument tell us about incomplete contracts?

Incomplete contracts is an ambiguous term, and has at least two different meanings: one that economists informally allude to in their introductions, another one that they formalize in their mathematical models, and the two are scandalously different. The first one refers to contracts that are silent on certain contingencies. The second one refers to contracts that are insufficiently state-contingent. These two concepts are often at odds with each other. For example, the null contract, which is one the most insufficently state-contingent contracts, is nevertheless complete in the first sense: "The null contract is complete in that it is absolutely clear what everybody's obligations are: nobody has any!" (Hart and Moore (1999))[6]

However, our study of Iredell's argument suggests that there may be a third meaning for incomplete contracts. A contract is more incomplete in this third sense if the judge conceives more (subjective) gaps in it, possibly as a result of an unfavorable "awareness check." To see how this third meaning differs from either of the previous two meanings, consider the famous case of ALCOA v. Essex Group, Inc.[7] ALCOA signed a long term contract with Essex, in which the price Essex was to pay ALCOA for its aluminum would be subject to a price escalator clause based in part on the wholesale price index for industrial commodities (WPI). When later on ALCOA found that the WPI did not rise as fast as its production cost, it reneged. It claimed that the event that the WPI failed to track their production cost was an unforeseen contingency. The judge accepted that argument, and released ALCOA from their obligation.

The long term contract signed by ALCOA would probably not qualify as an incomplete contract in the first sense: what else can be more complete than a function of a publicly available statistics such as the WPI? It is also neither a null contract, nor a constant contract. Nevertheless, it is incomplete in the third sense, because the judge conceived a gap in it—the judge considered the contract as silent on the contingency where the WPI failed to tract ALCOA's production cost.

Note that this gap is subjective rather than objective. Objectively, the contract contains no gap, because aluminum price was still well defined as a function of the WPI even in the contingency where the WPI failed to tract production cost. In this sense, the third meaning of incomplete contracts is a modification of the first, with objective gaps replaced by subjective ones. But this modification makes a big difference: while Hart and Moore

[6] Null contract has an important role in the literature of incomplete contracts. In particular, "foundations" of incomplete contracts often refer to theories of why contracting parties optimally choose null contracts over other contracts. See Che and Hausch (1999), Segal (1999), and Hart and Moore (1999). An exception is Spier (1992), who defines incomplete contracts as constant contracts: contracts that specify the same obligations for all contingencies.

[7] 499 F. Supp. 53 (W.D. Pa 1980).

(1999) are hard-pressed to find any contract that is incomplete in the first sense, contracts that are incomplete in the third sense abound. In fact, even artificial examples of incomplete contracts in legal textbooks are incomplete only in the third instead of the first sense. For instance, in Posner (1998), one of the examples of incomplete contracts involve a contract that states that the crew are to work on the ship until it arrives at the final destination. This contract allegedly is incomplete and contains the following gap: it is notably silent on whether the crew have the same obligation if a war breaks out in the destination country. Since the contract is actually no more silent on wars than on any other contingencies, such a gap is more a subjective one conceived by Posner. It also demonstrates that the third meaning of incomplete contracts is closer to what judges have in mind.[8]

The third meaning of incomplete contracts also bears more connections with the second than the first does. For example, constant contracts that are deemed incomplete in the second sense likely will also be seen as being silent on more contingencies by the judge, because the contracting parties will be perceived as having lower awareness types. However, this correlation is not perfect. Theoretically, even a constant contract can be written in a very complicated way, for example by enumerating a lot of contingencies while repeating the same transaction terms in each of these contingencies. Such a constant contract would not be deemed as incomplete in the third sense, because it signals a high awareness type of the contracting parties.

[8]Posner is currently a judge on the United States Court of Appeals for the Seventh Circuit.

Appendix A:

In this Appendix we shall characterize all the equilibria of the game.

J's choice between `legal`, `illegal`, and `fairness test` depends on her belief that an unlisted action being good. Let's use q to denote this belief. J would choose `legal` and `illegal` if $1-q<l$ and $q<l$, respectively; and choose `fairness test` otherwise. Let's use $q(\texttt{law};\texttt{aware})$ to denote J's belief that an unlisted action is good conditional on seeing law `law`, and if she believes that L's awareness type is `aware`. Our first observation is that $q(\texttt{law};\texttt{aware})$ is either strictly bigger than 1/2 (e.g., $q(\mathrm{BB};\mathrm{BB})=1>1/2$), or arbitrarily close to 1/2 and hence strictly bigger than any fixed l (with $l<1/2$) as $1/n$ goes to 0 (e.g., $\lim_{1/n\to 0} q(\mathrm{GG};\mathrm{GG})=1/2$). This is true even for potentially ambiguous terms of $q(\mathrm{B};\mathrm{BG})$ and $q(\mathrm{G};\mathrm{BG})$. For example, the exact value of $q(\mathrm{B};\mathrm{BG})$ may depend on whether J believes that the so-called "bad" action listed in the law is truly a bad action, or is actually a good action. Below, we shall always reserve the notation $q(\mathrm{B};\mathrm{BG})$ for the former interpretation. However, regardless of which interpretation we use, we will still have $\lim_{1/n\to 0} q(\mathrm{B};\mathrm{BG})\geq 1/2$.[9] Hence $\sigma_J^*(\texttt{law})\neq\texttt{illegal}$, because, upon seeing any law `law`, J's equilibrium belief $\beta_J^*(\texttt{law})$ must be some convex combination of all these different $q(\texttt{law};\texttt{aware})$'s.

There are two terms of the form $q(\texttt{law};\texttt{aware})$ that will prove particularly important. Let

$$\begin{aligned}
q^{**} &:= \lim_{1/n\to 0} q(\mathrm{BB};\mathrm{BB}) = \lim_{1/n\to 0} q(\mathrm{B};\mathrm{BB});\\
q^{*} &:= q(\mathrm{B};\mathrm{B});\ \text{and}\\
\bar{q} &:= Prob(\texttt{aware}=\mathrm{BB}|\texttt{aware}=\mathrm{BB}\text{ or }\mathrm{B})q^{**} + Prob(\texttt{aware}=\mathrm{B}|\texttt{aware}=\mathrm{BB}\text{ or }\mathrm{B})q^{*}.
\end{aligned}$$

It is easy to check that

$$1/2<q^*<\bar{q}<q^{**}<1,$$

which is intuitive. A BG-type legislator is a high-type legislator. If, even after he has thought hard, he has thought of only one bad action, then the posterior probability that there exists

[9] For example, if the so-called "bad" action listed in the law is truly the bad action that L is aware of (which will be the interpretation we reserve for the notation $q(\mathrm{B};\mathrm{BG})$), then we can calculate $q(\mathrm{B};\mathrm{BG})$ as follows. If $m=1$, the probability that a high-type legislator is of awareness type BG is approximately 3/4, the probability that nature will pick an unlisted action is approximately 1/2, and the probability that nature will pick a good unlisted action is also approximately 1/2. If $m=2$, then the probability that a high-type legislator is of awareness type BG is approximately 7/12, the probability that nature will pick an unlisted action is approximately 3/4, and the probability that nature will pick a good unlisted action is approximately 1/2. Therefore

$$\lim_{1/n\to 0} q(\mathrm{B};\mathrm{BG}) = \frac{\frac{1}{2}\times\frac{3}{4}\times\frac{1}{2}+\frac{1}{2}\times\frac{7}{12}\times\frac{1}{2}}{\frac{1}{2}\times\frac{3}{4}\times\frac{1}{2}+\frac{1}{2}\times\frac{7}{12}\times\frac{3}{4}} = \frac{32}{39}>\frac{1}{2}.$$

a second bad action must be lower. Hence it is more likely that an unlisted action is good. On the other hand, a B-type legislator is a low-type legislator. The fact that he has thought of only one bad action means nothing. The posterior probability that there exists a second bad action remains the same as the prior, and hence is not as likely that an unlisted action is good.

Let's use $\theta(\texttt{aware}) := \text{Prob}(m = 2|\texttt{aware})$ to denote L's posterior belief that there are two bad actions upon learning that his awareness type is `aware`. So, for example, $\theta(\text{BB}) = 1$ and $\theta(\text{G}) = 1/2$. Let

$$\theta^{**} := \lim_{1/n \to 0} \theta(\text{BG}); \text{ and}$$
$$\theta^{*} := \theta(\text{B}).$$

It is easy to check that

$$0 < \theta^{**} < \theta^{*} = 1/2.$$

Lemma 1 *In any equilibrium,* $\sigma_L^*(\text{BB}) = \text{BB}$, $\sigma_L^*(\text{GG}) = \sigma_L^*(\text{G}) = \emptyset$, $\sigma_J^*(\emptyset) = \texttt{fairness test}$, *and for any other law* `law`, $\sigma_J^*(\texttt{law}) \in \{\texttt{legal}, \texttt{fairness test}\}$.

Proof: We have already proved that $\sigma_J^*(\texttt{law}) \in \{\texttt{legal}, \texttt{fairness test}\}$ for any law `law`. It is also easy to check that

$$q(\emptyset; \text{BB}) = q(\emptyset; \text{BG}) = q(\emptyset; \text{GG}) = q(\emptyset; \text{B}) = q(\emptyset; \text{G}) = 1/2.$$

Therefore, since $l < 1/2$, we have $\sigma_J^*(\emptyset) = \texttt{fairness test}$ in any equilibrium.

Consider a legislator with awareness type GG (respectively G). He believes that nature will choose an action that he is unaware of with probability $1 - 2/2n$ (respectively $1 - 1/2n$); and, conditional on that event, the chosen action will be a good action with probability $(n-2)/(2n-2)$ (respectively $(n-1)/(2n-1)$), which is arbitrarily close to $1/2$ when $1/n$ goes to 0. Since $1 - p < 1/2$, he would prefer that J applies the `fairness test` to any of these unlisted actions instead of ruling them as `legal`. Writing a barebones law achieves just that. Writing any longer law will cost at least c, with a benefit of at most $[2/2n][1-p]$ (respectively at most $[1/2n][1-p]$), which is achieved only if $\sigma_J(\text{GG}) = \texttt{fairness test}$ (respectively only if $\sigma_J(\text{G}) = \texttt{fairness test}$). When c, $1/n$, and $1/cn$ are close enough to 0, he may as well write a barebones law.

Notice that we have proved more than that $\sigma_L^*(\text{GG}) = \sigma_L^*(\text{G}) = \emptyset$ in any equilibrium. We have also proved that if a GG- or G-type legislator deviates to any more complicated law, such a deviation cannot be rationalized by any belief. This observation will be useful when we later on apply the intuitive criterion to pin down J's equilibrium beliefs $\beta_J^*(\text{BG})$ and $\beta_J^*(\text{B})$.

Next consider a legislator with awareness type BB. We shall show that $\sigma_L^*(\text{BB}) = \text{BB}$ in any equilibrium. If the legislator writes law BB, the worst case scenerio is that $\sigma_J(\text{BB}) =$ `fairness test`, which results in an expected loss of $(1-p)/2+2c$. If he writes any law other than BB, the best he can hope for is that all actions that he is unaware of (which he knows must all be good) will be ruled as `legal`. But this good outcome can come only with the cost that at least one of the two bad actions he is aware of will be ruled as `legal` as well, because by assumption he is no longer listing all the bad actions in the law. Therefore the best case scenerio is that his expected loss will be $1/4 + c$, which is achieved if $\sigma_J(\text{B}) =$ `legal`. Since $(1-p)/2 < 1/4$, when c is close enough to 0, he cannot do worse by writing law BB.

Once again, notice that we have proved more than that $\sigma_L^*(\text{BB}) = \text{BB}$ in any equilibrium. We have also proved that if a BB-type legislator deviates to any other law, such a deviation cannot be rationalized by any belief. This observation will be useful when we later on apply the intuitive criterion to pin down J's equilibrium beliefs $\beta_J^*(\text{BG})$ and $\beta_J^*(\text{B})$. ■

Lemma 2 *In any equilibrium,* $\sigma_J^*(\text{BB}) =$ `legal`.

Proof: We know that $\sigma_J^*(\text{BB}) \neq$ `illegal`. So let's suppose $\sigma_J^*(\text{BB}) =$ `fairness test`. If $\sigma_L^*(\text{BG}) \neq \text{BB}$, then we would have had $\beta_J^*(\text{BB})(\{\text{BB}\}) = 1$, and hence $\sigma_J^*(\text{BB}) =$ `legal`, a contradiction. Therefore we have $\sigma_L^*(\text{BG}) = \text{BB}$. Consider J's out-of-equilibrium belief $\beta_J^*(\text{BG})$. A BG law can only be written by a legislator with awareness type BB, BG, or GG. But we have already seen that it is irrational for a BB- or GG-type legislator to deviate to law BG. So if we can prove that it can be rational for a BG-type legislator to deviate to law BG, then the intuitive criterion would dictate that $\beta_J^*(\text{BG})$ puts probability 1 on the awareness type BG.

A BG-type legislator's deviation to law BG is surely rational if he believes that $\sigma_J(\text{BG}) =$ `fairness test`. Is that belief consistent with the belief that J is rational? Since $\lim_{1/n\to 0} q(\text{BG};\text{GG}) = 1/2$, $\sigma_J(\text{BG}) =$ `fairness test` can be rationalized by a belief $\beta_J(\text{BG})$ that puts high enough probability on the awareness type GG. Therefore, L's belief that $\sigma_J(\text{BG}) =$ `fairness test` is a legitimate belief. Hence, the intuitive criterion is applicable, and we have $\beta_J^*(\text{BG})(\{\text{BG}\}) = 1$.

Since $q(\text{BB};\text{BB}) = q(\text{BG};\text{BB}) = 1 > q(\text{BB};BG) = q(\text{BG};BG)$, the lower the probability J assigns to the awareness type BB, the more tempted she would be to choose `fairness test` over `legal`. Since $\beta_J^*(\text{BG})(\{\text{BB}\}) = 0 < \beta_J^*(\text{BB})(\{\text{BB}\})$, $\sigma_J^*(\text{BB}) =$ `fairness test` implies $\sigma_J^*(\text{BG}) =$ `fairness test` as well. But then a BG-type legislator can profit from deviating to law BG, contradicting our assumption that $\sigma_L^*(\text{BG}) = \text{BB}$. This proves that $\sigma_J^*(\text{BB}) =$ `legal`. ■

Lemma 3 *In any equilibrium,* $\sigma_L^*(\text{BG}) \in \{\text{BG}, \text{B}, \emptyset\}$ *and* $\sigma_L^*(\text{B}) \in \{\text{B}, \emptyset\}$.

Proof: That $\sigma_L^*(\text{B}) \neq \text{G}$ is easy, for $\sigma_L(\text{B}) = \text{G}$ is clearly dominated by $\sigma_L(\text{B}) = \emptyset$ regardless of $\sigma_J^*(\text{G})$. The argument that $\sigma_L^*(\text{BG}) \neq (\text{GG}, \text{G})$ is similar.

Suppose $\sigma_L^*(\text{BG}) = \text{BB}$. Since $\sigma_J^*(\text{BB}) = \texttt{legal}$, the only reason that a BG-type legislator does not deviate to writing law BG is that $\sigma_J^*(\text{BG}) = \texttt{fairness test}$, and he prefers that any actions he is not aware of be ruled legal instead of being subjected to the `fairness test`. In other words, if it were up to L, he would have mandated that J choose $\sigma_J(\text{BG}) = \texttt{legal}$ instead of $\sigma_J(\text{BG}) = \texttt{fairness test}$. Recall that the legislator does not internalize all the litigation costs (he internalizes only the $1-p$ part but not the e part). Therefore, if a BG-type legislator prefers $\sigma_J(\text{BG}) = \texttt{legal}$ to $\sigma_J(\text{BG}) = \texttt{fairness test}$, J must also prefer $\sigma_J(\text{BG}) = \texttt{legal}$ to $\sigma_J(\text{BG}) = \texttt{fairness test}$, provided that she puts probability 1 on the awareness type BG after seeing law BG. But she indeed does. Using exactly the same argument as in the proof of Lemma 2, the intuitive criterion indeed dictates that $\beta_J^*(\text{BG})(\{\text{BG}\}) = 1$. This proves that $\sigma_L^*(\text{BG}) \neq \text{BB}$. ■

Lemma 4 *In any equilibrium, if* $\sigma_L^*(\text{BG}) = \text{BG}$*, then* $\sigma_L(\text{B}) = \text{B}$*.*

Proof: Suppose $\sigma_L^*(\text{BG}) = \text{BG}$ and $\sigma^*(\text{B}) = \emptyset$. If $\sigma_J^*(\text{B}) = \texttt{fairness test}$, a B-type legislator would have deviated to writing law B. Therefore $\sigma_J^*(\text{B}) = \texttt{legal}$. If $\sigma_J^*(\text{BG}) = \texttt{legal}$ as well, a BG-type legislator would have deviated to writing law B. There $\sigma_J^*(\text{BG}) = \texttt{fairness test}$. Since $\beta_J^*(\text{BG})(\{\text{BG}\}) = 1$, $\sigma_J^*(\text{BG}) = \texttt{fairness test}$ implies $l < 1 - q^{**}$. Since $1 - q^* < 1 - q^{**}$, we have $l < 1 - q^*$ as well. Now, when J sees an out-of-equilibrium law B, the intuitive criterion dictates that she must put probability 1 on either it is written by a B-type legislator, or it is written by a BG-type legislator *and* the listed action is the bad action he is aware of (the detailed argument is similar to that in the proof of Lemma 2, and hence is omitted). So her belief that any unlisted action is good must be some mixture of q^* and q^{**}. This implies $\sigma_J^*(\text{B}) = \texttt{fairness test}$, a contradiction. ■

Lemma 5 *In any equilibrium, if* $\sigma_L^*(\text{BG}) = \emptyset$*, then* $\sigma_L^*(\text{B}) = \emptyset$*.*

Proof: Suppose $\sigma_L^*(\text{BG}) = \emptyset$ and $\sigma_L^*(\text{B}) = \text{B}$. If $\sigma_J^*(\text{B}) = \texttt{fairness test}$, then a BG-type legislator would have deviated to writing law B. Therefore $\sigma_J^*(\text{B}) = \texttt{legal}$. A B-type legislator's loss from writing law B is then approximately $\theta^*/4 + c$; whereas his loss from writing a barebones law is $1-p$. Thus, $\sigma_L^*(\text{B}) = \text{B}$ then implies $\theta^*/4 \leq 1-p$. Since $\theta^{**} < \theta^*$, we have $\theta^{**}/4 < 1-p$ as well, and hence a BG-type legislator can profit from deviating to writing law B as well, a contradiction. ■

Lemmas 3-5 narrow the set of possible equilibria down to four possibilities:

- $\sigma_L^*(\text{BG}) = \text{BG}$ and $\sigma_L^*(\text{B}) = \text{B}$; we call this a communicative equilibrium. It is the only equilibrium where a BG-type legislator writes down every action, good or bad, into the law. However, he writes down the good action not because he wants to explain the boundary between good and bad in greater detail—this benefit would have been easily outweighted by the writing cost. Instead, he is more motivated by the desire to signal his higher awareness.

- $\sigma_L^*(\text{BG}) = \sigma_L^*(\text{B}) = \text{B}$; we call this a regular equilibrium. The legislator simply lists all the bad action(s) he is aware of.

- $\sigma_L^*(\text{BG}) = \text{B}$ and $\sigma_L^*(\text{B}) = \emptyset$ (in which case we must also have $\sigma_J^*(\text{B}) = \texttt{legal}$, otherwise the B-type legislator would have deviated to writing law B); we call this a loophole equilibrium. A legislator of awareness type B refrains from listing the bad action he is aware of, because he is aware that there may be a second bad action that he is not yet aware of, and he does not want the judge to rule that second bad action as `legal`.

- $\sigma_L^*(\text{BG}) = \sigma_L^*(\text{B}) = \emptyset$ (in which case we must also have $\sigma_J^*(\text{B}) = \texttt{legal}$, otherwise the B-type legislator would have deviated to writing law B); we call this a strong loophole equilibrium. A strong loophole equilibrium has much the same flavor as a loophole equilibrium.

We shall give the parameter range for each of these equilibria to exist. In the following propositions, the ranges are always described with strict inequalities. If we replace those strict inequalities with weak inequalities, we will get necessary conditions for existence. So the conditions presented below should be understood as sufficient and almost necessary conditions.

Proposition 1 *A communicative equilibrium exists when* $1 - q^{**} < 1 - p$ *and* $l < 1 - q^*$.

Proof: The only reason that a BG-type legislator would not deviate to writing law B is that $\sigma_J^*(\text{BG}) \neq \sigma_J^*(\text{B})$. Since $1 - q^{**} < 1 - q^*$, $\sigma_J^*(\text{BG}) = \texttt{fairness test}$ would have implied that $\sigma_J^*(\text{B}) = \texttt{fairness test}$ as well. So we must have $\sigma_J^*(\text{BG}) = \texttt{legal}$ and $\sigma_J^*(\text{B}) = \texttt{fairness test}$, which will be the case when $1 - q^{**} < l < 1 - q^*$. The only possible profitable deviation we need to consider is that a BG-type legislator deviating to writing law B. Since the difference in writing costs is vanishingly small, and the probability that nature picks the good action that he is aware of is also vanishingly small, his benefit of deviating is approximately $(1 - q^{**}) - (1 - p)$ times the probability that nature picks an action that he is unaware of, where $1 - q^{**}$ is his loss from having any such an action being ruled as `legal`, and $1 - p$ is his loss from having any such an action subject to the `fairness test`. Therefore, when $1 - q^{**} < 1 - p$, such a deviation is not profitable. Since $1 - p < 1 - p + e =: l$, the condition simplies into the one in the proposition. ■

Proposition 2 *A regular equilibrium exists when either (1)* $1 - p < 1 - q^{**}$ *and* $l < 1 - \bar{q}$*; or (2)* $\theta^*/4 < 1 - p$ *and* $1 - \bar{q} < l$.

Proof: There are two possible subcases for a regular equilibrium. Case 1: $\sigma_J^*(\text{B}) = \texttt{fairness test}$, which will be case if $l < 1 - \bar{q}$. The only possible profitable deviation we need to consider is that a BG-type legislator deviating to writing law BG, which is profitable

only if $\sigma_J^*(BG) = \texttt{legal}$. By almost exactly the same argument as in the proof of Proposition 1, when $1 - p < 1 - q^{**}$, such a deviation is not profitable.

Case 2: $\sigma_J^*(\text{B}) = \texttt{legal}$, which will be the case if $1 - \bar{q} < l$. Since the out-of-equilibrium belief $\beta_J^*(\text{BG})(\{\text{BG}\}) = 1$ never violates the intuitive criterion, and since $1 - \bar{q} < l$ implies $1 - q^{**} < l$, we can specify that $\sigma_J^*(\text{BG}) = \texttt{legal}$ as well. In that case the only possible profitable deviation we need to consider is that a BG- or a B-type legislator deviating to writing a barebones law. Such a deviation would not be profitable for the BG-type legislator if $\theta^{**}/4 < 1 - p$, where $\theta^{**}/4$ is the loss from having the second bad action being ruled as `legal`, multiplied by the probabilities that such a second bad action indeed exists (θ^{**}) and that it will be picked by nature (1/4). Similarly, such a deviation would not be profitable for a B-type legislator if $\theta/4 < 1 - p$. Since $\theta^{**} < \theta^*$, the condition simplies into the one in the proposition. ■

Proposition 3 *A loophole equilibrium exists when $\theta^{**}/4 < 1 - p < \theta^*/4$ and $1 - q^{**} < l$.*

Proof: As we argued earlier, we must have $\sigma_J^*(\text{B}) = \texttt{legal}$ in a loophole equilibrium, which can be the case when $1 - q^{**} < l$. Since the out-of-equilibrium belief $\beta_J^*(\text{BG})(\{\text{BG}\}) = 1$ never violates the intuitive criterion, we can specify that $\sigma_J^*(\text{BG}) = \texttt{legal}$ as well. Then the only possible profitable deviations we need to consider are that a BG-type legislator deviating to writing a barebones law, and a B-type legislator deviating to writing law B. By almost exactly the same argument as in the proof of Proposition 2, the first deviation is not profitable when $\theta^{**}/4 < 1 - p$, and the second deviation is not profitable when $1 - p < \theta^*/4$. ■

Proposition 4 *A strong loophole equilibrium exists when $1 - p < \theta^{**}/4$ and $1 - q^{**} < l$.*

Proof: The proof is almost exactly the same as the proof of Proposition 3. ■

Proof of Theorem 1: That the loophole equilibrium is the unique equilibrium in the stated parameter range follows from Propositions 1-4.

Consider the new game where it is now possible to write the law Bl. Consider the parameter range in Proposition 3, where the loophole equilibrium exists in the old game. To prove that the loophole equilibrium remains an equilibrium in the new game, it suffice to specify that $\sigma_J^*(\text{Bl}) = \texttt{legal}$ as well, which can be supported by out-of-equilibrium belief $\beta_J^*(\text{Bl})(\{BG\}) = 1$. To prove that such a belief does not violate the intuitive criterion, it suffices to prove that it can be rational for a BG-type legislator to deviate to writing law Bl.

We claim that a BG-type legislator would find it profitable to deviate to writing law Bl if he believes that $\sigma_J(\text{Bl}) = \texttt{fairness test}$. Given this belief, and since both the writing cost and the probability that nature picks the good action he is aware of are both vanishingly small, his benefit of deviating to $\sigma_L(\text{BG}) = \text{Bl}$ is approximately $(1 - q^{**}) - (1 - p)$ times the

probability that nature picks an action that he is not aware of. It can be easily checked that

$$\theta^*/4 < 1 - q^{**} < 1 - q^*, \tag{1}$$

and hence, in the stated parameter range, we have $1 - p < 1 - q^{**}$. Therefore the deviation will indeed be profitable if he believes that $\sigma_J(\text{Bl}) = \texttt{fairness test}$. The remaining question is whether this belief is legitimate. The answer is yes, because $\sigma_J(\text{Bl}) = \texttt{fairness test}$ if the judge puts high enough probability on the awareness type GG when she sees the law Bl. This finishes the proof that the loophole equilibrium remains an equilibrium in the new game.

Next consider the parameter range in Theorem 1, where the loophole equilibrium is the unique equilibrium in the old game. Since Lemmas 1-3 continue to hold in the new game, if there is a new equilibrium in the new game, it must involve law Bl being written by either a BG- or a B-type legislator. Since, in the stated parameter range, $1 - p$ is smaller than both $1 - q^{**}$ and $1 - q^*$ according to (1), both a BG- and a B-type legislator would love to have any action he is unaware of subject to the `fairness test`. Since a Bl law is more expensive to write than a B law, neither a BG- nor a B-type legislator would have written law Bl if $\sigma_J^*(\text{B}) = \texttt{fairness test}$. Therefore we must have $\sigma_J^*(\text{B}) = \texttt{legal}$. Once again, if $\sigma_J^*(\text{Bl}) = \texttt{legal}$ as well, no legislator would have written law Bl. So we must have $\sigma_J^*(\text{Bl}) = \texttt{fairness test}$. But then both a BG- and a B-type legislator would have written law Bl, which prompts the judge to believe that any action that is not against the law is good with probability approximately $\bar{q}$. Since $1 - \bar{q} < l$ in the stated parameter range, the judge's best response is to rule any not-explicitly-prohibited action as `legal`, a contradiction. ■

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